

# QED3, Entanglement Computation, ...

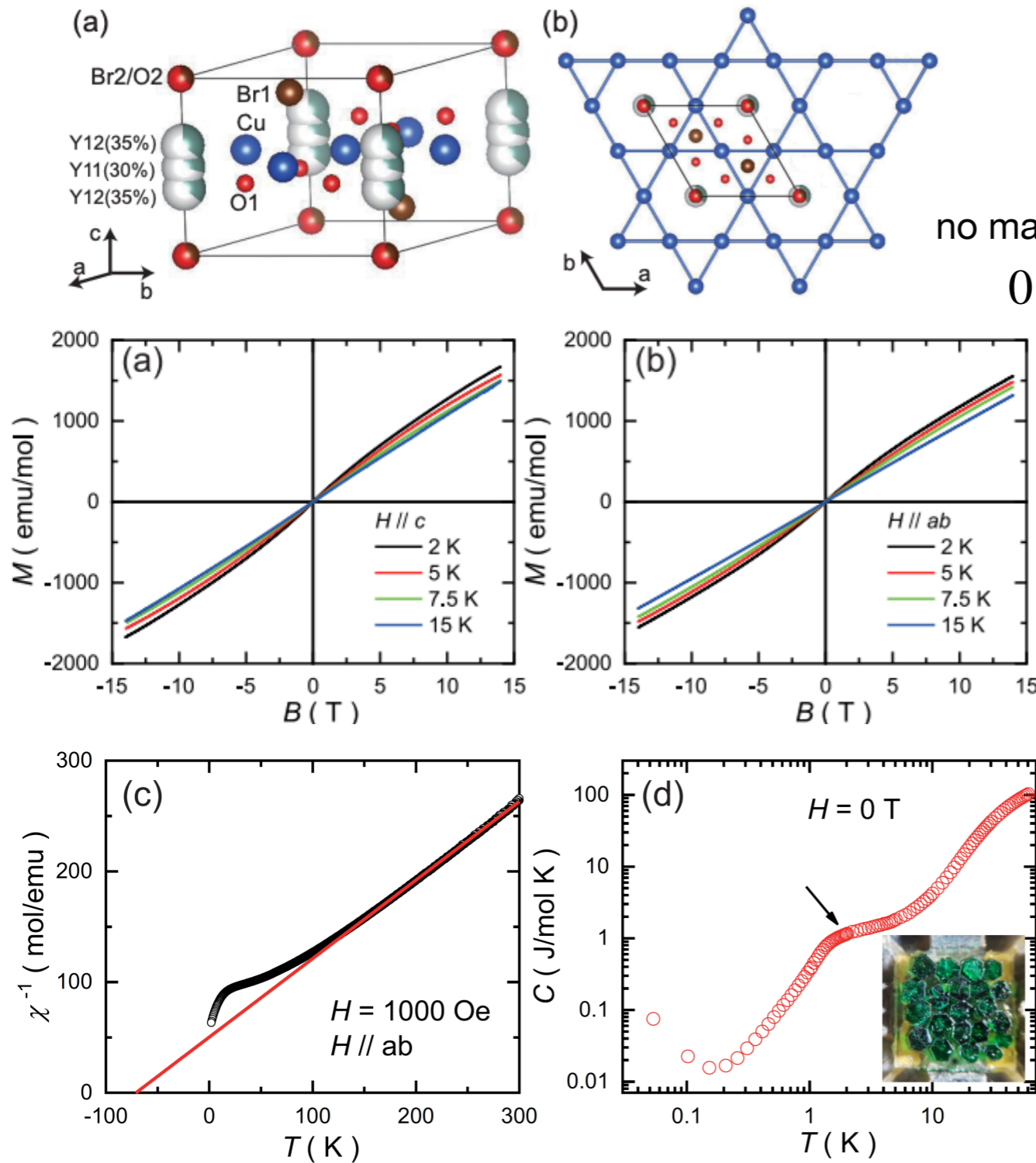
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ZI YANG MENG

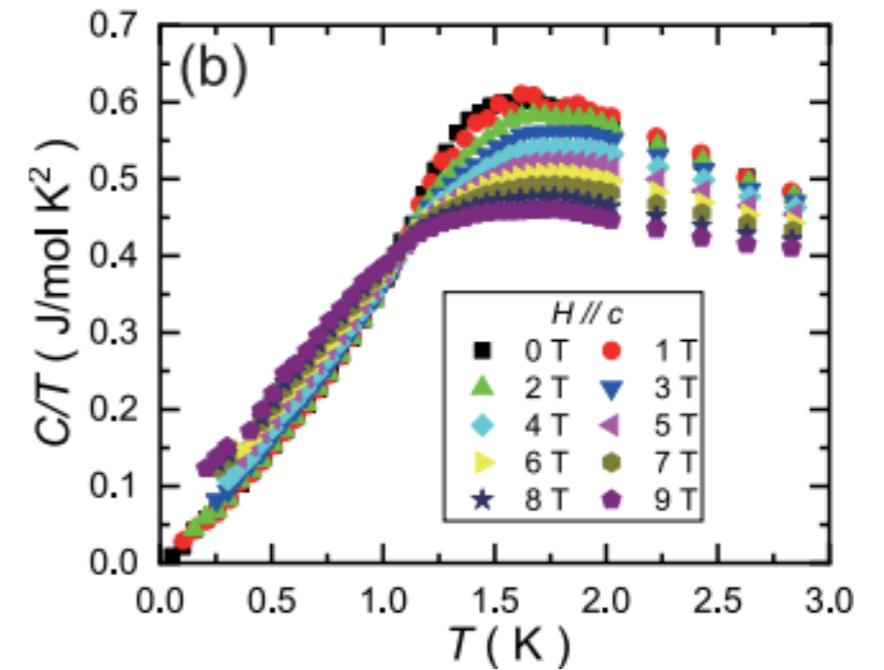
孟子楊

<https://quantummc.xyz/>

# YCu<sub>3</sub>(OH)<sub>6</sub>Br<sub>2</sub>[Br<sub>x</sub>(OH)<sub>1-x</sub>] (YCu<sub>3</sub>-Br)

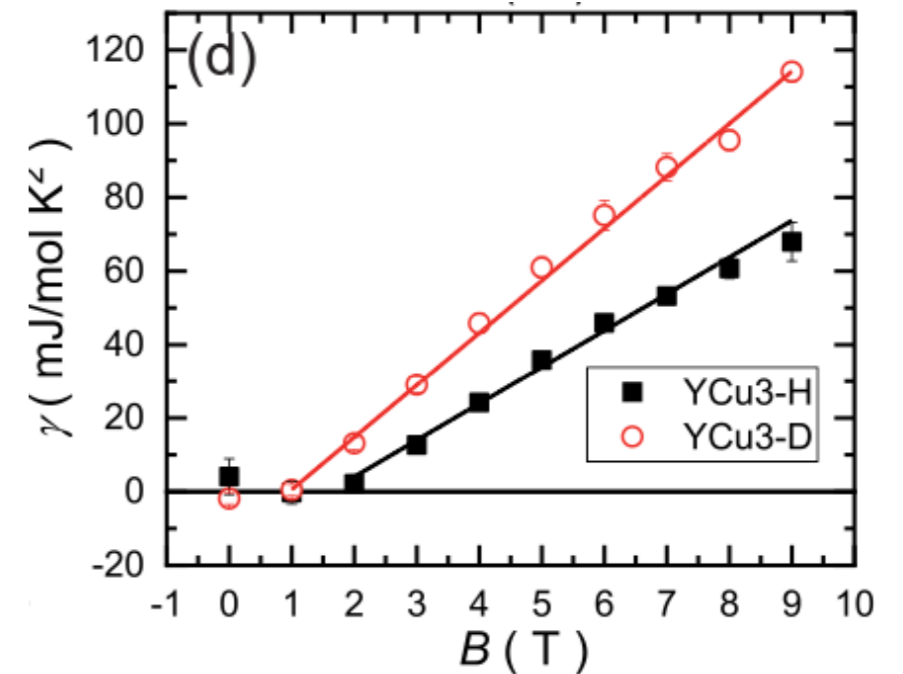


no mag. order  
0.05K



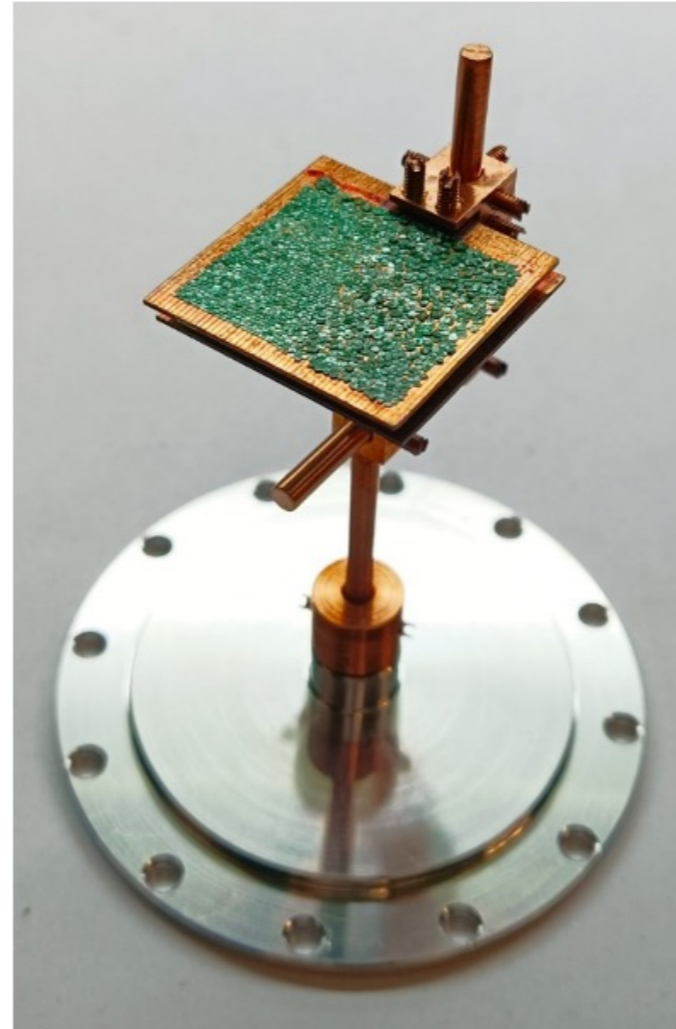
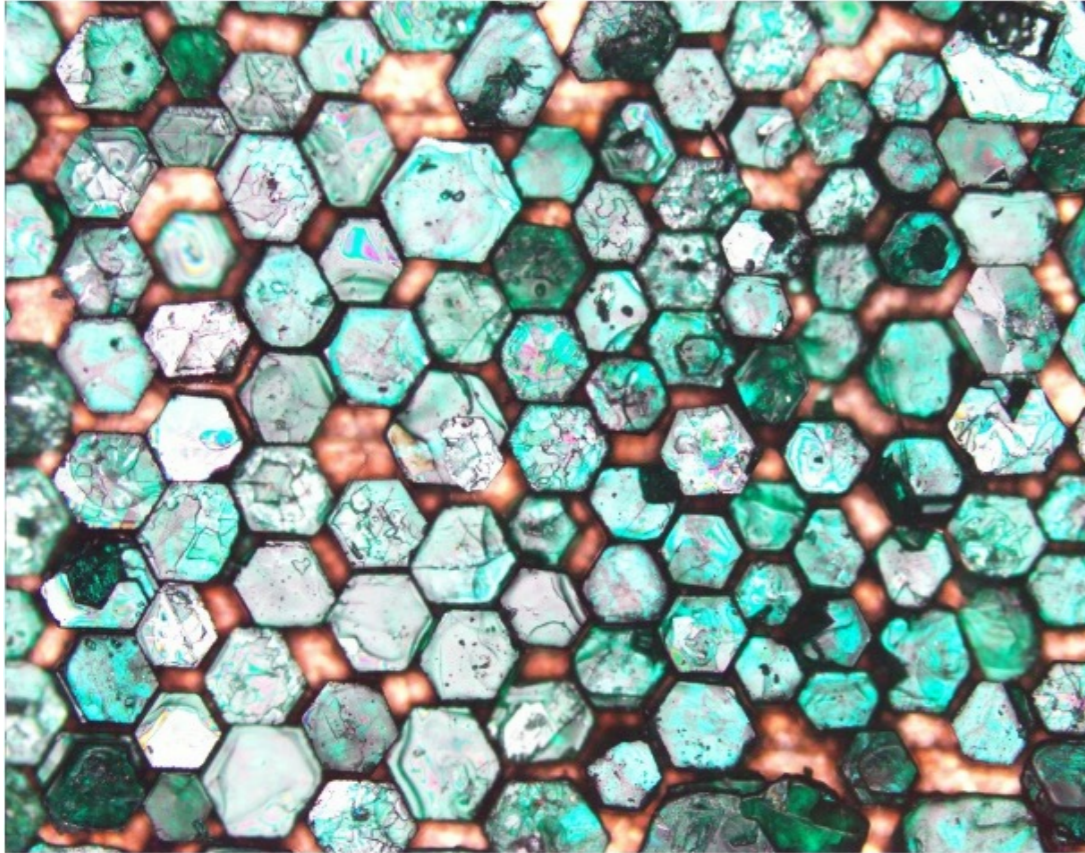
$$C/T = \gamma + \alpha T$$

$$k_B T \ll \mu_B B$$



Possible Dirac quantum spin liquid in a kagome quantum antiferromagnet YCu<sub>3</sub>(OH)<sub>6</sub>Br<sub>2</sub>[Br<sub>x</sub>(OH)<sub>1-x</sub>],  
Zeng, Ma, ..., ZYM, Shiliang Li,  
[Phys. Rev. B 105, L121109 \(2022\)](https://doi.org/10.1103/PhysRevB.105.L121109)

# $\text{YCu}_3(\text{OH})_6\text{Br}_2[\text{Br}_x(\text{OH})_{1-x}]$ (YCu<sub>3</sub>-Br)

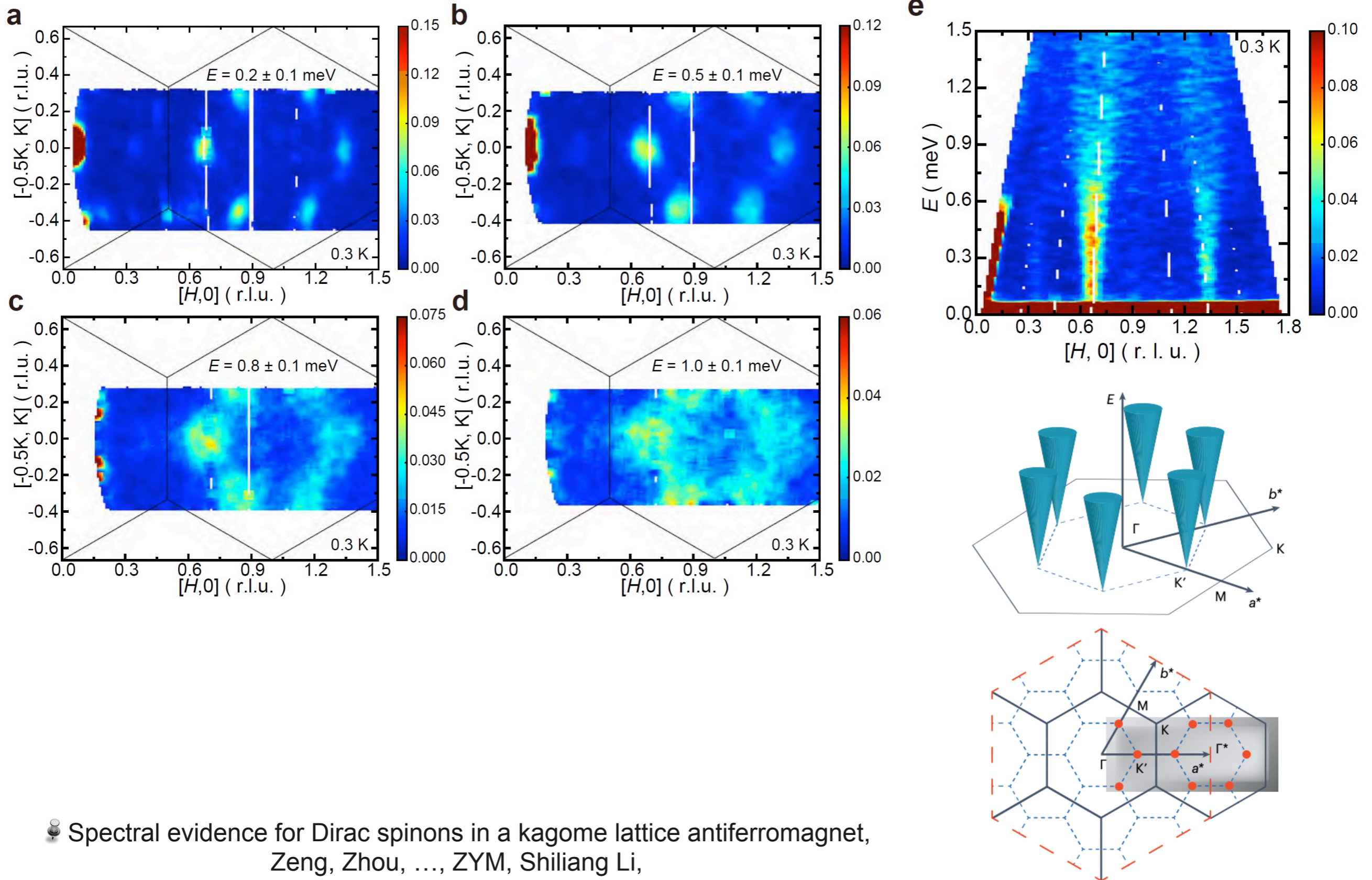


Sample #1: ~ 5000 single crystals, ~ 0.5 g @ AMATERAS, J-Parc

Sample #2: ~ 800 single crystals, ~ 0.5 g @ AMATERAS, J-Parc & THALES, ILL

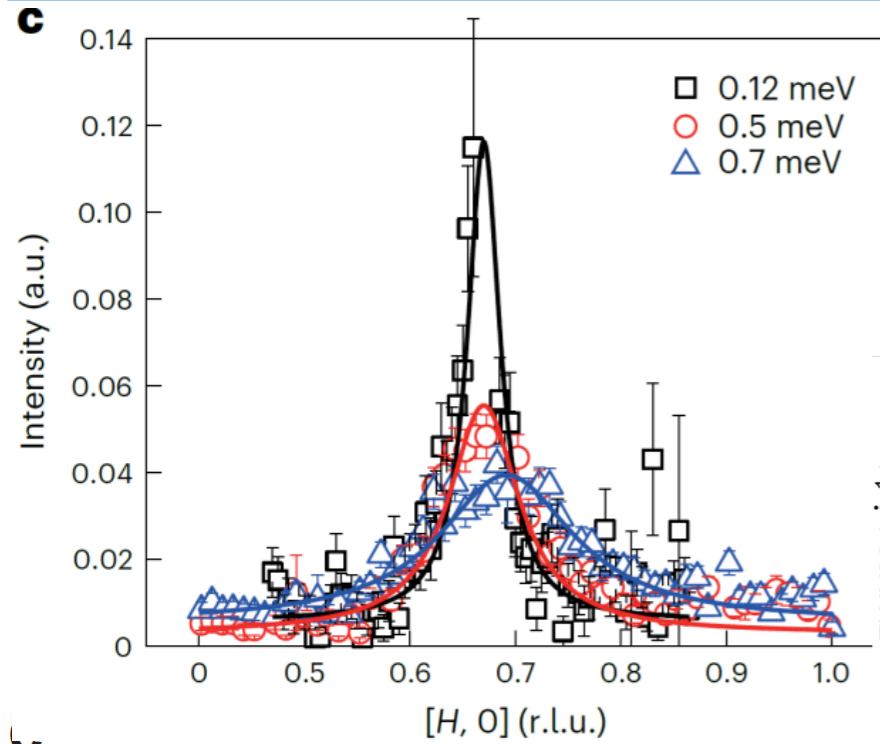
🔍 Spectral evidence for Dirac spinons in a kagome lattice antiferromagnet,  
Zeng, Zhou, ..., ZYM, Shiliang Li,  
[Nat. Phys. 20, 1097 \(2024\)](https://doi.org/10.1038/s41567-024-01097-1)

# YCu<sub>3</sub>(OH)<sub>6</sub>Br<sub>2</sub>[Br<sub>x</sub>(OH)<sub>1-x</sub>] (YCu<sub>3</sub>-Br or YCOB)

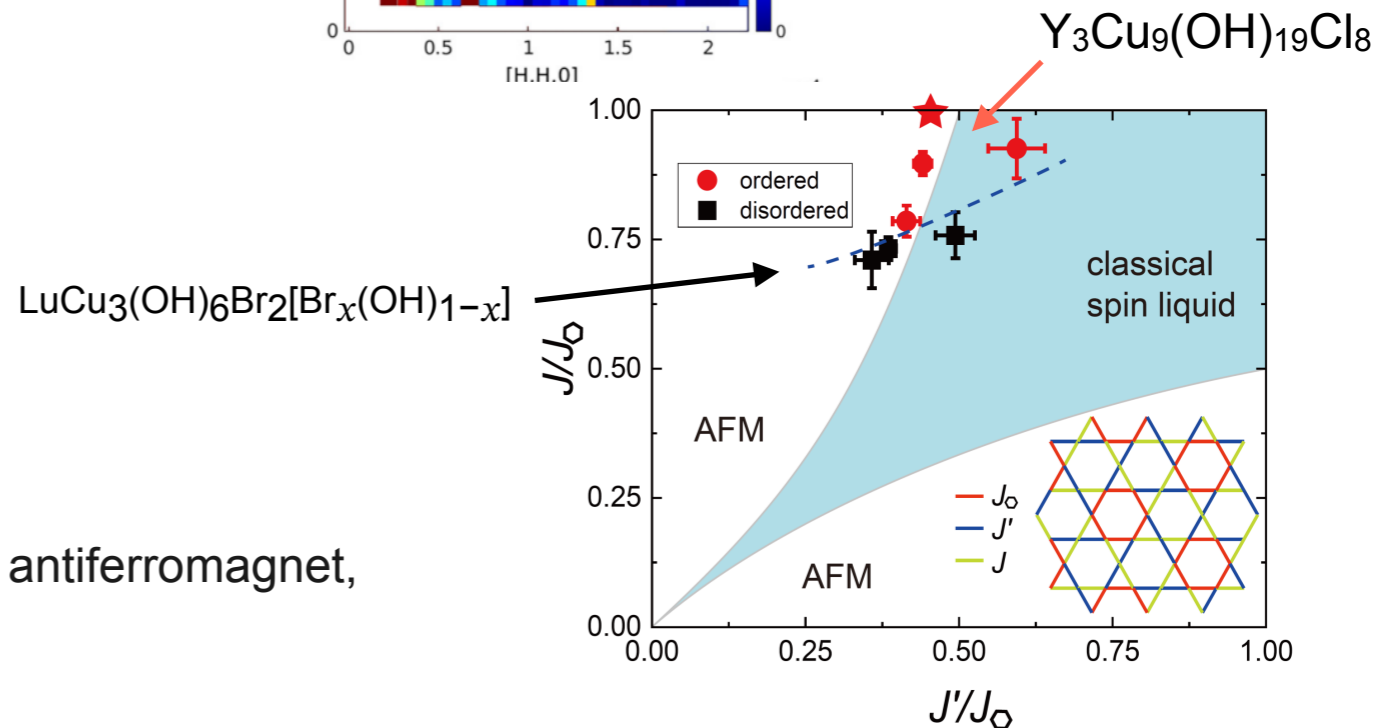
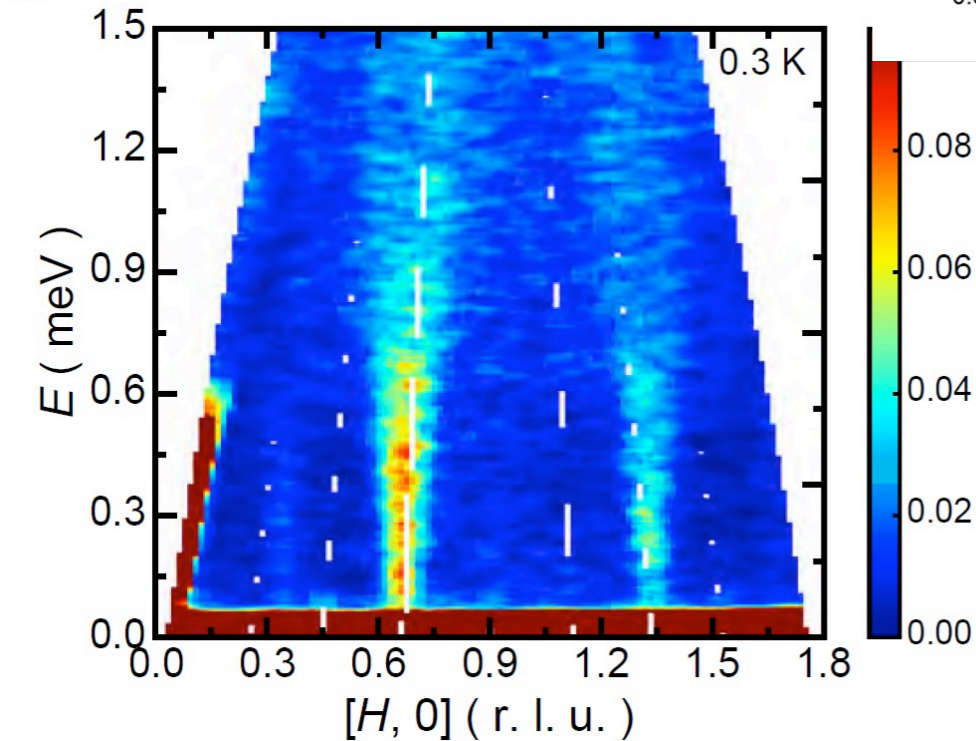
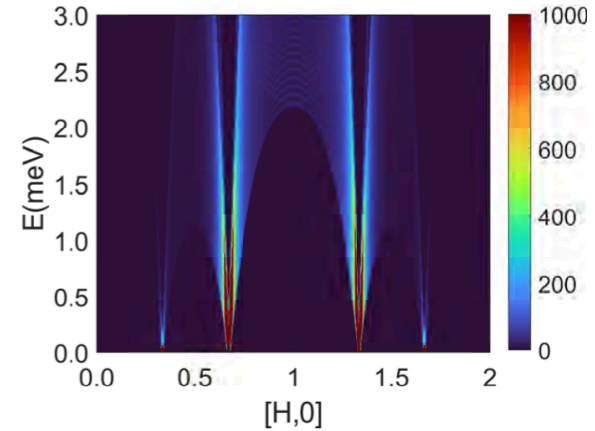
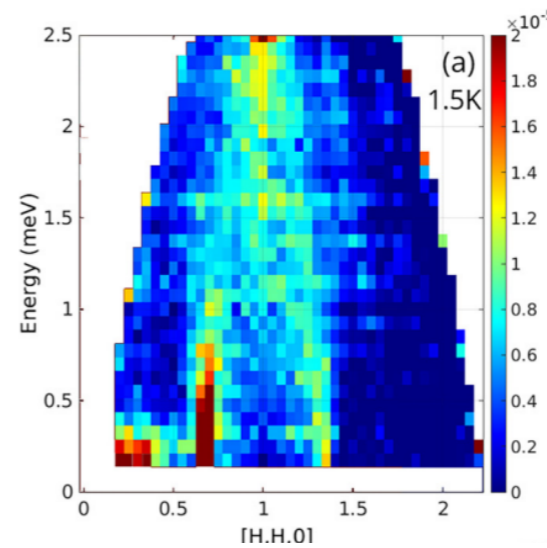
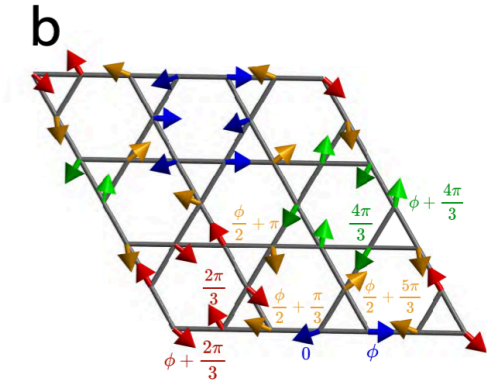
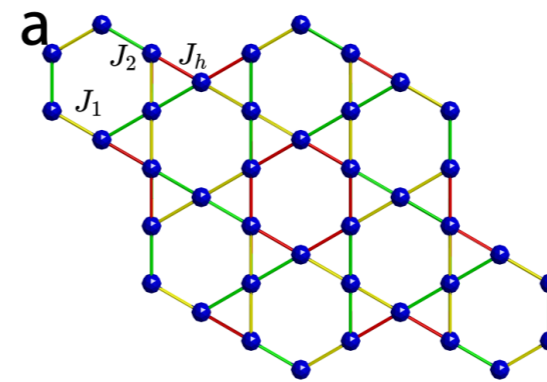
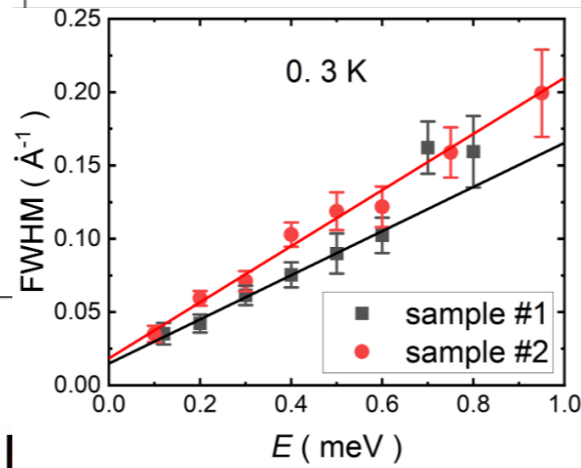


Spectral evidence for Dirac spinons in a kagome lattice antiferromagnet,  
 Zeng, Zhou, ..., ZYM, Shiliang Li,  
[Nat. Phys. 20, 1097 \(2024\)](https://doi.org/10.1038/s41567-024-01097-1)

# YCu<sub>3</sub>(OH)<sub>6</sub>Br<sub>2</sub>[Br<sub>x</sub>(OH)<sub>1-x</sub>] (YCu<sub>3</sub>-Br)



• Y<sub>3</sub>Cu<sub>9</sub>(OH)<sub>19</sub>Cl<sub>8</sub>, Y-kapellasite, *Phys. Rev. B* 107, 125156 (2023)





Spectral evidence for Dirac spinons in a kagome lattice antiferromagnet, Zeng, Zhou, ..., ZYM, Shiliang Li, *Nat. Phys.* 20, 1097 (2024)

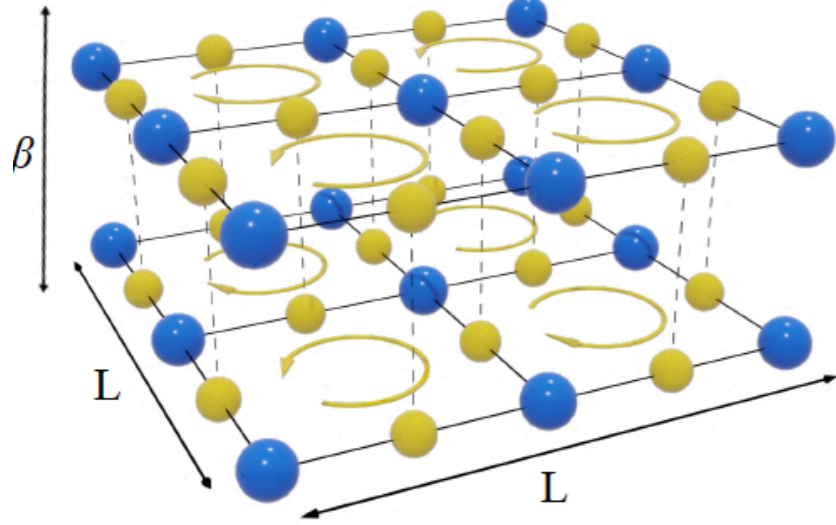
Antiferromagnetic Order and Possible Quantum Spin Liquid in LuCu<sub>3</sub>(OH)<sub>6</sub>Br<sub>2</sub>[Br<sub>x</sub>(OH)<sub>1-x</sub>], Z. Li, ..., Shiliang Li, *Chin. Phys. Lett.* 42, 027504 (2025)

# Monte Carlo Study of Lattice Compact Quantum Electrodynamics with Fermionic Matter: The Parent State of Quantum Phases

Xiao Yan Xu,<sup>1,\*</sup> Yang Qi,<sup>2-4,†</sup> Long Zhang,<sup>5</sup> Fakher F. Assaad,<sup>6</sup> Cenke Xu,<sup>7</sup> and Zi Yang Meng<sup>8,9,10,11,‡</sup>

 Phys. Rev. X 9, 021022 (2019)

 Fermion  
 Gauge Field



$$S = S_B + S_F = \int_0^\beta d\tau (L_B + L_F) \quad i, j \in L \times L$$

$$L_B^{nc} = \frac{1}{JN_f} \sum_{\langle i,j \rangle} \frac{(\phi_{ij,\tau+1} - \phi_{ij,\tau})^2}{\Delta\tau^2} + K \sum_{\square} \cos\left(\sum_{\langle i,j \rangle \in \square} \phi_{ij,\tau}\right) \quad L = 16, 20$$

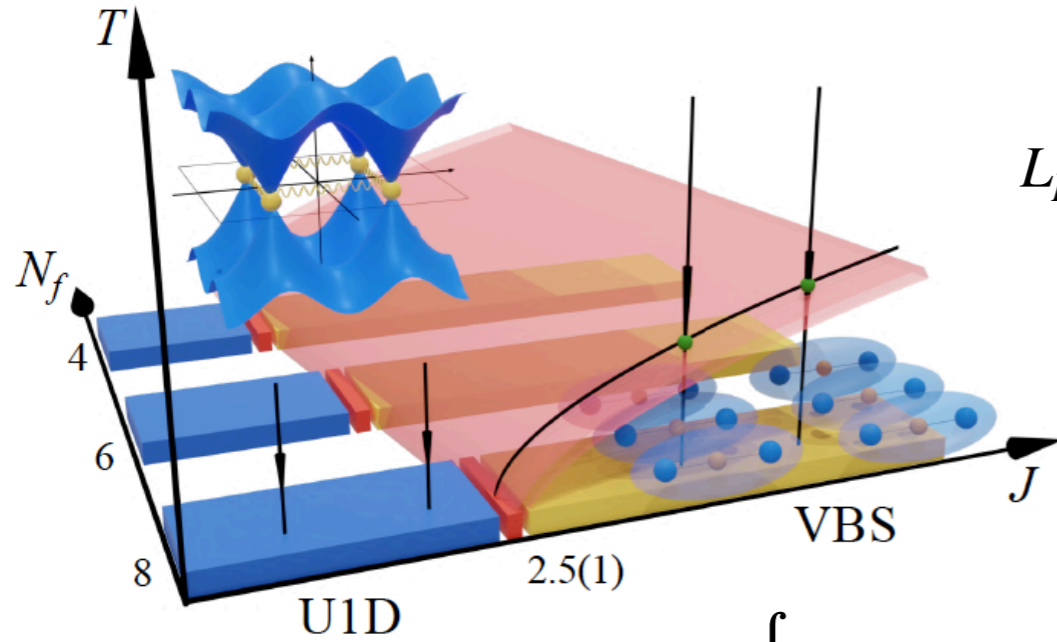
$$\sim E^2 \quad (\nabla \times a)^2 \sim B^2 \quad \tau \in [0, \beta = \frac{1}{T}]$$

$$L_B^c = \frac{1}{JN_f} \sum_{\langle i,j \rangle} \frac{1 - \cos(\phi_{ij,\tau+1} - \phi_{ij,\tau})}{\Delta\tau^2} + K \sum_{\square} \cos\left(\sum_{\langle i,j \rangle \in \square} \phi_{ij,\tau}\right) \quad \beta = 2L, 4L$$

$$\Delta\tau = 0.1$$

$$L_F = \sum_{\langle i,j \rangle, \alpha} \psi_{i,\tau,\alpha}^\dagger (\partial_\tau \delta_{ij} - te^{i\phi_{ij,\tau}}) \psi_{j,\tau,\alpha} + h.c. \quad \alpha \in 2, 4, \dots, N_f$$

$$O(\beta \times (L \times L)^3) \sim O(L^7) \quad N_f = 2, \dots, 8, 10$$



$$Z = \int D(\phi, \psi^\dagger, \psi) e^{-(S_B + S_F)} = \int D\phi e^{-S_B} \mathbf{Tr}[e^{-S_F}] = \int D\phi e^{-S_B} [\det(\mathbf{1} + \prod_{\tau=1}^{\beta} B_\tau(\phi_\tau))]^{N_f}$$

# Monte Carlo Study of Lattice Compact Quantum Electrodynamics with Fermionic Matter: The Parent State of Quantum Phases

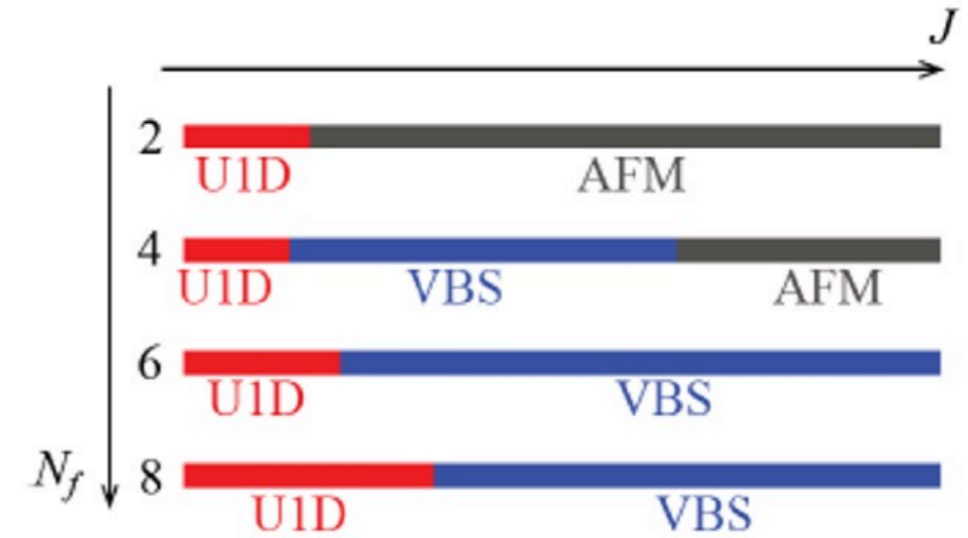
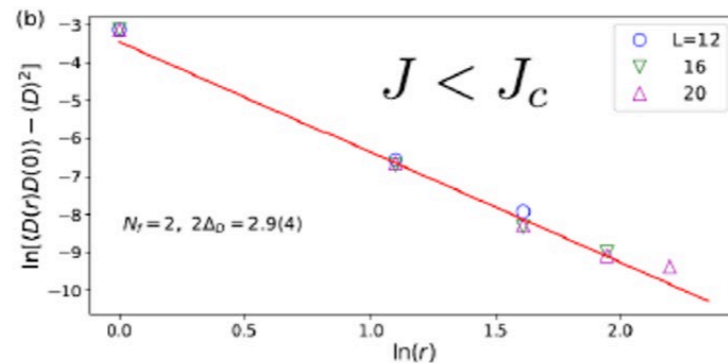
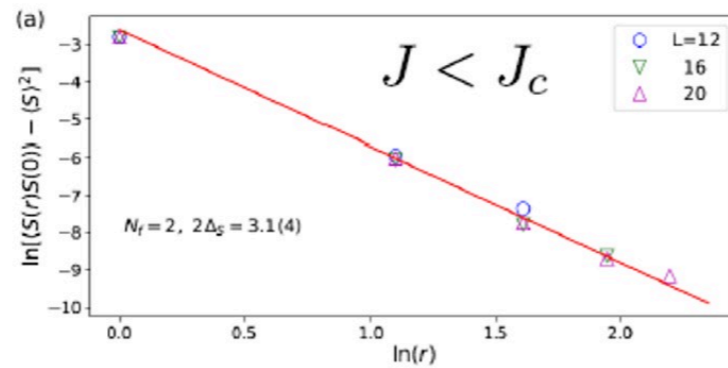
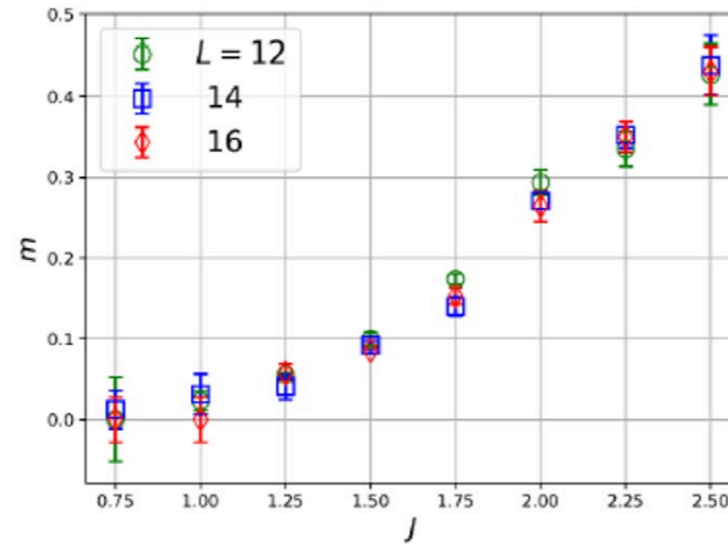
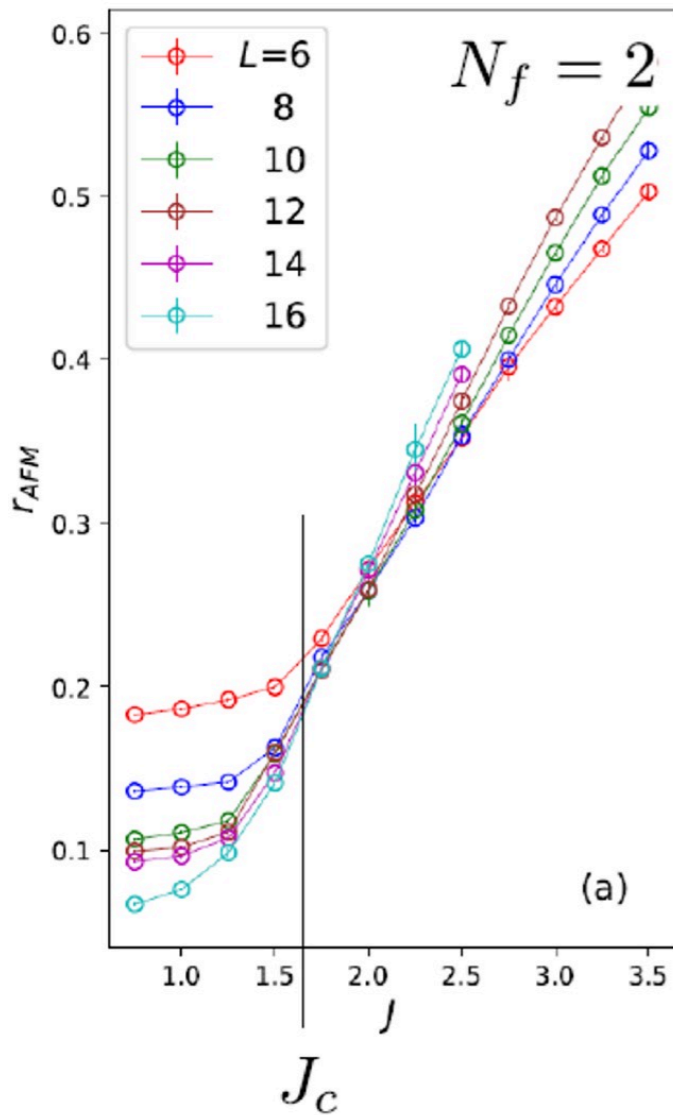
Xiao Yan Xu,<sup>1,\*</sup> Yang Qi,<sup>2-4,†</sup> Long Zhang,<sup>5</sup> Fakher F. Assaad,<sup>6</sup> Cenke Xu,<sup>7</sup> and Zi Yang Meng<sup>8,9,10,11,‡</sup>

 Phys. Rev. X 9, 021022 (2019)

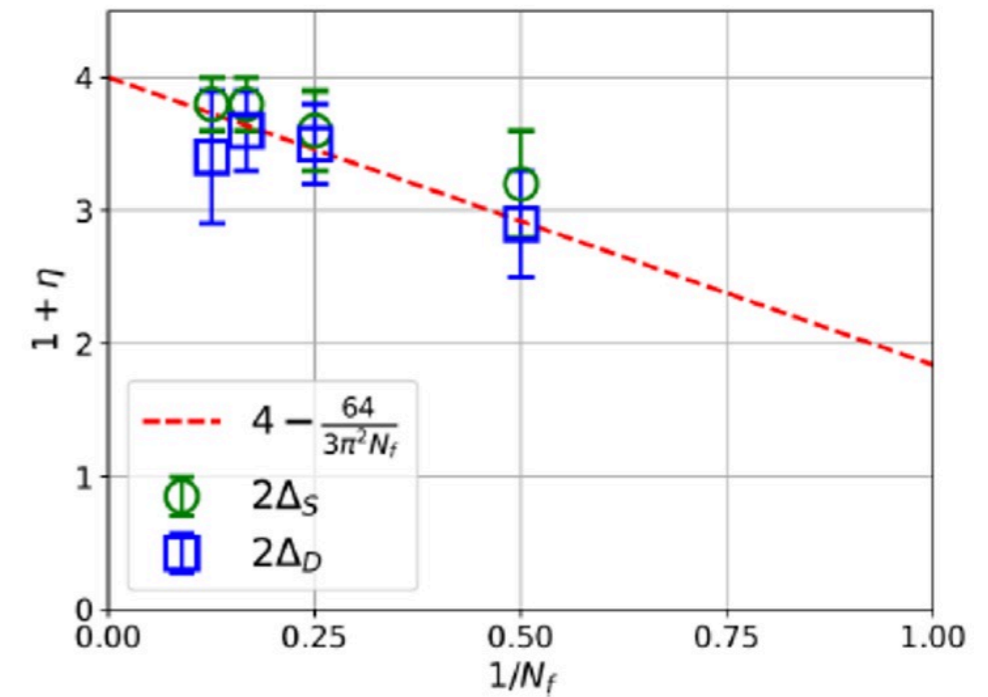
$$S_{\beta}^{\alpha}(i) = \psi_{i\alpha}^{\dagger} \psi_{i\beta} - \frac{1}{N_f} \delta_{\alpha\beta} \sum_{\gamma} \psi_{i\gamma}^{\dagger} \psi_{i\gamma}$$

$$\chi_S(\mathbf{k}) = \frac{1}{L^4} \sum_{ij} \sum_{\alpha\beta} \langle S_{\beta}^{\alpha}(i) S_{\alpha}^{\beta}(j) \rangle e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)}$$

$$r_{\text{AFM}} = 1 - \frac{\chi_S(\mathbf{X} + \delta\mathbf{q})}{\chi_S(\mathbf{X})}$$



$$C(\tau) = \langle \sin(\sum_{b \in \square} \phi_b(\tau)) \cdot \sin(\sum_{b \in \square} \phi_b(0)) \rangle \sim \exp(-m\tau)$$



# Monte Carlo Study of Lattice Compact Quantum Electrodynamics with Fermionic Matter: The Parent State of Quantum Phases

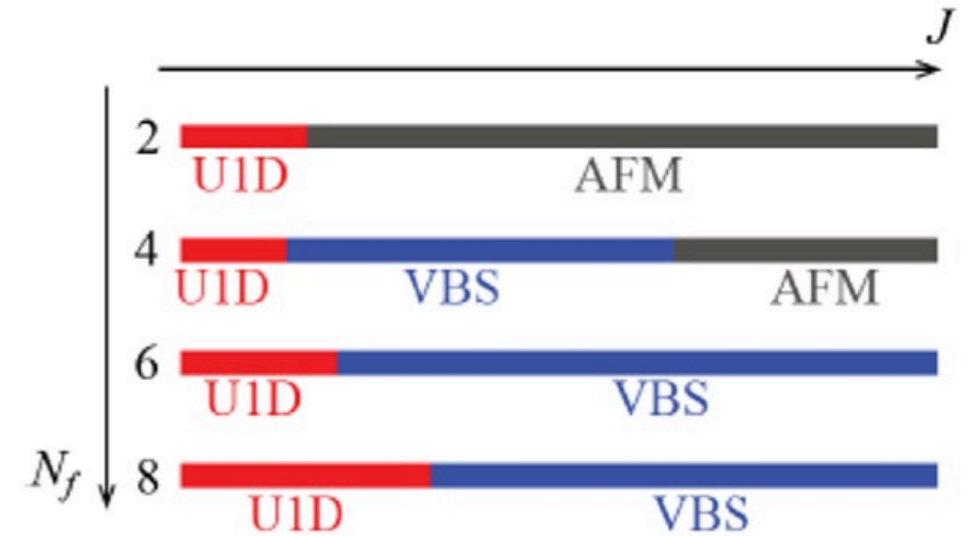
Xiao Yan Xu,<sup>1,\*</sup> Yang Qi,<sup>2-4,†</sup> Long Zhang,<sup>5</sup> Fagher F. Assaad,<sup>6</sup> Cenke Xu,<sup>7</sup> and Zi Yang Meng<sup>8,9,10,11,‡</sup>

 Phys. Rev. X 9, 021022 (2019)

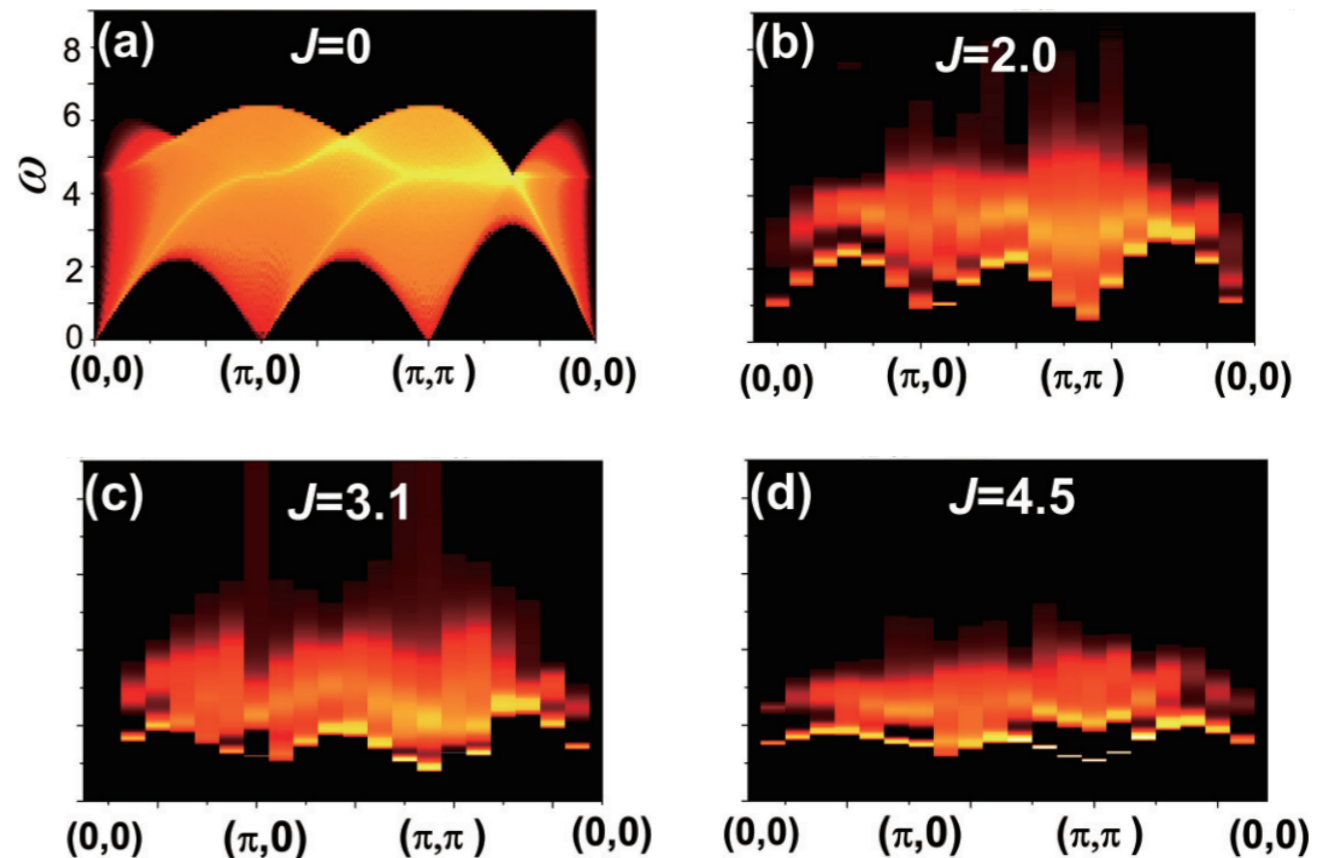
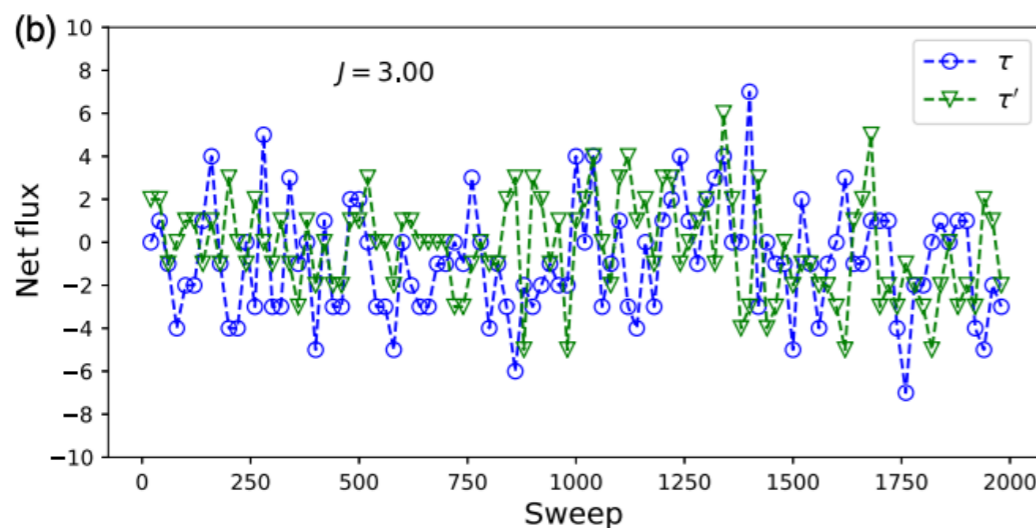
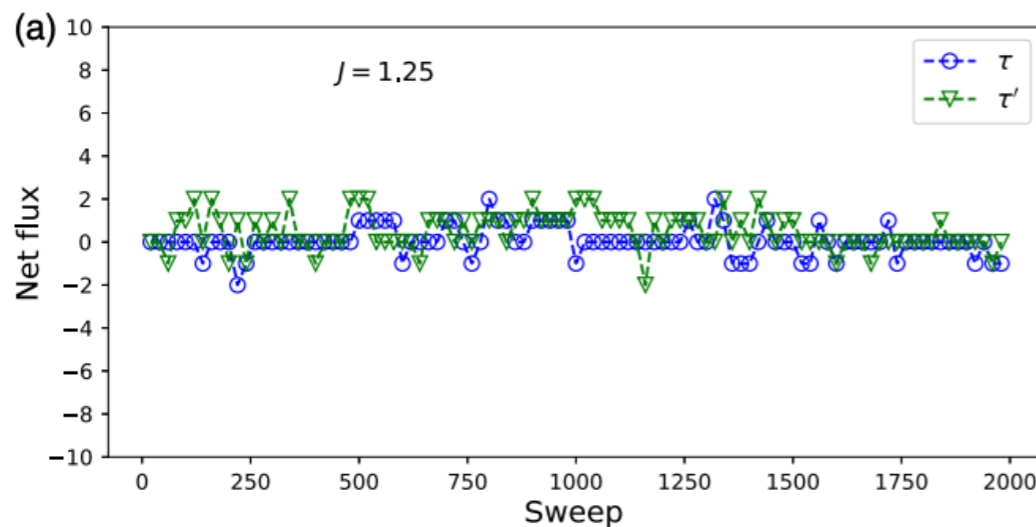
$$\sum_{b \in \square} \phi_b(\tau) = \Phi_{\square}(\tau) + 2\pi m_{\square}(\tau) \quad \Phi_{\square}(\tau) \in [0, 2\pi)$$

$$\text{Net flux} \quad M(\tau) = \sum_{\square} m_{\square}(\tau)$$

$$N_f = 2, \quad J_c \sim 1.6(2), \quad L = 12, \quad \tau' - \tau = 8\Delta\tau$$



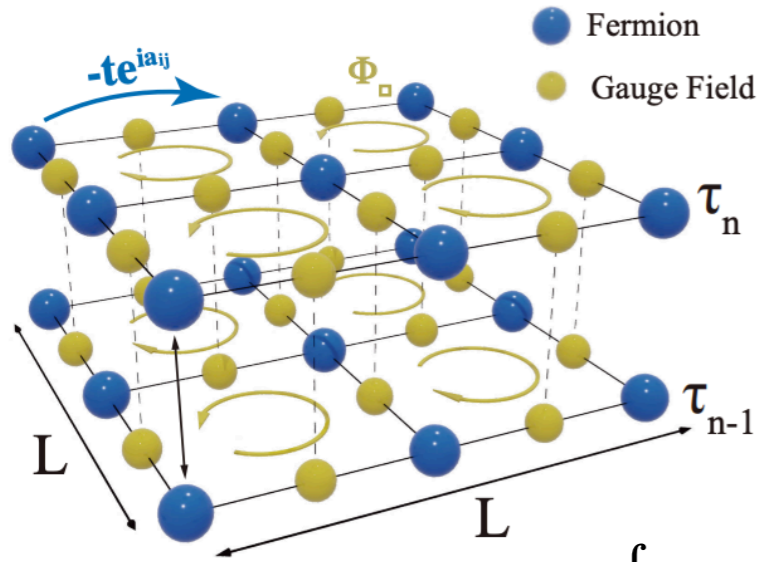
$$N_f = 8, \quad J_c \sim 2.5(1), \quad L = 14$$



# Emergent gauge flux in QED<sub>3</sub> with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

Chuang Chen,<sup>1,2</sup> Urban F. P. Seifert,<sup>3</sup> Kexin Feng,<sup>1</sup> Oleg A. Starykh,<sup>4</sup> Leon Balents,<sup>5,6,7</sup> and Zi Yang Meng<sup>1</sup>

 arXiv: 2508.08528



$$S = \sum_{i,n} \left[ \psi_i^\dagger(\tau_n) (\psi_i(\tau_n) - \psi_i(\tau_{n-1})) - \frac{1}{2} B \psi_i^\dagger(\tau_n) \sigma^z \psi_i(\tau_n) \right]$$

$$- t \sum_{\langle ij \rangle, n} \left[ e^{ia_{ij}(\tau_n)} \psi_i^\dagger(\tau_n) \psi_j(\tau_n) + \text{h.c.} \right]$$

$$+ \frac{1}{J} \sum_{\langle ij \rangle, n} [a_{ij}(\tau_n) - a_{ij}(\tau_{n-1})]^2$$

$$i, j \in L \times L$$

$$K = 0 \quad L = 16, 20$$

$$\tau \in [0, \beta = 1/T]$$

$$Z = \int D(\phi, \psi^\dagger, \psi) e^{-(S_B + S_F)} = \int D\phi e^{-S_B} \text{Tr}[e^{-S_F}] = \int D\phi e^{-S_B} [\det(\mathbf{1} + \prod_{\tau=1}^{\beta} B_\tau(\phi_\tau, B))]^{N_f} \quad \beta = 2L$$

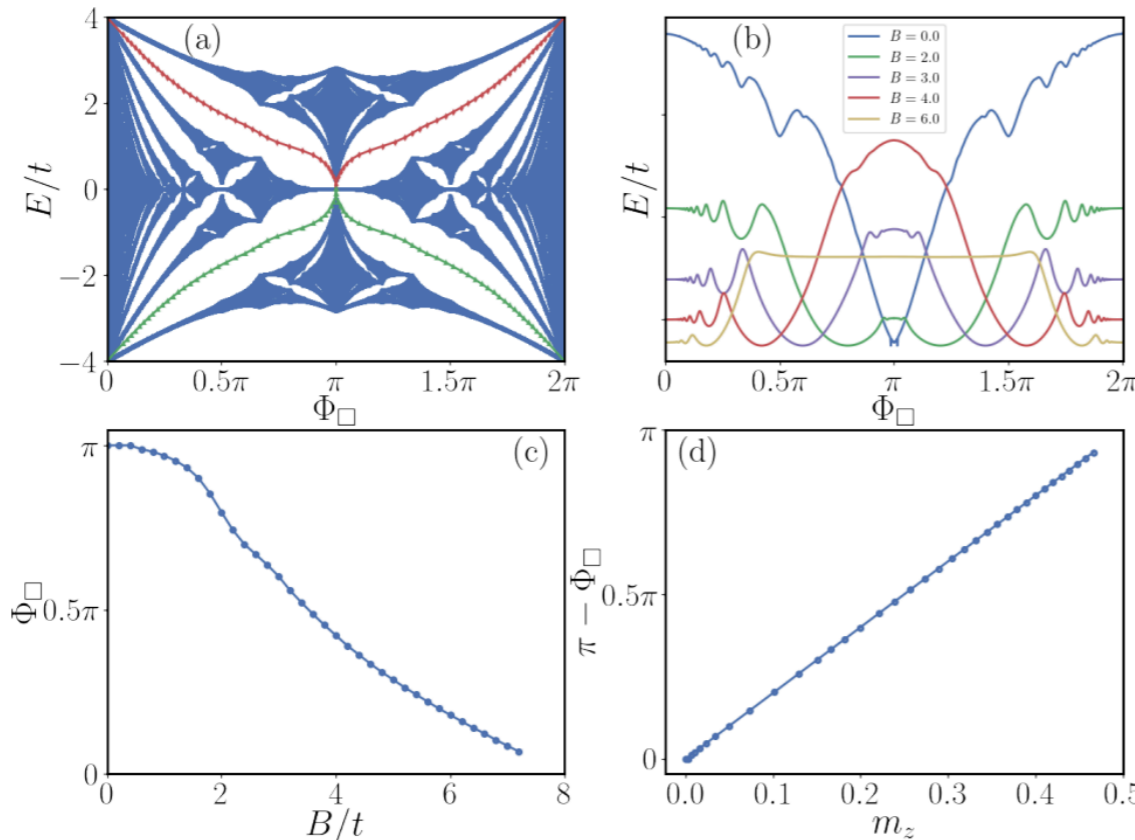
$J \rightarrow 0$  : mean-field limit, temporal gauge fluctuations frozen

$$\Delta\tau = 0.1$$

$$N_f = 2$$

$$O(\beta \times (L \times L)^3) \sim O(L^7)$$

$L = 32$

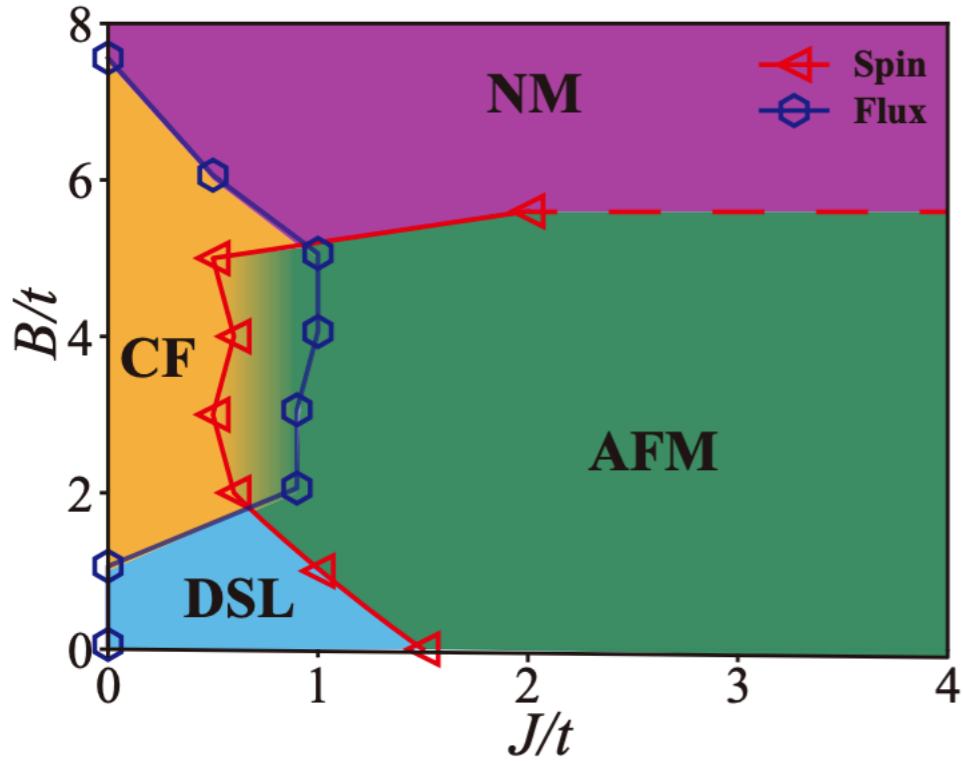


$$\Phi_{\square}(\tau) = \nabla \times a_{\square}(\tau) = \sum_{b \in \square} \phi_b(\tau)$$

# Emergent gauge flux in QED<sub>3</sub> with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

Chuang Chen,<sup>1,2</sup> Urban F. P. Seifert,<sup>3</sup> Kexin Feng,<sup>1</sup> Oleg A. Starykh,<sup>4</sup> Leon Balents,<sup>5,6,7</sup> and Zi Yang Meng<sup>1</sup>

 [arXiv: 2508.08528](https://arxiv.org/abs/2508.08528)



$$\text{CF: } \chi = \langle \sin(\sum_{b \in \square} \phi_b) \rangle = \langle \sin(\nabla \times a) \rangle \neq 0$$

emergent gauge flux

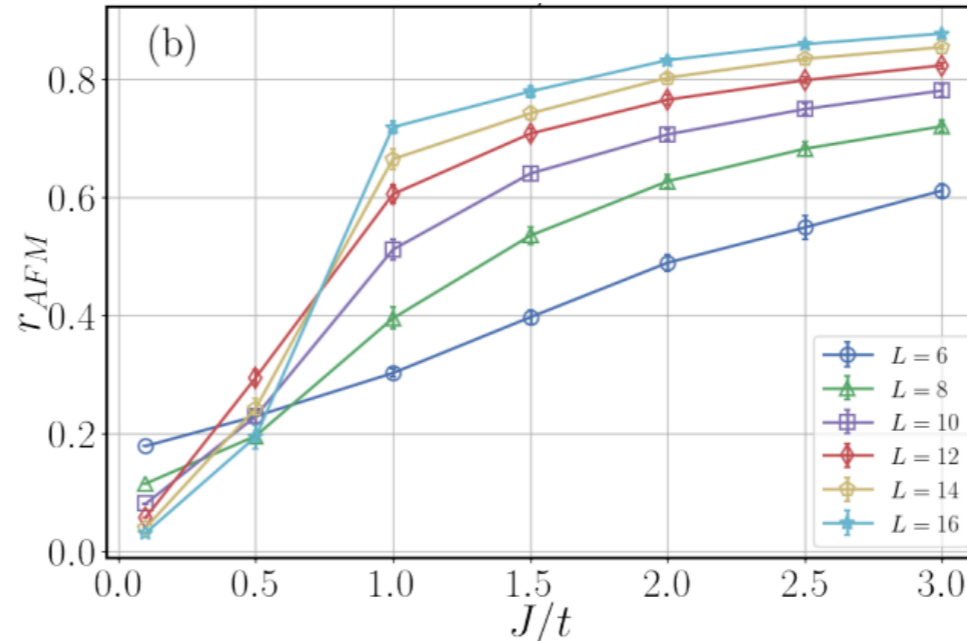
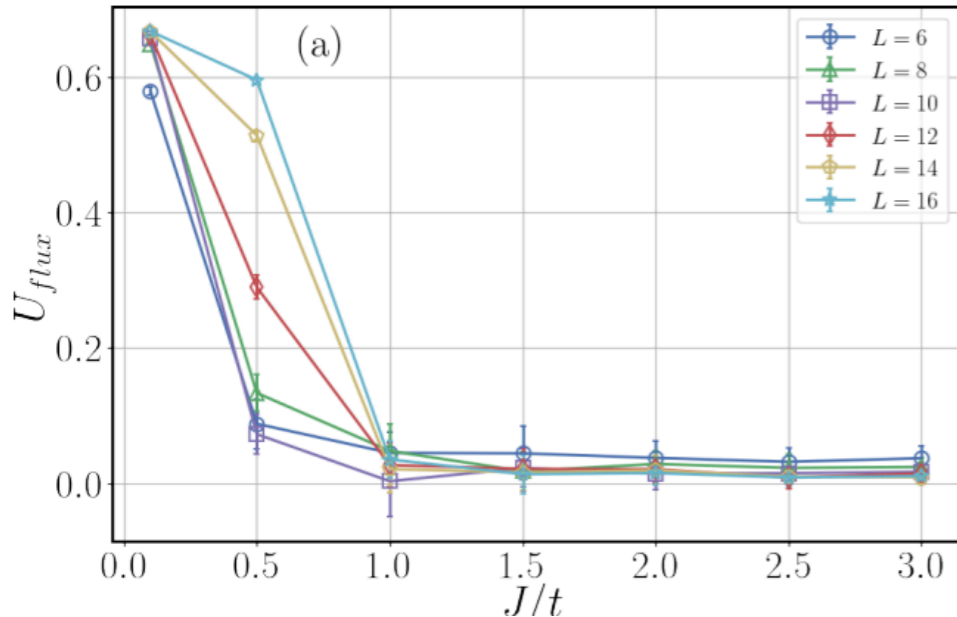
$$S^\pm(\mathbf{q}) = \frac{1}{N} \sum_{ij} \langle \sin(\Phi_{\square,i}) \sin(\Phi_{\square,j}) \rangle e^{-i\mathbf{q} \cdot \mathbf{r}_{ij}} \quad \mathbf{q} = \Gamma$$

$$\text{AFM: } N^+ = (-1)^i \langle \psi_i^\dagger \sigma^+ \psi_i \rangle \neq 0 \quad \text{in-plane AFM order}$$

$$S^\pm(\mathbf{q}) = \frac{1}{N} \sum_{ij} \langle S_i^+ S_j^- + h.c. \rangle e^{-i\mathbf{q} \cdot \mathbf{r}_{ij}} \quad \mathbf{q} = M = (\pi, \pi)$$

$$N^+ = \lim_{N \rightarrow \infty} \sqrt{\frac{1}{N} S^\pm(M)}$$

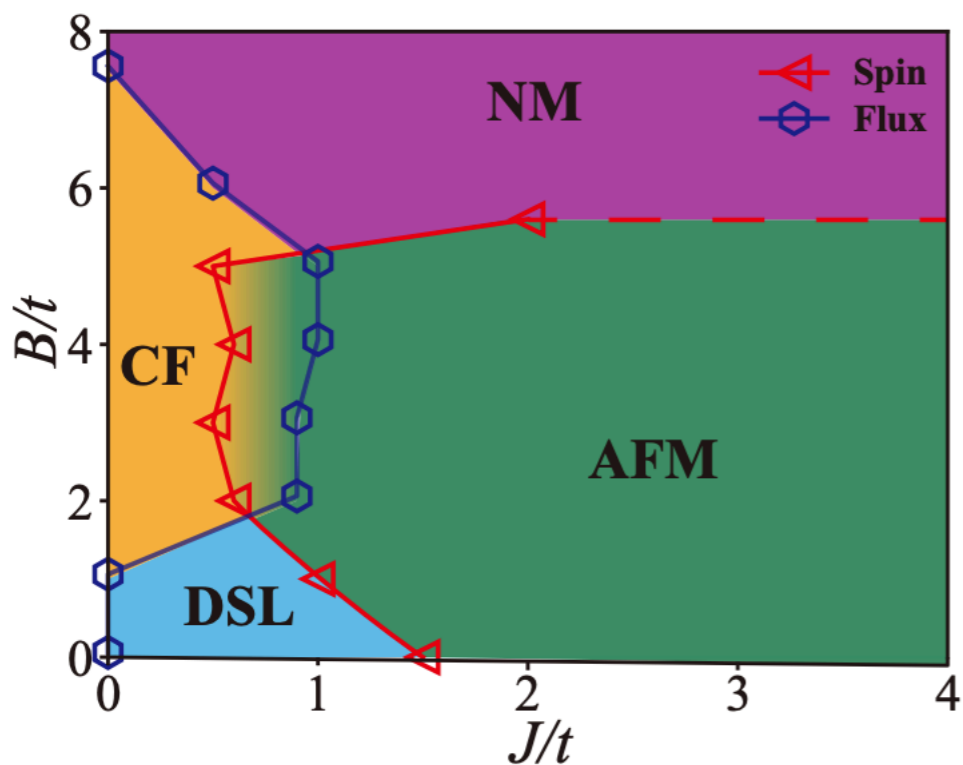
$B/t = 2$



# Emergent gauge flux in QED<sub>3</sub> with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

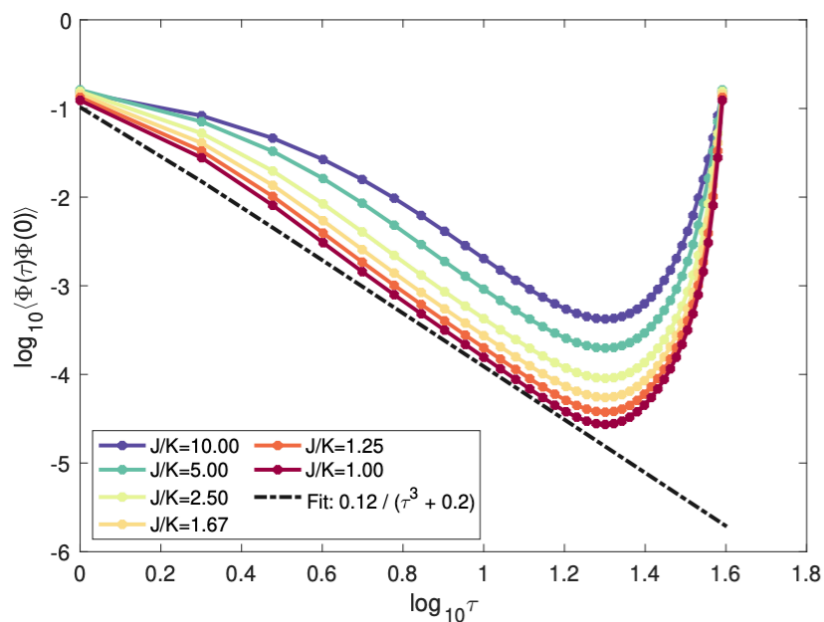
Chuang Chen,<sup>1,2</sup> Urban F. P. Seifert,<sup>3</sup> Kexin Feng,<sup>1</sup> Oleg A. Starykh,<sup>4</sup> Leon Balents,<sup>5,6,7</sup> and Zi Yang Meng<sup>1</sup>

 [arXiv: 2508.08528](https://arxiv.org/abs/2508.08528)



$$C_\chi(\tau) \sim \tau^3$$

in free photon theory and in AFM



$$\text{CF: } \chi = \langle \sin(\sum_{b \in \square} \phi_b) \rangle = \langle \sin(\nabla \times a) \rangle \neq 0$$

emergent gauge flux

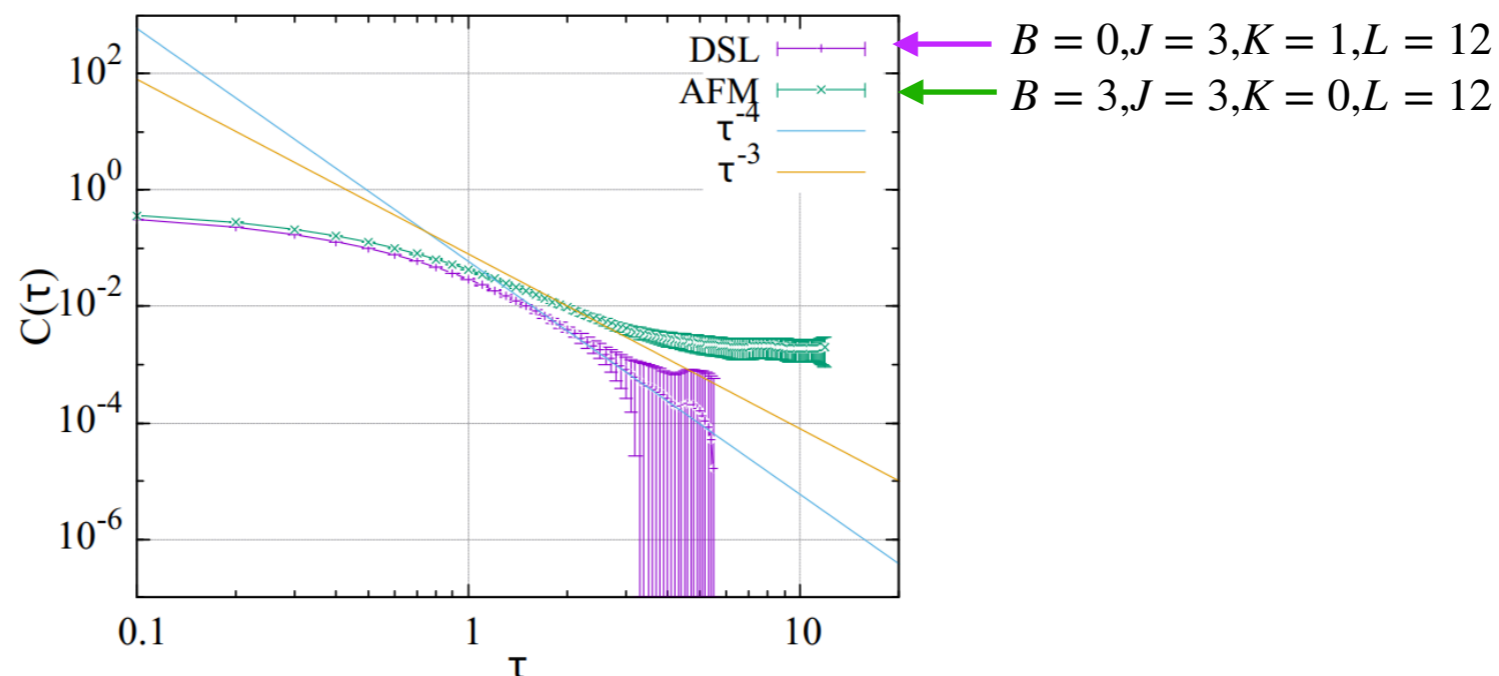
$$\text{AFM: } N^+ = (-1)^i \langle \psi_i^\dagger \sigma^+ \psi_i \rangle \neq 0$$

in-plane AFM order

$$C_\chi(\tau) = \frac{1}{N_\square} \sum_{\square} \langle \sin(\sum_{b \in \square} \phi_b(\tau)) \cdot \sin(\sum_{b \in \square} \phi_b(0)) \rangle$$

$$C_\chi(\tau) \sim \tau^4$$

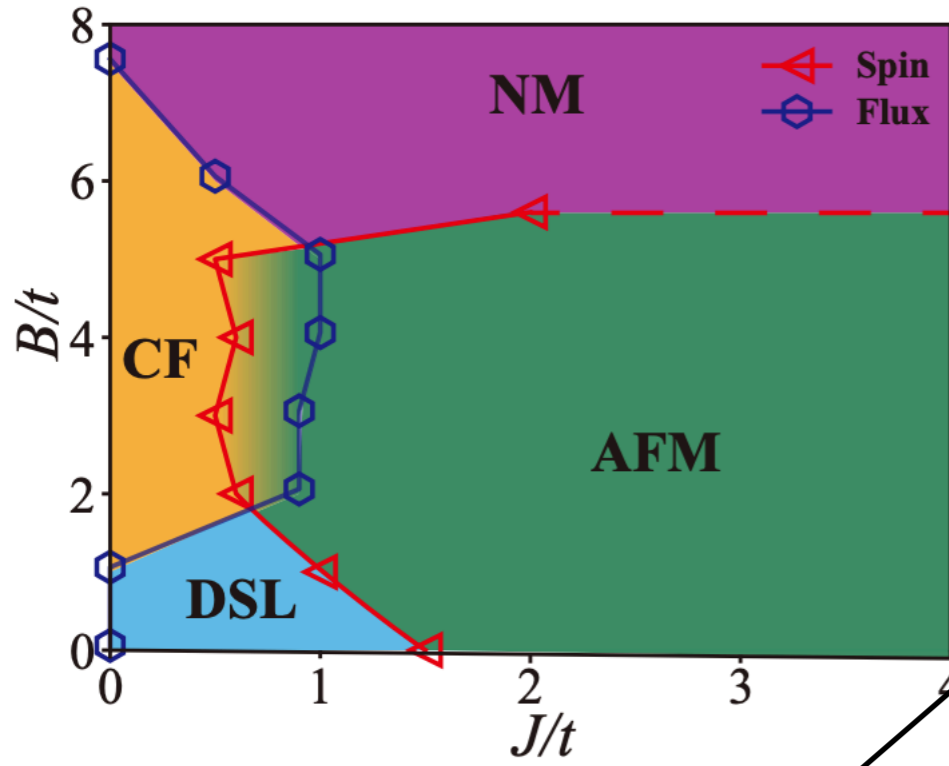
in DSL, conserved currents



# Emergent gauge flux in QED<sub>3</sub> with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

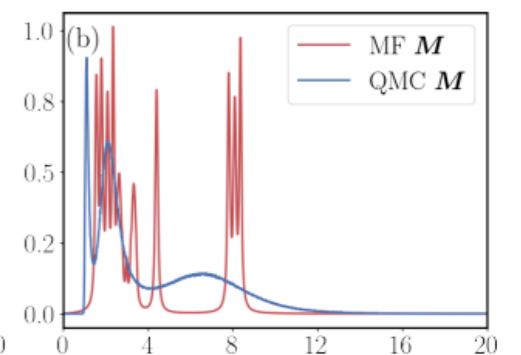
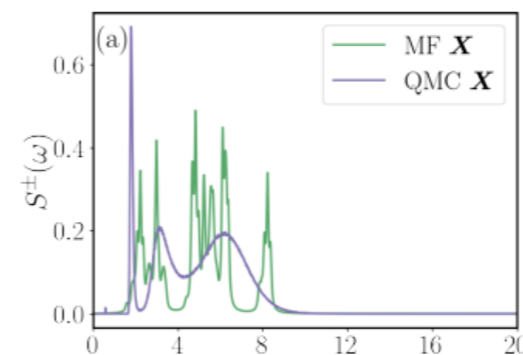
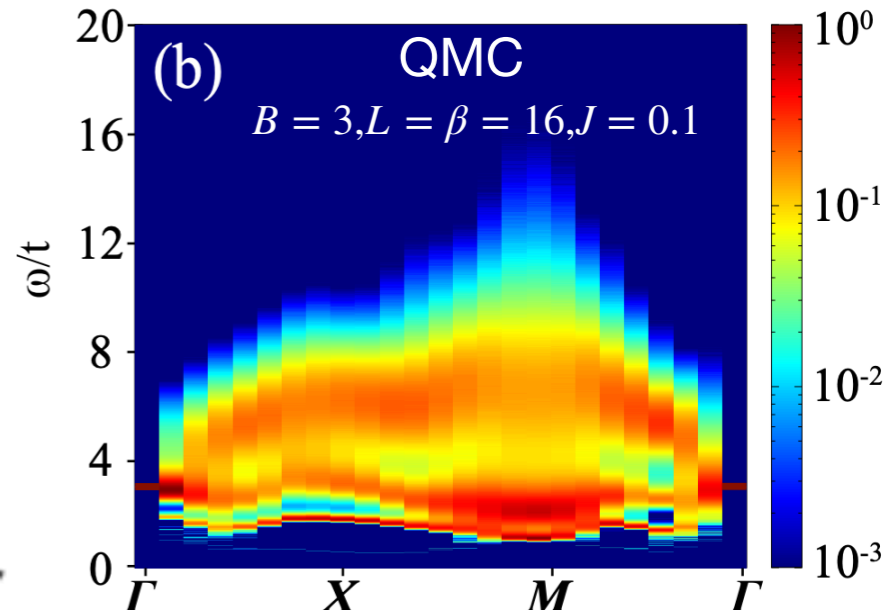
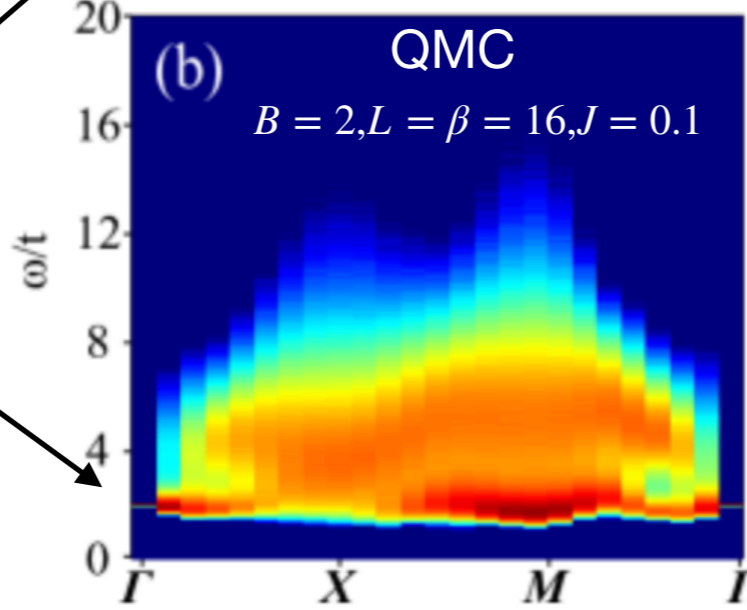
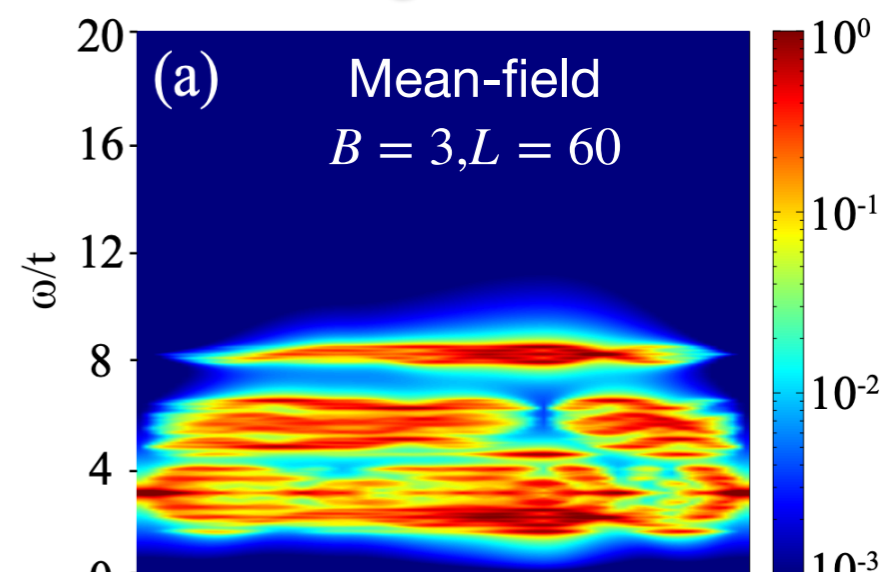
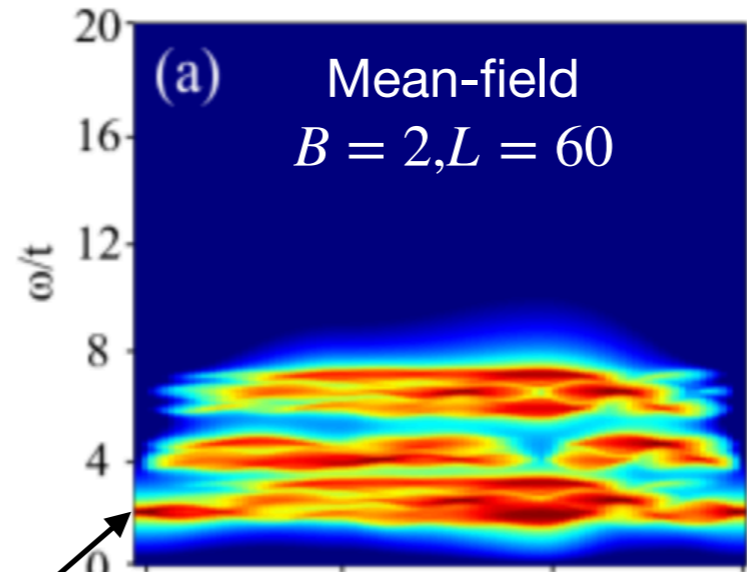
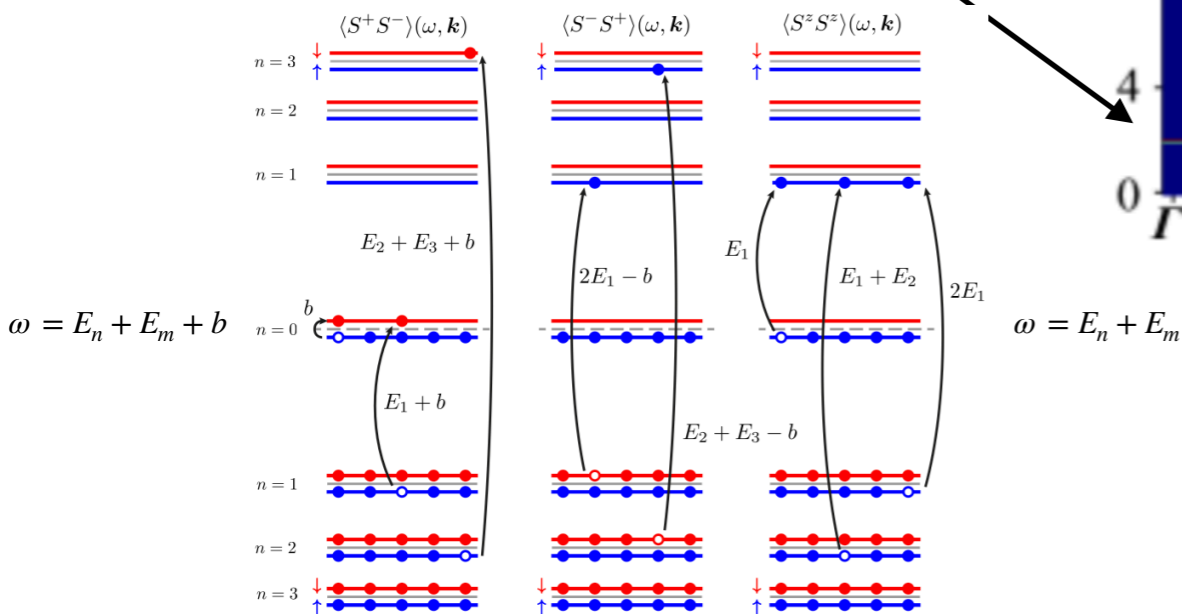
Chuang Chen,<sup>1,2</sup> Urban F. P. Seifert,<sup>3</sup> Kexin Feng,<sup>1</sup> Oleg A. Starykh,<sup>4</sup> Leon Balents,<sup>5,6,7</sup> and Zi Yang Meng<sup>1</sup>

 [arXiv: 2508.08528](https://arxiv.org/abs/2508.08528)



$S^\pm(\mathbf{q}, \omega)$

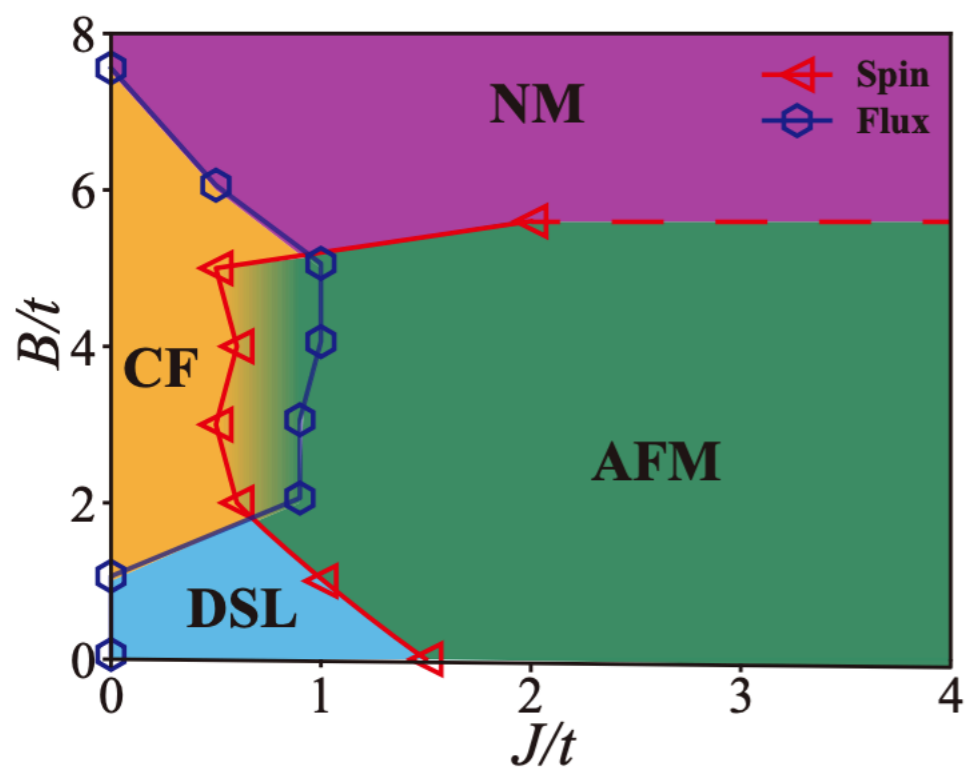
Larmor pole at  $\omega = B$



# Emergent gauge flux in QED<sub>3</sub> with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

Chuang Chen,<sup>1,2</sup> Urban F. P. Seifert,<sup>3</sup> Kexin Feng,<sup>1</sup> Oleg A. Starykh,<sup>4</sup> Leon Balents,<sup>5,6,7</sup> and Zi Yang Meng<sup>1</sup>

 arXiv: 2508.08528



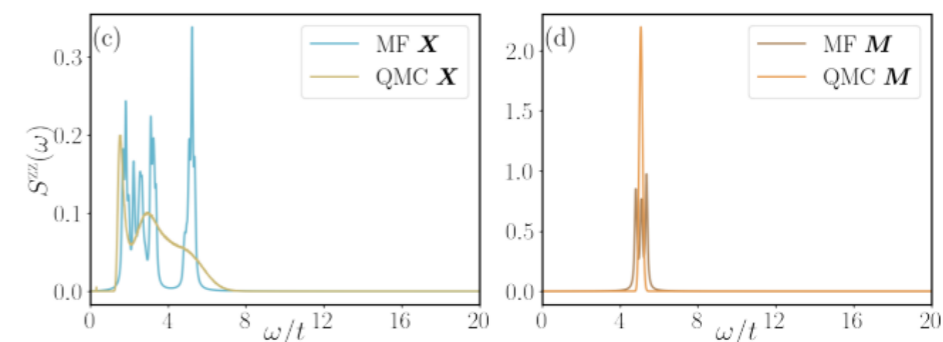
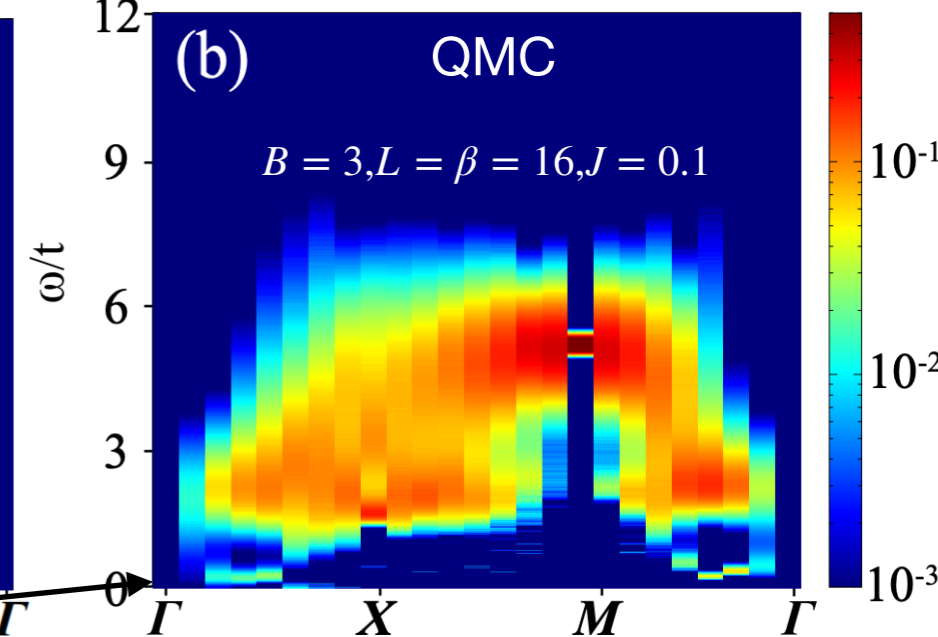
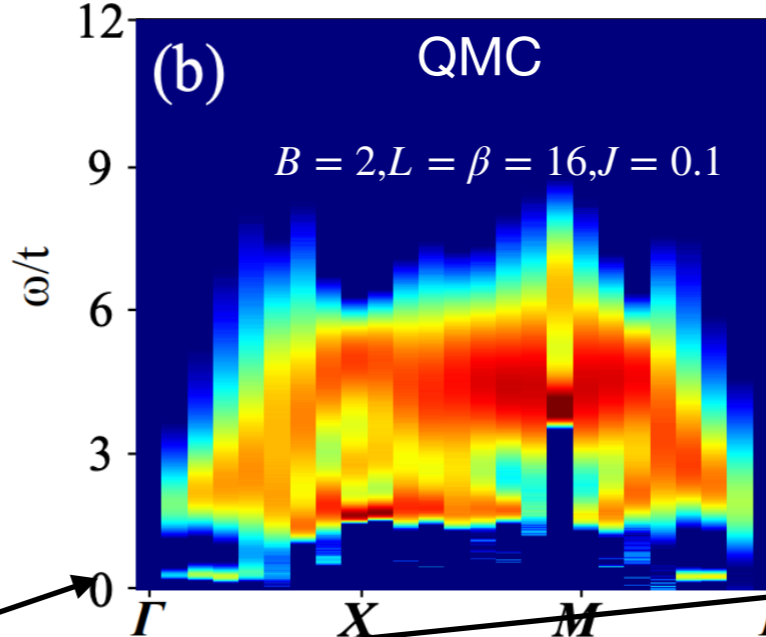
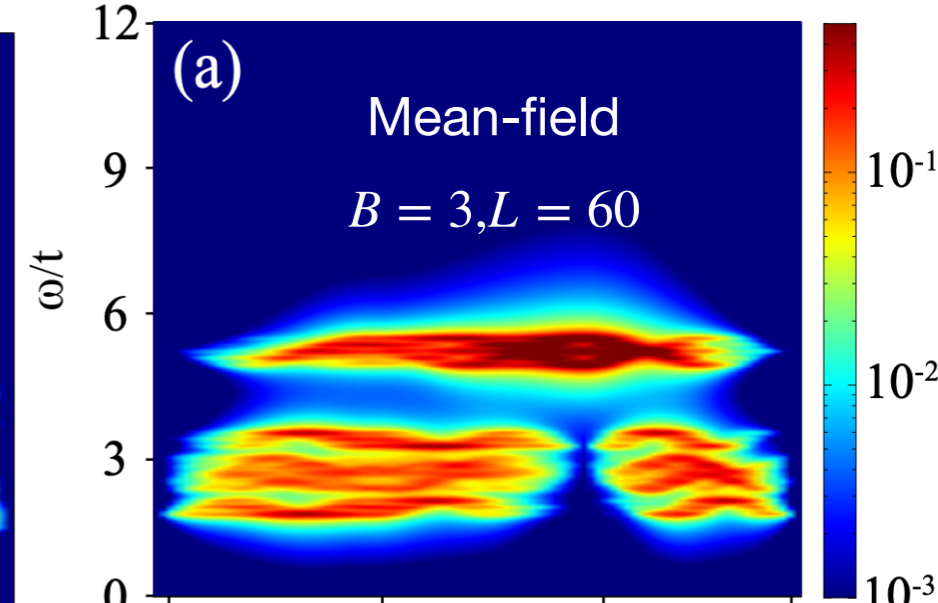
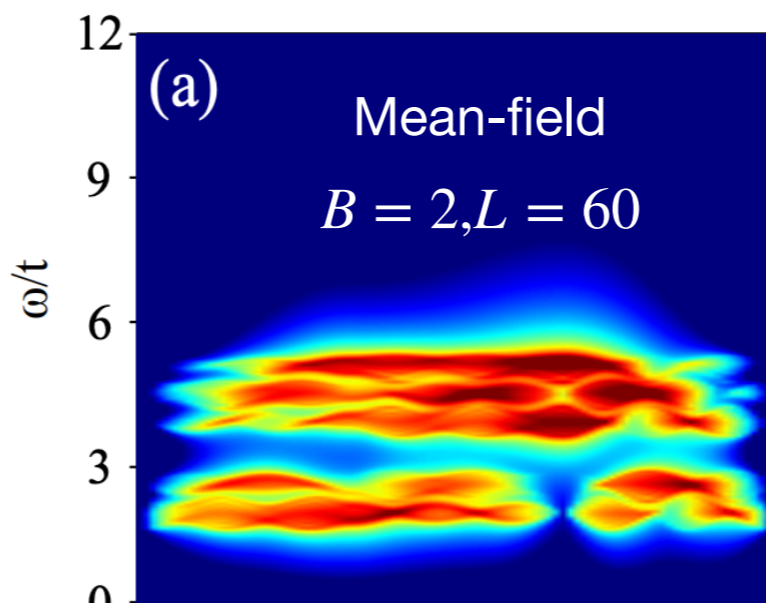
$$S^{zz}(\mathbf{q}, \omega)$$

$$S^z \sim \nabla \times a^c$$

$$\langle S^z S^z \rangle_{\omega, \mathbf{q}} \sim \frac{1}{2} \chi c q \delta(\omega - cq)$$

spin corr. probes emergent magnetic flux

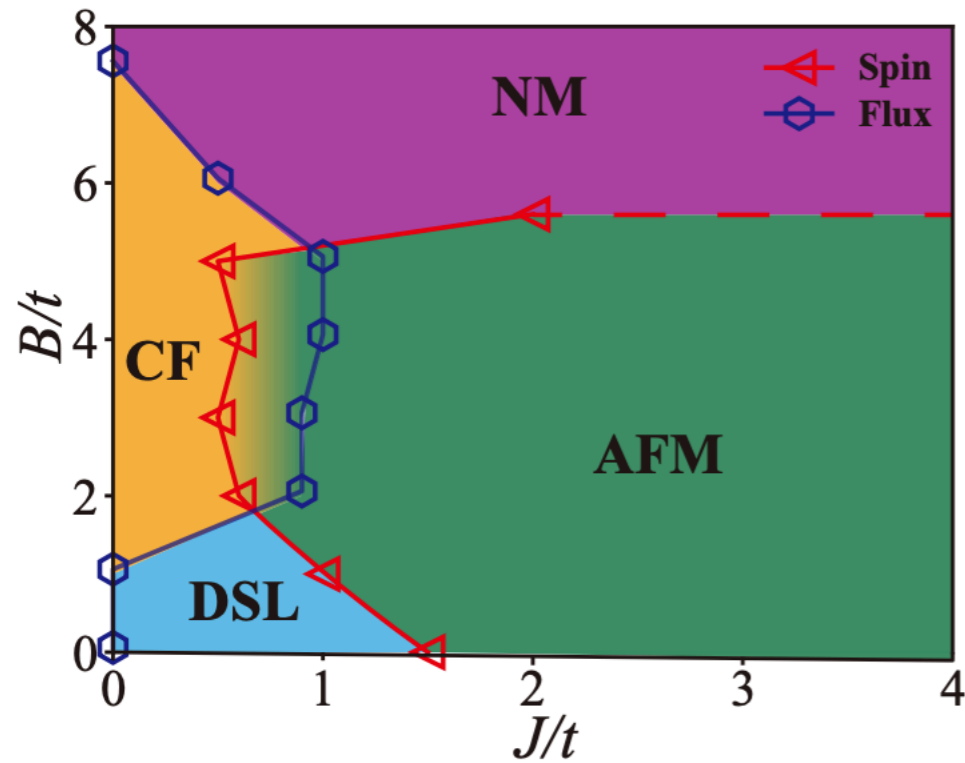
gapless gauge photon



# Emergent gauge flux in QED<sub>3</sub> with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

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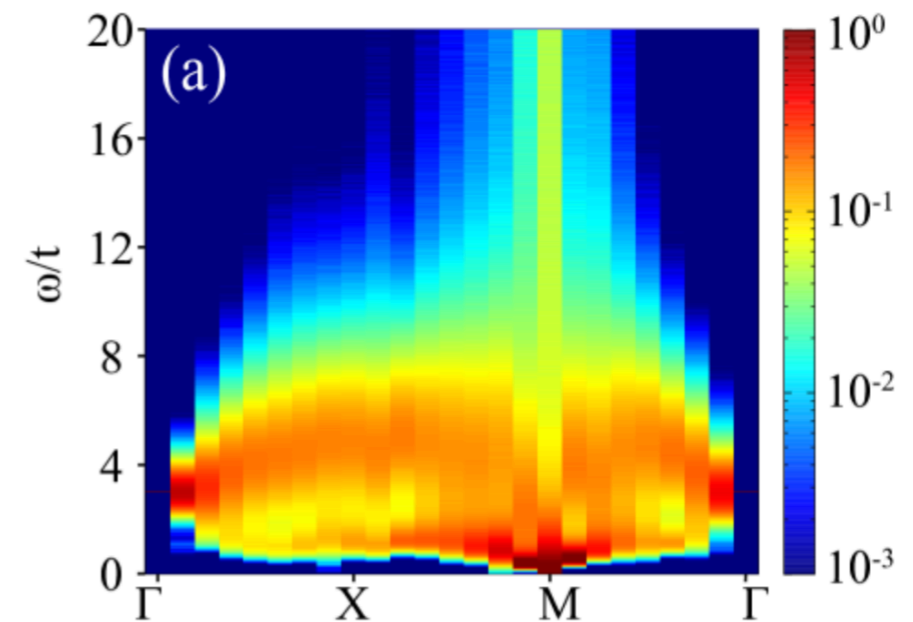
 [arXiv: 2508.08528](https://arxiv.org/abs/2508.08528)



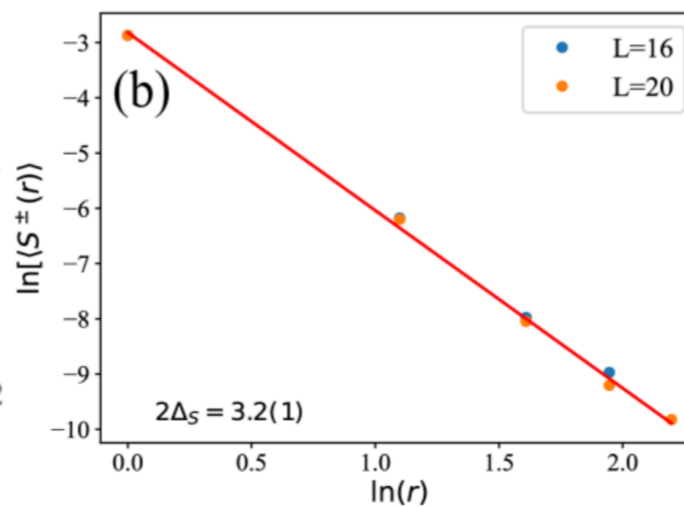
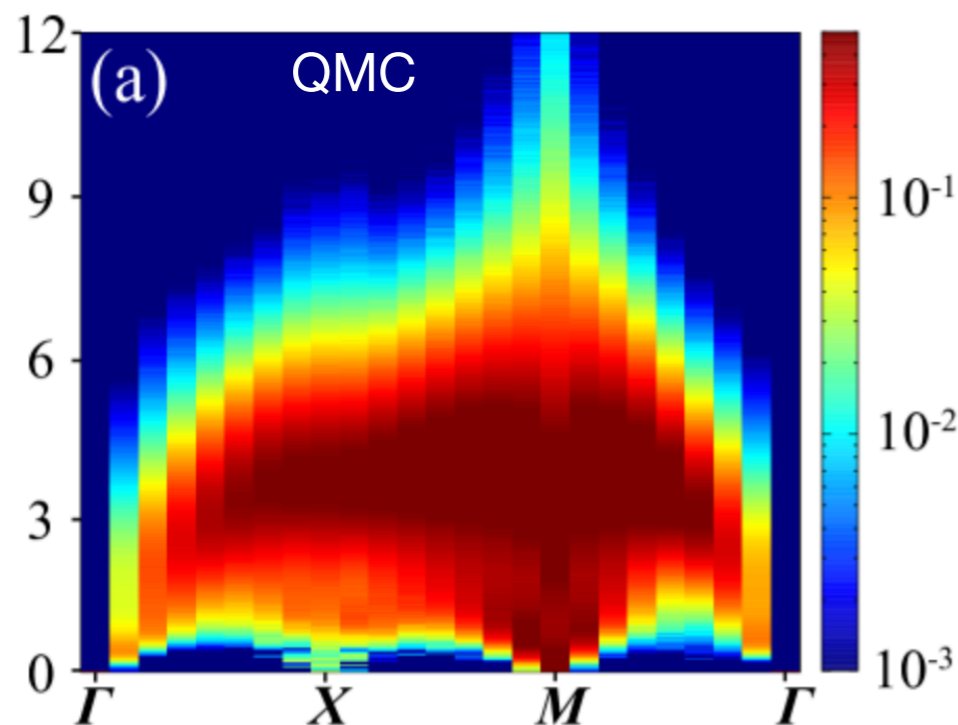
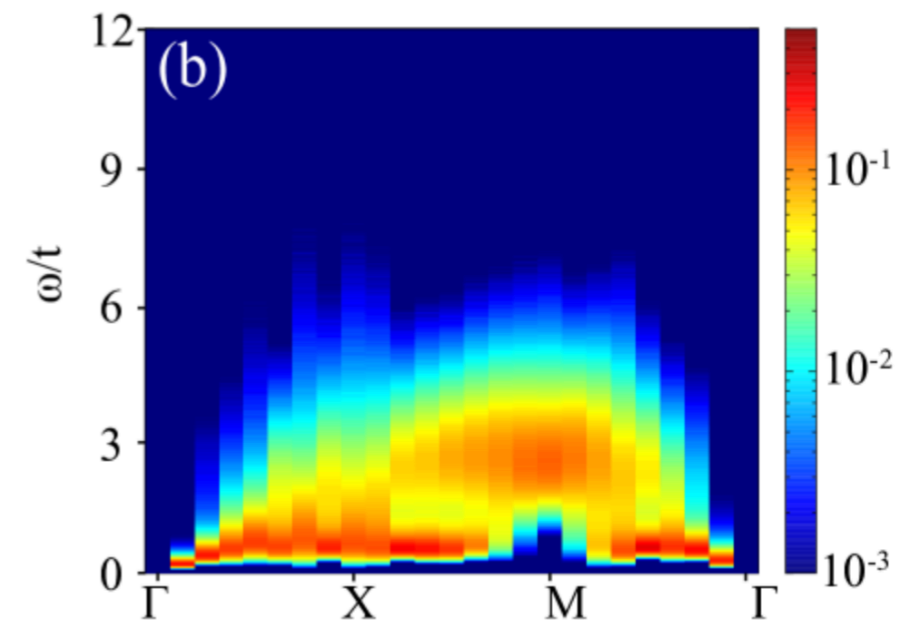
- fermion bilinears or mass terms: a singlet  $M_s = \bar{\Psi}\Psi$  and a set of adjoints  $M_a = \bar{\Psi}T_a\Psi$ , where  $a = 1 \dots 15$  range over the SU(4) generators. *A priori*, these two sets have independent scaling dimensions,  $\Delta_s$  and  $\Delta_{\text{adj}}$ , respectively. An estimate from Ref. [5] is  $\Delta_s \approx 2.3$ ,  $\Delta_{\text{adj}} \in (1.4, 1.7)$ .

$$S^\pm(\mathbf{q}, \omega) \quad B = 0, L = \beta = 16, J = 1$$

$$S^\pm(\mathbf{q}, \omega) \quad B = 3, L = \beta = 16, J = 3$$



$$S^{zz}(\mathbf{q}, \omega) \quad B = 3, L = \beta = 16, J = 3$$



# Quantum Fisher Information as a Thermal and Dynamical Probe in Frustrated Magnets: Insights from Quantum Spin Ice

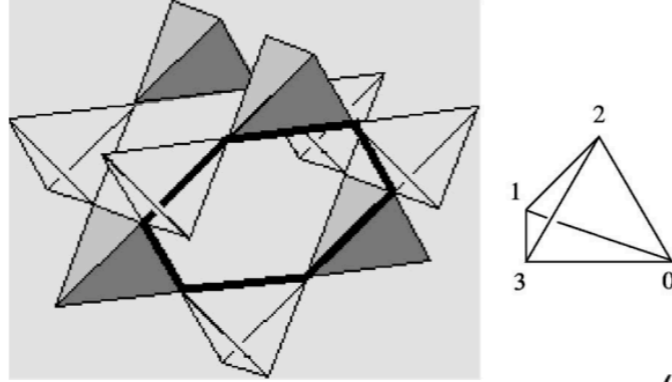
Chengkang Zhou,<sup>1,\*</sup> Zhengbang Zhou,<sup>2,\*</sup> Félix Desrochers,<sup>2,3</sup> Yong Baek Kim,<sup>2</sup> and Zi Yang Meng<sup>1</sup>


 [arXiv: 2510.14813](https://arxiv.org/abs/2510.14813)

$$\mathcal{H} = \mathcal{H}_I + \mathcal{H}'$$


$$\mathcal{H}_I = \frac{J_z}{2} \sum_t (S_t^z)^2$$

$$\mathcal{H}' = \frac{J_\perp}{2} \sum_{\langle ij \rangle} (S_i^+ S_j^- + \text{H.c.})$$



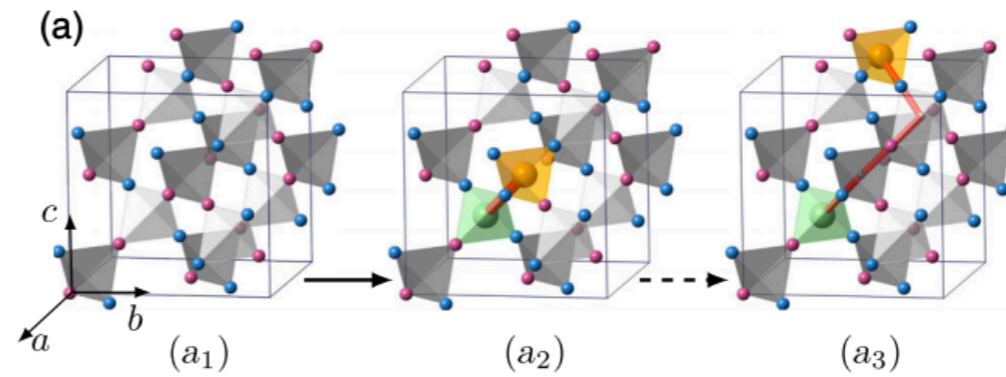
 Dynamics of Topological Excitations in a Model Quantum Spin Ice, Huang, Deng, Wang, Meng, PRL 120, 167202 (2018)

$$\mathcal{H} = \sum_{\langle i,j \rangle} -J_\pm (S_i^+ S_j^- + \text{H.c.}) + J_z S_i^z S_j^z$$

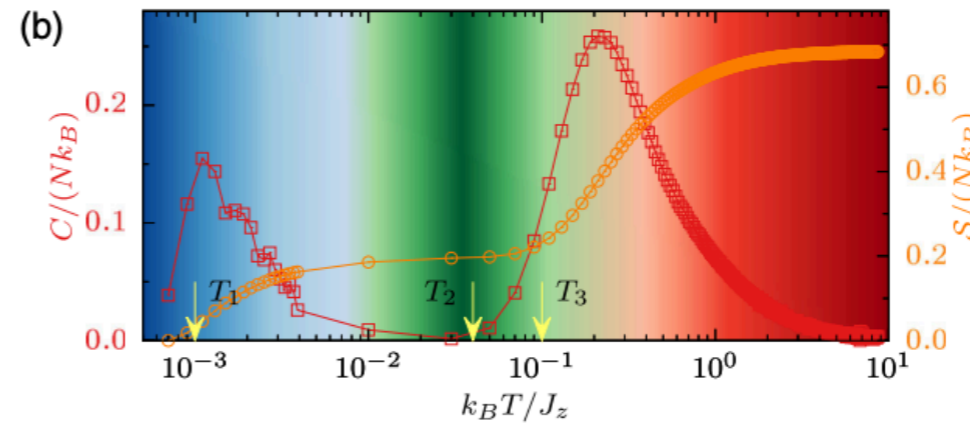
 Pyrochlore photons: The U(1) spin liquid in a S=1/2 three-dimensional frustrated magnet, Hermele, Fisher, Balents, PRB 69, 064404 (2004)

$$\mathcal{H}_p = \frac{U}{2} \sum_{\langle rr' \rangle} (n_{rr'} - 1/2)^2 - K \sum_{\square} \cos(\phi_1 - \phi_2 + \phi_3 - \phi_4 + \phi_5 - \phi_6)$$

$$\mathcal{H}_p = \frac{U}{2} \sum_{\langle rr' \rangle} e_{rr'}^2 - K \sum_{\square} \cos\left(\sum_{rr' \in \square} a_{rr'}\right)$$



$$J_{\pm,c}/J_z = 0.052$$




$$J_\pm/J_z = 0.046$$

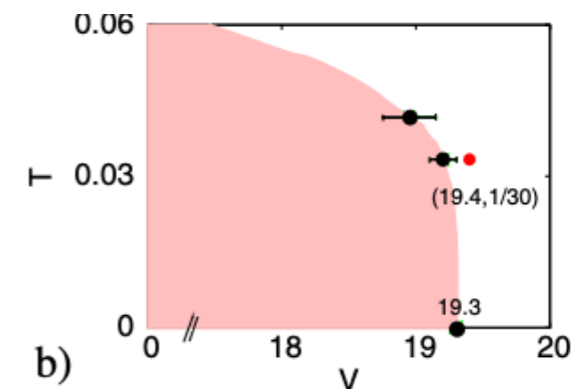
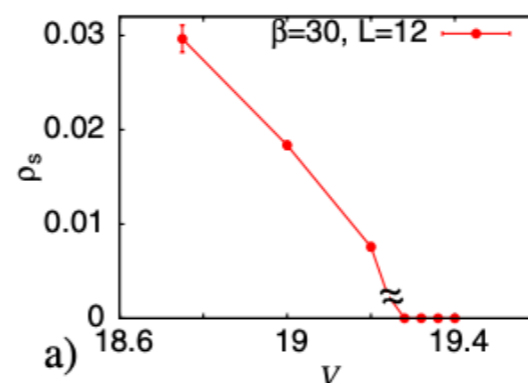
$$T_1 = 0.001 J_z$$

$$T_2 = 0.04 J_z$$

$$T_3 = 0.1 J_z$$

 Unusual Liquid State of Hard-Core Bosons on the Pyrochlore Lattice, Banerjee, Isakov, Damle, Kim, PRL 100, 047208 (2008)


$$H = \sum_{\langle ij \rangle} [V(n_i - 1/2)(n_j - 1/2) - t(b_i^\dagger b_j + b_i b_j^\dagger)]$$



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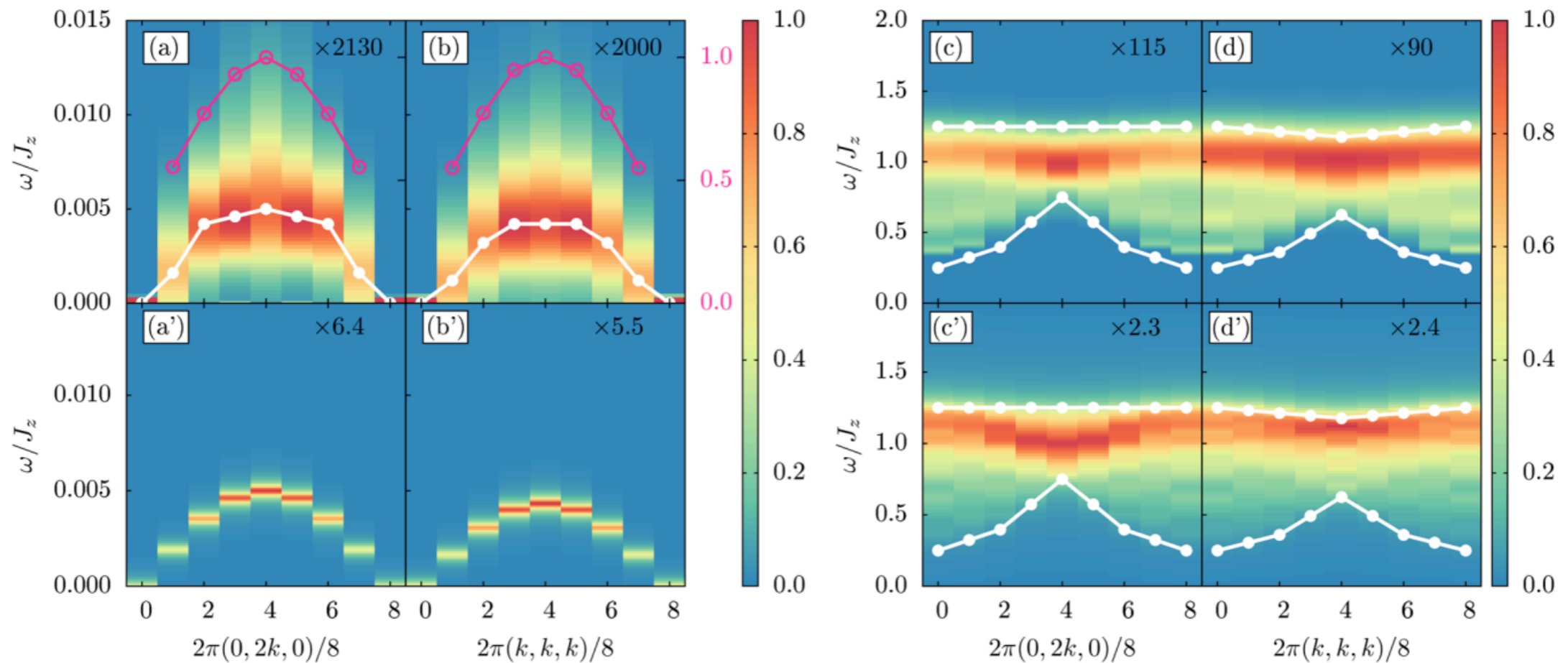
 arXiv: 2510.14813

 Dynamics of Topological Excitations in a Model Quantum Spin Ice,  
Huang, Deng, Wang, Meng, PRL 120, 167202 (2018)

$$\mathcal{H} = \sum_{\langle i,j \rangle} -J_{\pm}(S_i^+ S_j^- + \text{H.c.}) + J_z S_i^z S_j^z.$$

$$S_{\alpha\beta}^{zz}(\mathbf{q}, \tau) = \langle S_{-\mathbf{q},\alpha}^z(\tau) S_{\mathbf{q},\beta}^z(0) \rangle$$

$$S_{\alpha\beta}^{+-}(\mathbf{q}, \tau) = \langle S_{-\mathbf{q},\alpha}^+(\tau) S_{\mathbf{q},\beta}^-(0) \rangle$$



$$J_{\pm}/J_z = 0.046, \quad T_1 = 0.001J_z, \quad 4 \times 8 \times 8 \times 8$$

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 arXiv: 2510.14813

$$\rho = \sum_n p_n |n\rangle\langle n| \quad F_Q[\rho, O] = 2 \sum_{m,n} \frac{(p_m - p_n)^2}{p_m + p_n} |\langle m|O|n\rangle|^2$$

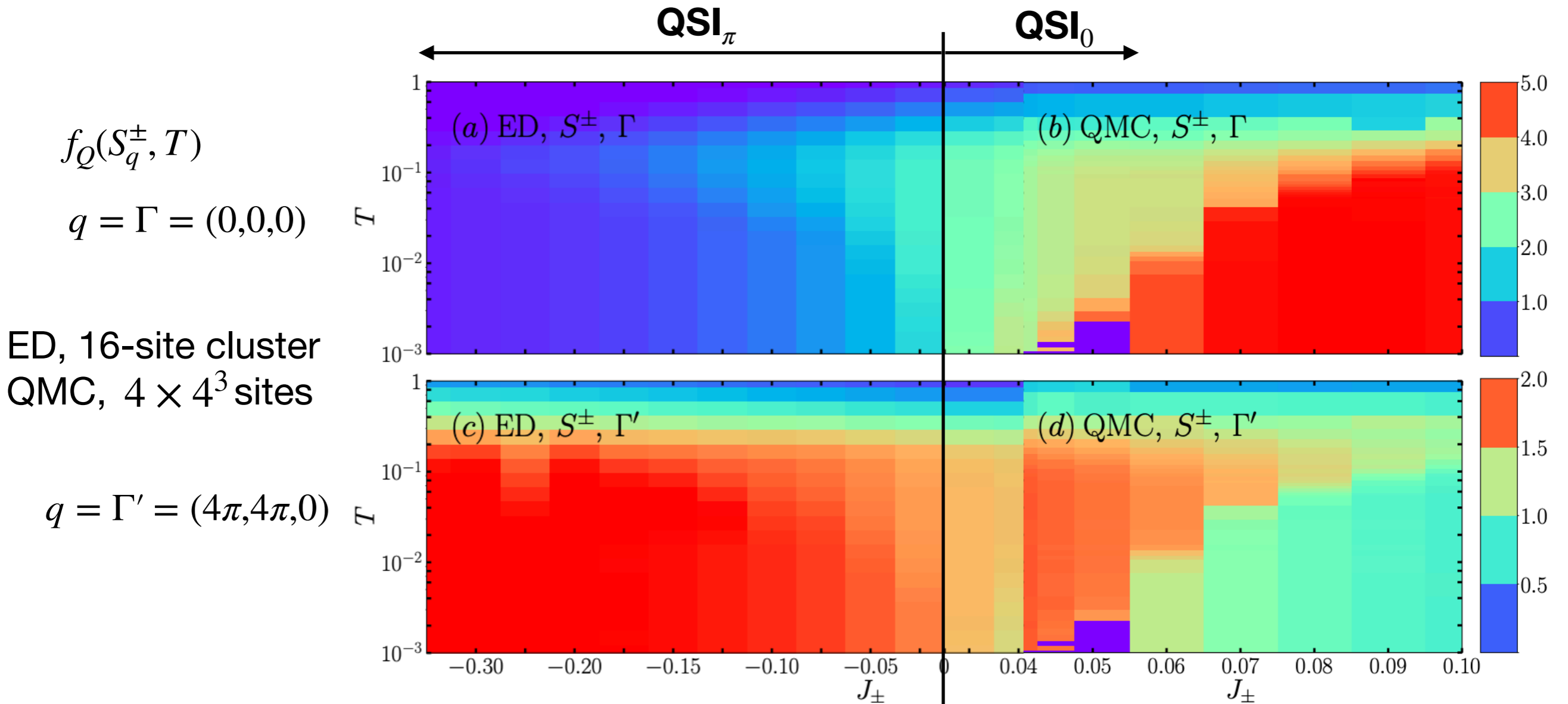
$$O = S_{\mathbf{q}}^\alpha := \sum_i S_{\mathbf{R}_i}^\alpha e^{i\mathbf{q}\cdot\mathbf{R}_i}$$

$$f_Q(O) := F_Q[\rho, O]/N, \quad \Delta\lambda = \lambda_{\max} - \lambda_{\min}$$

$$f_Q(S_{\mathbf{q}}^\alpha, T) = 4 \int_0^\infty d\omega \tanh\left(\frac{\omega}{2T}\right) (1 - e^{-\omega/T}) A^\alpha(\mathbf{q}, \omega)$$

$$\text{nQFI}(O) := \frac{f_Q(O)}{(\Delta\lambda)^2} > m \implies \text{entanglement depth} \geq m + 1$$

$$A^\alpha(\mathbf{q}, \omega) := \frac{1}{2\pi N} \int dt \langle S_{\mathbf{q}}^{\alpha\dagger}(t) S_{\mathbf{q}}^\alpha(0) \rangle e^{i\omega t}$$



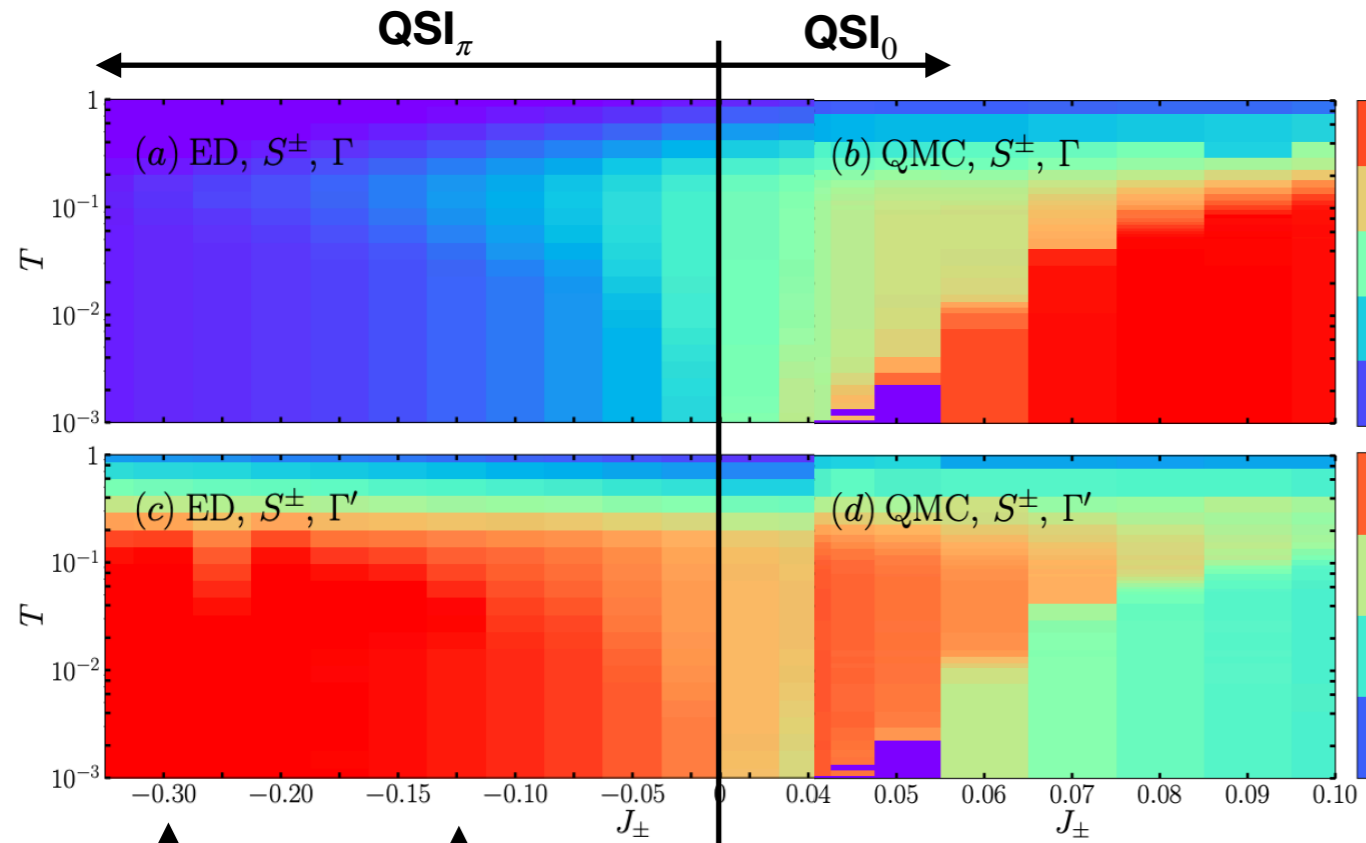
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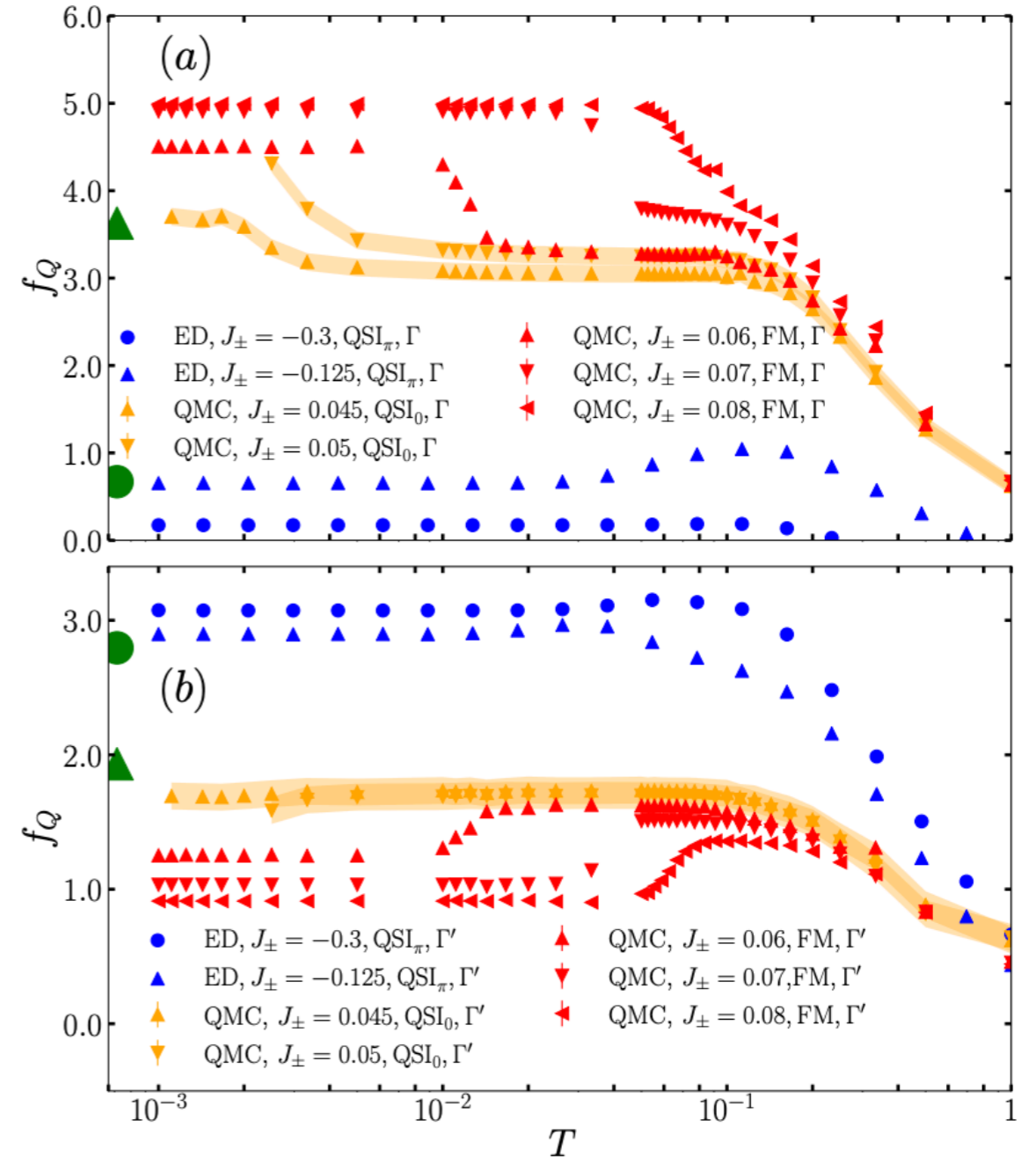
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$$f_Q(S_{\mathbf{q}}^\alpha, T) = 4 \int_0^\infty d\omega \tanh\left(\frac{\omega}{2T}\right) (1 - e^{-\omega/T}) A^\alpha(\mathbf{q}, \omega)$$

$$A^\alpha(\mathbf{q}, \omega) := \frac{1}{2\pi N} \int dt \langle S_{\mathbf{q}}^{\alpha\dagger}(t) S_{\mathbf{q}}^\alpha(0) \rangle e^{i\omega t}$$



$Ce_2Zr_2O_7$      $Ce_2Hf_2O_7$



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Ce-based dipolar–octupolar (DO) pyrochlore  $Ce_2Zr_2O_7$

from local to global frame

 [arXiv: 2510.14813](https://arxiv.org/abs/2510.14813)

neutron scattering couples to dipolar moments  $S^\pm$

$$\frac{d^2\sigma}{d\Omega d\omega} \propto \frac{1}{N} \sum_{\mu\nu} \left( \hat{\mathbf{z}}_\mu \cdot \hat{\mathbf{z}}_\nu - \frac{(\hat{\mathbf{z}}_\mu \cdot \mathbf{Q})(\hat{\mathbf{z}}_\nu \cdot \mathbf{Q})}{|\mathbf{Q}|^2} \right) \langle \tau_{-\mathbf{q},\mu}^z(t) \tau_{\mathbf{q},\nu}^z(0) \rangle := A^{\text{DO}}(\mathbf{q}, t)$$

$$f(S_q^{\text{DO}}, T) \quad q = \Gamma' = (4\pi, 4\pi, 0)$$

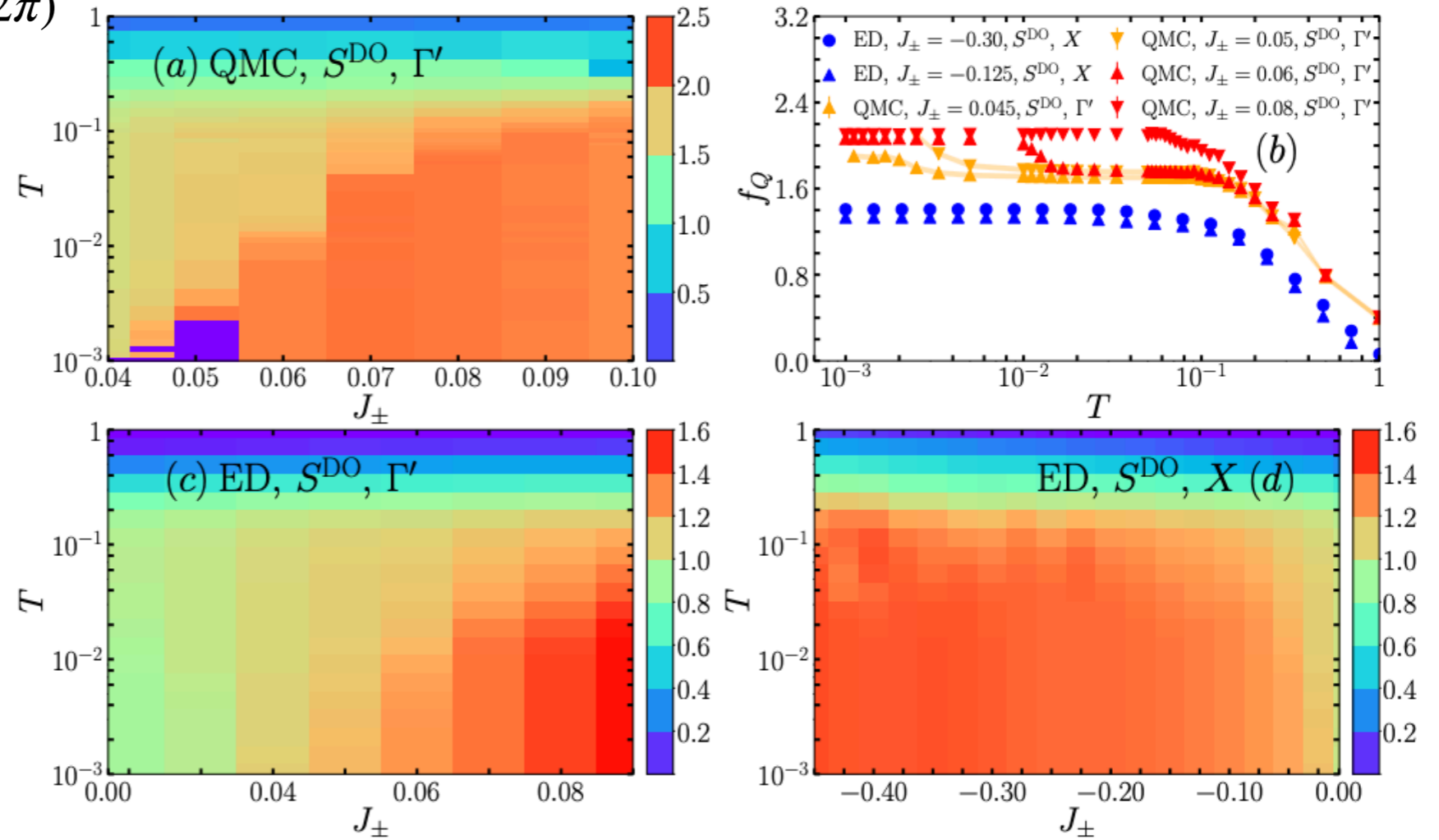
$$f_Q(S_q^\alpha, T) = 4 \int_0^\infty d\omega \tanh\left(\frac{\omega}{2T}\right) (1 - e^{-\omega/T}) A^\alpha(\mathbf{q}, \omega)$$

$$q = X = (0, 0, 2\pi)$$

$$\text{nQFI}(S_q^{\text{DO}}) = \frac{3}{2} f_Q(S_q^{\text{DO}})$$

$$\text{nQFI}(S_{\Gamma'}^{\text{DO}}, T) \gtrsim 2.2$$

**QSI**<sub>0</sub> > 3 partite entangled



$$\text{nQFI}(S_X^{\text{DO}}, T) \gtrsim 2.3$$

**QSI** <sub>$\pi$</sub>  > 3 partite entangled