西湖大學
WESTLAKE UNIVERSITY

Bipartite Entanglement and Surface Criticality: The Extra Contribution of the Nonordinary Edge in Entanglement

报告人：严正

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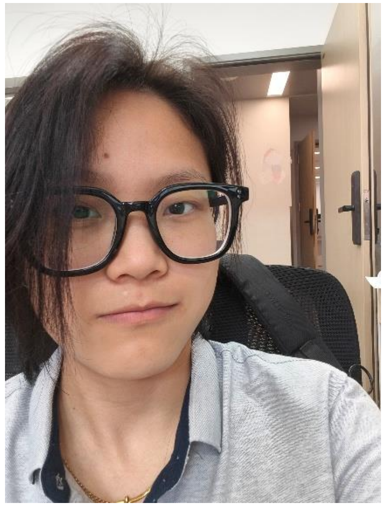
合作者[Collaborators]

博士生:[Ph.D. students]



博士后:[Post Doc.]

教授:[Professor]

CONTENT

研究背景和动机[Research background and motivation.]

临界性的一般性描述：
[general description of criticality]

热涨落驱动经典相变，量子涨落驱动量子相变。[Thermal fluctuations drive classical phase transitions, while quantum fluctuations drive quantum phase transitions.]

- 朗道：用**序参量**区分破缺不同对称性的相。[Landau uses **order parameters** to distinguish phases with different broken symmetries.]
- 连续场论：Landau-Ginzburg-Wilson 哈密顿：

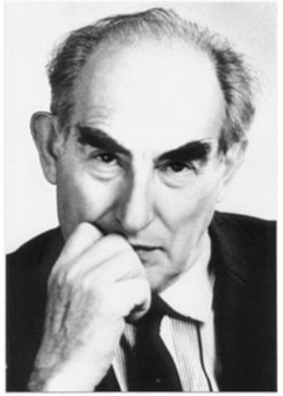
$$H(\Phi) = \int \mathrm{d}V((\nabla\Phi)^2 + s\Phi^2 + u(\Phi^2)^2)$$

$$\mathcal{Z} = \int \mathcal{D}\Phi \ \mathrm{e}^{-H(\Phi)}$$

- Wilson：重整化群理论[Renormalization Group]

普适类：只由系统对称性与维度决定

[Universality class: only determined by symmetry and dimension of the system]



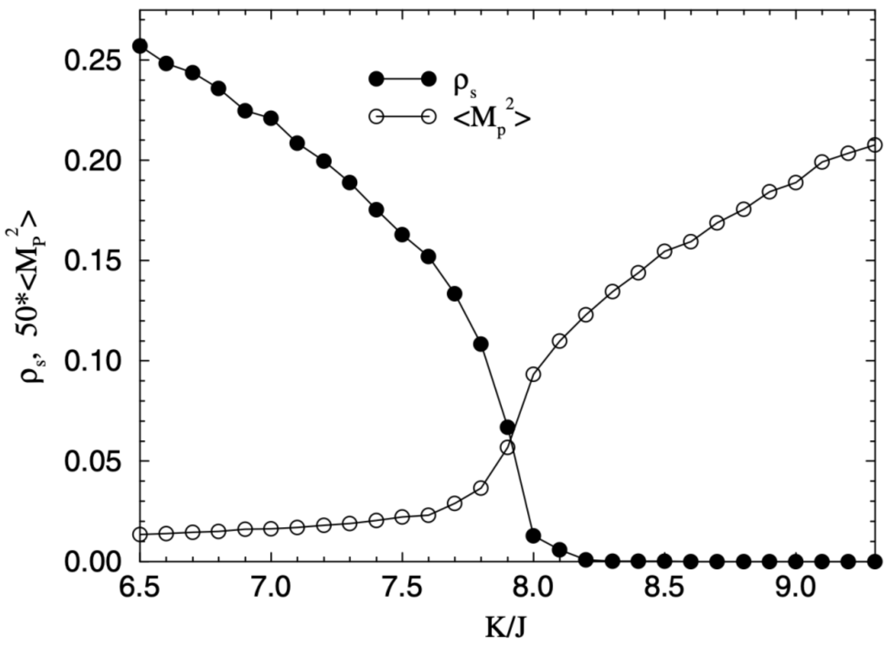
研究背景和动机[Research background and motivation.]

去禁闭量子临界性：
[deconfined quantum criticality]

最初的数值发现。[The earliest numerical discovery.]

- S=1/2 XY Model with an additional four-spin ring exchange term K.
- Increasing K leads to the emergence of a striped bond plaquette order.

朗道-金兹堡-威尔森范式期待一级相变的出现，但这却被违反了。[Landau-Ginzburg-Wilson Paradigm expects a first order phase transition here, but it is violated.]

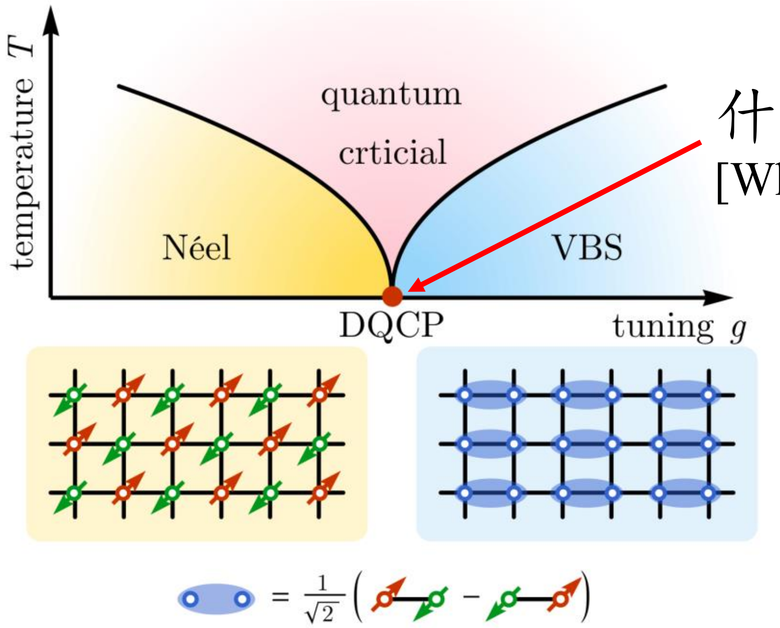


A. W. Sandvik, et al, ***Phys. Rev. Lett.*** 89, 247201 (2002).

研究背景和动机[Research background and motivation.]

去禁闭量子临界性：
[deconfined quantum criticality]

去禁闭量子相变是两个破坏了无关的对称性的相（例如：Nèel与VBS）之间发生的**连续**相变。[A deconfined quantum phase transition is a **continuous** quantum phase transition between two phases that break unrelated symmetries (e.g., Nèel AFM and VBS).]



什么普适类？
[What Universality class?]

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[2] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Quantum criticality beyond the landau-ginzburg-wilson paradigm, Phys. Rev. B **70**, 144407 (2004).

[3] A. W. Sandvik, Evidence for deconfined quantum criticality in a two-dimensional heisenberg model with four-spin interactions, Phys. Rev. Lett. **98**, 227202 (2007).

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[7] Y. Q. Qin, Y.-Y. He, Y.-Z. You, Z.-Y. Lu, A. Sen, A. W. Sandvik, C. Xu, and Z. Y. Meng, Duality between the deconfined quantum-critical point and the bosonic topological transition, Phys. Rev. X **7**, 031052 (2017).

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[11] J. D'Emidio, A. A. Eberharter, and A. M. Läuchli, Diagnosing weakly first-order phase transitions by coupling to order parameters, SciPost Phys. **15**, 061 (2023).

[12] Z. Deng, L. Liu, W. Guo, and H.-Q. Lin, Diagnosing quantum phase transition order and deconfined criticality via entanglement entropy, Phys. Rev. Lett. **133**, 100402 (2024).

[13] M. Song, J. Zhao, M. Cheng, C. Xu, M. Scherer, L. Janssen, and Z. Y. Meng, Evolution of entanglement entropy at su(<i>n</i>) deconfined quantum critical points, Science Advances **11**, eadr0634 (2025), https://www.science.org/doi/pdf/10.1126/sciadv.adr0634.

[14] A. Nahum, Note on wess-zumino-witten models and quasiuniversality in $2+1$ dimensions, Phys. Rev. B **102**, 201116 (2020).

[15] R. Ma and C. Wang, Theory of deconfined pseudocriticality, Phys. Rev. B **102**, 020407 (2020).

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[20] J. Takahashi, H. Shao, B. Zhao, W. Guo, and A. W. Sandvik, So(5) multicriticality in two-dimensional quantum magnets (2024), arXiv:2405.06607 [cond-mat.str-el].

[21] Y. D. Liao, G. Pan, W. Jiang, Y. Qi, and Z. Y. Meng, The teaching from entanglement: 2d su(2) antiferromagnet to valence bond solid deconfined quantum critical points are not conformal (2023), arXiv:2302.11742 [cond-mat.str-el].

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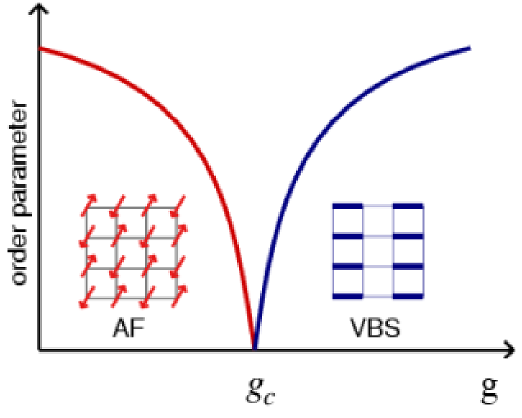
Deconfined quantum criticality

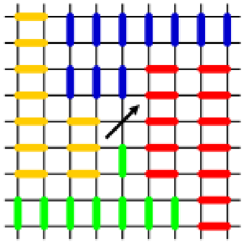
Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 2004

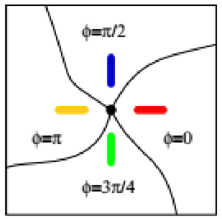
- Field theory describes continuous transition between two unrelated ordered states: AFM-VBS

$$\boxed{H = J\sum_{\langle i,j\rangle} \mathbf{S}_i\cdot\mathbf{S}_j + g\ [\text{other symmetry preserving interactions}]}$$



- g small: AFM, breaks O(3) symmetry
- g large: valence-bond solid, breaks Z_4 symmetry





- Topological defects (Spinons) bind together in the VBS state (confinement) and condensate in the Neel state, deconfine at the critical point
- leading to a continuous phase transition in a New universality

Levin and Senthil, PRB 2004

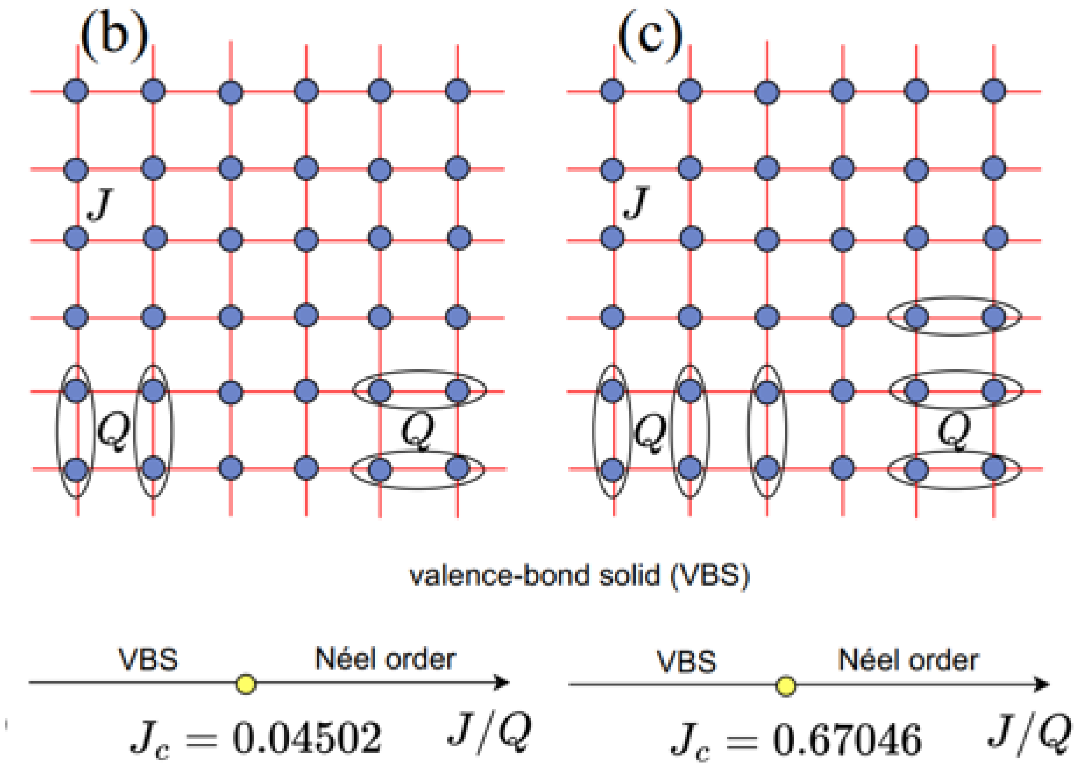
- AFM and VBS order parameters are not fundamental degrees of freedom, but emerge from the spinon field

- Convincing in SU(N) field theory for large N, not clear for small N (esp. N=2).

研究背景和动机[Research background and motivation.]

去禁闭量子临界性：
[deconfined quantum criticality]

J-Q模型：[model]



$J-Q_2$: $$H = -J\sum_{\langle ij\rangle} P_{i,j} - Q\sum_{\langle ijkl\rangle} P_{ij}P_{kl},$$

$J-Q_3$: $$H = -J\sum_{\langle ij\rangle} P_{i,j} - Q\sum_{\langle ijklmn\rangle} P_{ij}P_{kl}P_{mn},$$

$$P_{ij} = \frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j$$

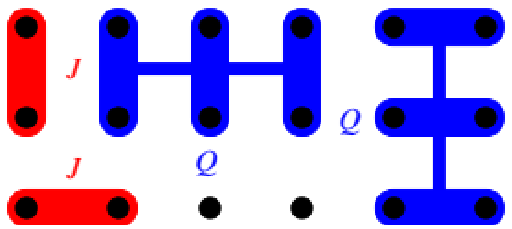
研究背景和动机[Research background and motivation.]

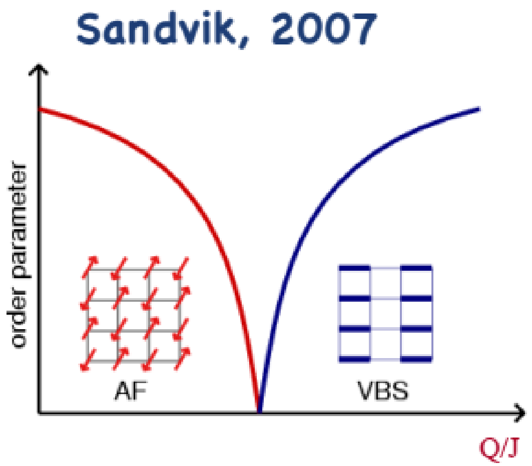
- Spin-1/2 J-Q model is a designer Hamiltonian for DQC physics

$$\boxed{H = -J\sum_{\langle i,j\rangle} C_{ij} - Q\sum_{\langle ijklmn\rangle} C_{ij}C_{kl}C_{mn}}$$

Heisenberg exchange= singlet-projector

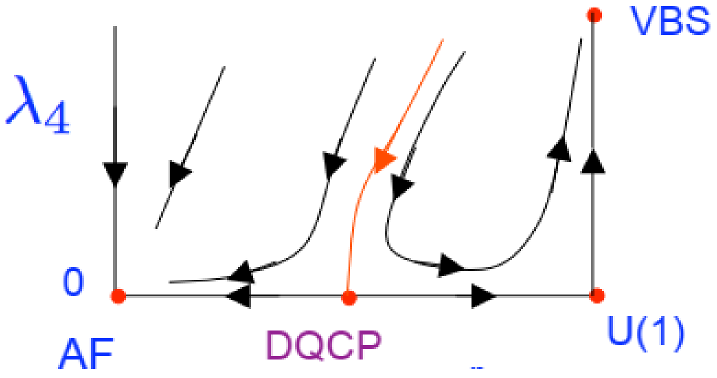
$$\boxed{C_{ij} = \frac{1}{4} - \mathbf{S}_i\cdot\mathbf{S}_j}$$





- The Q terms are multi-spin interactions that favor the Valence bond solid state, while the Heisenberg J term favors the AF Neel state.

- The model exhibits VBS-AFM transition, showing signatures of the DQC, with the transition appearing continuous
  - Scaling violation $\rho_s(L)L \neq \text{const.}$ is explained by an unconventional two-length scale scaling
  - Z_4 各向异性是dangerously irrelevant



Shao, Guo, Sandvik. Science 2016

研究背景和动机[Research background and motivation.]

纠缠熵与凝聚态物理：
[Entanglement entropy and condensed matter physics]

- 标准的可观测量（结构因子，Binder累积量，关联长度）在Nèel-VBS附近难以解释。[Standard observables (structure factors, Binder cumulants, correlation lengths) become difficult to interpret near the Nèel-VBS transition.]
- 纠缠熵通过非局域的观测能直接捕捉量子上的关联，对微观细节更加不敏感。[Entanglement Entropy (EE) is less sensitive to microscopic details and directly captures quantum correlations across a spatial cut.]

在量子蒙卡模拟中，我们无法测量von Neumann纠缠熵，我们使用二阶Rènyi 纠缠熵。[In QMC simulations, the von Neumann EE is out of reach, we use the Rènyi EEs.]

$$S_2 = \ln Tr(\rho_A^2), \qquad \rho_A = Tr_{\bar{A}} e^{-\beta H} = e^{-\beta H_A}.$$

研究背景和动机[Research background and motivation.]

纠缠熵与凝聚态物理：
[Entanglement entropy and condensed matter physics]

Physical state	Entropy	Example
Gapped (brok. disc. sym.)	$aL^{d-1} + \ln(\mathrm{deg})$	Gapped XXZ
$d = 1$ CFT	$\frac{c}{3} \ln L$	$s = \frac{1}{2}$ Heisenberg chain
$d \geq 2$ QCP	$aL^{d-1} + \gamma_{\mathrm{QCP}}$	Wilson–Fisher O(N)
Ordered (brok. cont. sym.)	$aL^{d-1} + \frac{n_G}{2} \ln L$	Superfluid, Néel order
Topological order	$aL^{d-1} - \gamma_{\mathrm{top}}$	$\mathbb{Z}_2$ spin liquid

N. Laflorencie, **Physics Reports** 646, 1 (2016)

研究背景和动机[Research background and motivation.]

纠缠熵与临界性：
[Entanglement entropy and criticality]

任何现有的共形场论都认为纠缠熵满足该标度关系，并且$a > 0$，这与数值结果冲突。[All CFT predictions for the EE of a critical system with corners is this equation, with $a > 0$, which disagrees with the numerical results.]

$$S(l) = Al - a \ln l + \text{constant},$$

J. Zhao, et al, ***Phys. Rev. Lett.*** 128, 010601 (2022).

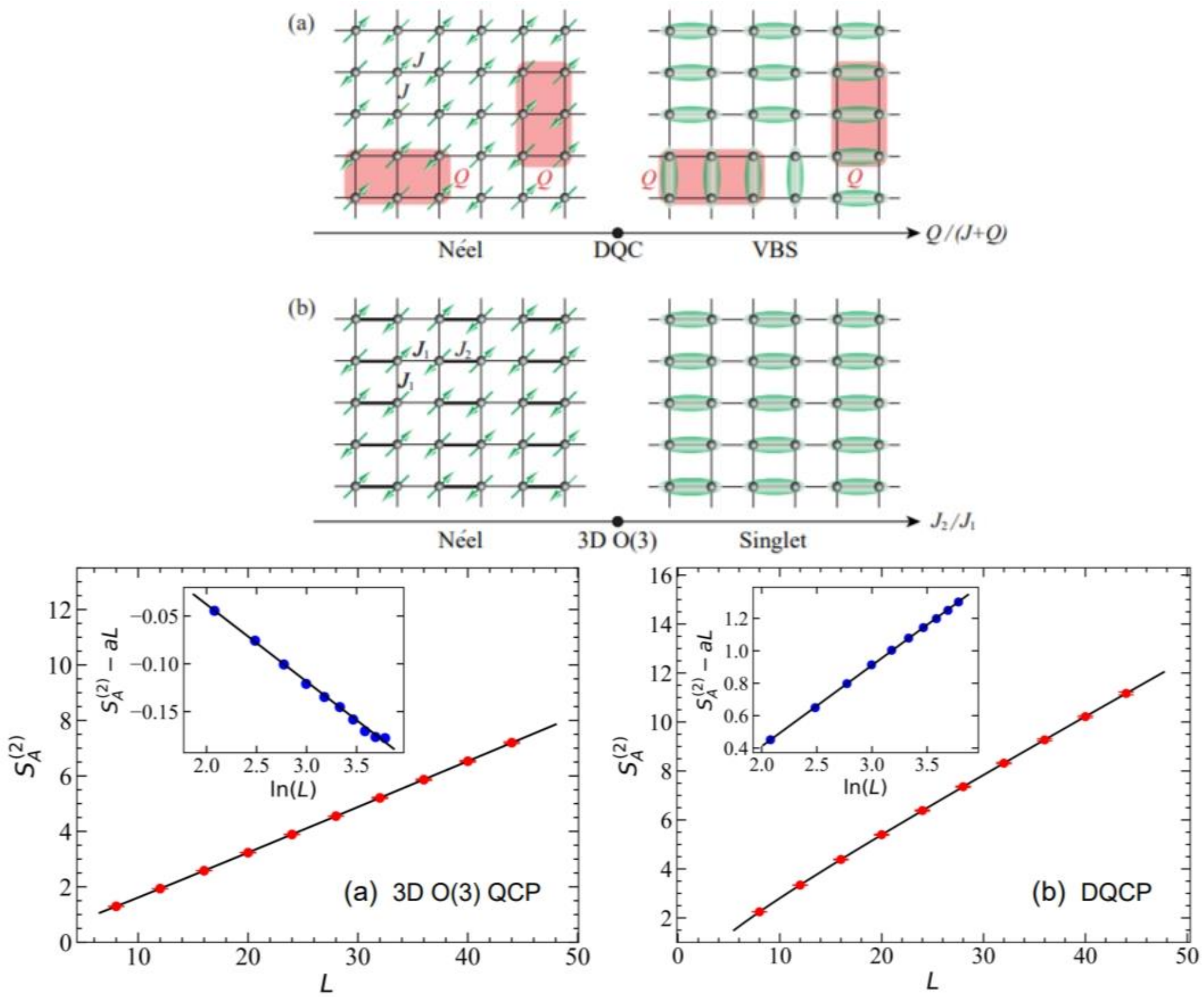


FIG. 3. $S_A^{(2)}(L)$ with even-B boundary of entangling region for (a) the $L \times L$ square lattice $H_{J_1-J_2}$ model at 3D O(3) QCP and (b) the H_{J-Q_3} model at DQCP with pinning field $\delta = 0.05$. The fitting result for $H_{J_1-J_2}$ is $S_A^{(2)}(L) = 0.168(1)L - 0.081(4)\ln(L) - 0.124(7)$. The fitting result for H_{J-Q_3} is $S_A^{(2)}(L) = 0.224(1)L + 0.49(1)\ln(L) - 0.58(2)$. Insets show the $S_A^{(2)} - aL$ versus $\ln(L)$ such that the sign of the log-corrections manifest. It is clear that the DQC log-correction accquires a opposite sign compared with the O(n) ones. This is in constrast with the positivity requirement of EE for unitary CFTs.

研究背景和动机[Research background and motivation.]

纠缠切割与边缘临界性：
[Entanglement bipartition and surface criticality]

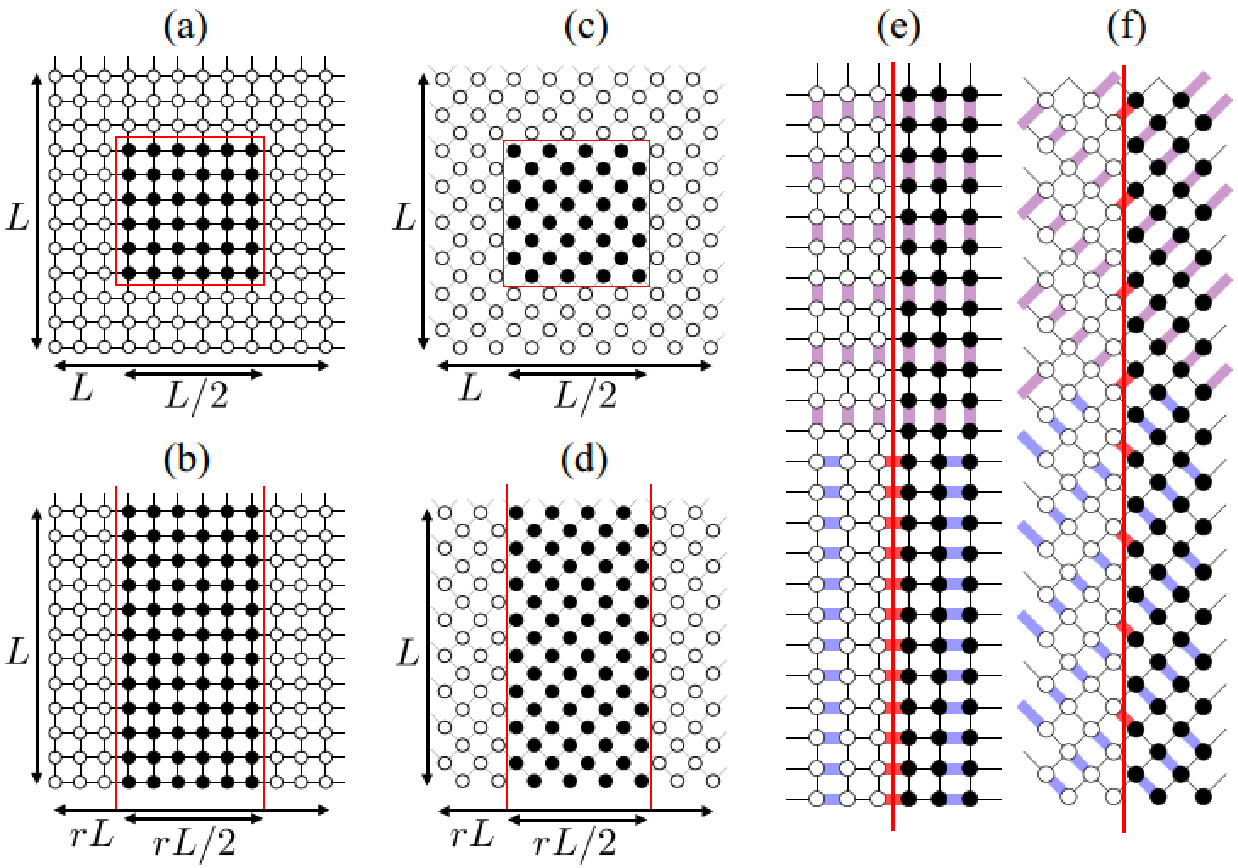
任何现有的共形场论都认为纠缠熵满足该标度关系，并且$a > 0$，这与数值结果冲突。[All CFT predictions for the EE of a critical system with corners is this equation, with $a > 0$, which disagrees with the numerical results.]

$$S(l) = Al - a \ln l + \text{constant},$$

Goldstone Mode Correction:

$$S_n(L) = aL^{d-1} + \frac{N_G}{2} \ln\left(\frac{\rho_s}{c} L^{d-1}\right) + \gamma_{\text{ord}}$$

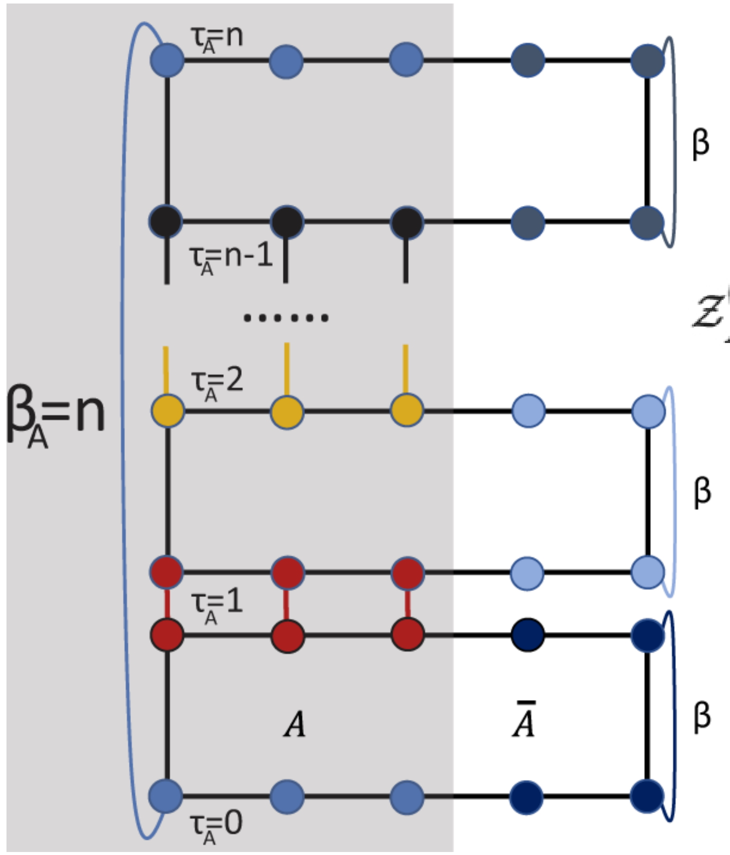
Z. Deng, et al, ***Phys. Rev. Lett.*** 133, 026502 (2024).



斜切以确保不人为破坏对称性。[Use tilted cut to avoid destroying symmetry manually.]

J. Emidio, et al, ***Phys. Rev. Lett.*** 133, 100402 (2024).

纠缠谱与纠缠哈密顿量信息的提取：
[Entanglement spectrum and entanglement Hamiltonian]



$$\mathcal{Z}_A^{(n)} = \mathrm{Tr}[\rho_A^n] = \mathrm{Tr}[e^{-n\mathcal{H}_A}].$$

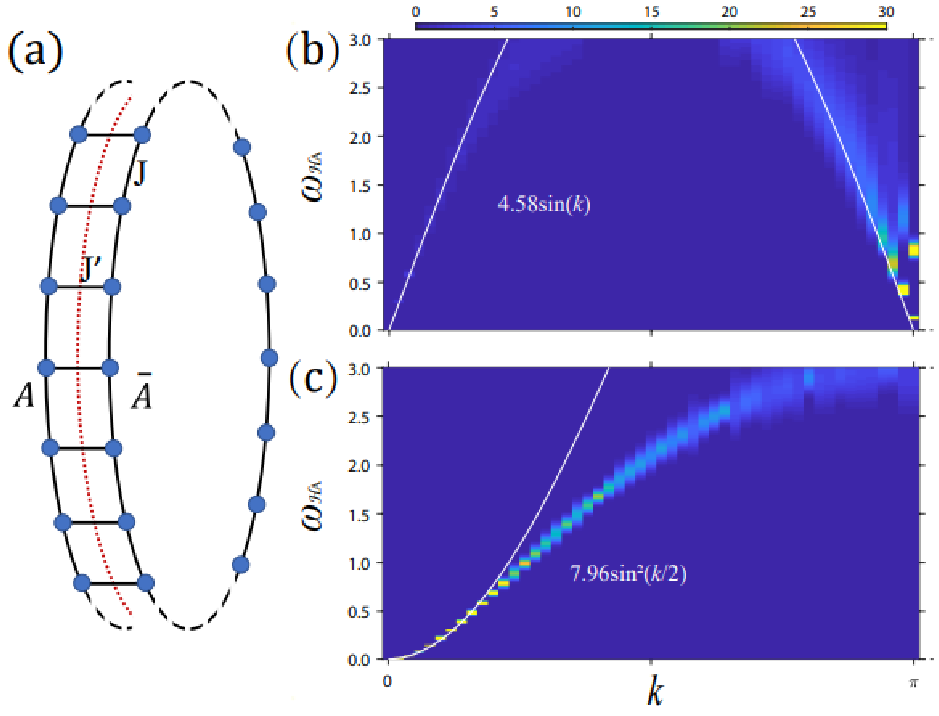
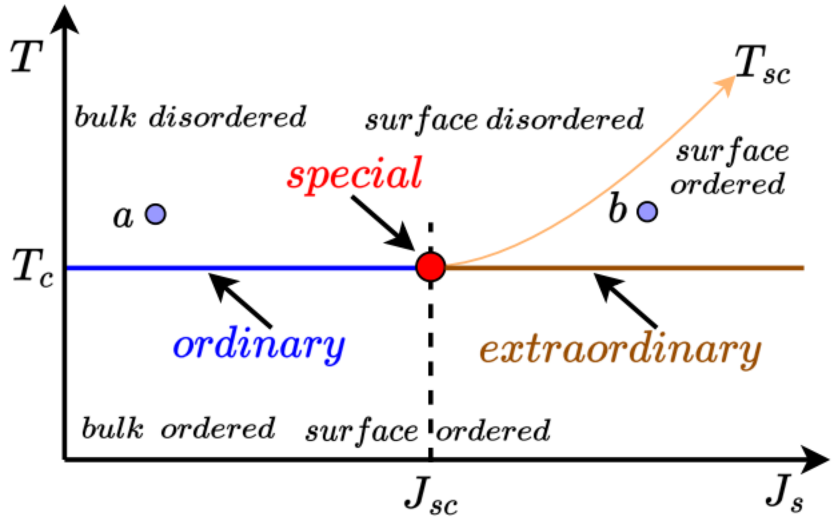


FIG. 2. **ES of Heisenberg ladder.** (a) Heisenberg spin ladder. The red dashed line cut it into two entangled constituents, A and $\bar{A}$. (b) The low-lying ES with $L = 100$, $J' = 1.732$, $J = 1$ and $\beta = 100$, $\beta_A = 200$. The white line is fitting to the data with the dispersion $4.58\sin(k)$. (c) The low-lying ES with $L = 100$, $J' = 1.732$, $J = -1$ and $\beta = 100$, $\beta_A = 800$. The white line is fitting to the data with the dispersion $7.96\sin^2(k/2)$.

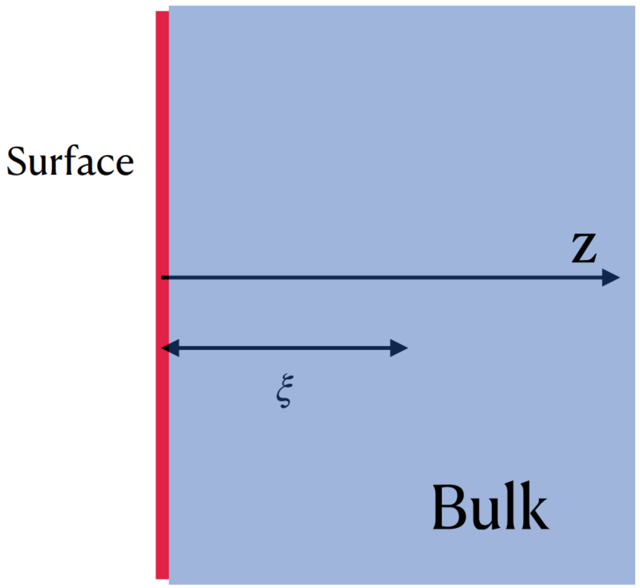
ZY, ZY Meng, Nature communications 14, 2360 (2023)

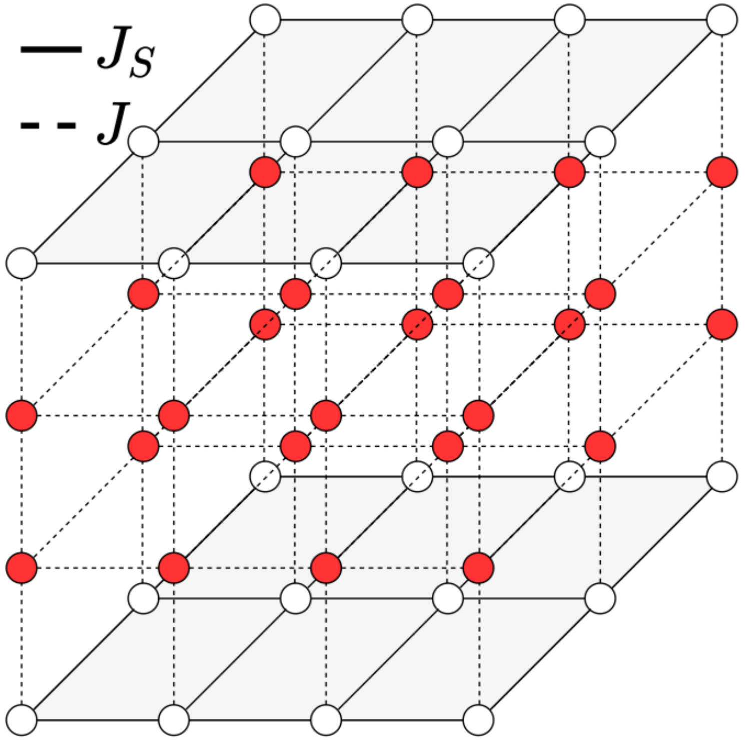
Surface Critical Behavior(SCB)

For classical models, by tuning surface couplings, the surfaces may exist three possible phase transition: ordinary, special and extraordinary phase transition.



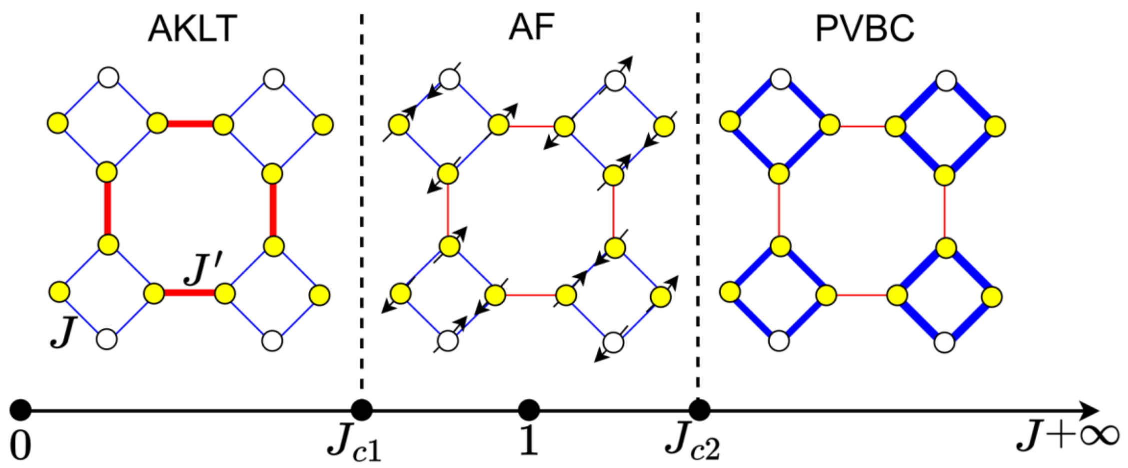
Binder, PRB 9, 2194 (1974);
Binder, Phase and Critical Phenomena,1983.



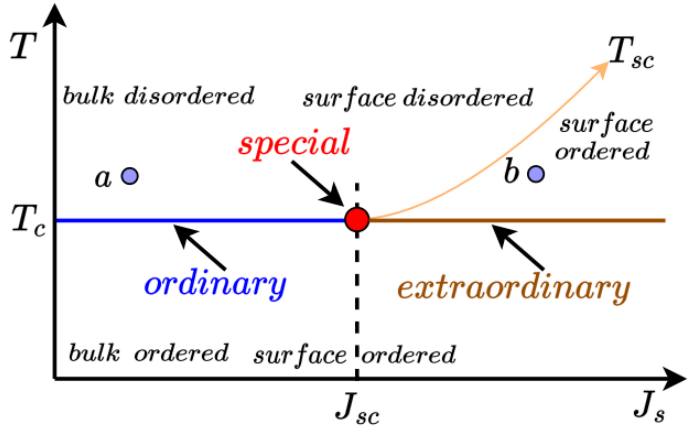


SCBs of QCP: pioneering work by Zhang and Wang

Decorated square (DS) lattice



$$H = J\sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle i,j\rangle'} \mathbf{S}_i \cdot \mathbf{S}_j$$



- Ordinary SCB at 3D O(3) critical point J_{c2} with surface critical exponents

$$\eta_{||} = 1.327(25) \qquad \eta_{\perp} = 0.680(8)$$

- Nonordinary/special SCB at 3D O(3) critical point J_{c1} with different surface critical exponents

$$\eta_{||} = -0.449(5) \qquad \eta_{\perp} = -0.2090(15)$$

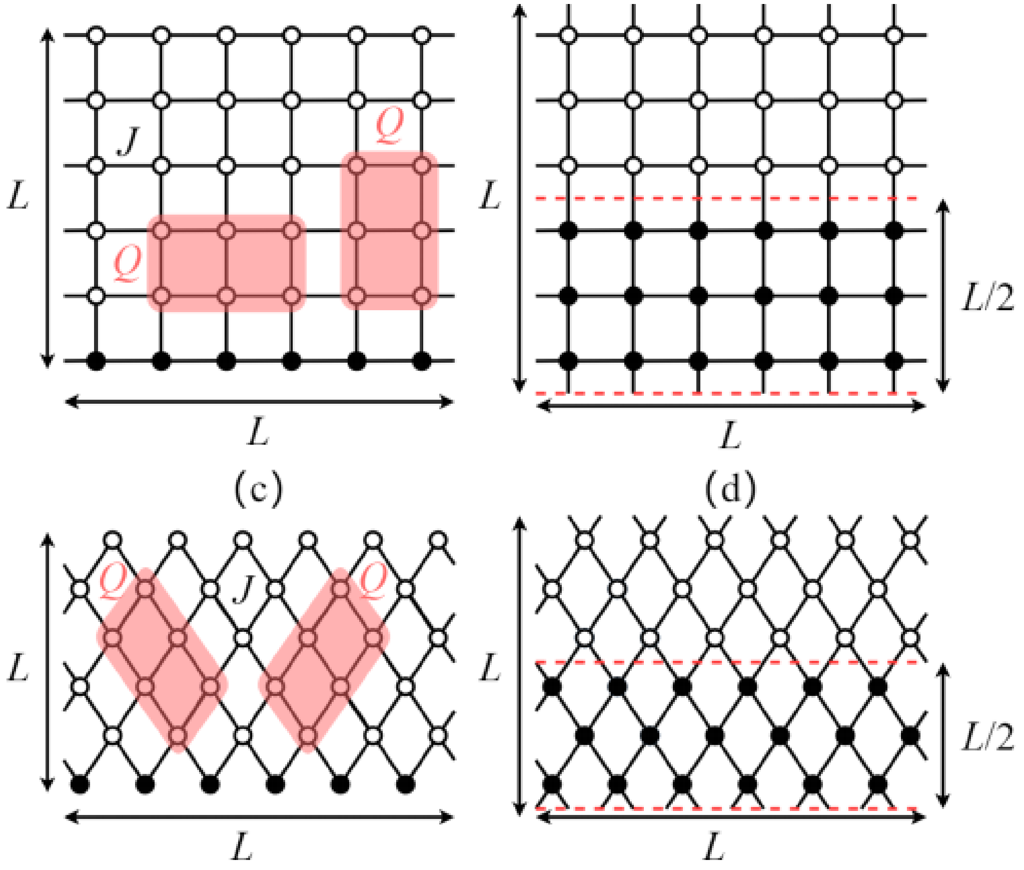
- The gapless surface states of the AKLT-like (symmetry-protected topological, SPT) phase induce special transition.

Zhang, PRL 118, 87201 (2017).

研究内容和结论[Research content and conclusion]

纠缠切割与边缘临界性：
[Entanglement bipartition and surface criticality]

我们首次将纠缠与表面临界行为建立联系，发现不同的表面临界行为，会诱导不同的纠缠谱，从而影响纠缠熵的测量，这一点超出了共形场论的预测。[We establish, for the first time, a connection between entanglement and surface critical behavior, and find that different surface critical behaviors lead to distinct entanglement spectra, thereby affecting the measurement of entanglement entropy—a phenomenon that goes beyond the predictions of conformal field theory.]



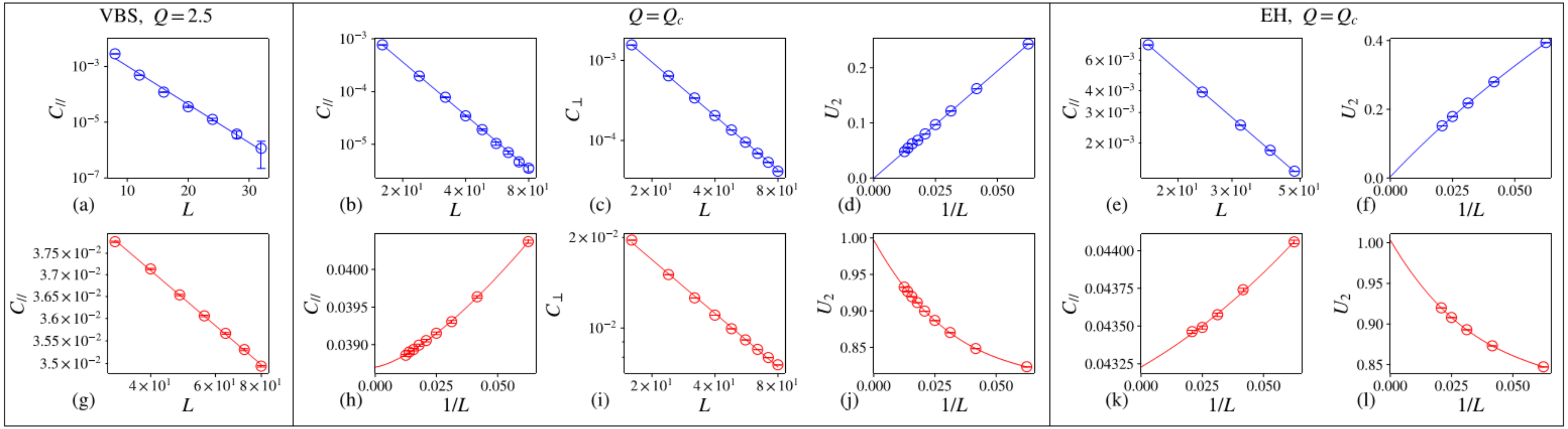
Y. Zhu, Z. Liu, Zhe Wang, Y. C. Wang and **ZY**, Phys. Rev. Lett. 136, 046501 (2026)

研究内容和结论[Research content and conclusion]

纠缠切割与边缘临界性：
[Entanglement bipartition and surface criticality]

正切
Standard

斜切
Tilted



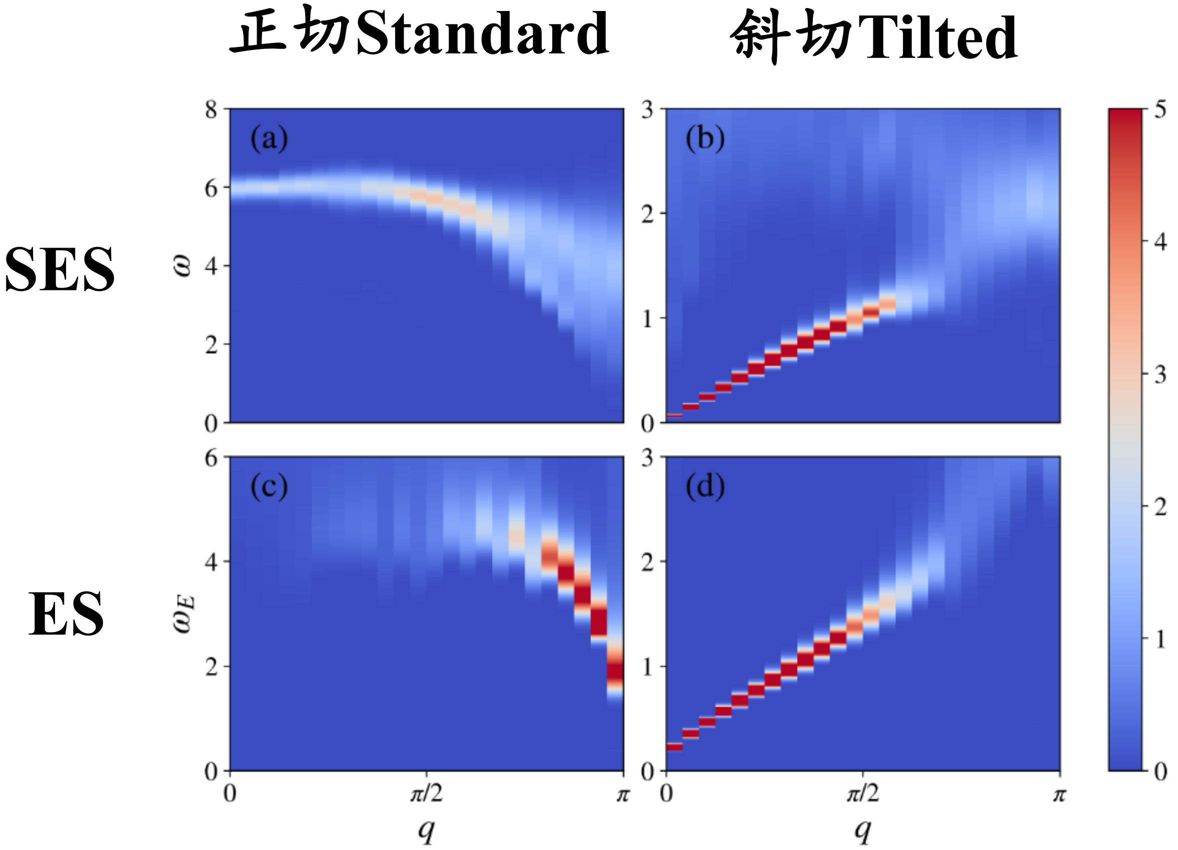
在斜切情况下，一个额外的序在临界点被引入了。我们推测此时边缘上发生了一个extraordinary相变。[In tilted cut, an additional order is induced at the critical point. We conject that an extraordinary phase transition occurs at the surface.]

研究内容和结论[Research content and conclusion]

纠缠切割与边缘临界性：
[Entanglement bipartition and surface criticality]

我们测试了Li-Haldane-Poilblanc 猜想对于JQ模型是否仍然成立：边缘激发谱与纠缠谱的对应关系。[We check the Li-Haldane-Poilblanc conjecture for JQ model – the correspondence between Surface Excitation Spectra (SES) and Entanglement Spectra (ES).]

定性上，我们可以认为猜想对该模型适用，并且两种切割方式会带来很大的不同。[Qualitatively, we can argue that the conjecture holds for this model, and that the two ways of cut show a big difference.]



研究内容和结论[Research content and conclusion]

纠缠切割与边缘临界性：
[Entanglement bipartition and surface criticality]

SCB class at 3D O(3) critical point [9–16]	$\eta_{\parallel}$	$\eta_{\perp}$
Spe.	-0.56(1)	-0.259(3)
Ord.	1.36(6)	0.69(3)
SCB class at 3D O(2) critical point[14, 17]	$\eta_{\parallel}$	$\eta_{\perp}$
Spe.	-0.35(1)	
Ord.	1.43(1)	0.69(2)
SCB class at 3D Ising critical point[17, 18]	$\eta_{\parallel}$	$\eta_{\perp}$
Spe.	-0.27(1)	
Ord.	1.52(1)	0.82(8)
SCB class at DQCP (obtained in this work)	$\eta_{\parallel}$	$\eta_{\perp}$
Ord.	2.36(2)	1.305(10)

我们还在正切下找到一组新的满足标度律的临界指数，我们认为此时发生了一个ordinary相变。[We also find a new set of critical exponents for standard cut satisfying the scaling law, which we argue is an ordinary phase transition.]

$$2\eta_{\perp} = \eta_{\parallel} + \eta \qquad \eta = 0.26(3)$$

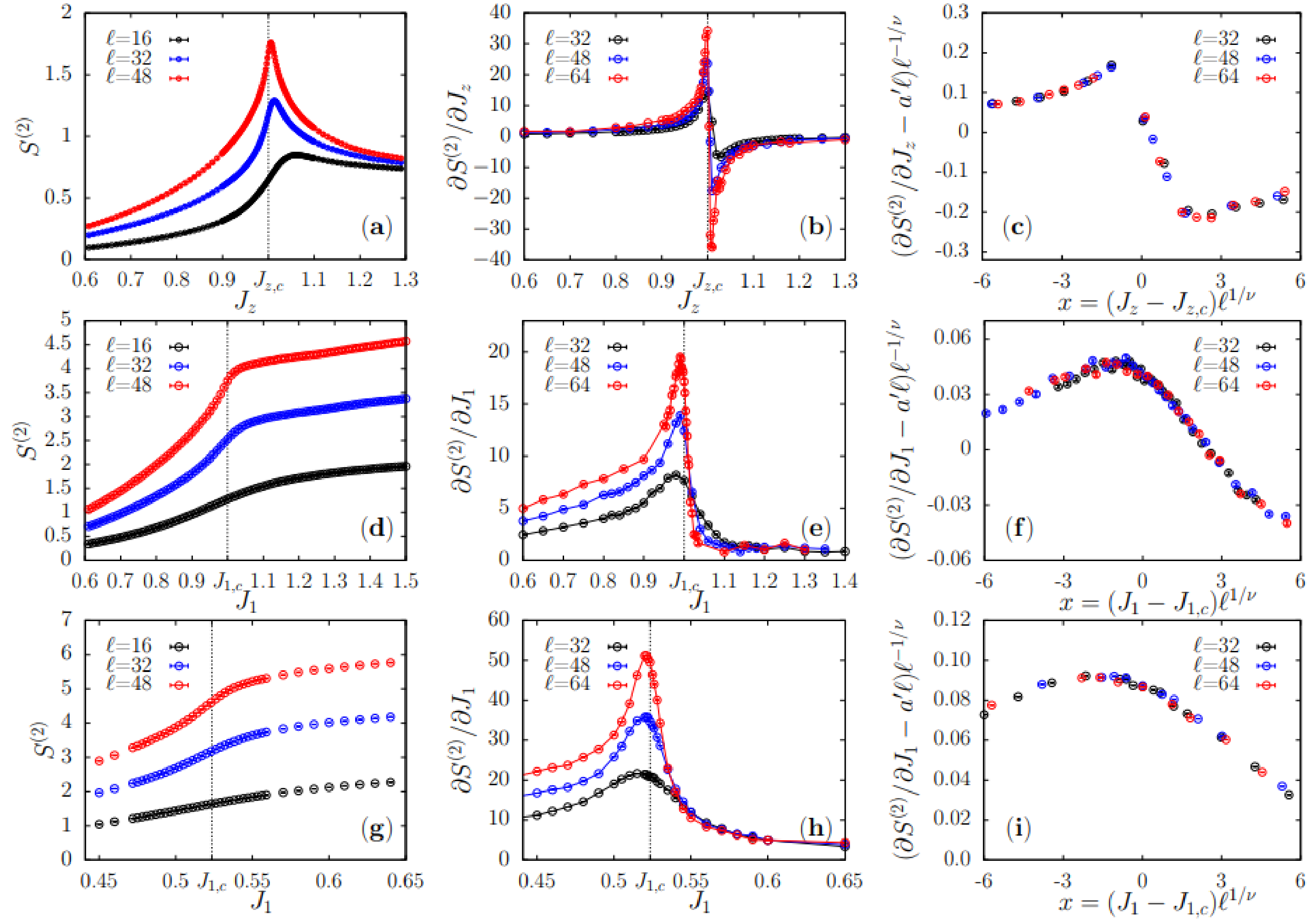
研究内容和结论[Research content and conclusion]

量子蒙卡计算纠缠熵和其导数：
[Entanglement entropy and its derivative by QMC]

$$\frac{dS^{(n)}}{dJ} = \frac{1}{1-n}\left[-n\beta \left\langle \frac{dH}{dJ} \right\rangle_{Z_A^{(n)}} + n\beta \left\langle \frac{dH}{dJ} \right\rangle_{Z} \right]$$

$$\frac{dS^{(2)}}{dJ_1} = \left[2\beta \left\langle \frac{H_{J_1}}{J_1} \right\rangle_{Z_A^{(2)}} - 2\beta \left\langle \frac{H_{J_1}}{J_1} \right\rangle_{Z} \right]$$

Wang, Wang, Ding, Mao and ***ZY***, ***Nat Commun*** 16, 5880 (2025).



$$\frac{\partial S(\ell, g)}{\partial g} = a'(g)\ell + \tilde{S}_0'(x)\ell^{1/\nu}.$$

Wang, Deng, Liu, Wang, Ding, Zhang, Guo, **ZY**
Chinese Physics Letters 42, 110712 (2025)

研究内容和结论[Research content and conclusion]

Universal and non-universal contributions of entanglement under different bipartitions

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(Dated: January 21, 2026)

Entanglement entropy (EE) is a fundamental probe of quantum phases and critical phenomena, which was thought to reflect only bulk universality for a long time. Very recently, people realized that the microscopic geometry of the entanglement cut can induce distinct entanglement-edge modes, whose coupling to bulk critical fluctuations may alter the scaling of the EE. However, this perception is very qualitative and lacks quantitative consideration. Here, we investigate this problem through high-precision quantum Monte Carlo simulations combined with the analysis of scaling theory to build a quantitative understanding. By considering three distinct bipartitions corresponding to three surface criticality types, we reveal a striking dependence of the constant term γ on the microscopic cut at the quantum critical point. Notably, cuts that generate extra gapless edge modes yield a sign reversal in γ compared to those producing gapped edges. We explain this behavior via a modified scaling form that incorporates contributions from both bulk and surface critical modes. Furthermore, we demonstrate that the derivative of EE robustly extracts the bulk critical point and exponent ν regardless of the cut geometry, providing a reliable diagnostic of bulk universality in the presence of strong surface effects. Our work for the first time establishes a direct quantitative connection between surface criticality and entanglement scaling, challenging the conventional view that EE solely reflects bulk properties and offering a refined framework for interpreting entanglement in systems with boundary-sensitive criticality.

arXiv:2601.12365

数值结果[Numerical results]

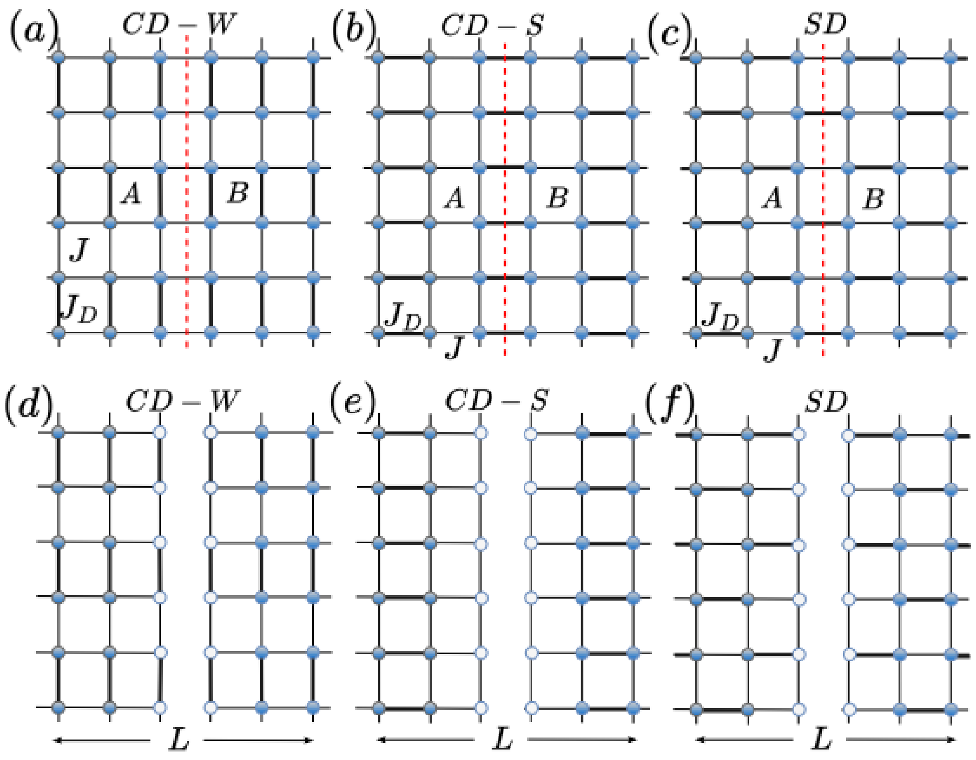

FIG. 1. Columnar and staggered dimerized spin-1/2 Heisenberg models on the square lattice, with couplings J and $J_D > J$ on alternating bonds. (a,d) Entanglement/physical boundary cuts weak bonds (CD-W). (b,e) Cuts strong bonds (CD-S). (c,f) Cuts staggered bonds in the staggered model (SD). The system size is $L \times L$; subsystem A is a $(L/2) \times L$ cylinder, separated from environment B by dashed lines. Physical boundaries (open circles) denote true open edges.

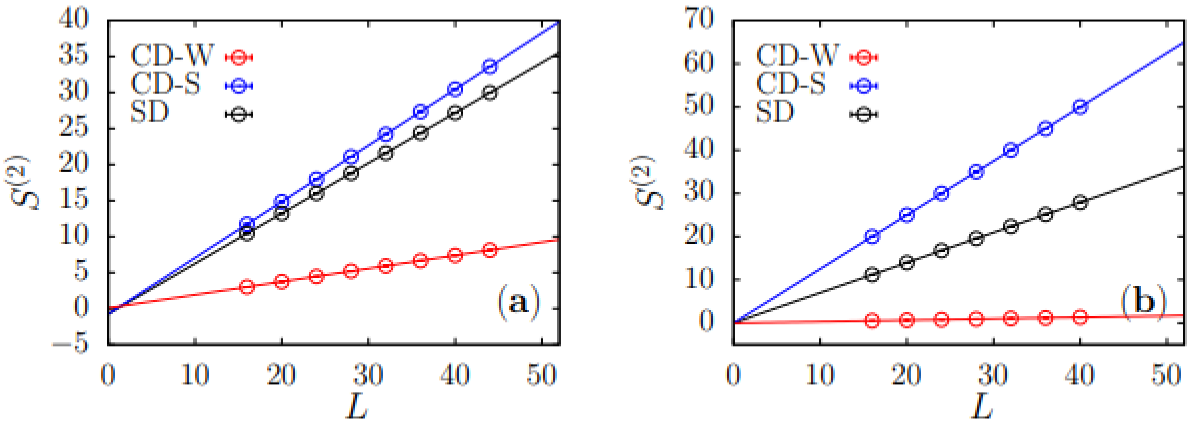

FIG. 2. Second Rényi EE $S^{(2)}$ under three bipartition schemes versus system size L, at the QCP J_c (a) or in the gapped dimer phase at $J_c/2$ (b). The fitting results are listed in Table I.

TABLE I. Fitting results for the data in Fig. 2(a) ($J = J_c$) and Fig. 2(b) ($J = J_c/2$), using the form $S^{(2)}(L) = aL + \gamma$.

Cuts	$a(J = J_c)$	$\gamma(J = J_c)$	$a(J = J_c/2)$	$\gamma(J = J_c/2)$
CD-W	0.183(1)	0.069(4)	0.033(1)	0.000(4)
CD-S	0.781(1)	-0.764(9)	1.2498(2)	0.000(3)
SD	0.698(1)	-0.739(7)	0.6977(2)	0.001(2)

数值结果[Numerical results]

$$S(L,g) = a(g)\,L + \tilde{S}_0(x), \quad x = (g-g_c)\,L^{1/\nu}$$

$$\tilde{S}_0(x) \;\simeq\; \gamma\,|g - g_c|^{\nu} L$$

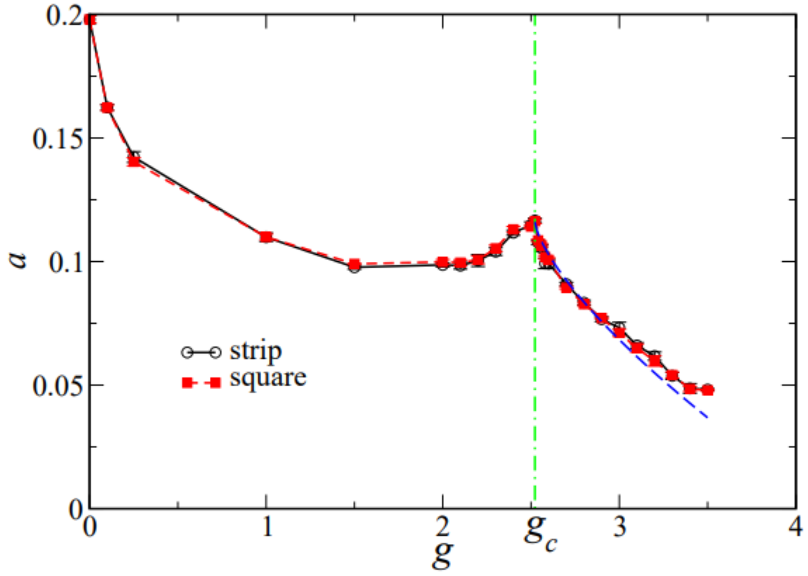

FIG. 5. (Color online) Area law prefactor a of the second Rényi entropy S_2 as a function of the coupling ratio g for strip and square shaped subregions. The dashed line is a fit to $a - a(g_c) = r\xi^{D-1} \propto |g-g_c|^{\nu}$ with the correlation length exponent $\nu = 0.71$.

Johannes Helmes and Stefan Wessel
Phys. Rev. B **89**, 245120 (2014)

$$S = C\frac{L^{d-1}}{a^{d-1}} + r\frac{L^{d-1}}{\xi^{d-1}}$$

Max A. Metlitski, Carlos A. Fuertes, and Subir Sachdev
Phys. Rev. B **80**, 115122 (2009)

Edge cases:

$$S(L,g) = a(g)\,L + \tilde{S}_b(x) - \tilde{S}_s(x),$$

$$S = a(g)\frac{L^{d-1}}{a^{d-1}} + (\gamma_b - \gamma_s)\frac{L^{d-1}}{\xi^{d-1}}.$$

$$\frac{\partial S(L,g)}{\partial g} = a'(g)L + \tilde{S}_0'(x)L^{1/\nu},$$

数值结果[Numerical results]

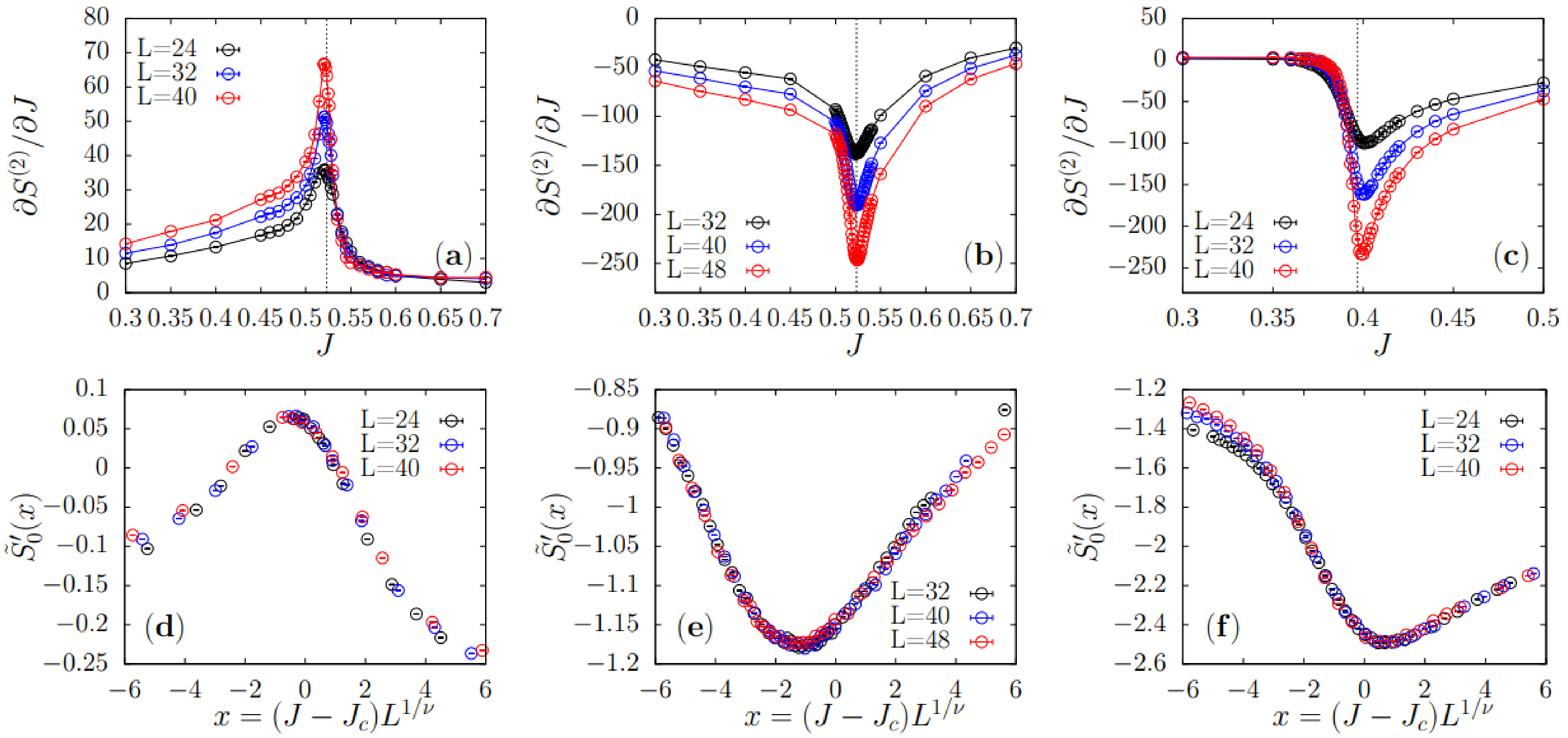

FIG. 3. The derivative of the second Rényi EE and the scaling function $\tilde{S}_0'(x)$ obtained from the data collapse analysis of the DEE near the transition. (a,d) Data of the columnar model with entanglement boundary cuts weak bonds (CD-W). (b,e) Data of the columnar model with entanglement boundary cuts strong bonds (CD-S). For columnar model $J_D = 1.0$ is fixed, while J is tuned around the QCP $J_c = 0.52337(3)$. (c,f) Data of the staggered model with entanglement boundary cuts staggered bonds (SD). For staggered model, $J_D = 1.0$ is fixed, while J is tuned around the QCP $J_c = 0.39692(1)$.

谢谢！Thanks!