

# Many-body physics of cavity embedded quantum matter

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# Why light-matter interaction in cavity QED?

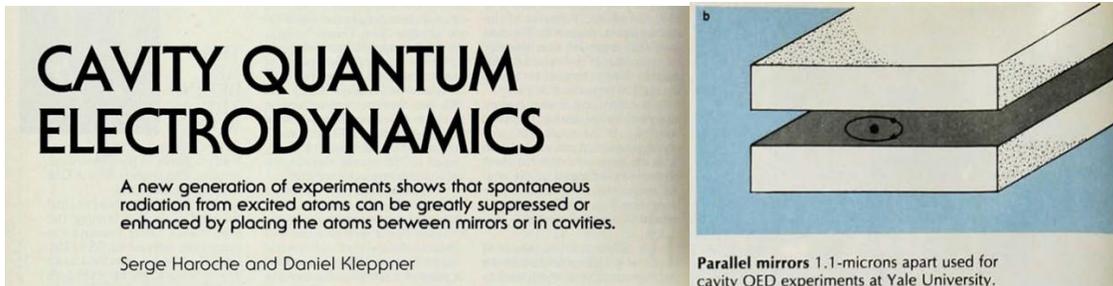
Fine structure constant

$$\alpha = \frac{e^2}{4\pi \epsilon_0 \hbar c} \approx \frac{1}{137}$$

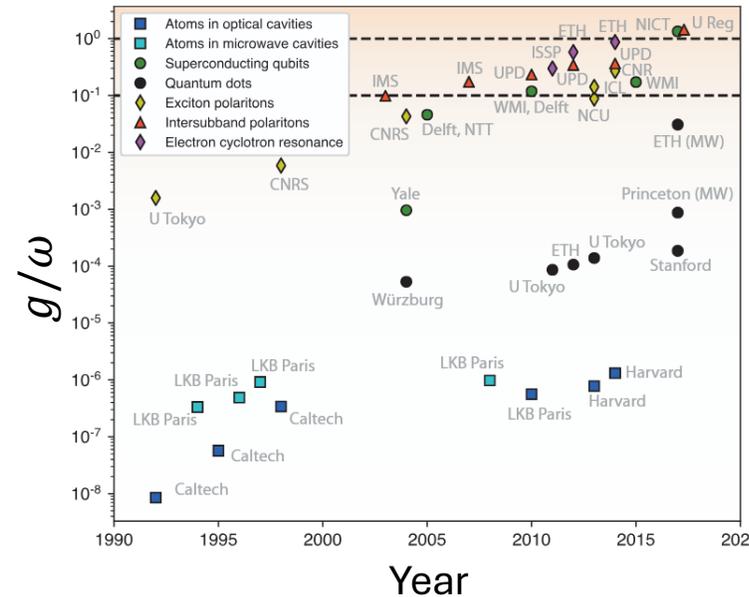
Need to enhance interactions

$$g \sim \vec{d} \cdot \vec{E}_{vac} \quad E_{vac} \sim \frac{1}{\sqrt{V}}$$

Microwave resonators



Physics Today (1989)



Review strong coupling:

Forn Diaz et al. , Rev. Mod. Phys. (2019)

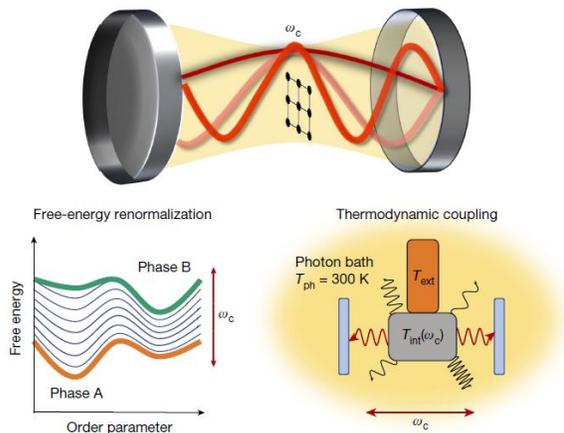
# New avenues: controlling matter

Schlawin et al. , Appl. Phys. Rev. (2022)  
Garcia-Vidal et al. , Science (2021)  
Mivehvar et. al. , Adv. Phys. (2021)

Single emitters  $\longrightarrow$  Many-body

Control quantum matter  
by cavity embedding?

Quantum materials



Subwavelength cavities

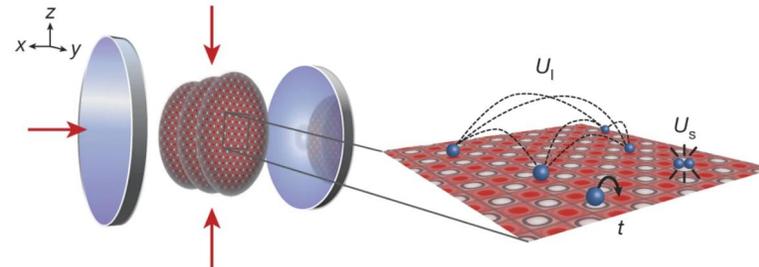
No external drive

Few control parameters

Fausti, Nature (2023)

Other exp: J. Faist (ETH Zurich),  
S. Zhang (HKU), D. Basov (Columbia)

Cold atom systems



Fabri-perot cavities

Out-of-equilibrium

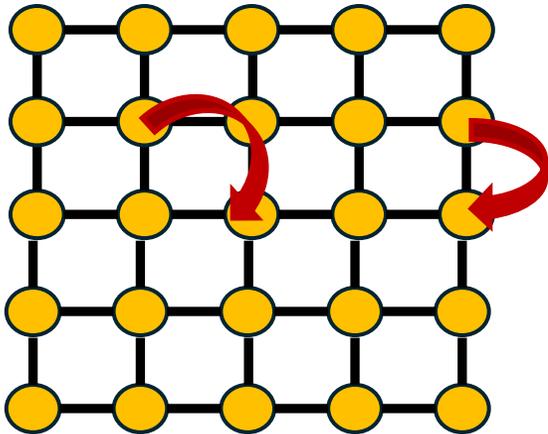
Highly controllable

Esslinger, Nature (2016)

# New hybrid many-body scenario

«Standard»

$$\hat{H} = \sum_{\langle ij \rangle} \hat{O}_i \hat{O}_j$$



Electrons, atoms, spins ...

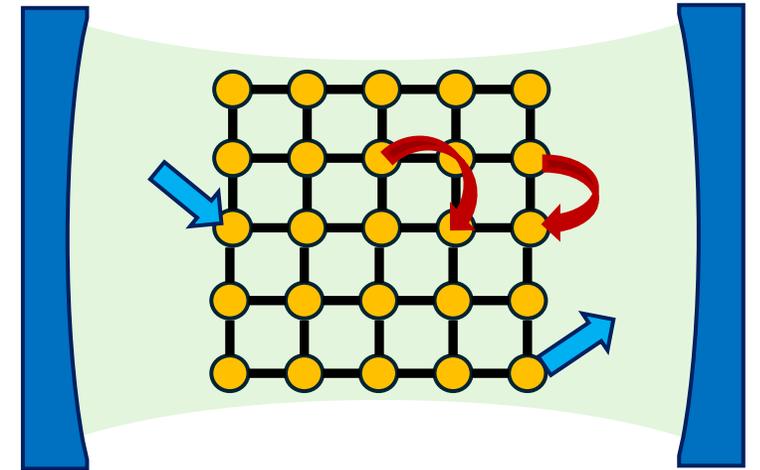
## Locality

- Correlation structure  
Eisert et al. RMP, 82,277 (2008)
- Dynamical properties  
Chen et al. RPP, 86, 116001 (2023)
- Topological protection  
X.-G. Wen, OUP (2007).

?

Cavity embedded

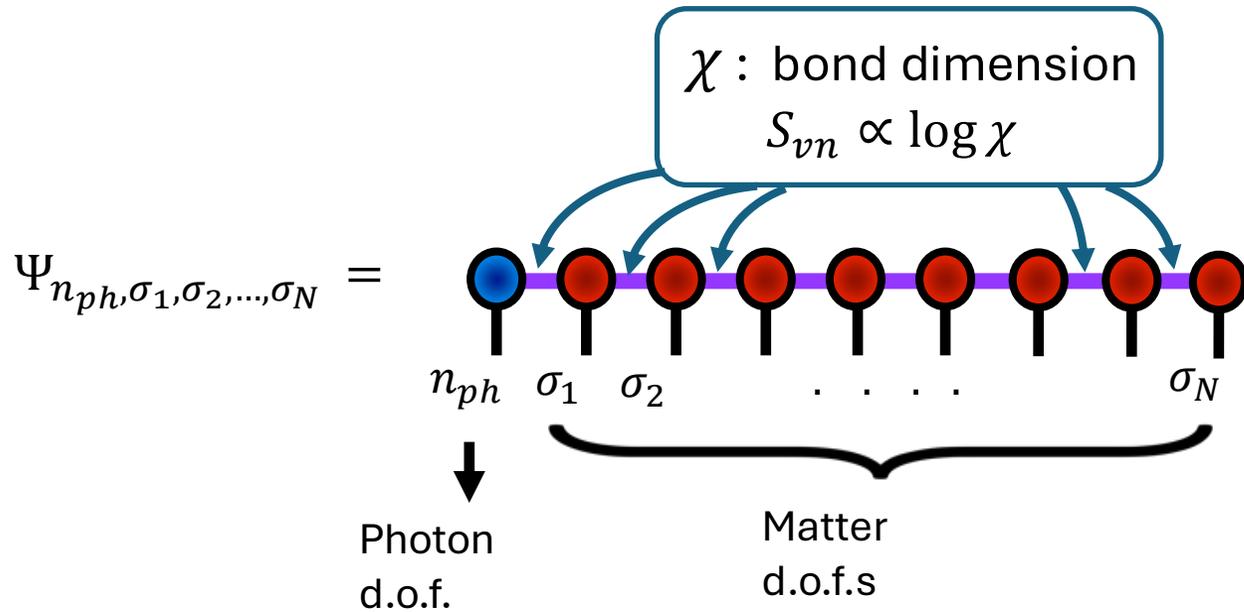
$$\hat{H} = \sum_{\langle ij \rangle} \hat{O}_i \hat{O}_j + \sum_{\alpha} \omega_{\alpha} \hat{a}_{\alpha}^{\dagger} \hat{a}_{\alpha} + \sum_{\alpha} g_{\alpha} \hat{O}_i (\hat{a}_{\alpha}^{\dagger} + \hat{a}_{\alpha})$$



Non-local cavity modes

# Methods: Matrix product states (MPS)

$$|\Psi\rangle = \sum_{n_{ph}, \{\sigma_i\}} \Psi_{n_{ph}, \sigma_1, \sigma_2, \dots, \sigma_N} |n_{ph}, \sigma_1, \sigma_2, \dots, \sigma_N\rangle$$



➤ Mean Field

$$|\Psi\rangle = |\psi_{ph}\rangle \prod_j |\phi_j\rangle$$

➤ Photon Mean Field

$$|\Psi\rangle = |\psi_{ph}\rangle |\psi_{mat}\rangle$$

➤ Polaritons

$$|1_{ph}, \{0\}\rangle + \sum_j \sigma_j^+ |0_{ph}, \{0\}\rangle$$

$\chi = 2$

- Refs:** C. Halati, A. Sheikhan, and C. Kollath PRR **2**, 043255 (2020)  
G. Chiriacò, M. Dalmonte, and T. Chanda PRB **106**, 155113 (2022)  
G. Passetti, C. J. Eckhardt, M. A. Sentef, and D. M. Kennes PRL **131**, 023601 (2023)  
Z. Bacciconi, G. M. Andolina, T. Chanda, G. Chiriacò, M. Schirò, M. Dalmonte SciPost Phys. 15, 113 (2023)

# Outline

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## I) Topology: FQH

Bacciconi et al.; PRX 15,021027 (2025)

- Introduction and motivation
- Stability of topology
- Emergent graviton-polaritons

## II) Non-local dynamics: Rydberg atom array

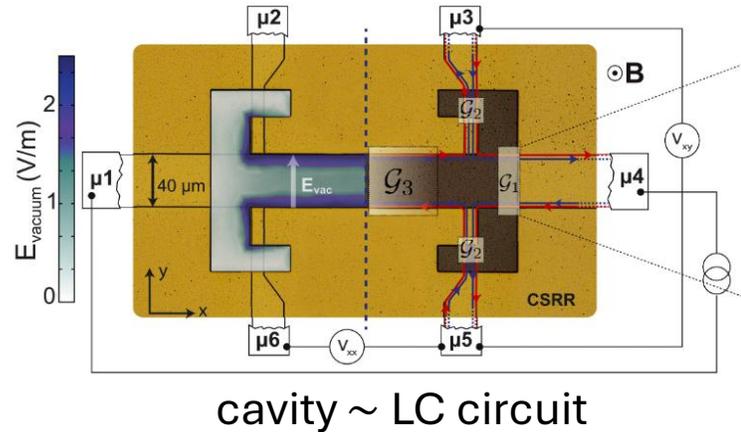
Bacciconi et al.; PRL 134,13604 (2025)

- Introduction
- Mixed local and non-local excitations

# Part I) Topology: Fractional Quantum Hall

1. *Theory of fractional quantum hall liquids coupled to quantum light and emergent graviton-polaritons*, Bacciconi, Xavier, Chanda, Carusotto, Dalmonte; PRX 15,021027 (2025)

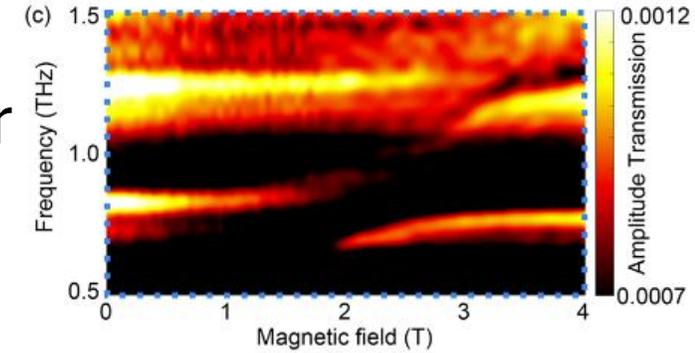
# Quantum Hall systems: experiments



## 2DEG + split-ring resonator

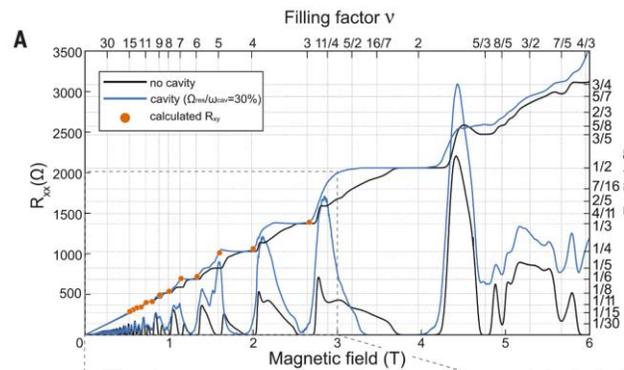
@ETH, J. Faist's group

Subwavelength cavity  $\lambda \gg L$



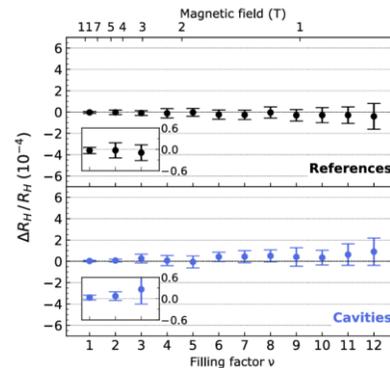
## Integer Quantum Hall

«Breakdown of topological protection»



Appugliese, Faist, et al., Science (2022)

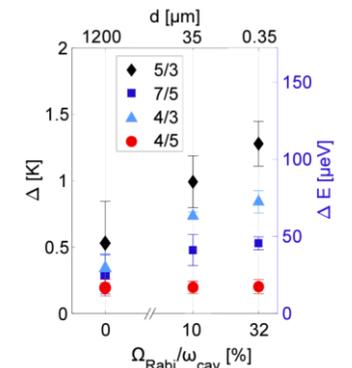
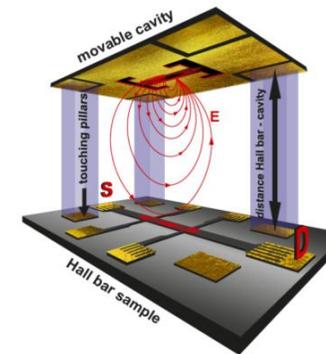
«Von Klitzing constant»



Enker, Faist, et al., PRX (2024)

## Fractional Quantum Hall

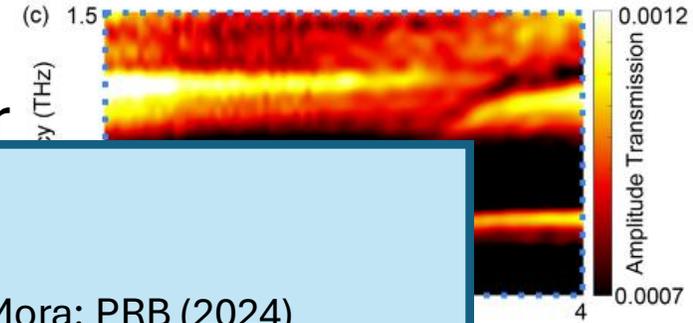
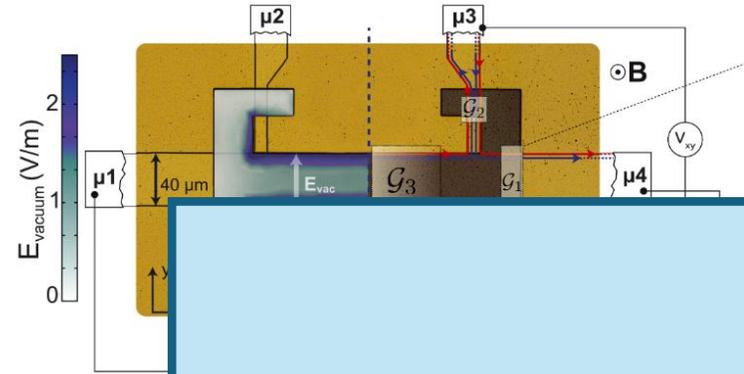
«Enhanced FQH gaps»



Enker, Faist et al. Nature (2025)

# Quantum Hall systems: experiments

2DEG + split-ring resonator



A. Non-locality and topology?

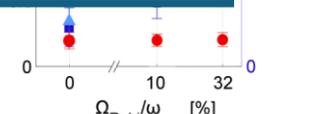
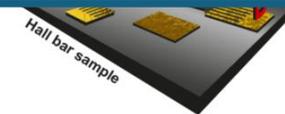
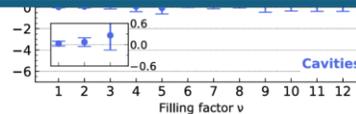
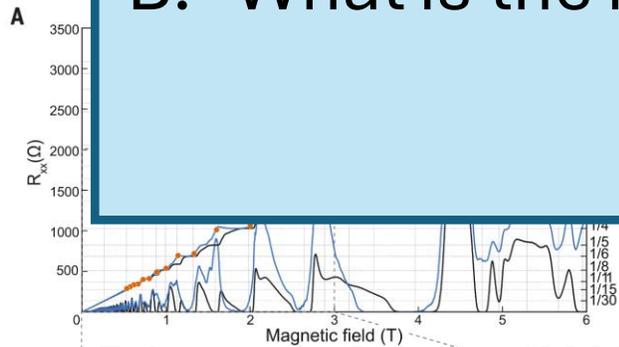
See also:

**ZB**, G.M. Andolina, C. Mora; PRB (2024)

G.M. Andolina, **ZB**, A. Nardin, M. Schirò, P. Rabl, D. De Bernardis, arxiv 2511.04744 (2025)

B. What is the key player in the FQH regime?

«Breakdown

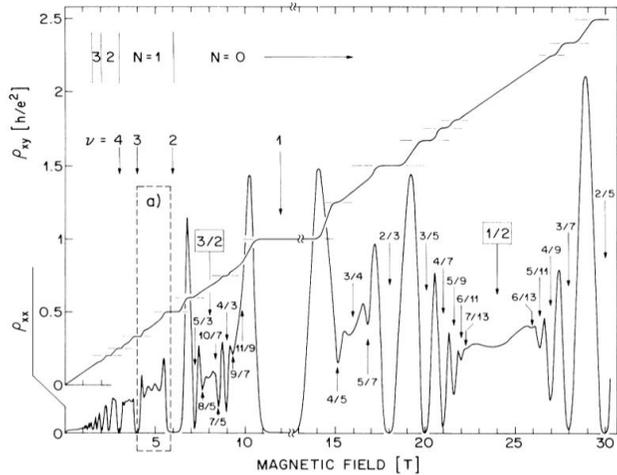


Appugliese, Faist, et al., Science (2022)

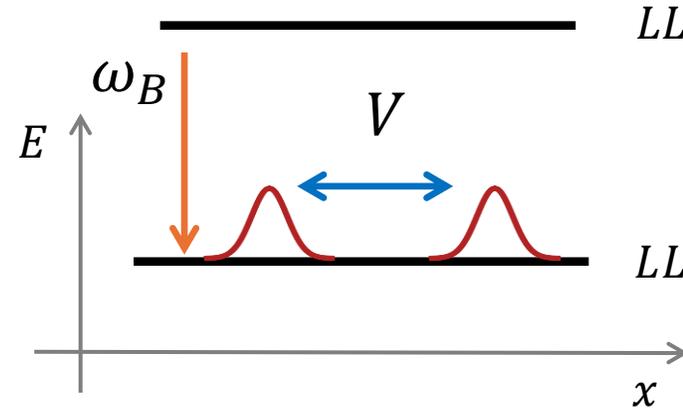
Enker, Faist, et al. , PRX (2024)

Enker, Faist et al. Nature (2025)

# Intro: Fractional Quantum Hall effect

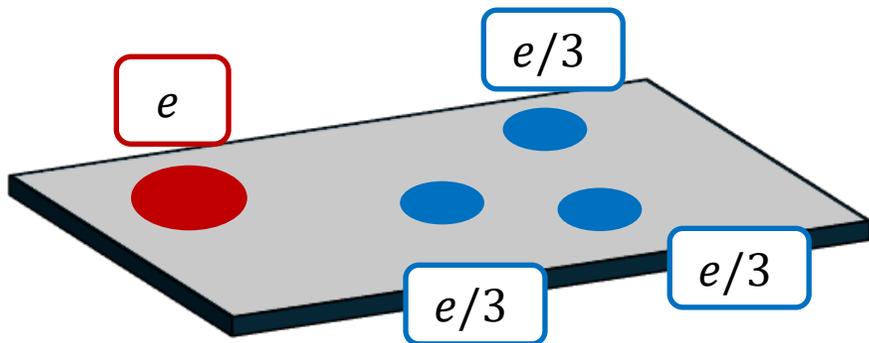


Laughlin, Störmer, Tsui  
Nobel Prize 1998



Interactions  
 $\omega_B \gg V$   
+  
Landau Level filling  
 $\nu$

Laughlin state  $\nu = 1/3$



1) Quantized Hall resistivity

$$\rho_{xy} = \frac{e^2}{h} \frac{1}{\nu}$$

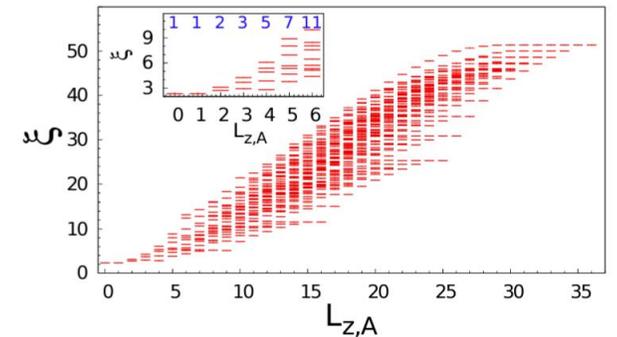
2) Fractionalized quasiparticles

$$e^* = \frac{e}{3} \quad \theta_{br} = \frac{2\pi}{3}$$

3) Entanglement structure

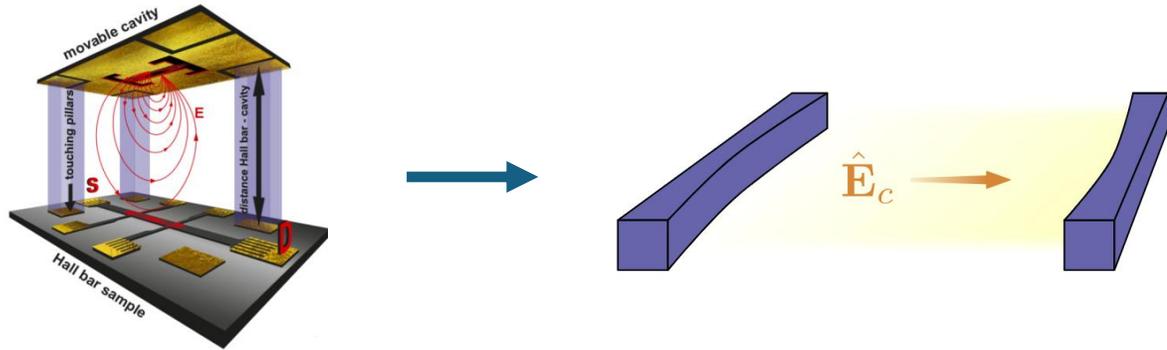
Li Haldane, PRL (2008)

N. Regnault, lecture notes (2015)



# Quantum light-matter coupling

Split-Ring resonator  $\simeq$  single cavity mode  
Ciuti; PRB (2018)



1) Frequency

$$\omega_c \hat{a}^\dagger \hat{a}$$

2) Mode structure

$$\hat{E}_c(\mathbf{r}) = (\hat{a}^\dagger + \hat{a}) \mathbf{E}_c(\mathbf{r})$$

$$V_c(\mathbf{r}) = \int_{r_0}^r d\mathbf{r} \cdot \mathbf{E}_c(\mathbf{r})$$

Dipole gauge

$$\hat{H}_{tot} = \hat{H}_{int} + \omega_c \hat{a}^\dagger \hat{a} + \omega_c (\hat{a}^\dagger + \hat{a}) \hat{P} + \omega_c \hat{P}^2$$

$$\hat{P} = \int d^2\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}) V_c(\mathbf{r})$$

Multipolar expansion

$$\hat{P} \simeq \mathbf{D} \cdot \mathbf{E}_c(\mathbf{r}_0) + \vec{\mathbf{Q}} \cdot \partial \mathbf{E}_c(\mathbf{r}_0)$$

Only inter LL  
 $\omega_B$

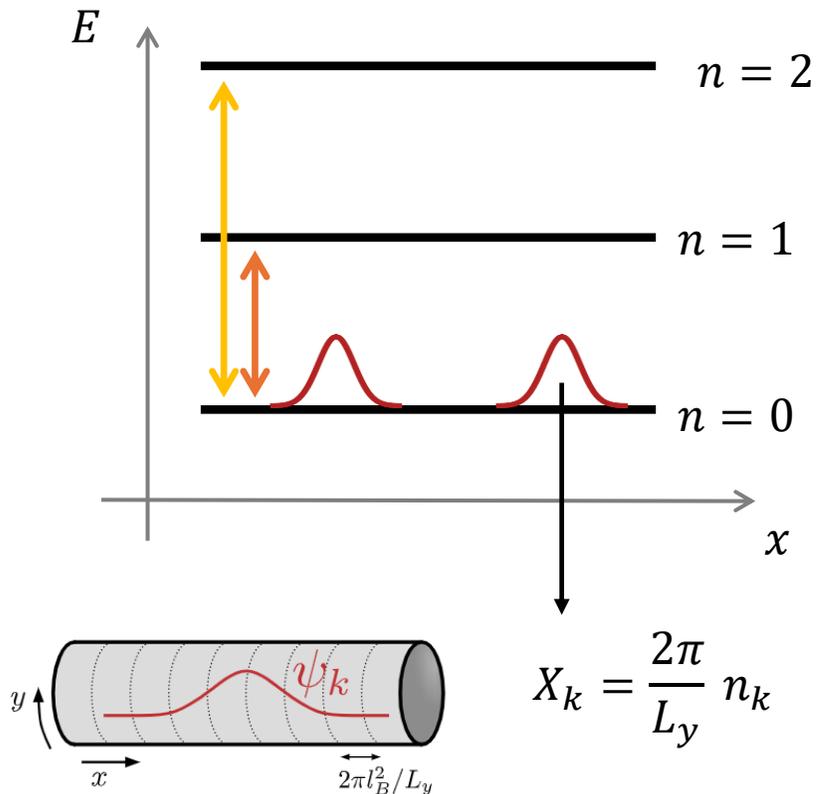
Also intra LL!  
 $V$

Dmytruk, Schirò; PRB (2021)

Li, Eckstein et al; PRB (2021)

# Quantum light-matter coupling: Landau Levels

Polarization operator



$$\hat{P} = \sum_k \sum_{n,n'} \chi_k^{n,n'} \hat{c}_{k,n}^\dagger \hat{c}_{k,n'}$$

Simplifying assumption:

$$\mathbf{E}_c(\mathbf{r}) = E_c(x) \mathbf{u}_x$$

$$\begin{aligned} \omega_c \chi_k^{n,n'} = & \underbrace{e l_B E_c(X_k)}_{\text{Inter-LL Dipole}} (\delta_{n,n+1} + \delta_{n,n-1}) + \\ & + \underbrace{e l_B^2 \partial_x E_c(X_k)}_{\text{Inter-LL Quadrupole}} (\delta_{n,n-2} + \delta_{n,n+2}) + \\ & + \underbrace{\chi(X_k)}_{\text{Intra-LL Quadrupole}} \delta_{n,n'} \end{aligned}$$

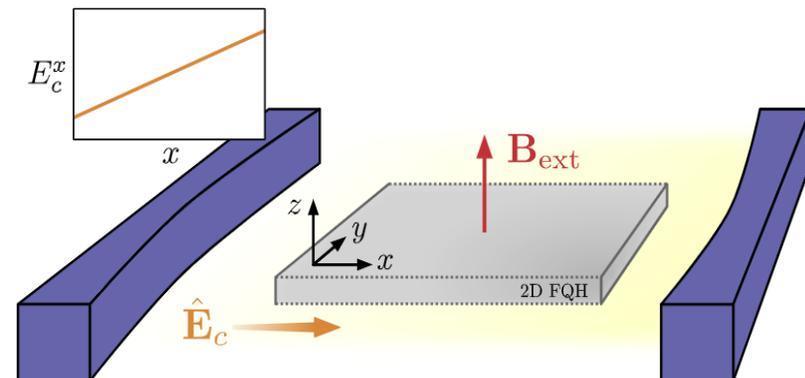
**No dipole intra-LL transitions!**  
(without disorder or edges)

# A toy model for $\nu = \frac{1}{3}$

1) Lowest Landau Level  $\longrightarrow$  No dipole transitions!

2) Uniform cavity gradient  $g = e l_B^2 \frac{\partial_x E_c}{\omega_c}$   $Q_{xx} = \int x^2 \rho(x)$

3) Haldane pseudopotential  $V(r) = V \nabla^2 \delta(r)$



Laughlin  $\nu = 1/3$

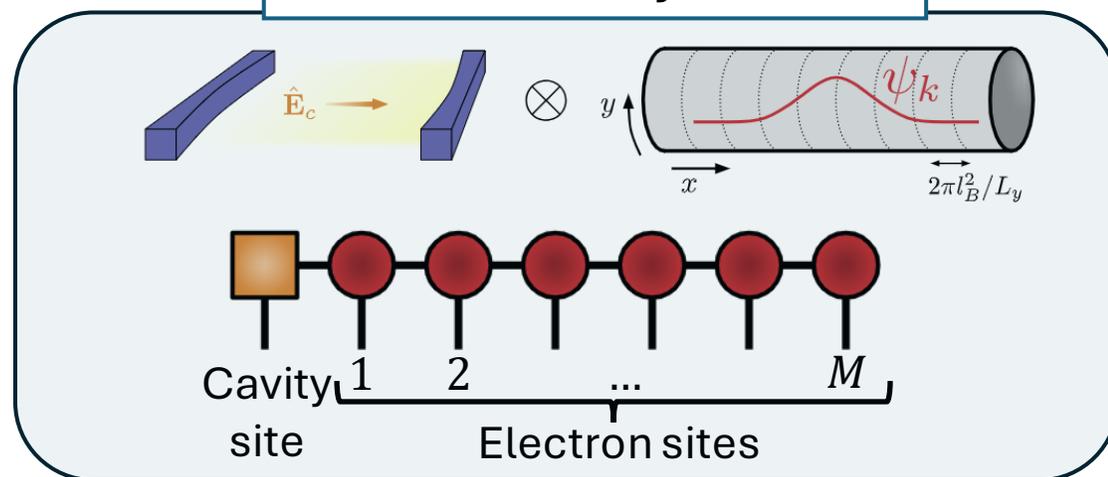
Cavity

$$\hat{H}_{tot} = \sum_q V_q \hat{\rho}_{-q} \hat{\rho}_q + \omega_c \hat{a}^\dagger \hat{a} +$$

$$+ i\omega_c g (\hat{a}^\dagger - \hat{a}) \hat{Q}_{xx} + \omega_c g^2 \hat{Q}_{xx}^2 + \text{Stark shifts}$$

Light-matter interaction

MPS for cavity+matter



# A toy model for $\nu = \frac{1}{3}$

1) Lowest Landau Level  $\longrightarrow$  No dipole transitions!

## IMPORTANT REMARK:

Single particle coupling      Collective coupling  $G = g\sqrt{N}$

$$g(a + \hat{a}^\dagger) \sum_{i=1}^N O_i$$

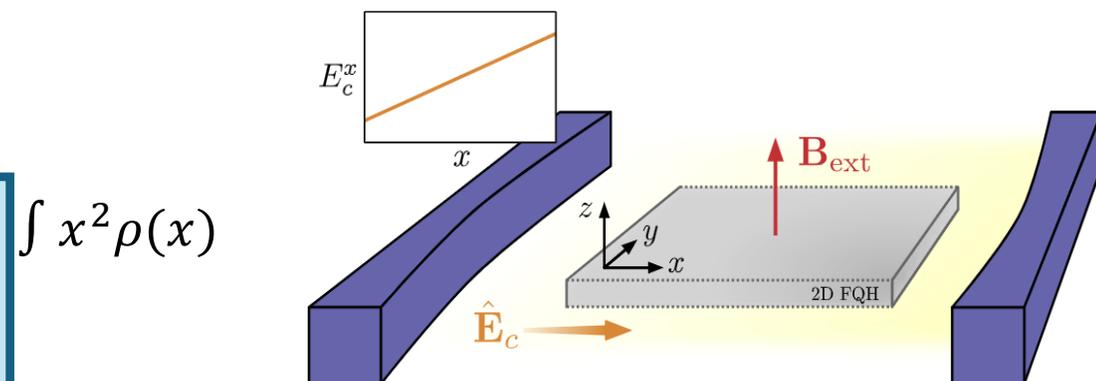
couples «everything»

$$\frac{G}{\sqrt{N}} (a + \hat{a}^\dagger) \sum_{i=1}^N O_i$$

q=0 collective modes!

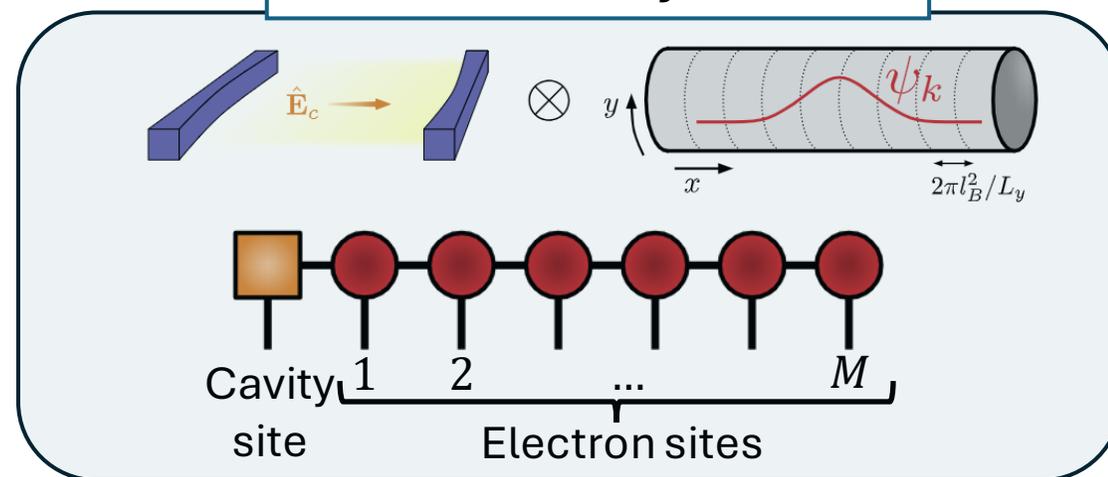
$$+ i\omega_c g (\hat{a}^\dagger - \hat{a}) \hat{Q}_{xx} + \omega_c g^2 \hat{Q}_{xx}^2 + \text{Stark shifts}$$

Light-matter interaction



$$\int x^2 \rho(x)$$

## MPS for cavity+matter



# Stability of Topology

# Stability of Topology: Hall resistivity

## Flux insertion argument

D. J. Thouless, JMP (1994)

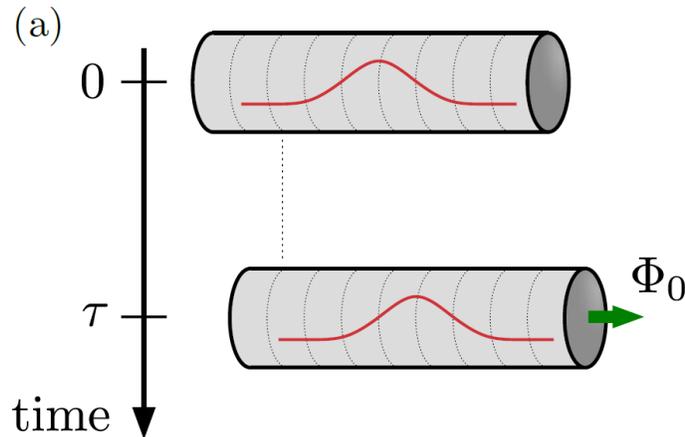
Zalatel et al., J. Stat. Mech (2015)

$$\Phi_{\text{ext}}(t) = \Phi_0 \frac{t}{\tau}$$



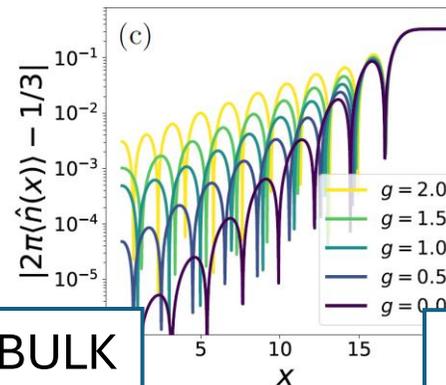
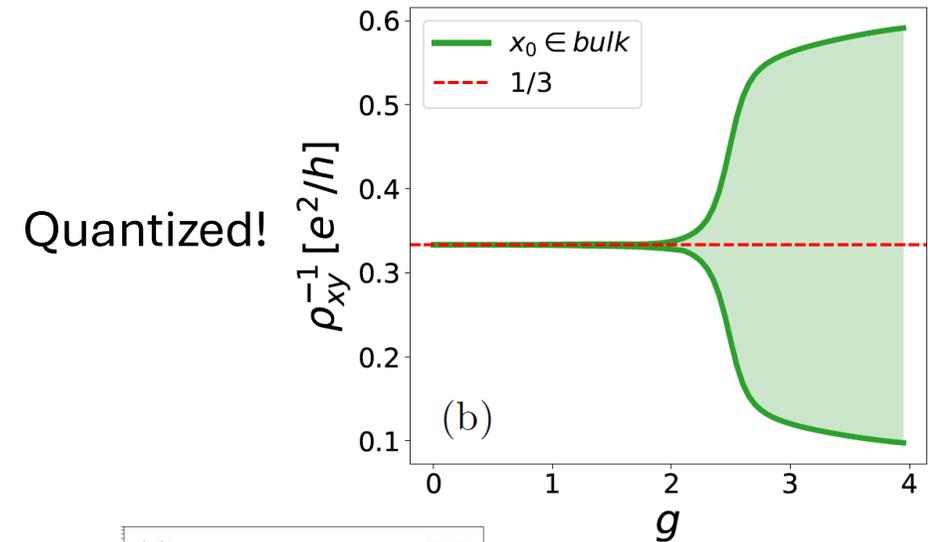
Rigid shift of Landau Level

$$\rho_{xy}(x_0) = \frac{E_{\text{ext}}^y}{J^x(x_0)} = \frac{1}{2\pi} \frac{1}{\langle \hat{n}(x_0) \rangle}$$



Bulk density determines the Hall response!

Do numerical experiment....



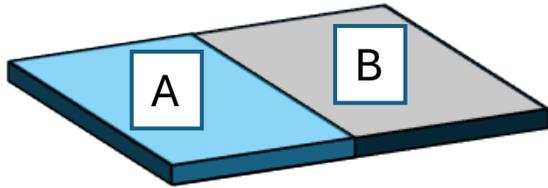
Exponential corrections

BULK

EDGE

# Stability of Topology: Entanglement spectrum

Bipartition of a system



$$\hat{\rho}_A = \text{Tr}_B [ |\Psi\rangle\langle\Psi| ]$$

$$\hat{\rho}_A = \exp(-\xi_j |j\rangle\langle j|)$$

If  $|\Psi\rangle$  has topological order

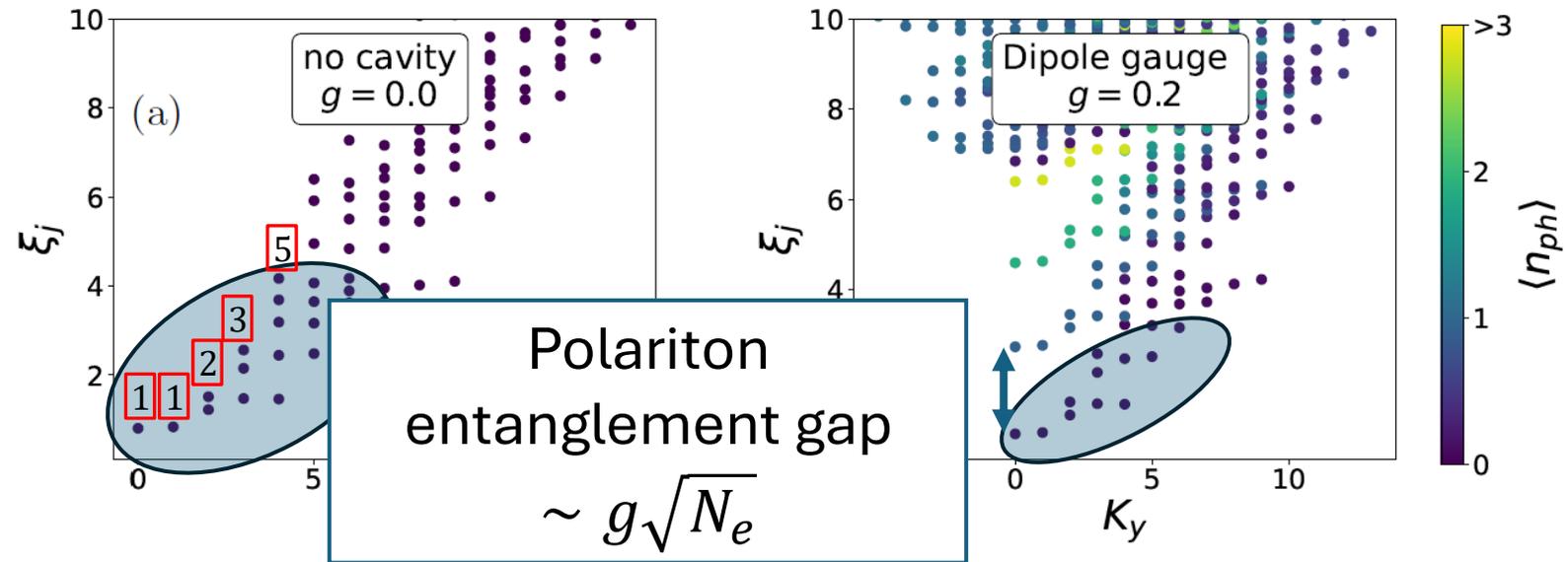
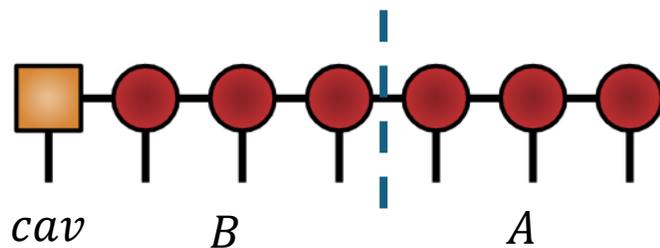


Low-energy of  $\xi_j$  is determined by edge structure

Li, Haldane, PRL (2008)

Bipartition of cavity+matter

$$\hat{\rho}_A = \text{Tr}_{B+cav} [ |\Psi\rangle\langle\Psi| ]$$

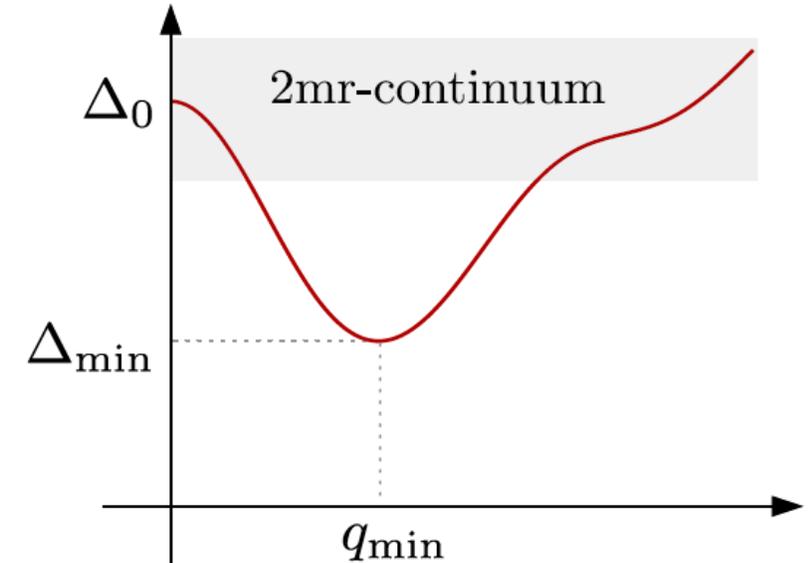
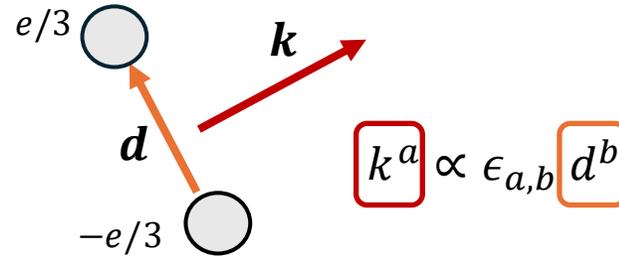


What is the cavity  
coupling to?

# What is the cavity coupling to?

## Magneton mode

Girvin, MacDonald, Platzman, PRB (1985)



## Graviton-mode $\Delta_0$

Haldane; PRL (2011)  
 Golkar, Nguyen, Son; JHEP (2013)  
 Liou, Haldane, Yang, Rezayi; PRL (2019)  
 Balram, Liu, Gromov, Papić; PRX (2022)

Fluctuations of emergent metric

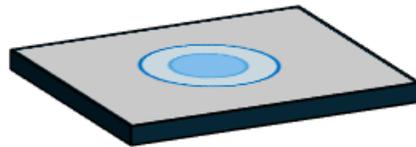
$$\psi_{1/3}(\{z_i\}) \sim (z_i - z_j)^3$$

$$z = \tilde{x} + i\tilde{y} \quad \tilde{R}^a = \Lambda^{a,b} R^a$$

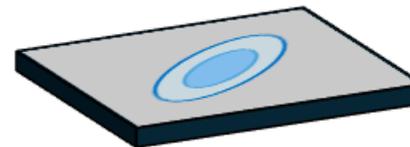
1) Long wavelength magnetorotors

2) No dipole, only quadrupole!

$$\Lambda^{a,b} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



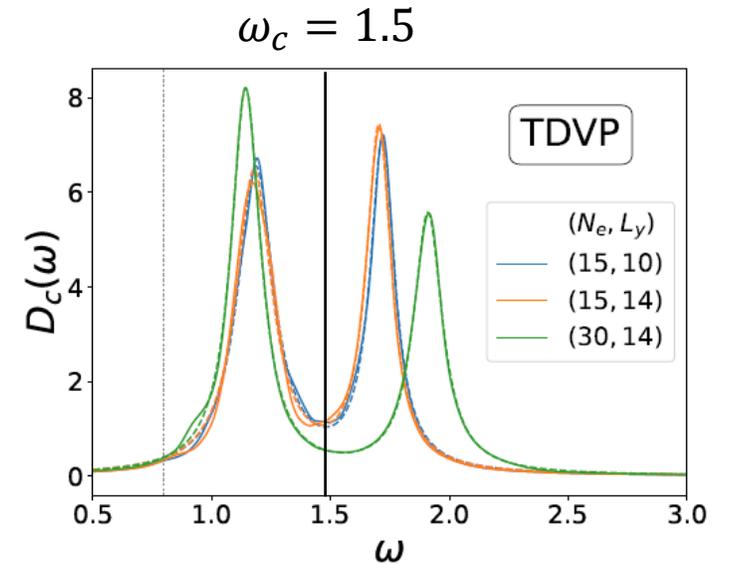
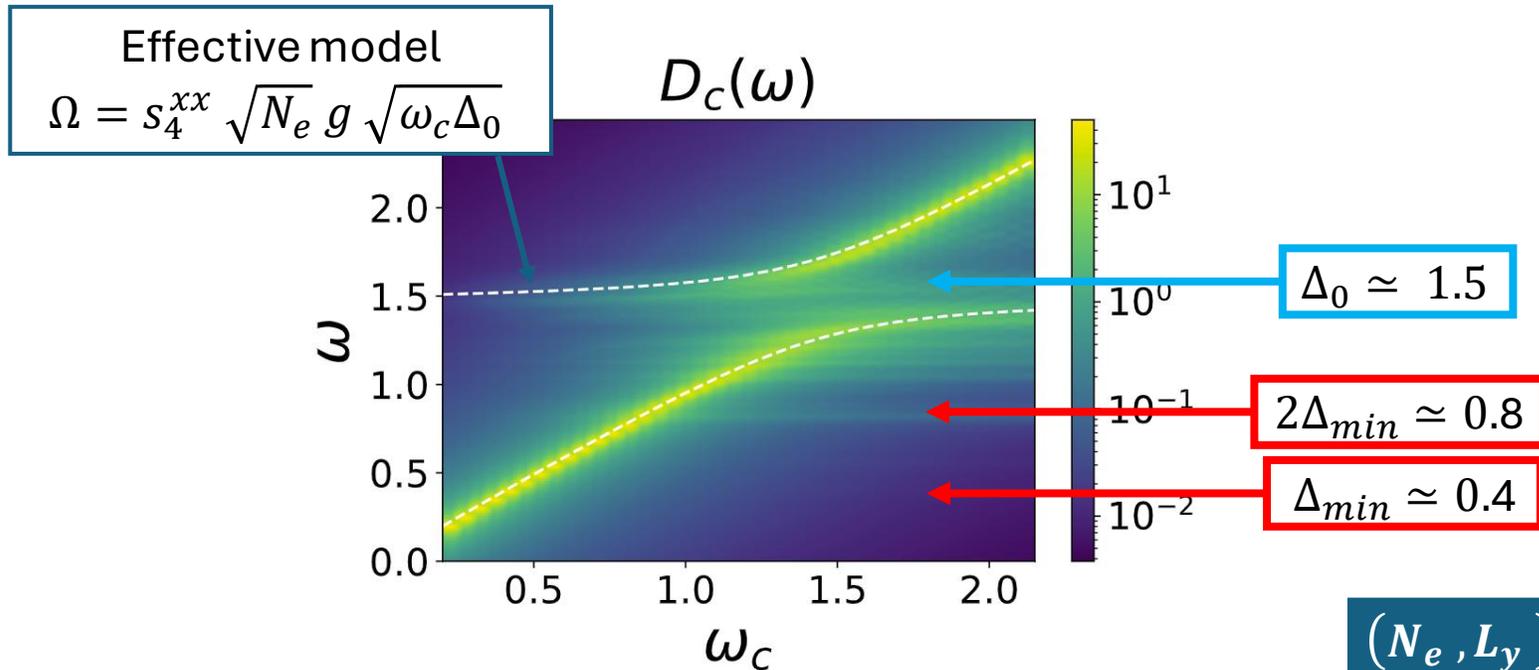
$$\Lambda^{a,b} = \begin{pmatrix} 2 & 0 \\ 0 & 0.5 \end{pmatrix}$$



«shape of liquid correlations»

# Emergent graviton-polaritons

Cavity density of states  $D_c(\omega) = \sum_n |\langle n | \hat{a}^\dagger | 0 \rangle|^2 \delta(\omega - E_n + E_0)$



Anticrossing at the Graviton-mode energy  
 +  
 2-MR continuum

$(N_e, L_y)$	(15, 10)	(15, 14)	(30, 14)	Effective model
$\Omega_{fit} / \Omega$	0.91	0.91	0.94	1

# Ultra-strong coupling regime

Aka where to look for ground state changes

# FQH geometry

vs

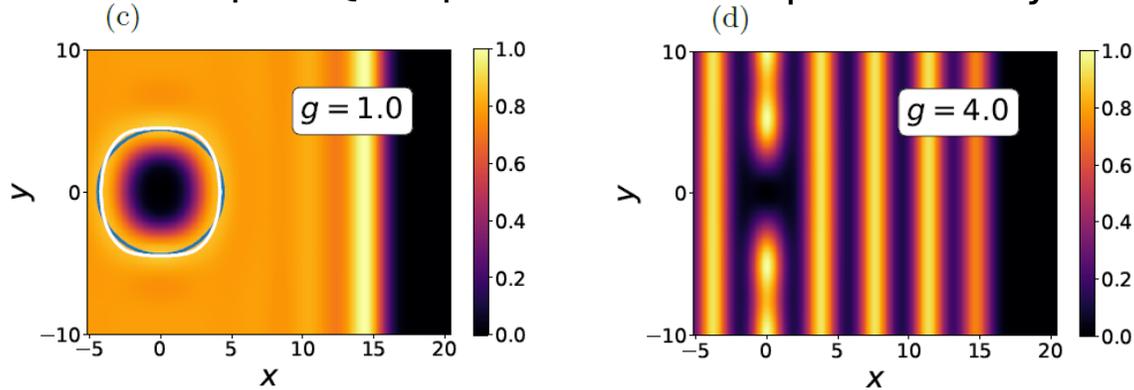
# Cavity-induced Stark shift

Vacuum fluctuations penalize quadrupole!

$$H_{pMF} = H_{int} + g^2 \omega_c (Q_{xx} - \langle Q_{xx} \rangle)^2$$

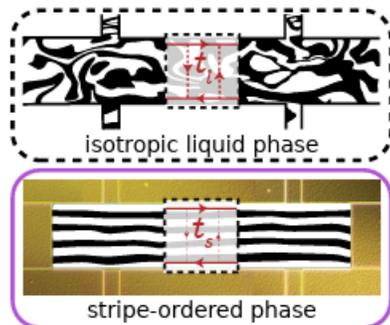
Anisotropic FQH liquid

Stripe instability



~ Cavity-oriented stripes

Graziotto, Faist et al.,  
arxiv2502.15490



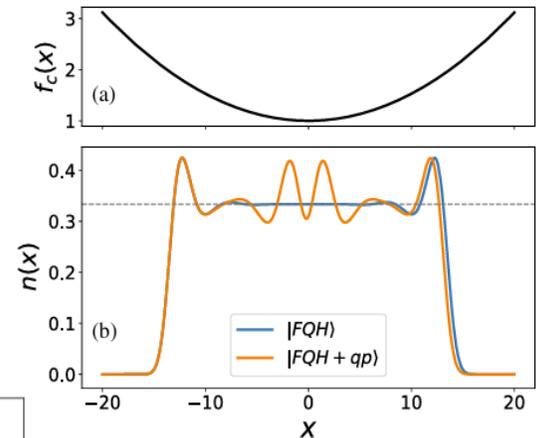
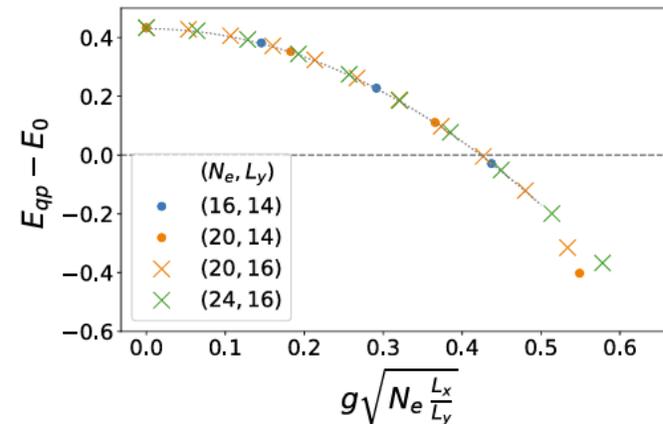
Reintroduce

Light-matter interaction

$$\text{Stark shifts} \sim \int dx \rho_{LLL}(x) \frac{E_c^2(x)}{2\omega_c}$$

Static potential

$$V_{eff} = l_B^2 \frac{E_c^2(x)}{2\omega_c}$$



Quasi-particle creation!

# FQH geometry

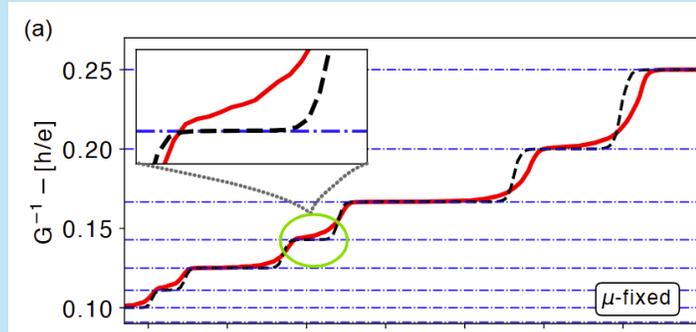
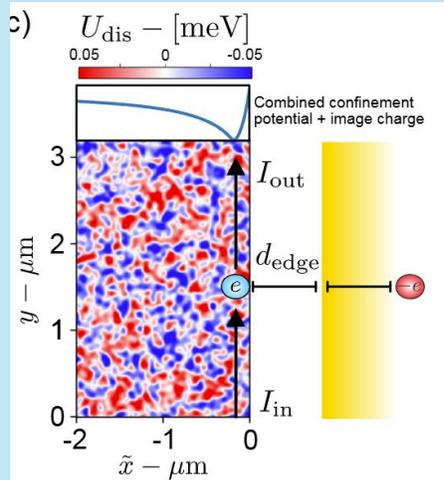
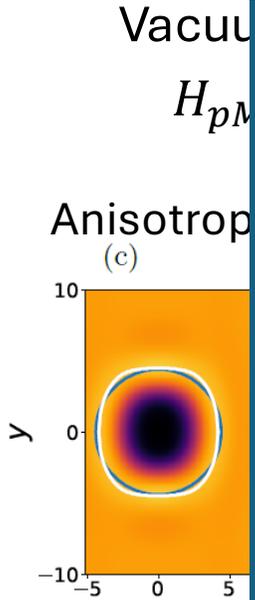
vs

# Cavity-induced Stark shift

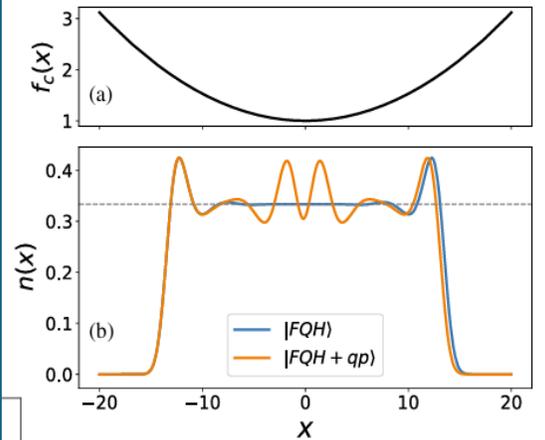
Light-matter interaction

$$E_s \sim \int dx \rho_{LLL}(x) \frac{E_c^2(x)}{2\omega_c}$$

At ultra-strong coupling, other off-resonant and electrostatics contributions can be relevant!



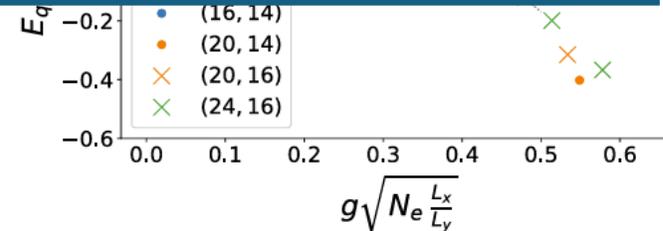
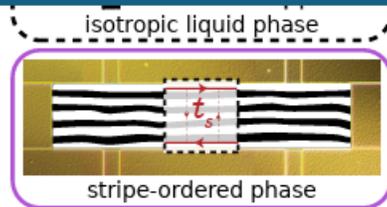
Andolina, Bacciconi, ..., De Bernardis, arxiv 2511.04744



Quasi-particle creation!

~ Cavity-oriented stripes

Graziotto, Faist et al., arxiv2502.15490

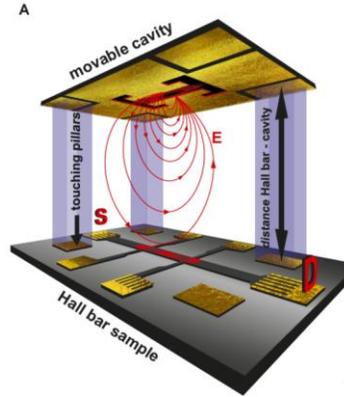


# Experimental remarks

## Cavity set-up

## Split-ring resonator

Appugliese, Faist, et al. , Science (2022)  
Enker, Faist, et al. , PRX (2024)  
Enker, Faist et al. Nature (2025)  
Xue, Zhang et al. arxiv (2025)



Resonance

$$\omega_c \simeq 2\pi \cdot 100 \text{ GHz} \simeq 0.4 \text{ meV}$$

Strong coupling

$$\partial_x E_c^x \simeq 10^5 - 10^8 \text{ V/m}^2$$
$$l_B \simeq 10 \text{ nm}$$

$$g = e l_B^2 \frac{\partial_x E_c}{\omega_c} \simeq 10^{-5} - 10^{-8}$$

$$\frac{\Omega}{\omega_c} \simeq g \sqrt{N_e} \simeq 10^{-2} - 10^{-5}$$

## What we did not consider

- Spin degree of freedom
- Off-resonant inter-LL mixing  
 $e l_B E_c$  vs  $e l_B^2 \partial_x E_c$
- Finite temperatures
- Electrostatic effects

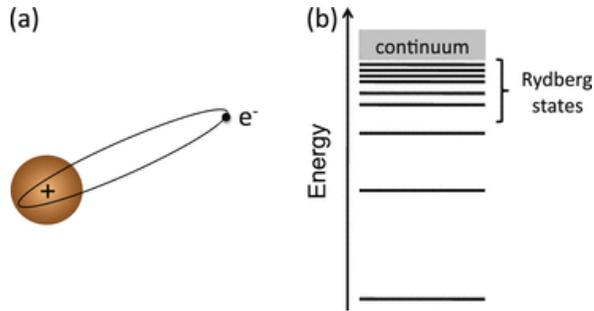
# Part II) Non-local dynamics: Rydberg arrays

2. *Local vs Non-local dynamics in cavity coupled Rydberg atom arrays*,  
**Bacciconi**, Xavier, Marinelli, Bhakuni, Dalmonte; PRL 134,13604 (2025)

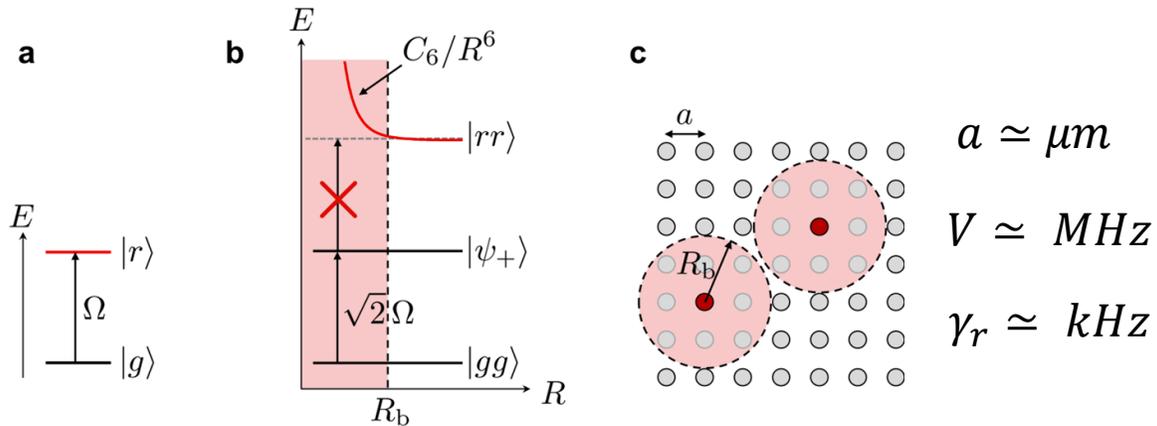
# Rydberg atom arrays: what and why

## What

Atoms with high principal quantum number



strong Van der Waals interactions

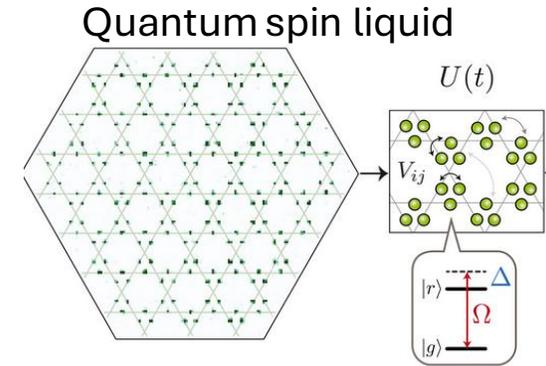


Broweys and Lahaye, Nat. Phys. (2020)

## Why

Quantum Simulation!

$$H \approx V \sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z + \Omega \sum_j \hat{\sigma}_j^x$$



Semeghini, Lukin et al; Science (2021)

- ✓ High interaction tunability
- ✓ Long coherence times
- ✓ Space and time resolution

Boston, Lukin  
 Los Angeles, Endres  
 Paris, Broweys  
 Munich, Zeiher  
 Wuhan, Li  
 ...

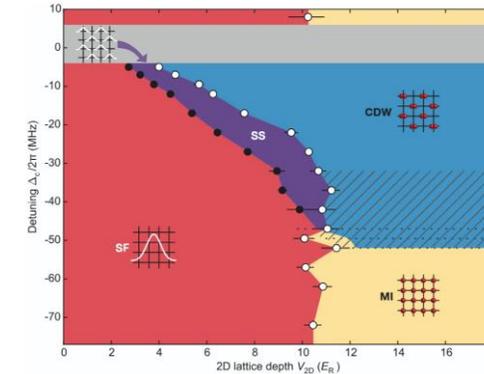
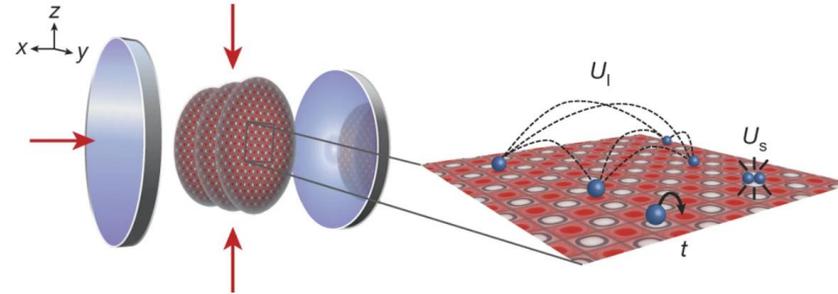
of interest for quantum computing!



# Cavity QED with atoms

Ultracold atomic gases in high finesse Fabri-Perot

Review: Mivehvar et. al., Adv. Phys. (2021)

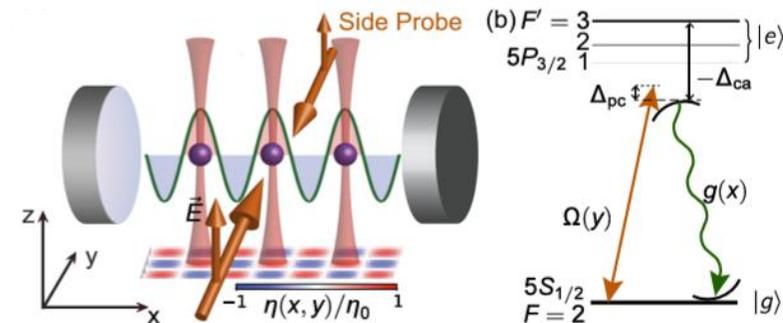


Esslinger, Nature (2016)



Atom in tweezers inside an **optical** cavity

Lukin (Harvard), Vuletic (MIT), Brantut (EPFL), Schleier-Smith (Stanford), Stamper-Kurn (Berkeley), Simon (Stanford) Leonard (Vienna), Welte (Stuttgart), Covey (Illinois), Schine (Maryland), Zeiher (MPQ), **Marinelli** (UniTs)



Stamper-Kurn et al. , PRL (2023)

# Cavity QED with rydberg atoms

## Our proposal: Tavis-Cumming-Ising model

### See other works

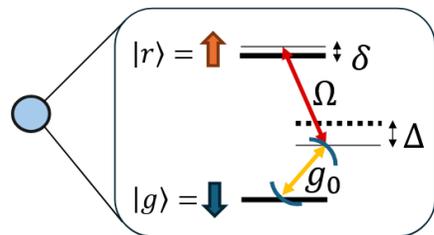
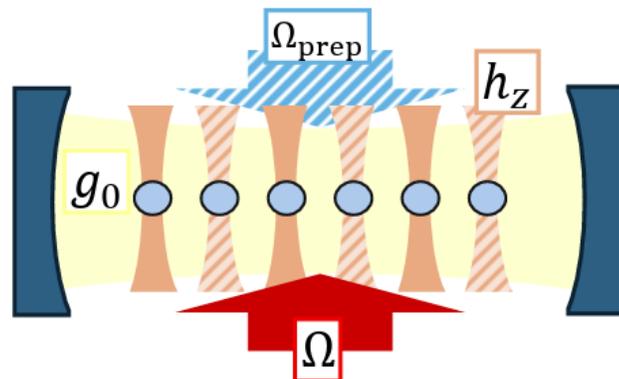
Zhang et al., PRL (2013)

Gelhausen et al., SciPost (2016)

Puel, Macrì, PRL (2024)

Hosseiniabadi, Marino et al. arXiv (2025)

Mann, Chang, et al. (2025)



$$\hat{H}_{eff} = V \sum_i \hat{\sigma}_i^Z \hat{\sigma}_{i+1}^Z + \sum_i h_i \hat{\sigma}_i^Z$$

Rydberg                      Local potential

$$+ \delta \hat{a}^\dagger \hat{a} + g \sum_i (\hat{a}^\dagger \hat{\sigma}_i^- + \hat{a} \hat{\sigma}_i^+)$$

Detuning                      2 photon transition

$$g = g_0 \Omega / \Delta \sim 0.1 \text{ MHz}$$

$$V \sim 1 \text{ MHz}$$

$$\kappa \sim 0.05 \text{ MHz}$$

# Low energy excitations ( $g \ll V$ )

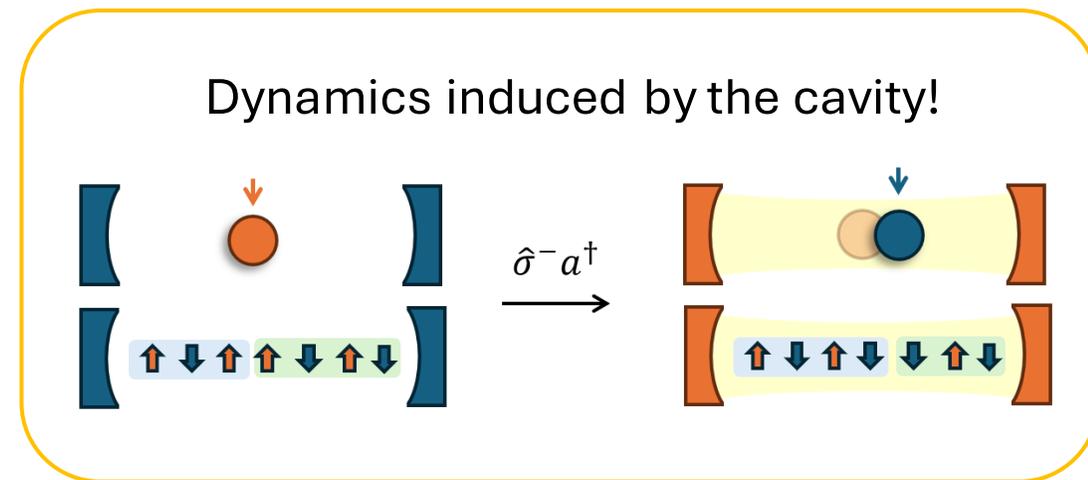
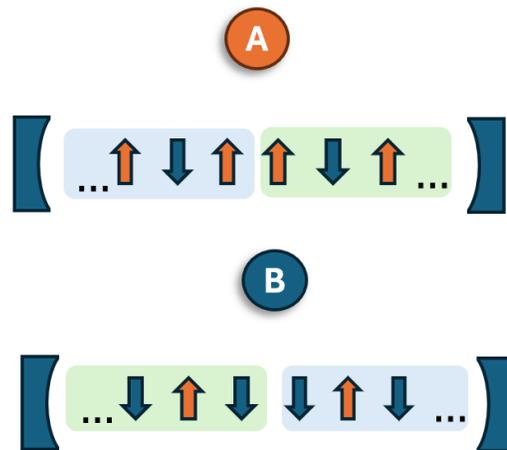
$$\hat{H} = V \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + h_s \sum_i (-1)^i \hat{\sigma}_i^z + \delta \hat{a}^\dagger \hat{a} + g \sum_i (\hat{a}^\dagger \hat{\sigma}_i^- + \hat{a} \hat{\sigma}_i^+)$$

$$|GS\rangle \sim \begin{array}{c} |\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \rangle |0_{ph}\rangle \\ \text{or} \\ |\downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \rangle |0_{ph}\rangle \end{array} \quad \Delta E = h_s N$$

Minimal excitation: Domain wall

$U(1)$  conservation

$$\hat{Q} = \hat{a}^\dagger \hat{a} + \sum_i \hat{n}_i$$



# Low energy excitations ( $g \ll V$ )

$$\hat{H} = V \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + h_s \sum_i (-1)^i \hat{\sigma}_i^z + \delta \hat{a}^\dagger \hat{a} + g \sum_i (\hat{a}^\dagger \hat{\sigma}_i^- + \hat{a} \hat{\sigma}_i^+)$$

$|GS\rangle \sim | \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \rangle |0_{ph}\rangle$  or  $| \downarrow \uparrow \downarrow \uparrow \downarrow \rangle |0_{ph}\rangle$

$\Delta E = h_s N$

**IMPORTANT REMARK (hopefully clearer now):**

Single particle coupling

Collective coupling  $G = g\sqrt{N}$

$$g(a + \hat{a}^\dagger) \sum_{i=1}^N O_i$$

couples «everything»

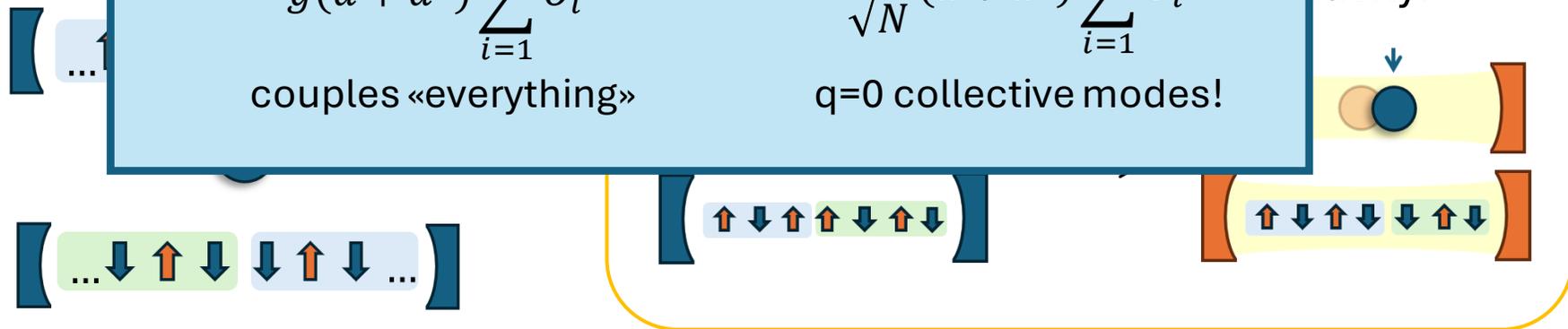
$$\frac{G}{\sqrt{N}} (a + \hat{a}^\dagger) \sum_{i=1}^N O_i$$

q=0 collective modes!

Minimal excitation: Domain wall

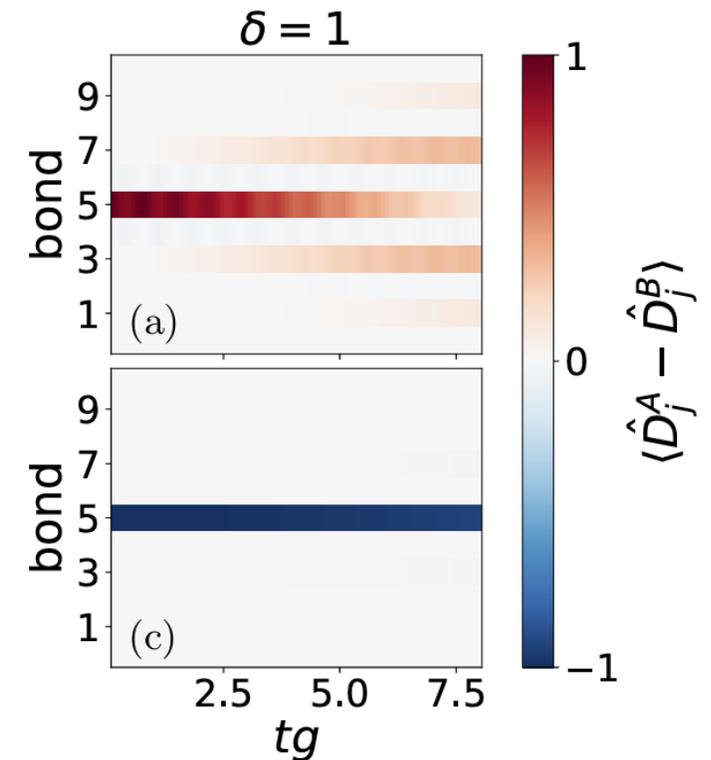
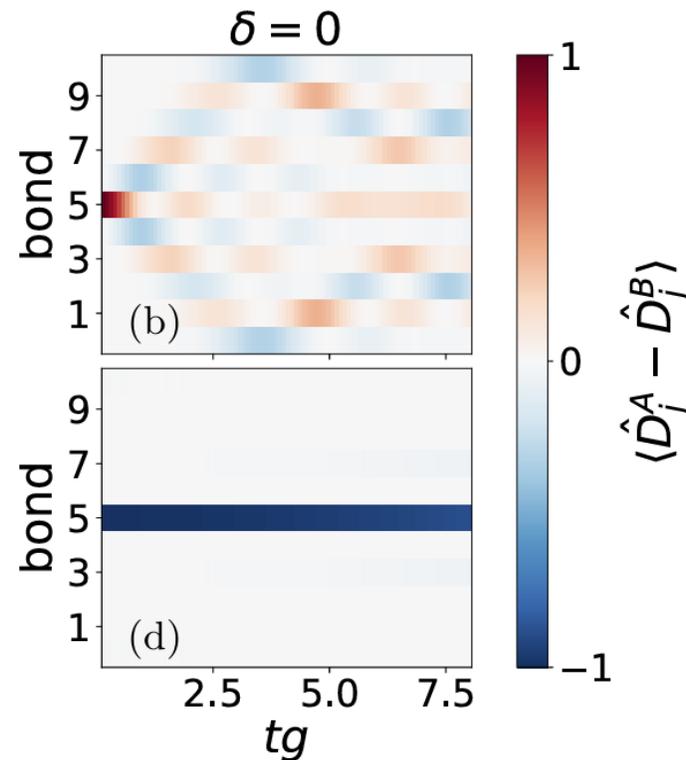
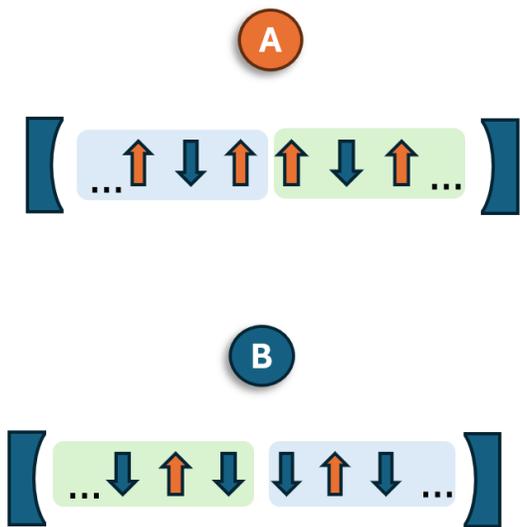
$U(1)$  conservation

$$\hat{Q} = \hat{a}^\dagger \hat{a} + \sum_i \hat{n}_i$$



# Domain wall dynamics: $h_s = 0$ $g = 0.1$

Domain wall density  $\hat{D}_j^A = |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$   $\hat{D}_j^B = |\downarrow\downarrow\rangle\langle\downarrow\downarrow|$



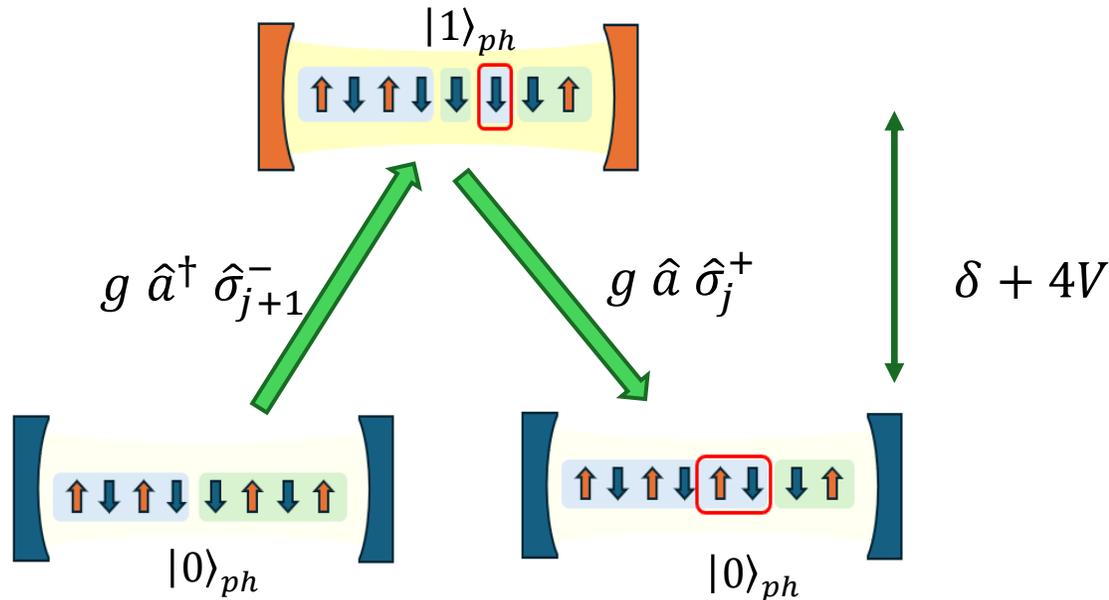
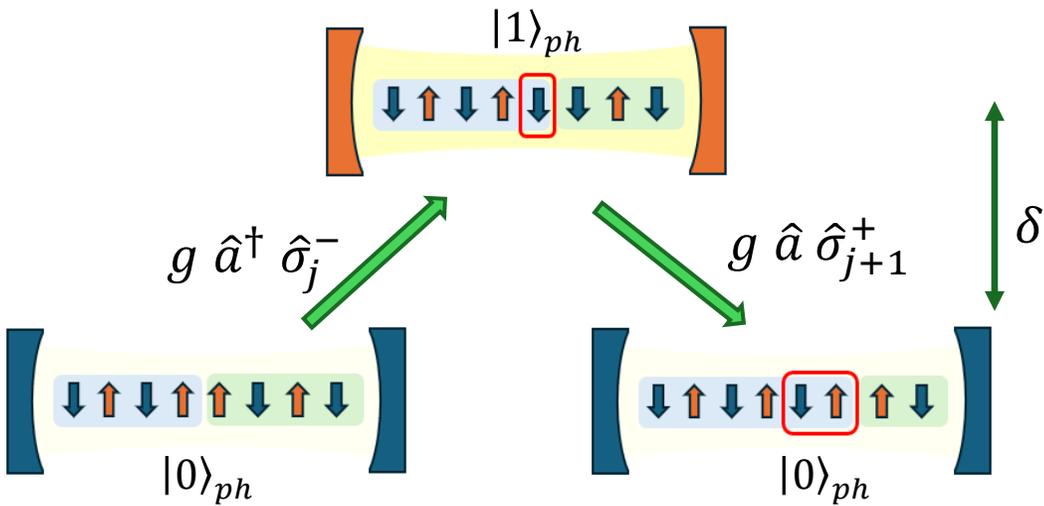
# Domain wall dynamics: perturbation theory

Type A domain wall 

$$J_A = \frac{g^2}{\delta}$$

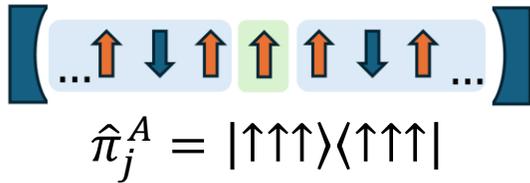
Type B domain wall 

$$J_B = \frac{g^2}{\delta + 4V}$$



# Composite excitations: mesons

Meson  $E_\pi = 4V + 2h_s$

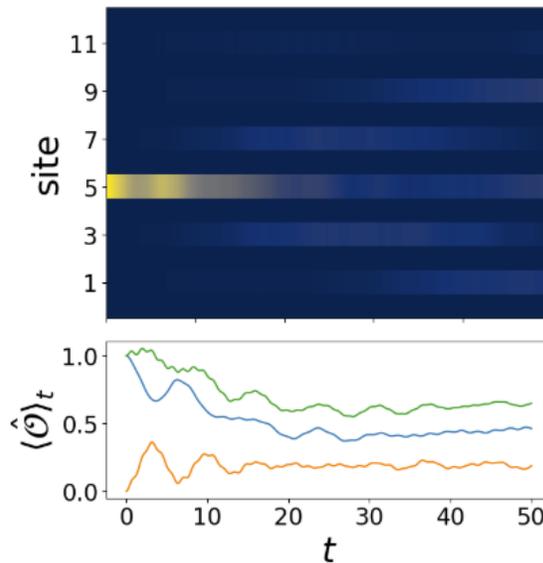


Meson-polaritons ( $G = g\sqrt{N}$ )

$$\hat{\sigma}_j^+ \hat{a} \begin{cases} |AFM\rangle |1_{ph}\rangle \\ \frac{1}{\sqrt{N/2}} \sum_j |\pi_j^A\rangle |0_{ph}\rangle \end{cases}$$

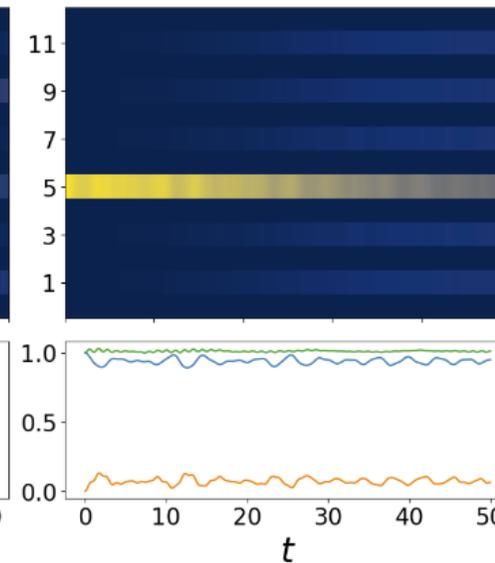
Not confined

$$h_s = 0 \quad \delta = 1.$$



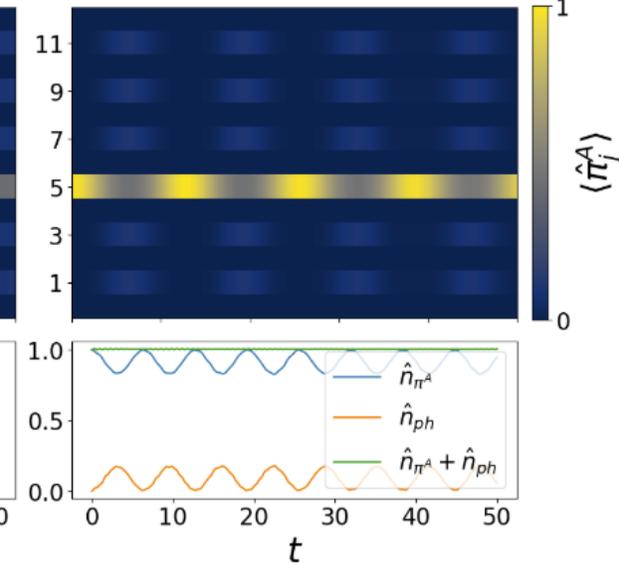
Confined  
Off resonance

$$h_s = 0.4 \quad \delta = 1.$$



Confined  
On resonance

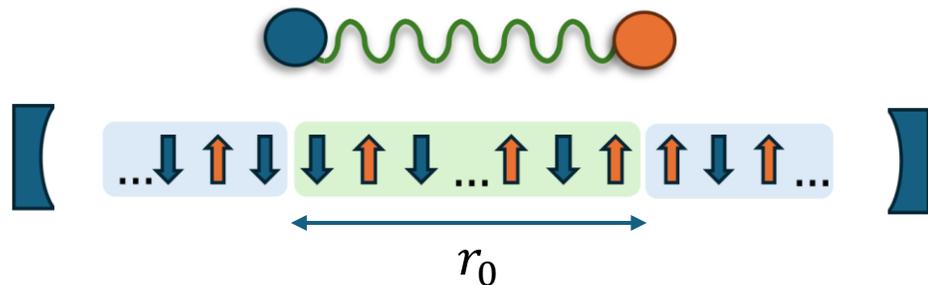
$$h_s = 0.4 \quad \delta = 4.8$$



# Composite excitations: strings

String

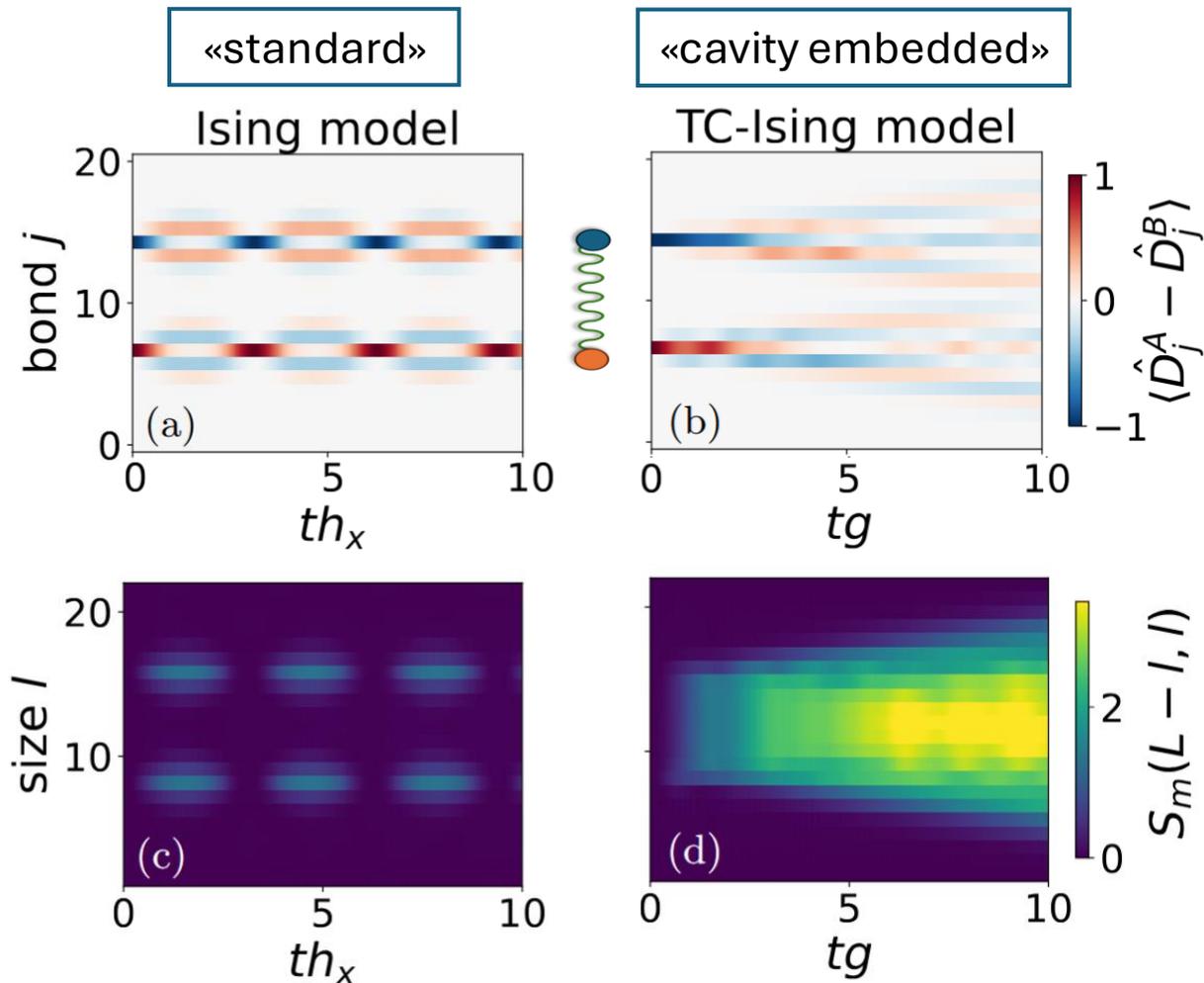
$$E_S = 4V + 2 r_0 h_s$$



Long range interaction between domain walls!

$$\text{Mutual information} \\ S_{vn}(A) + S_{vn}(B) - S_{vn}(AB)$$

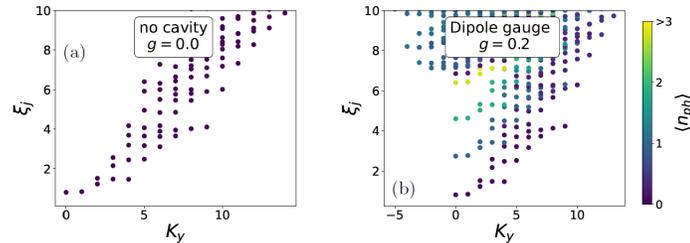
$$J_s^{loc} \propto e^{-r_0} \quad J_s^{cav} \propto \frac{g^2}{h_s + \delta}$$



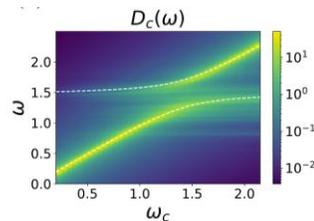
# Summary

## I) Topological matter: FQH

- Topology is stable under *some non-local* perturbations

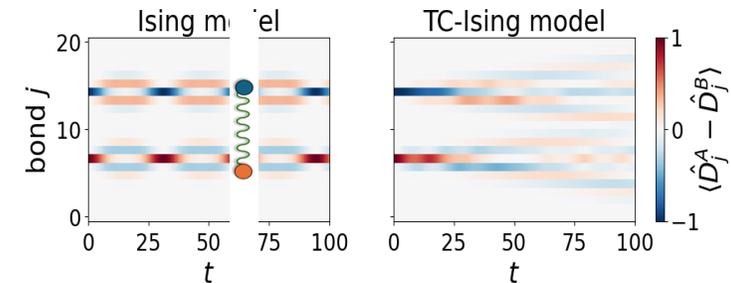


- Spectroscopy of graviton-polaritons excitations

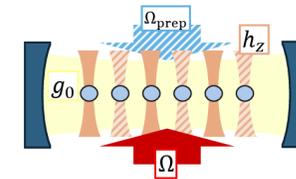


## II) Non local dynamics in Rydberg atom arrays

- Cavity modes generate **non-local** dynamics



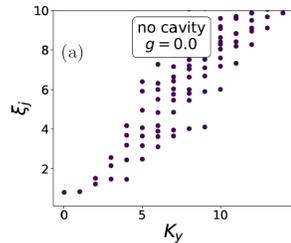
- *Simple* experimental scheme



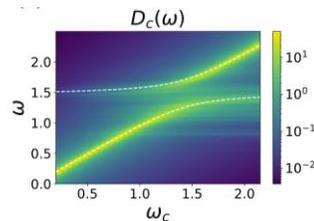
# Summary

## I) Topological matter: FQH

- Topology is stable under *some non-local* perturbations



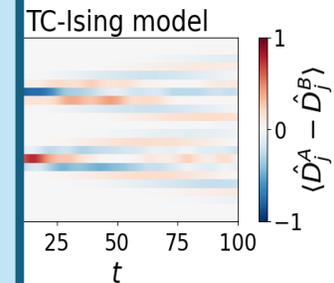
- Spectroscopy of ground state excitations



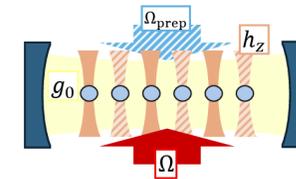
New phenomena from coexistence of local and non-local modes!

## II) Non local dynamics in Rydberg atom arrays

- Cavity modes generate **non-local** dynamics



- *Simple* experimental scheme



# Acknowledgements

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