Exotic metals,
fractionalization, and quantum criticality

Matthias Vojta
TU Dresden

Colloquium @ HKU, Dec 2025

Exotic metals, fractionalization, and quantum criticality

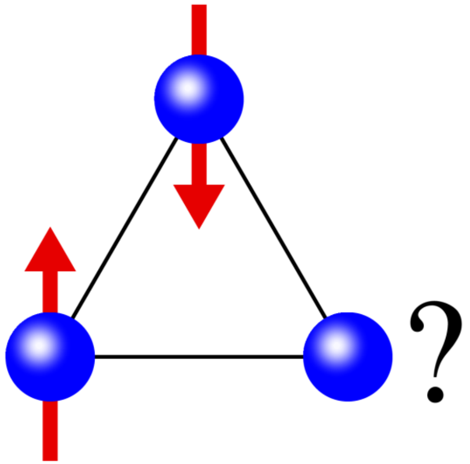
Tom Drechsler, Chi Chen (TU Dresden)
Matthew Bunney, Stephan Rachel (Melbourne)
Urban Seifert, Heike Eisenlohr (Cologne)
Seung-Sup Lee (Seoul)

1. Frustration & fractionalization: From insulators to metals
2. Fractionalized Fermi liquids and superconductors
3. Weakly doped Mott insulators: Cuprates and beyond
4. Mott criticality

TECHNISCHE UNIVERSITÄT DRESDEN

Deutsche Forschungsgemeinschaft DFG

Frustration



Frustration tends to suppress conventional magnetic order, promotes novel forms of order and disorder

The birth of spin liquids

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

(Received December 5, 1972; Invited**)

ABSTRACT

The possibility of a new kind of electronic state is pointed out, corresponding roughly to Pauling's idea of "resonating valence bonds" in metals. As observed by Pauling, a pure state of this type would be insulating; it would represent an alternative state to the Néel antiferromagnetic state for S = 1/2. An estimate of its energy is made in one case.

Anderson, Mater Res Bull **8**, 153 (1973)

On the ground state properties of the anisotropic triangular antiferromagnet

By P. Fazekas† and P. W. Anderson‡
Cavendish Laboratory, Cambridge, England

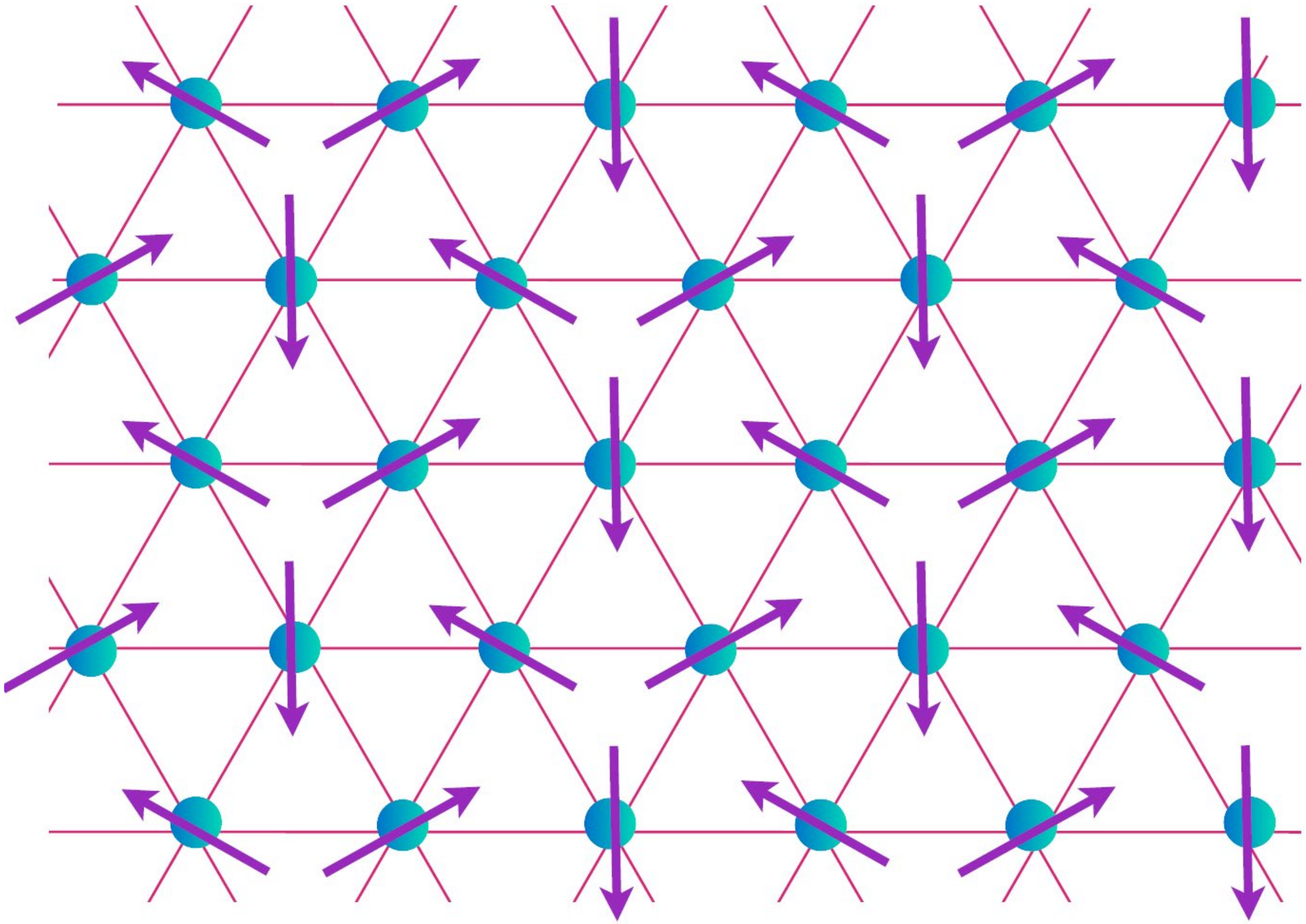
[Received 24 May 1974]

Abstract

Our aim is to present further evidence supporting a recent suggestion by Anderson (1973) that the ground state of the triangular antiferromagnet is different from the conventional three-sublattice Néel state. The anisotropic Heisenberg model is investigated. Near the Ising limit a peculiar, possibly liquid-like state is found to be energetically more favourable than the Néel-state. It seems to be probable that this type of ground state prevails in the anisotropy region between the Ising model and the isotropic Heisenberg model. The implications for the applicability of the resonating valence bond picture to the $S=\frac{1}{2}$ antiferromagnets are also discussed.

Fazekas /Anderson, Phil Mag **30**, 23 (1974)

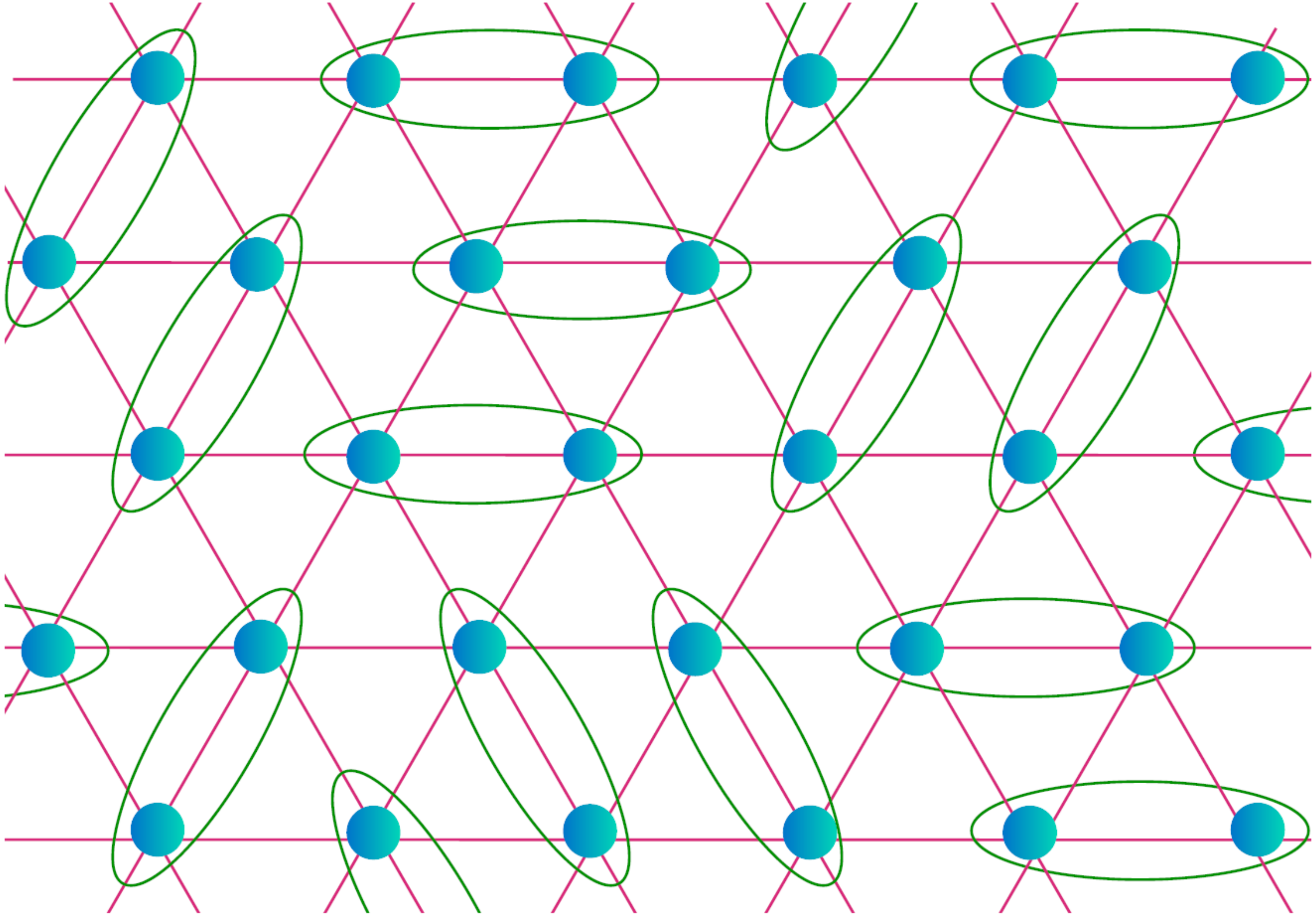
Triangular-lattice Heisenberg antiferromagnet



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Non-collinear 120° order
of nearest-neighbor model

Spin liquids: Resonating valence bonds

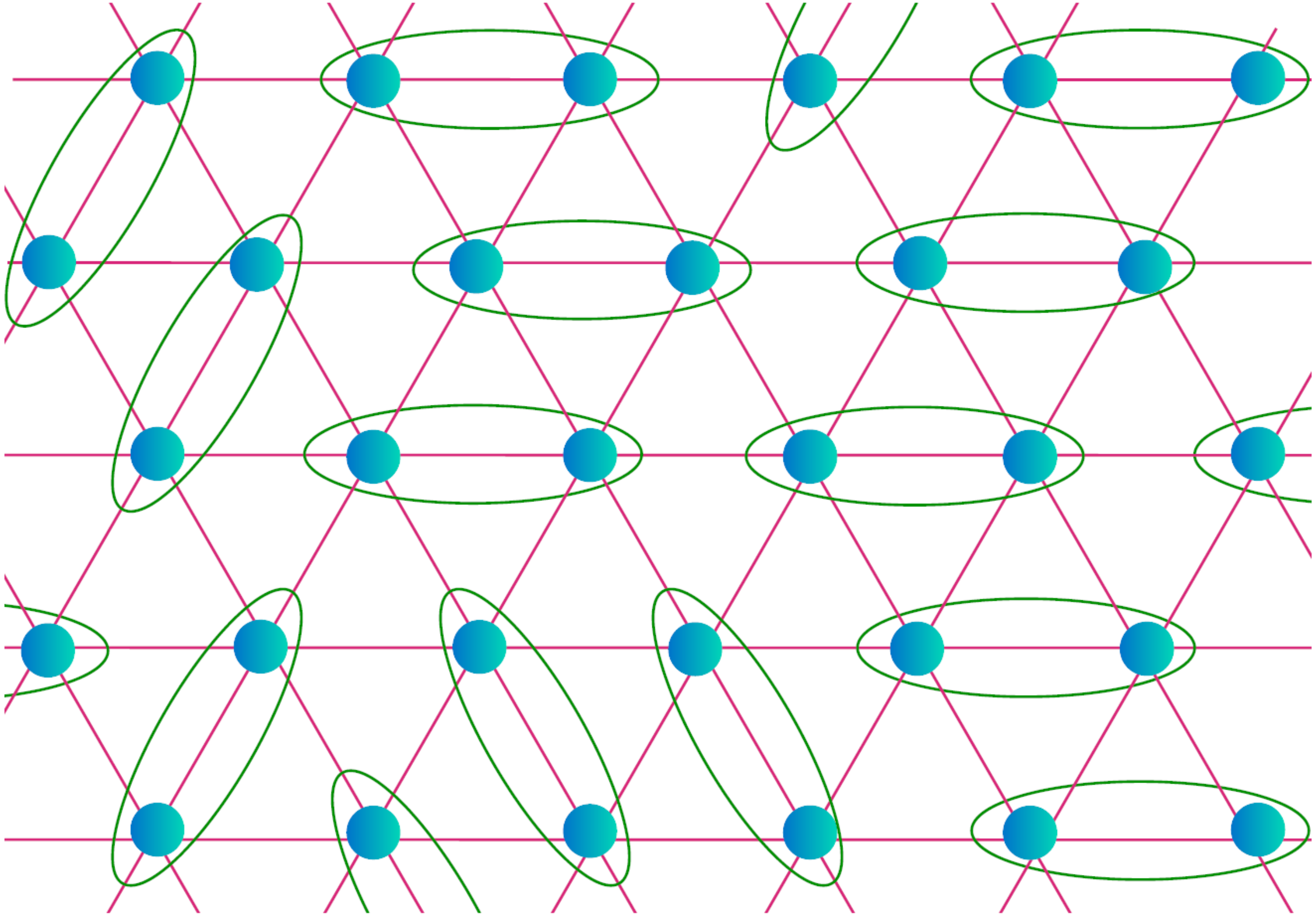


 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

Spin liquids: Resonating valence bonds



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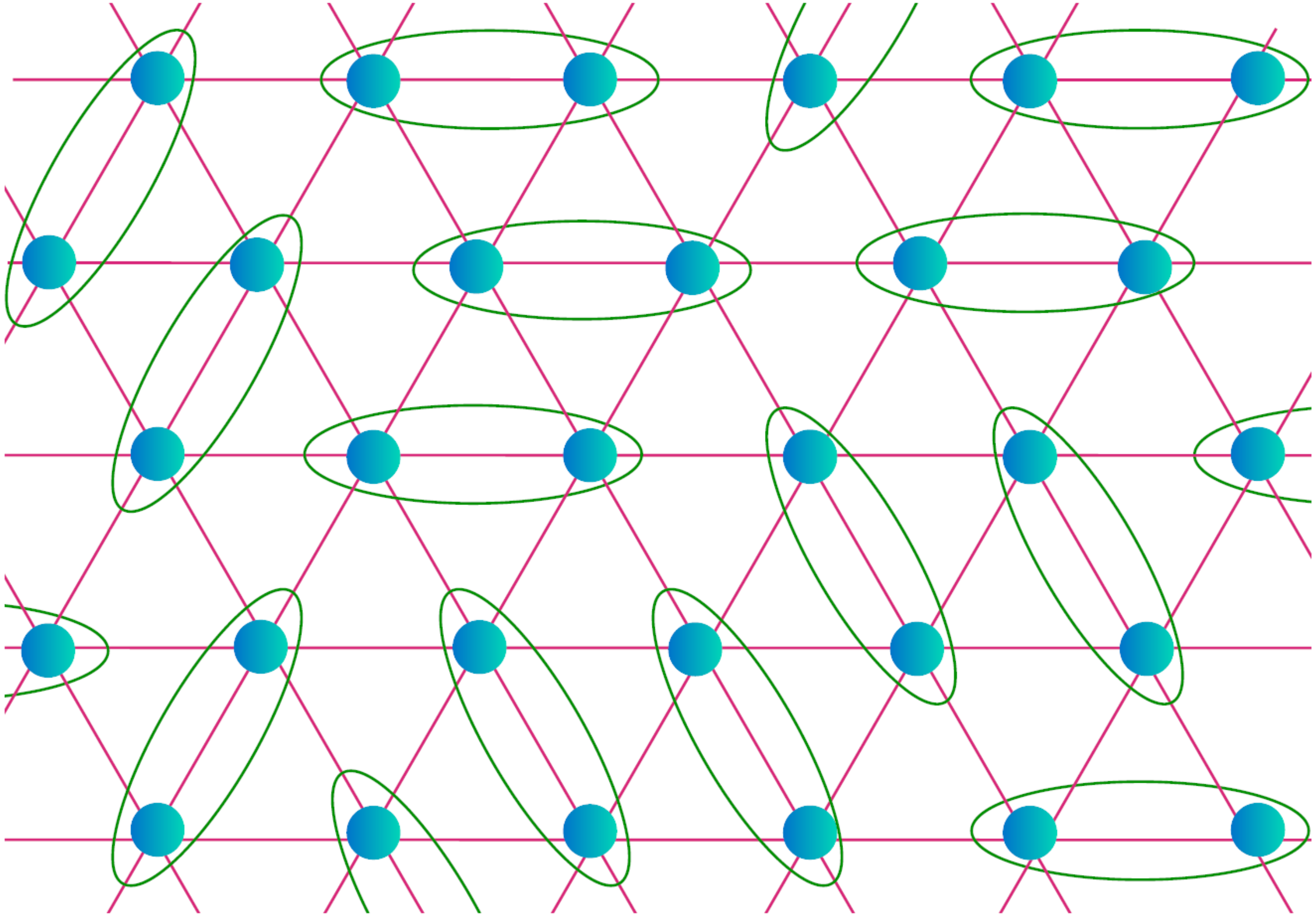

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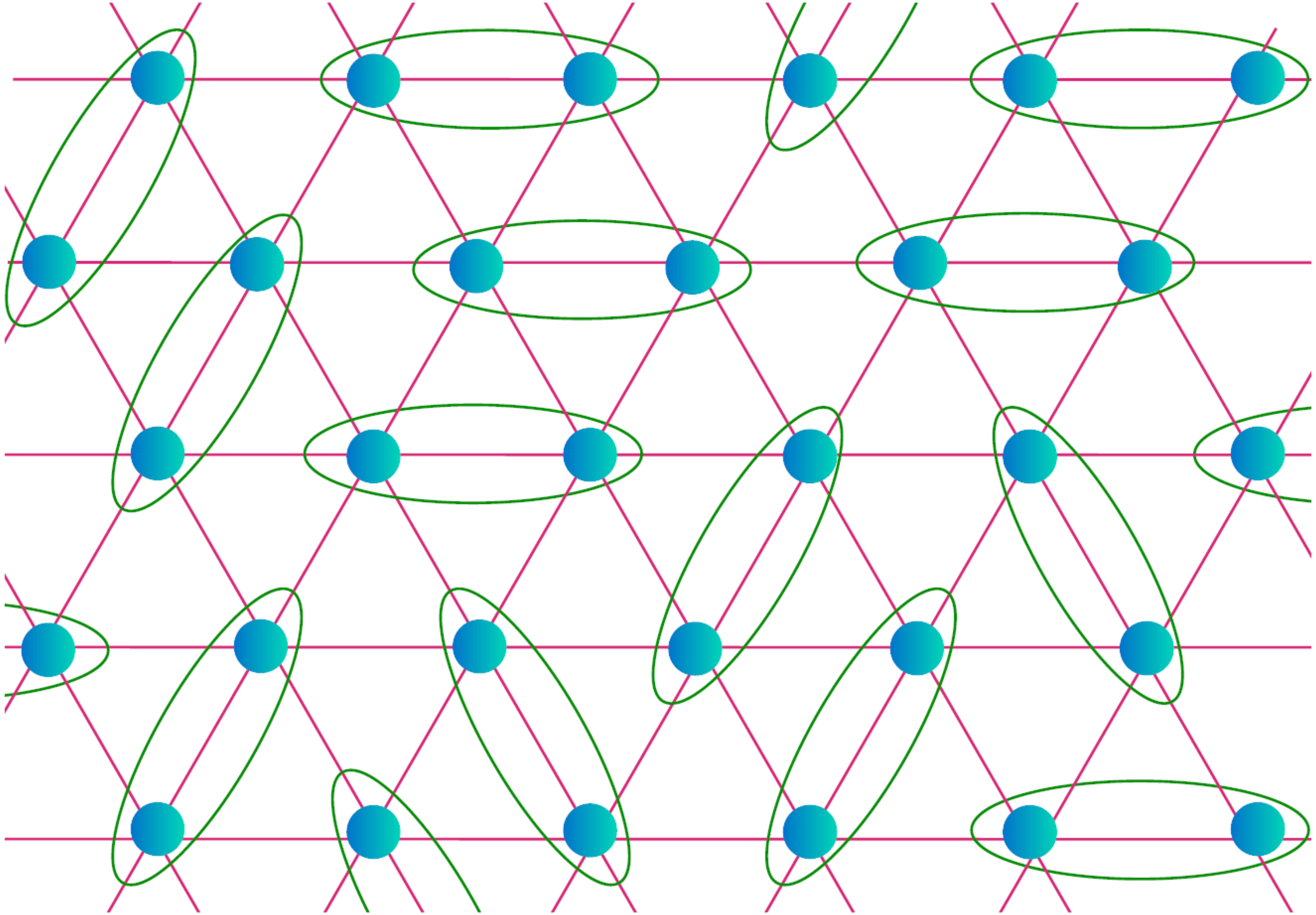

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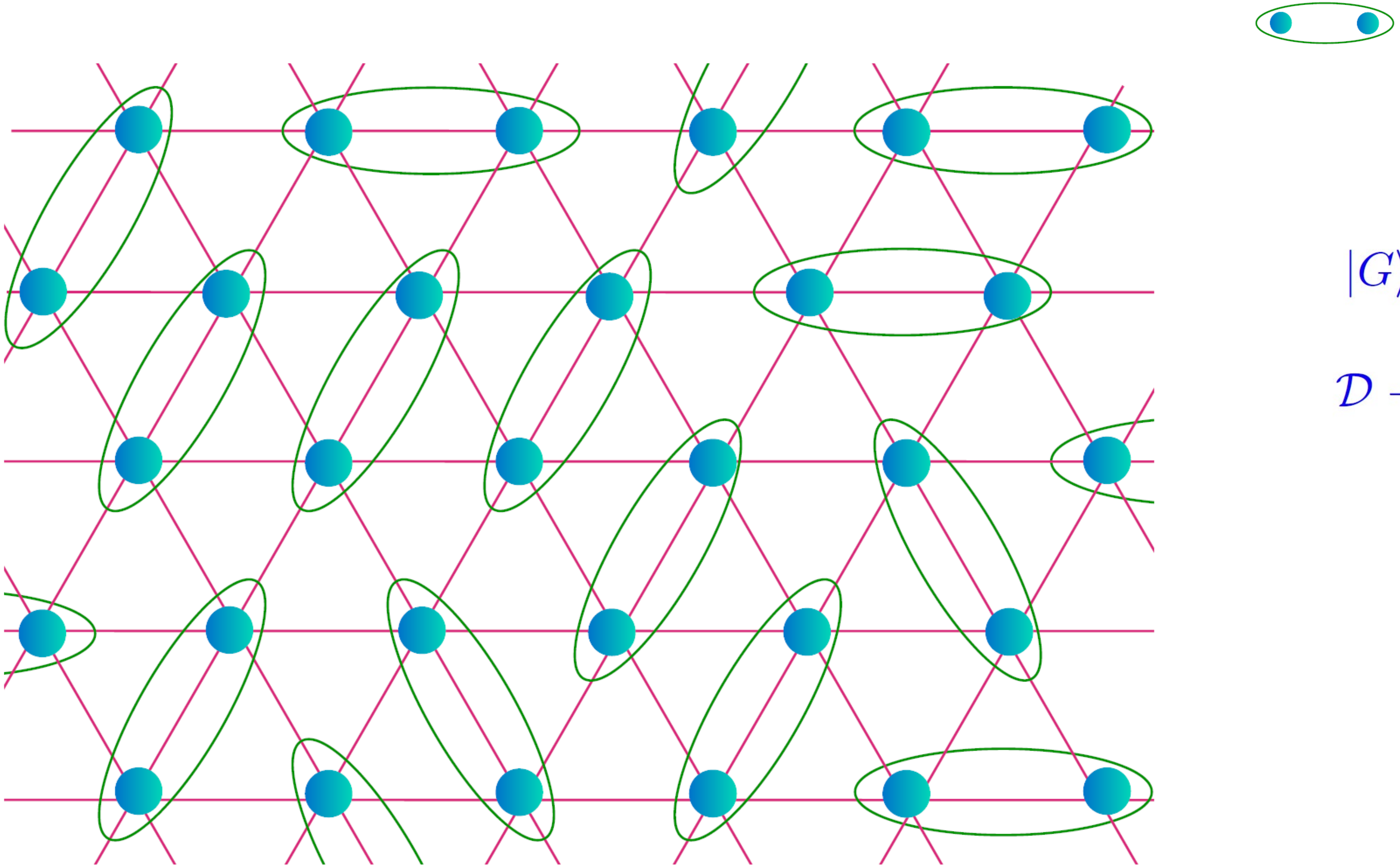
Spin liquids: Resonating valence bonds



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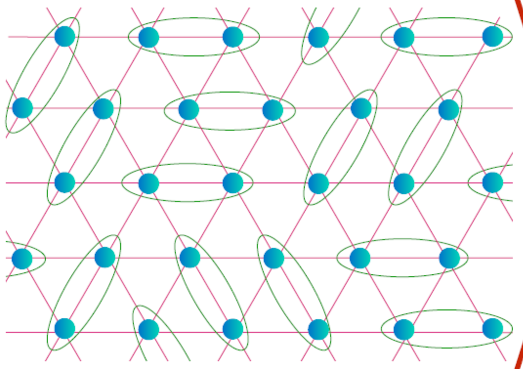
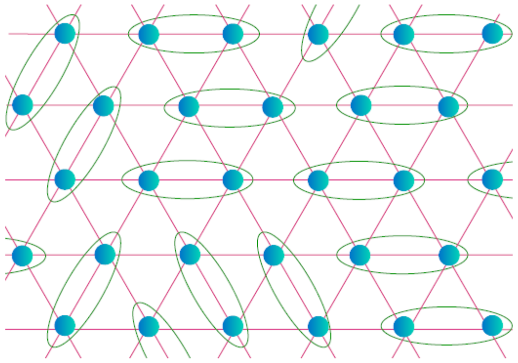
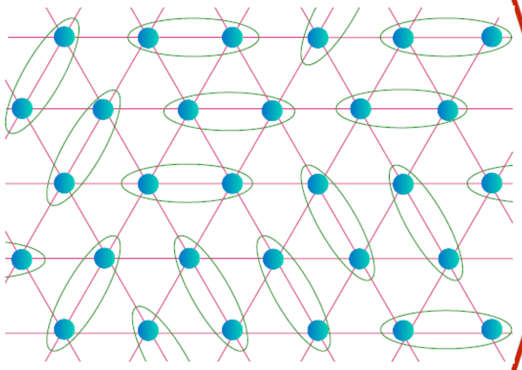
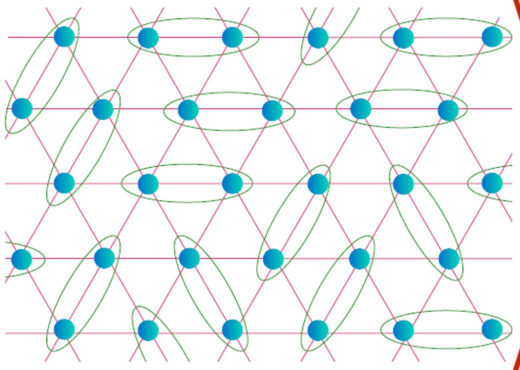
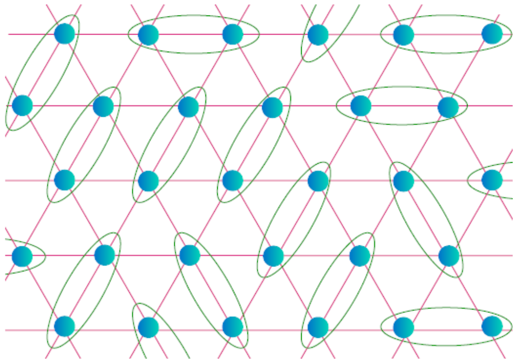
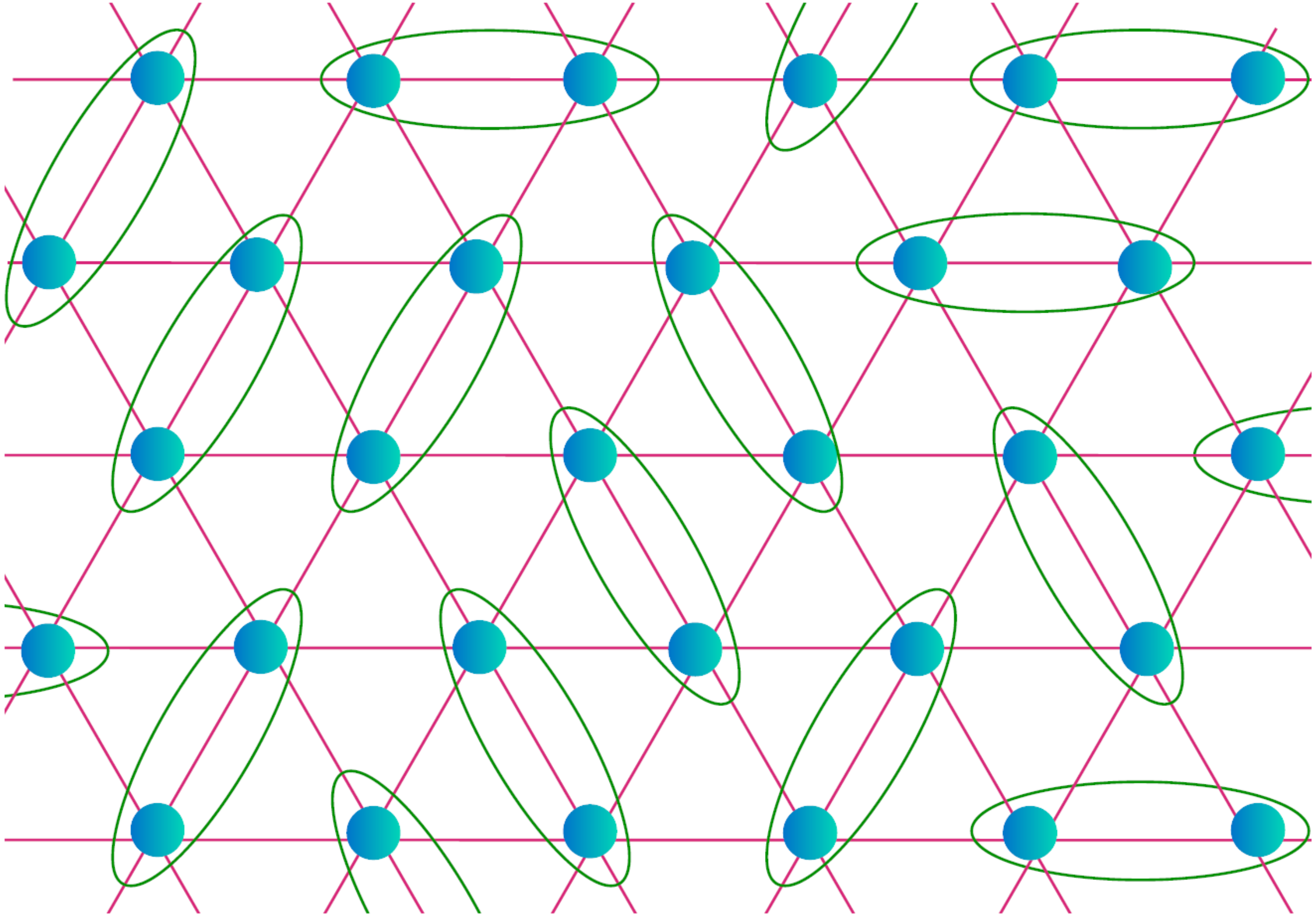
Spin liquids: Resonating valence bonds

$|\Psi\rangle = C_1 \Big|$  $\Big\rangle + C_2 \Big|$  $\Big\rangle$

$+ C_3 \Big|$  $\Big\rangle + C_4 \Big|$  $\Big\rangle + C_5 \Big|$  $\Big\rangle + \ldots$

Fractionalized spinon excitations



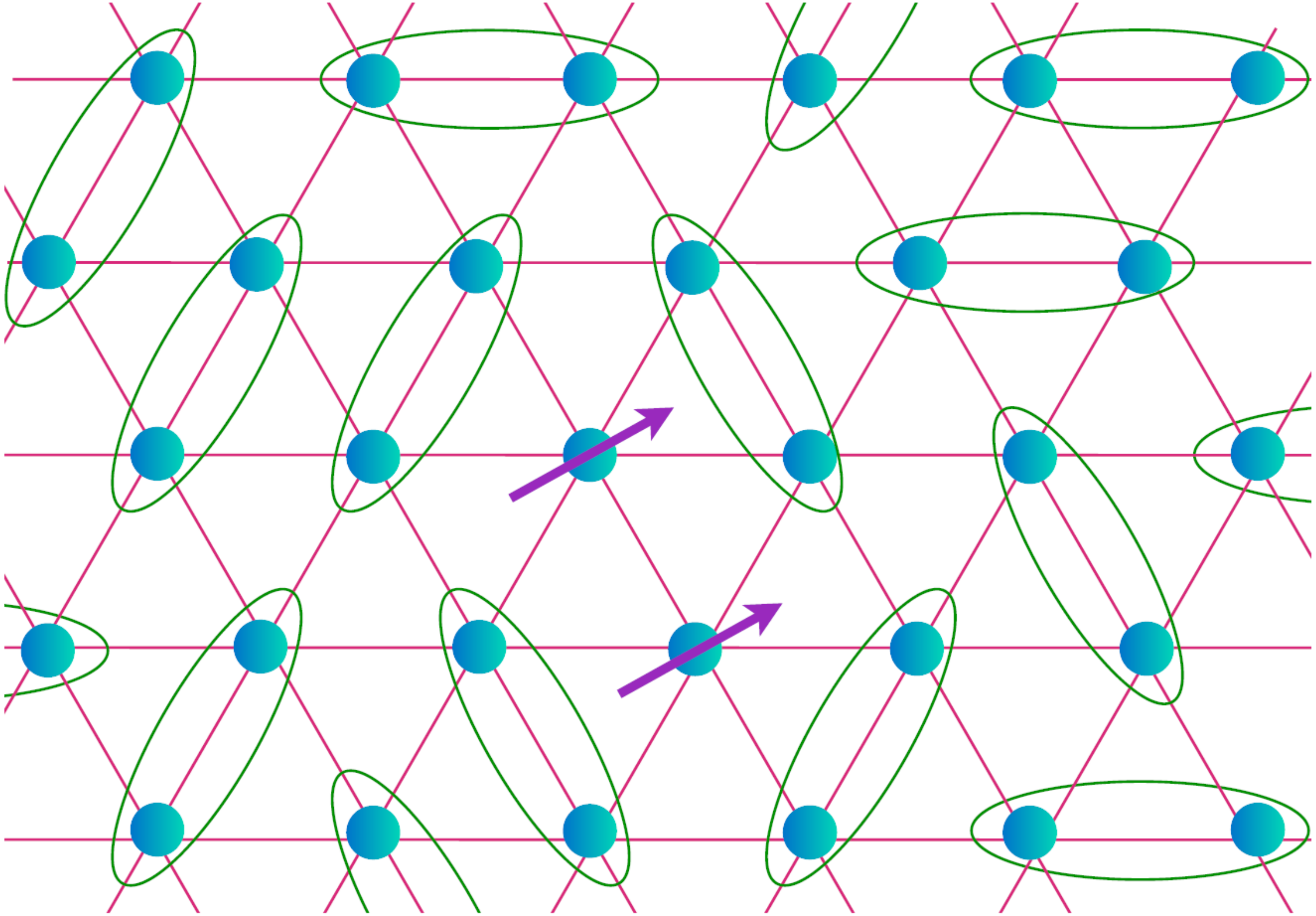
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Fractionalization
Excitations are particle-like,
but single excitation cannot
be created by local operators

Here:
Spin-1/2 spinons

Fractionalized spinon excitations

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



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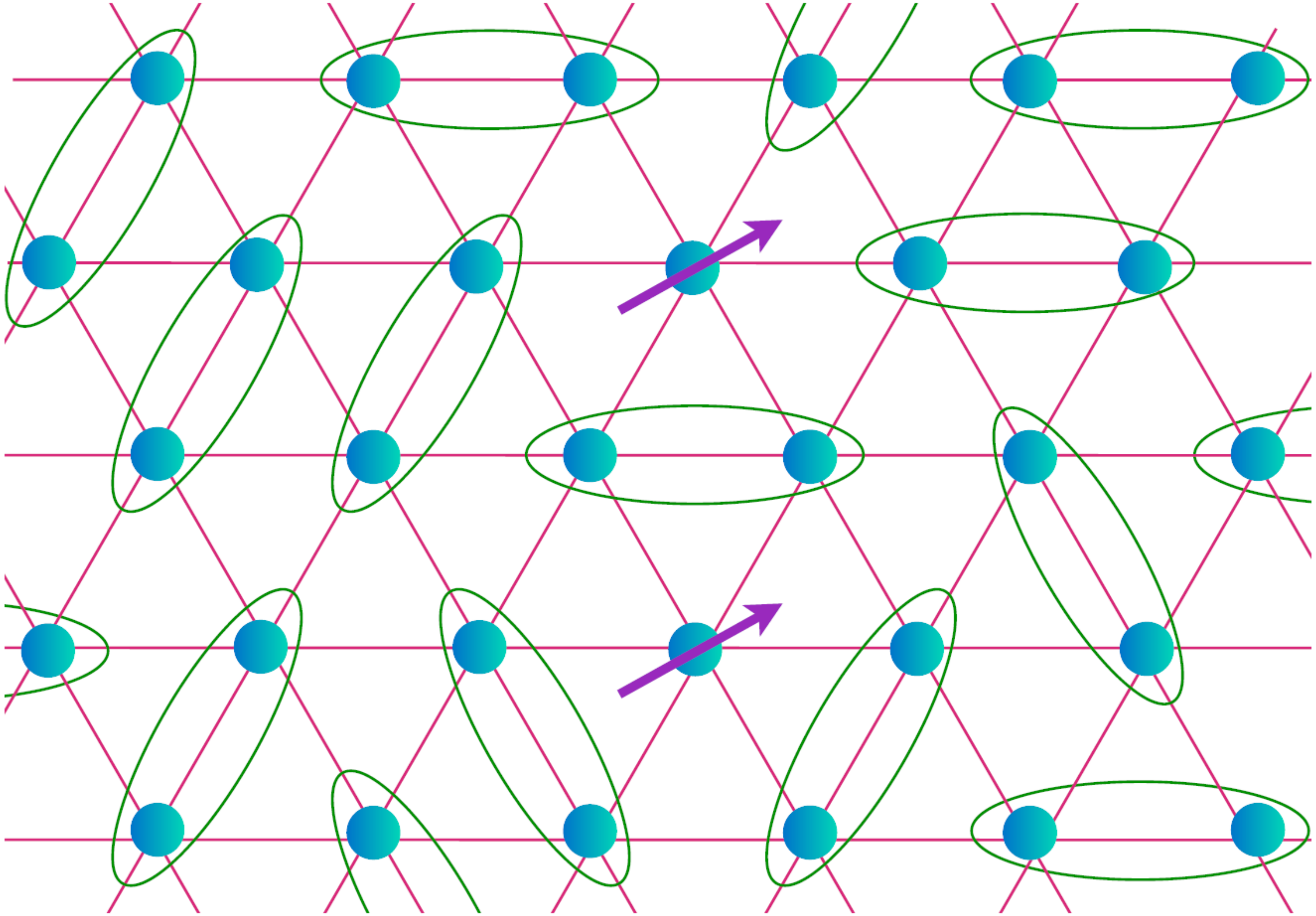

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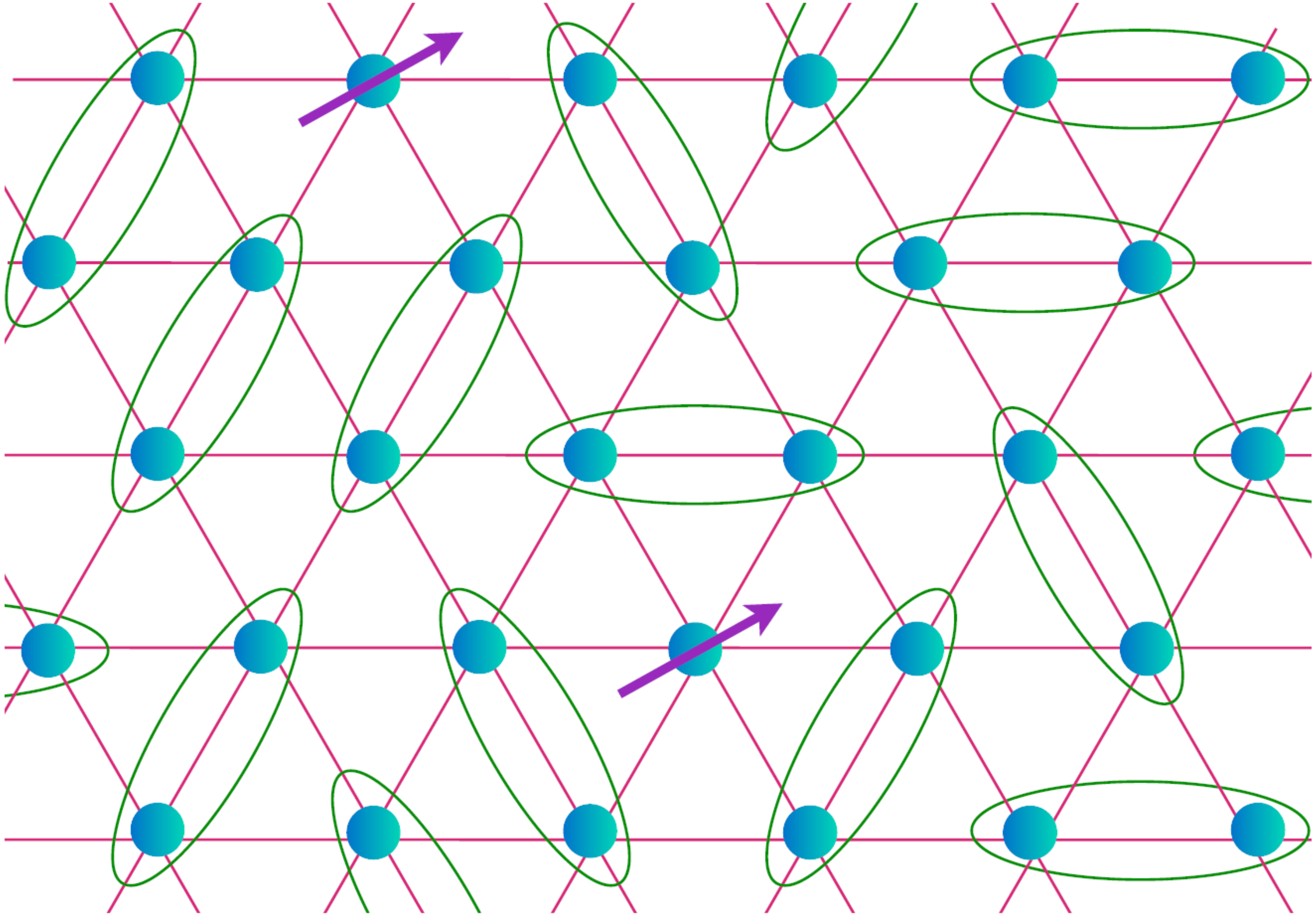

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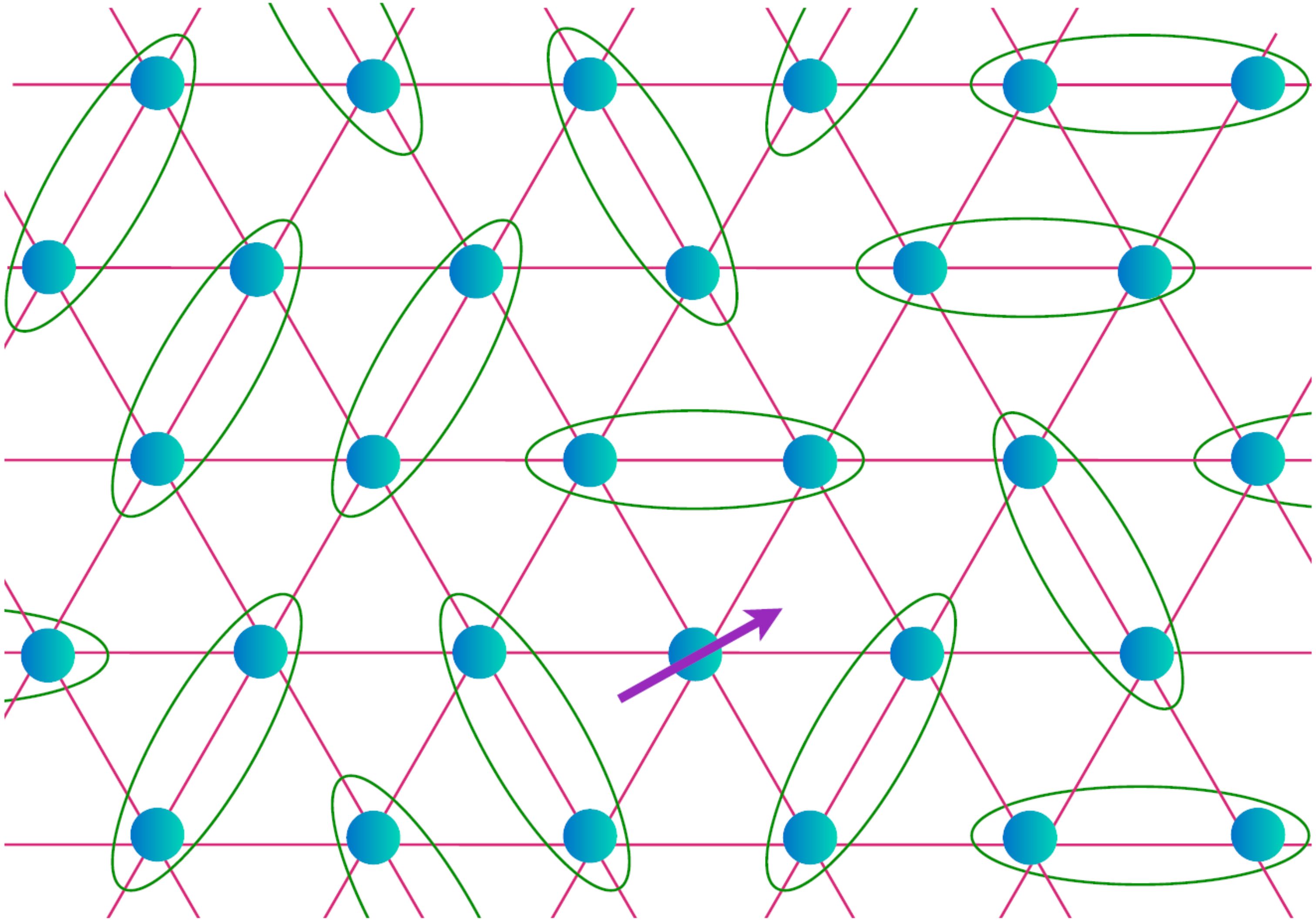

Fractionalization
Excitations are particle-like,
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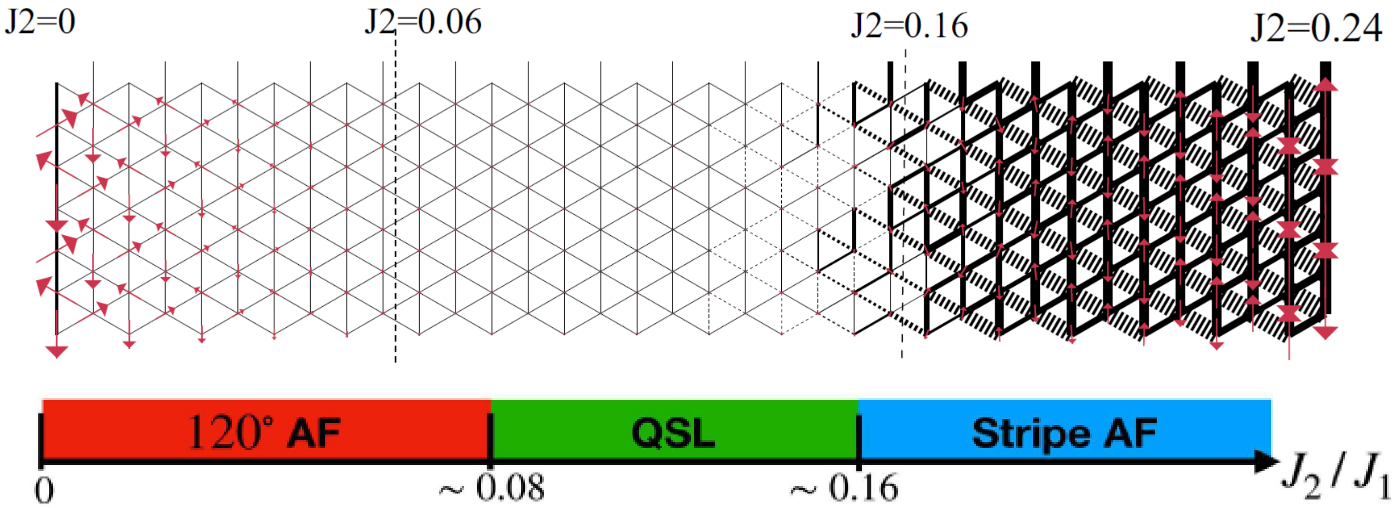
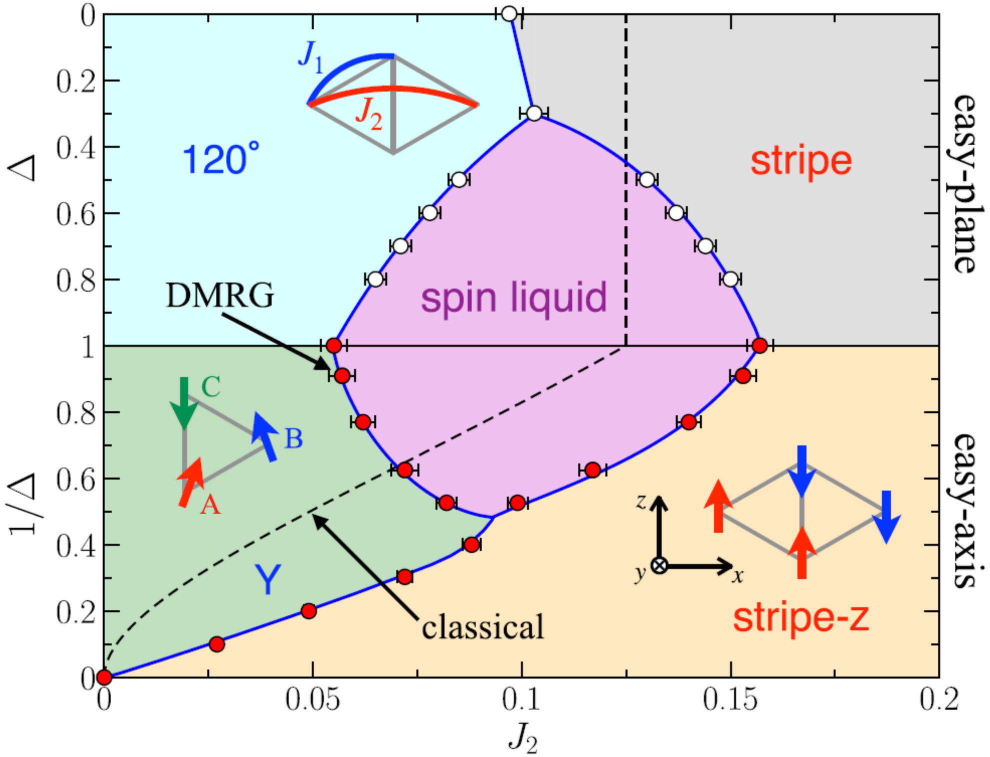
Here:
Spin-1/2 spinons

J_1-J_2 triangular-lattice Heisenberg model

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j$$



Zhu/White, PRB **92**, 041105(R) (2015)
Iqbal *et al*, PRB **93**, 144411 (2016)

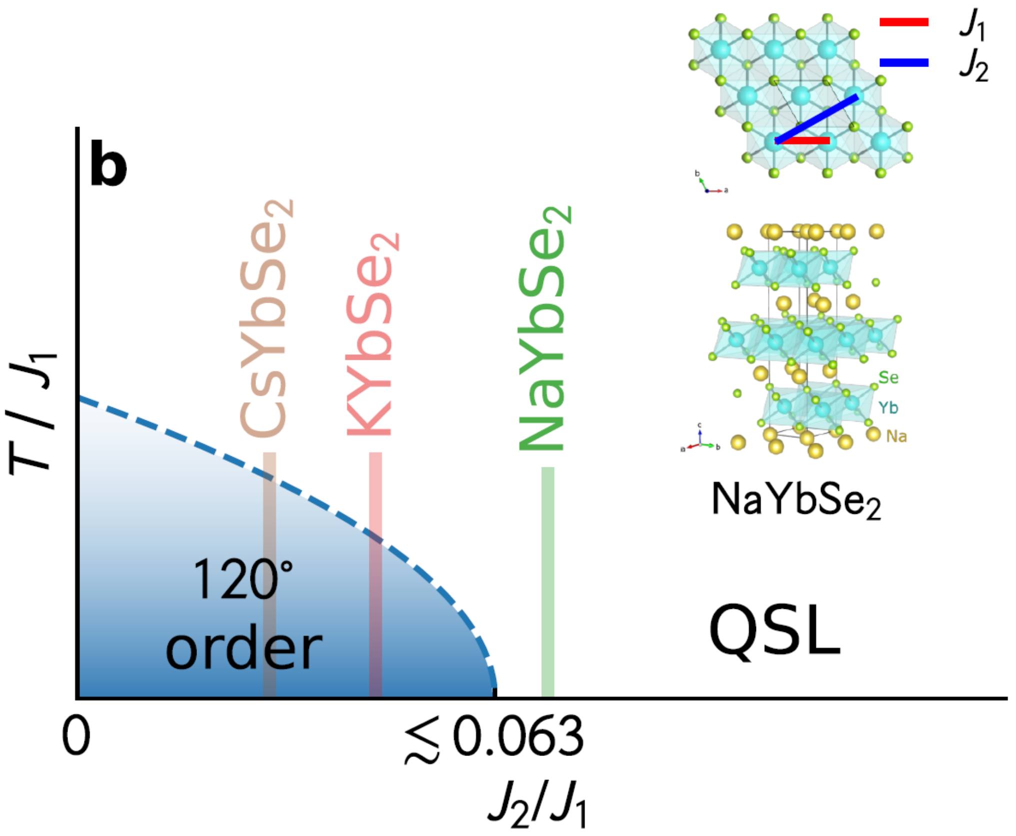


XXZ model with easy-axis/easy-plane anisotropy

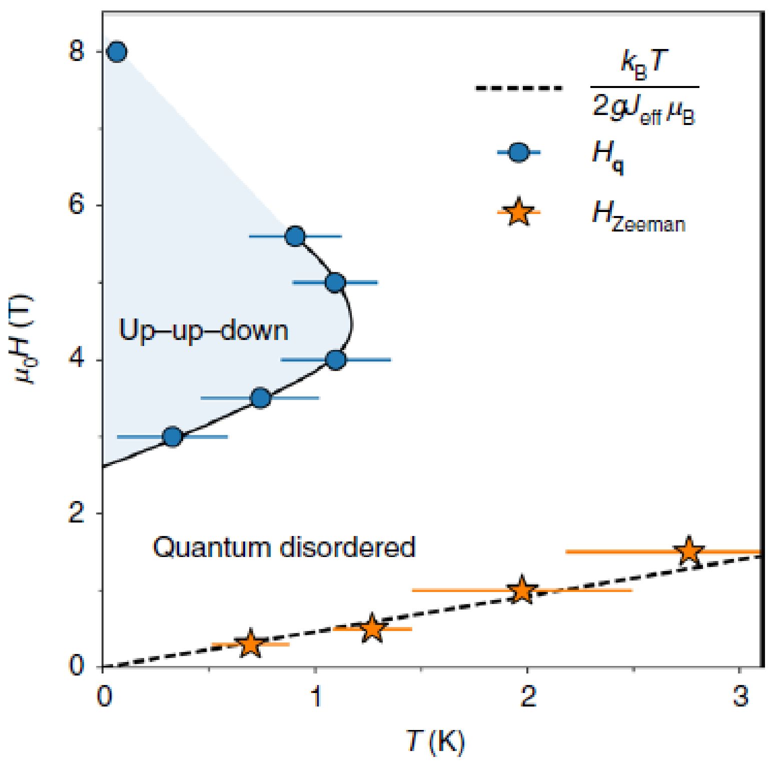
Gallegos *et al*, PRL **134**, 196702 (2025)

Realization of triangular-lattice quantum spin liquids

Rare-earth magnets $A\mathrm{YbSe_2}$, $A\mathrm{YbS_2}$, $A\mathrm{YbO_2}$



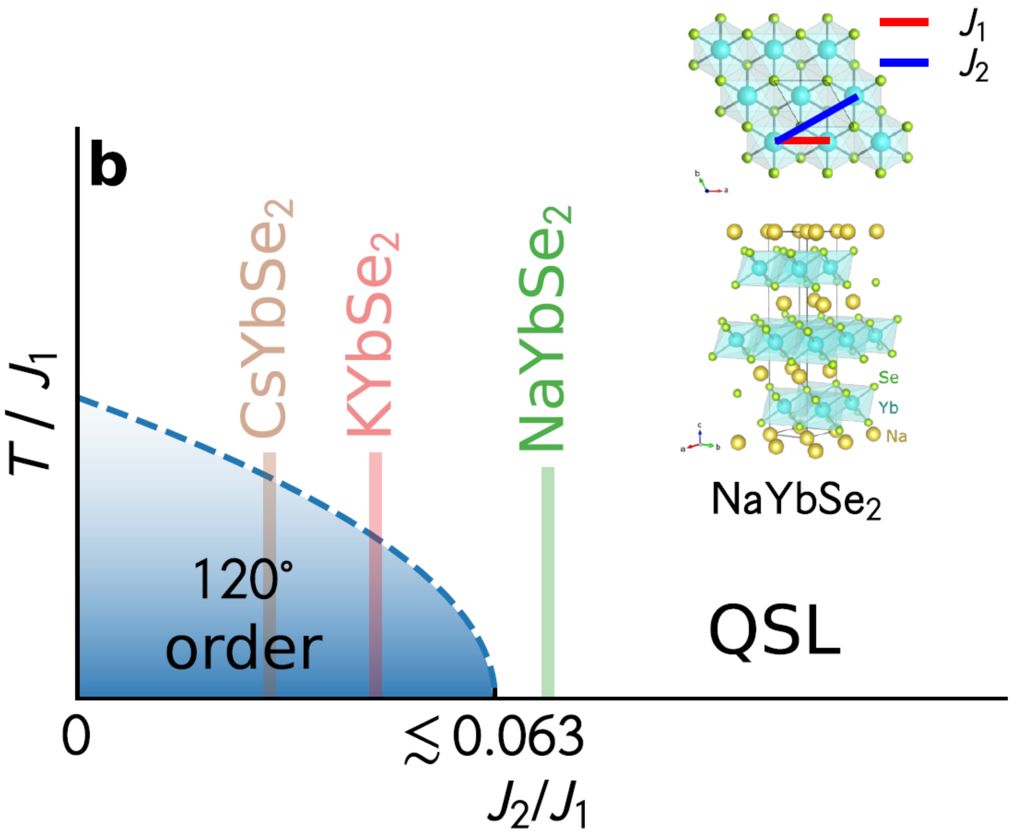
No zero-field order down to lowest T in $\mathrm{NaYbSe_2}$, $\mathrm{NaYbS_2}$, $\mathrm{NaYbO_2}$



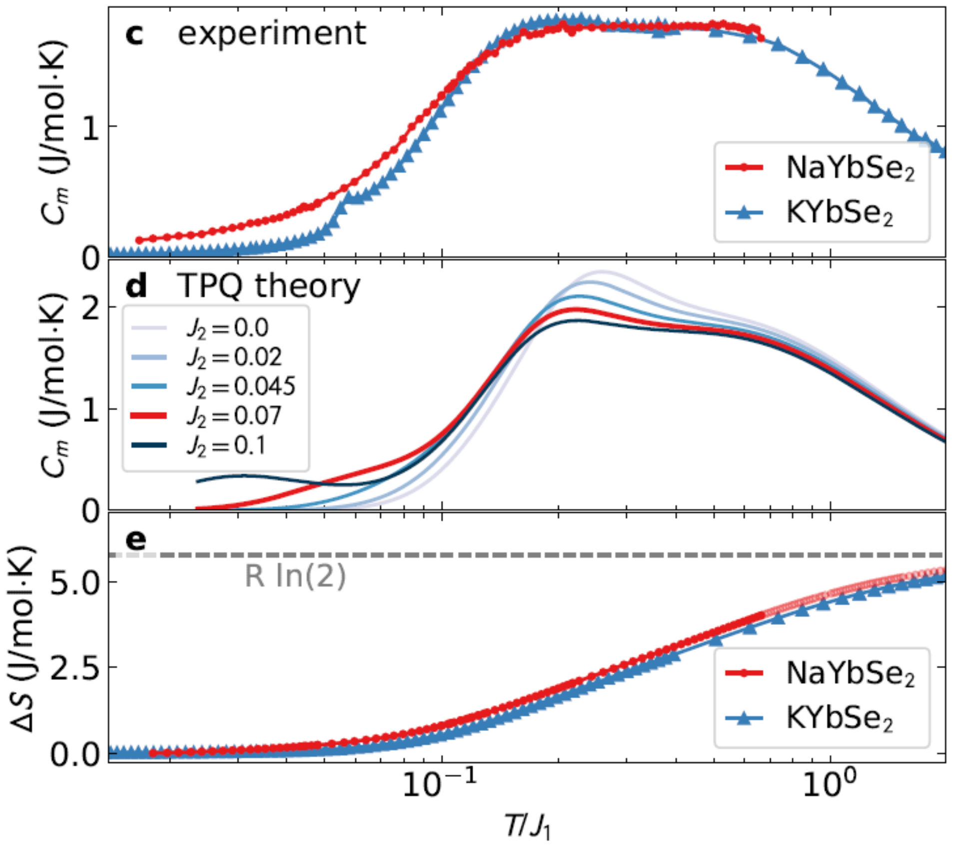
Bordelon *et al*, Nat Phys **15**, 1058 (2019)
Sarkar *et al*, PRB **99**, 241116(R) (2019)
Ding *et al*, PRB **100**, 144432 (2019)

Realization of triangular-lattice quantum spin liquids

Rare-earth magnets AYbSe$_2$, AYbS$_2$, AYbO$_2$



NaYbSe$_2$ appears well described by J_1-J_2 Heisenberg model with J_1=0.55 meV, J_2/J_1=0.07



Scheie/Tennant *et al*, arXiv:2406.17773

Realization of triangular-lattice quantum spin liquids

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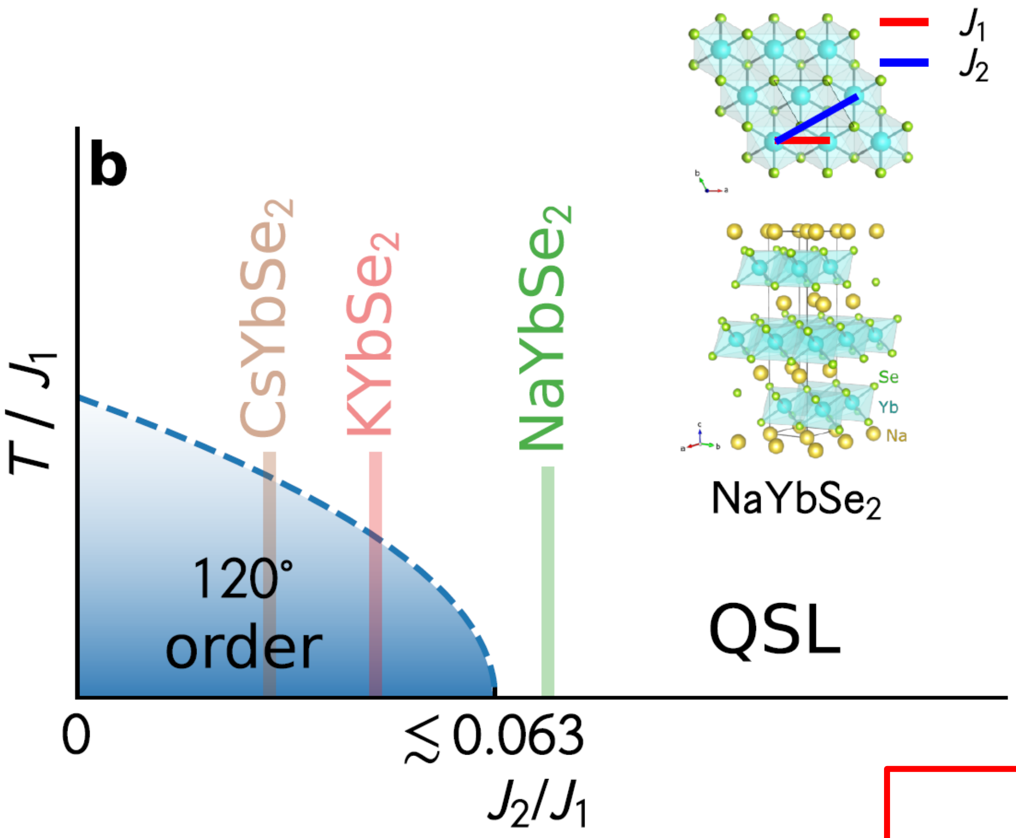

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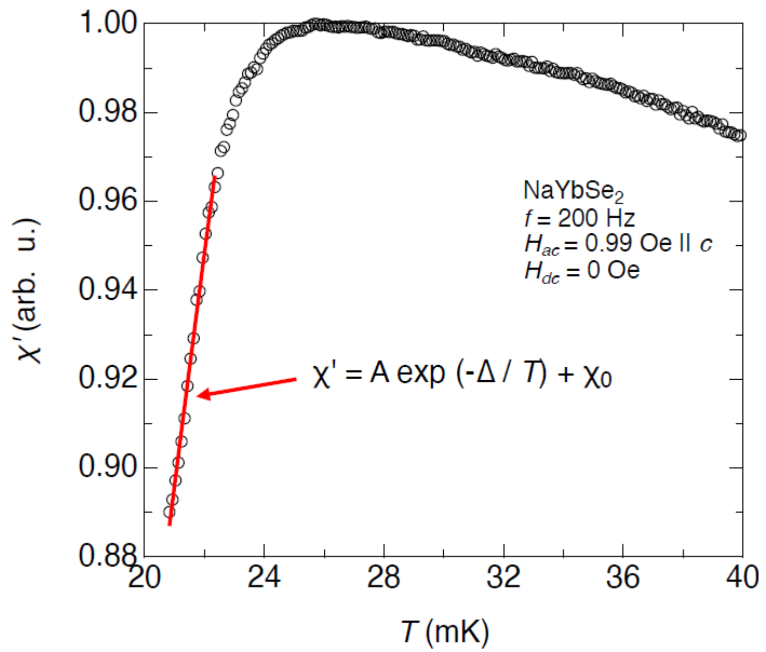


No order down to 20 mK,
Estimated spin gap 2.1 μeV

NaYbSe$_2$ realizes gapped Z_2 (RVB) spin liquid (?)

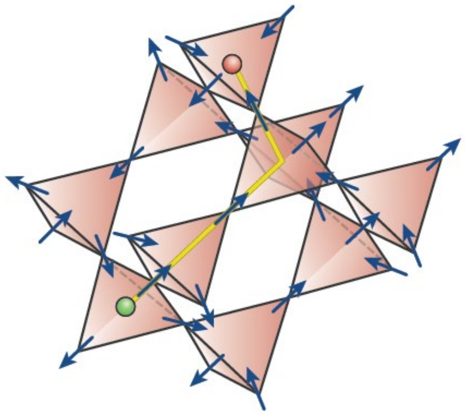
Scheie/Tennant *et al*, arXiv:2406.17773

Spin liquids (local-moment states w/o symmetry breaking order as $T \rightarrow 0$)

classical

- massively degenerate ground states, extensive residual entropy
  → sensitive to small perturbations
- emergent static gauge theory

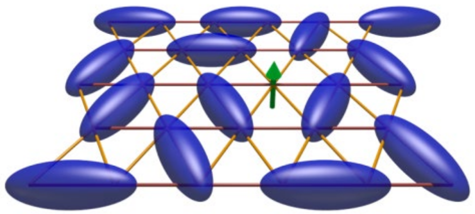
Examples:
- triangular-lattice Ising model
- spin ice



quantum

- small degeneracies (topological)
  → stable against small perturbations
- Long-range entanglement
- emergent dynamic gauge theory

Examples:
- resonating-valence bond state
- Kitaev honeycomb model

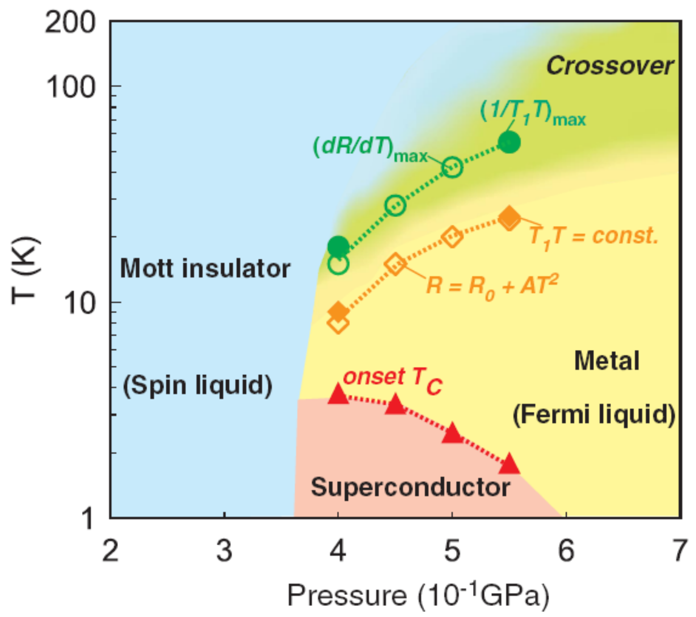


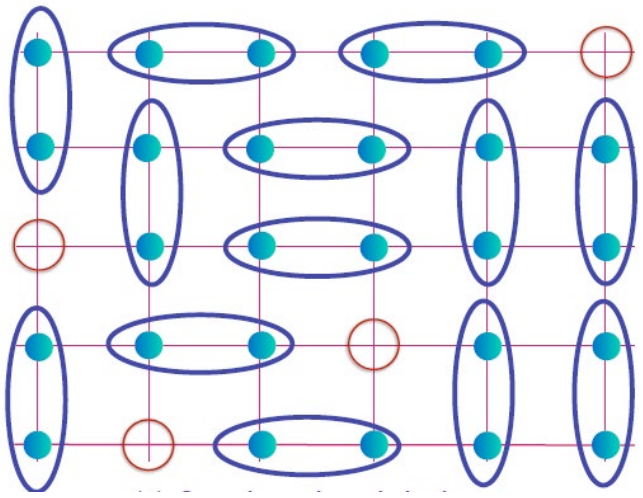
Frustration in metals

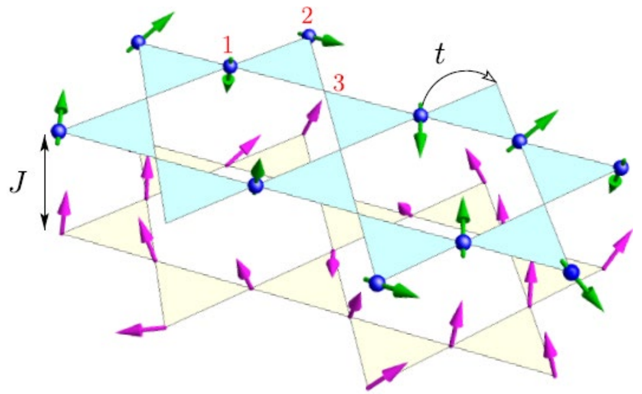
Hubbard models
on frustrated lattices
(half filling, near Mott transition)

Carrier-doped spin liquids
(*t*-*J* or Hubbard w/ *U*>>*t*)

Frustrated local moments
plus conduction electrons
(multiband setting)







κ-$(ET)_2Cu_2(CN)_3$

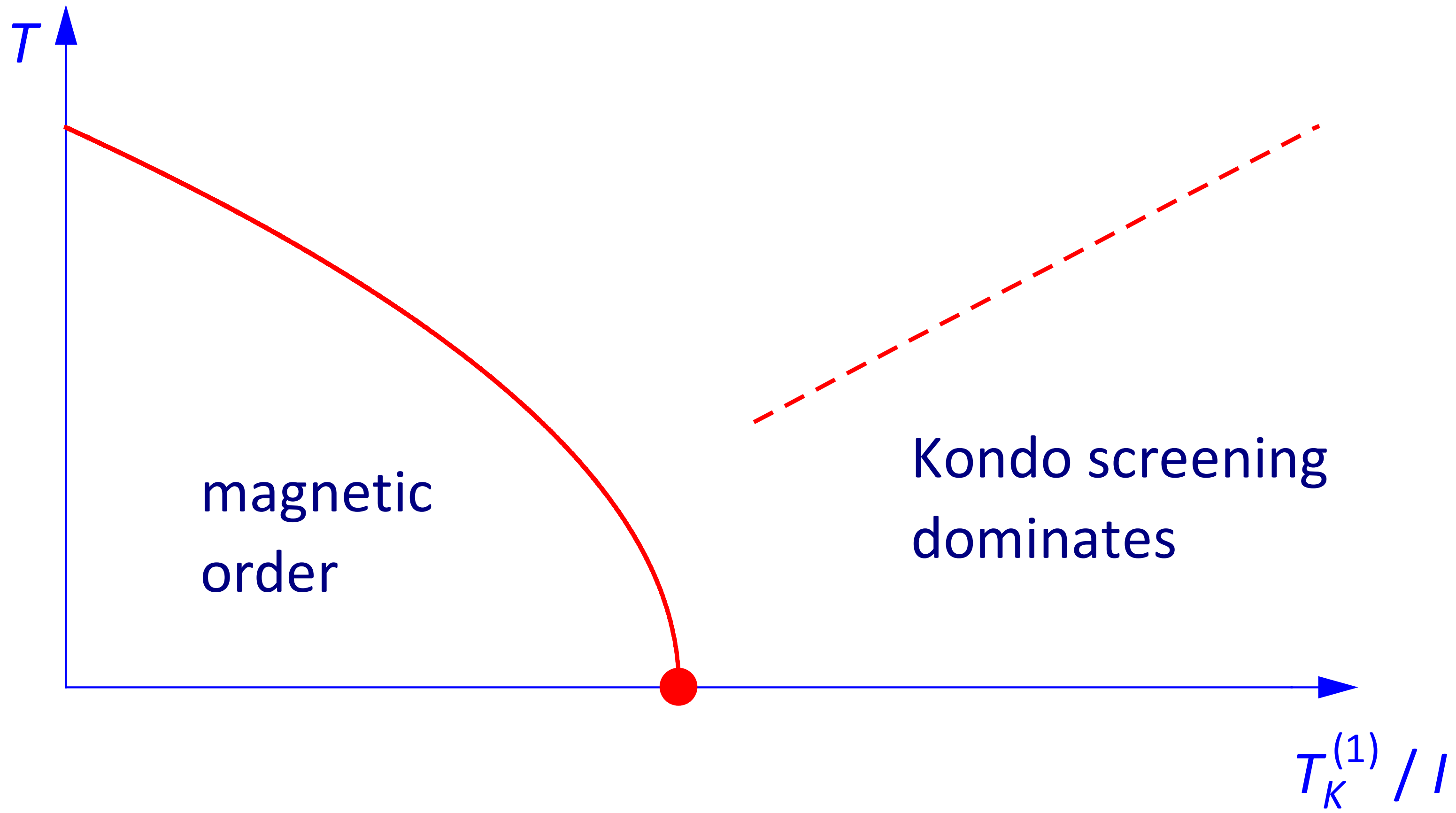
Underdoped cuprates?

**CeRhSn, CePdAl,
YbAgGe, YbPdAs,
$Pr_2Ir_2O_7$**

Fractionalized
Fermi liquids & superconductors

Doniach‘s phase diagram for *f*-electron metals

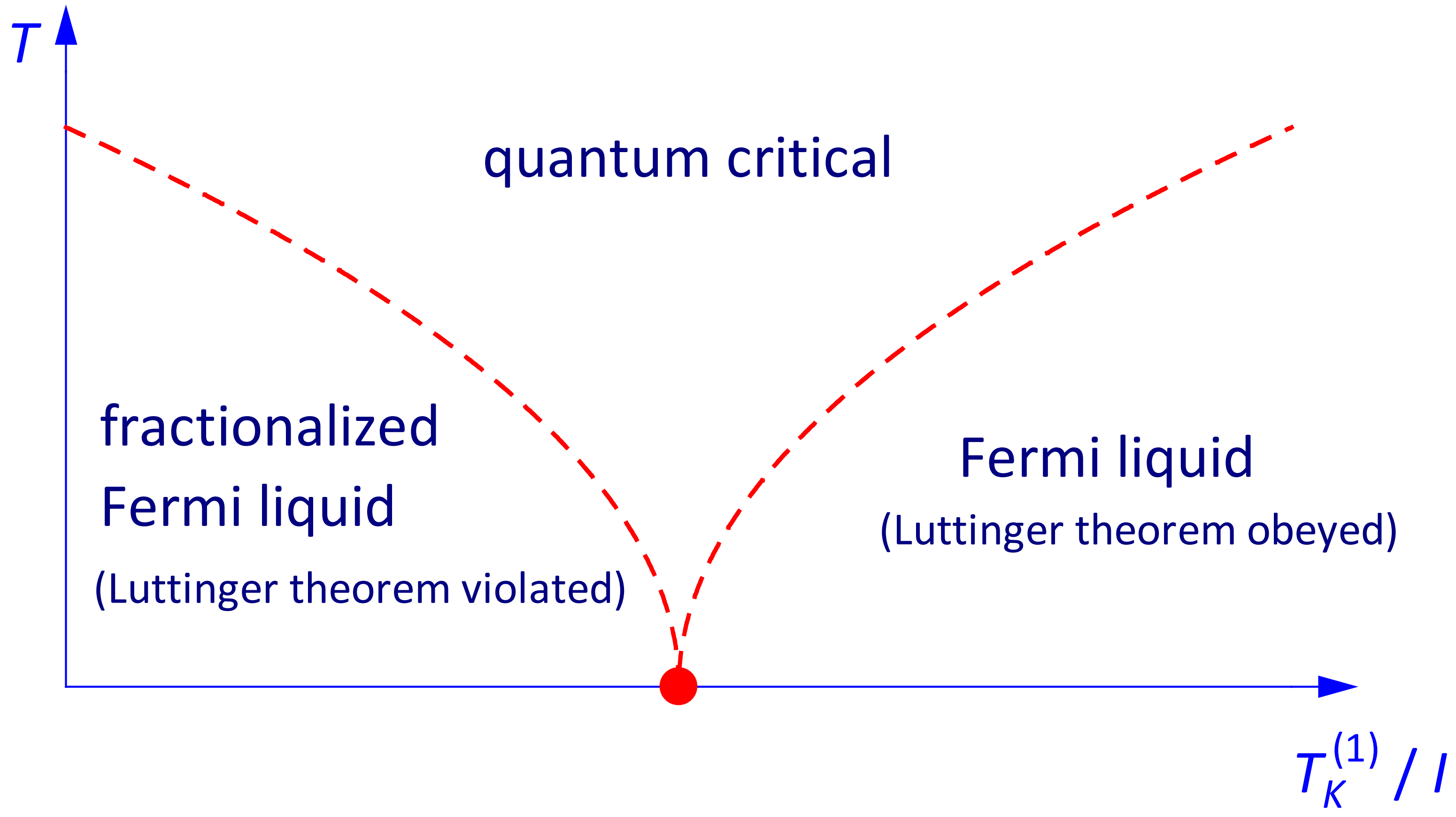
$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Competition between local-moment interactions and metallicity

Doniach + strong frustration

$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_{\mathrm{K}} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Senthil / Sachdev / Vojta, PRL **90**, 216403 (2003)
see also Si *et al.*, Coleman *et al.*

Reminder: Luttinger theorem

Momentum-space volume enclosed by Fermi surface

=

Total number of electrons per unit cell (mod 2)

$$\boxed{\mathcal{V}_{\mathrm{FL}} = K_d (n_{\mathrm{tot}} \bmod 2)}$$

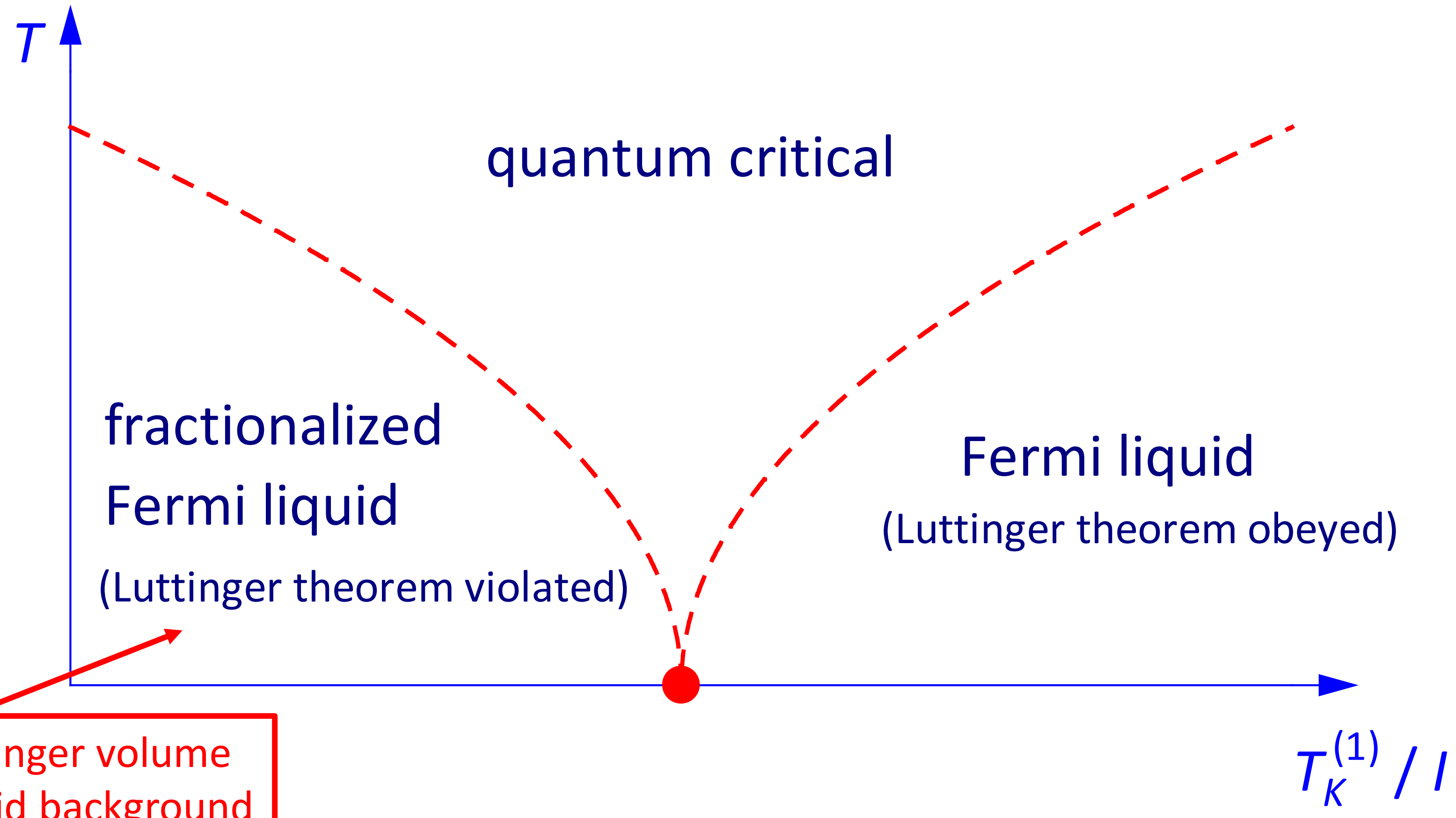
$$K_d = (2\pi)^d/(2v_0)$$

v_0 unit cell volume

Non-perturbative proof for Fermi liquids:
M. Oshikawa, PRL **84**, 3370 (2000)

Doniach + strong frustration

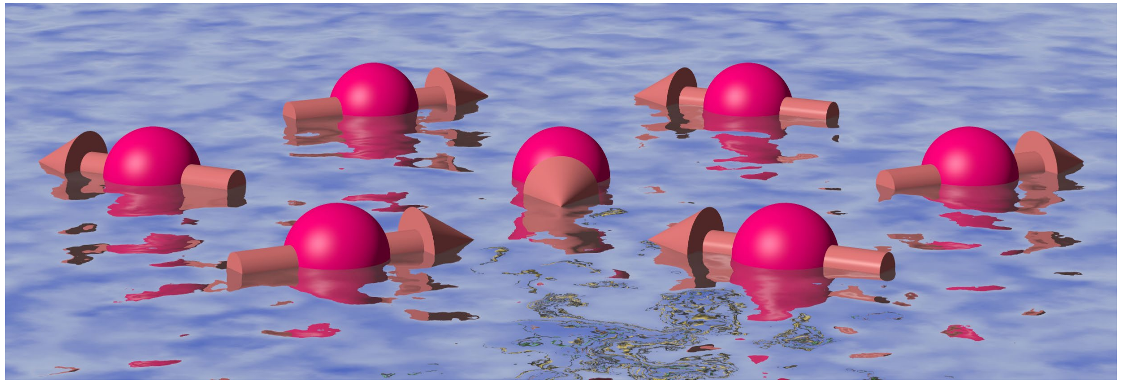
$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Violation of Luttinger volume requires spin-liquid background (i.e. non-trivial entanglement)

Senthil / Sachdev / Vojta, PRL **90**, 216403 (2003)
see also Si *et al.*, Coleman *et al.*

Fractionalized Fermi liquid



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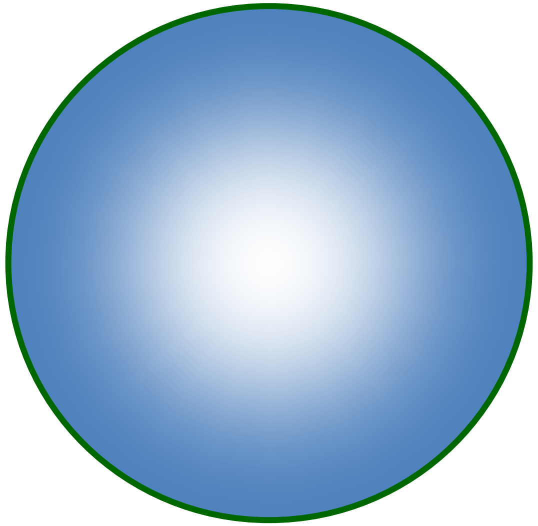


Local moments

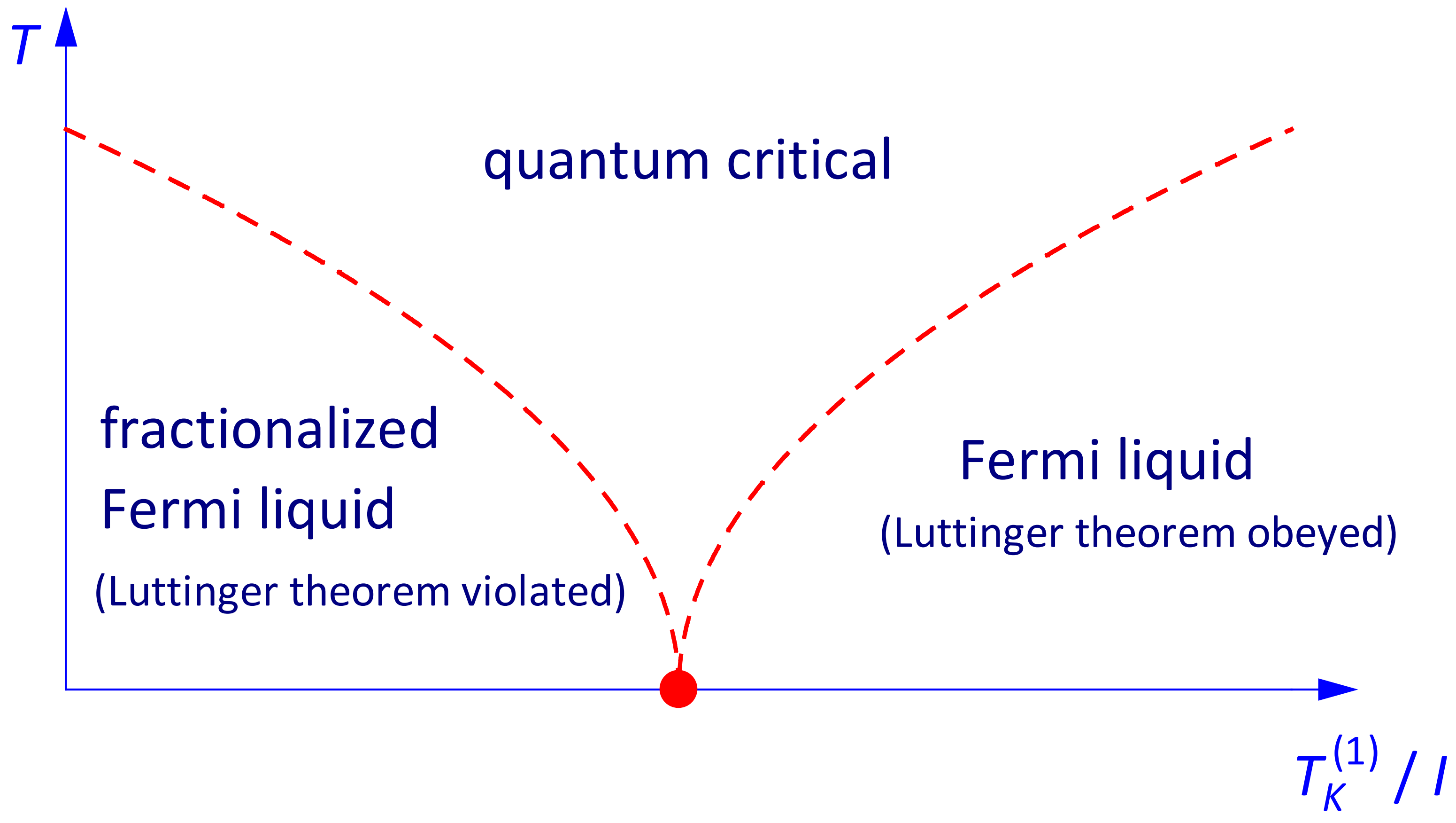
Conduction electrons

$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

ct.qmat

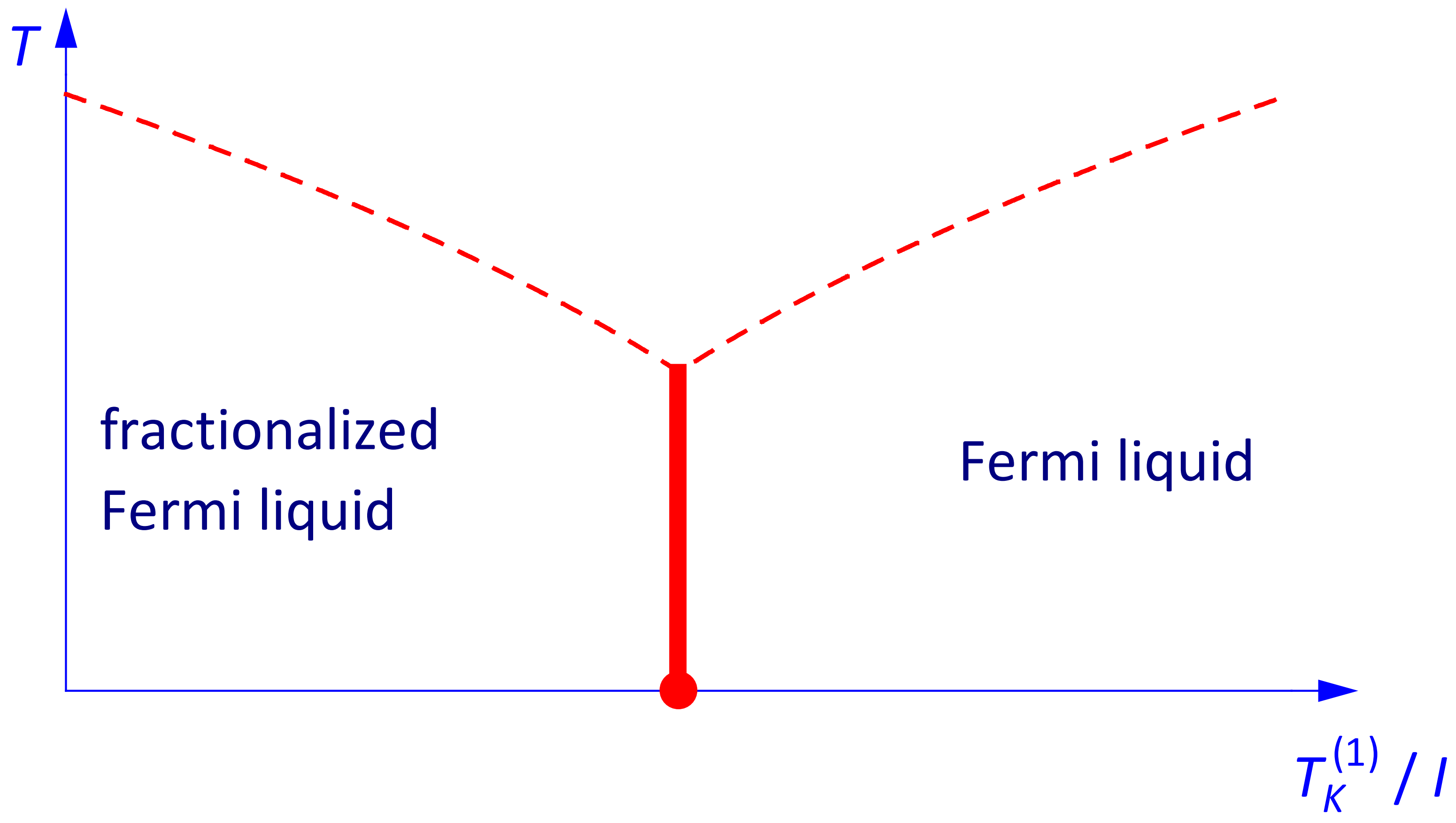
Kondo breakdown transitions

$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j$$



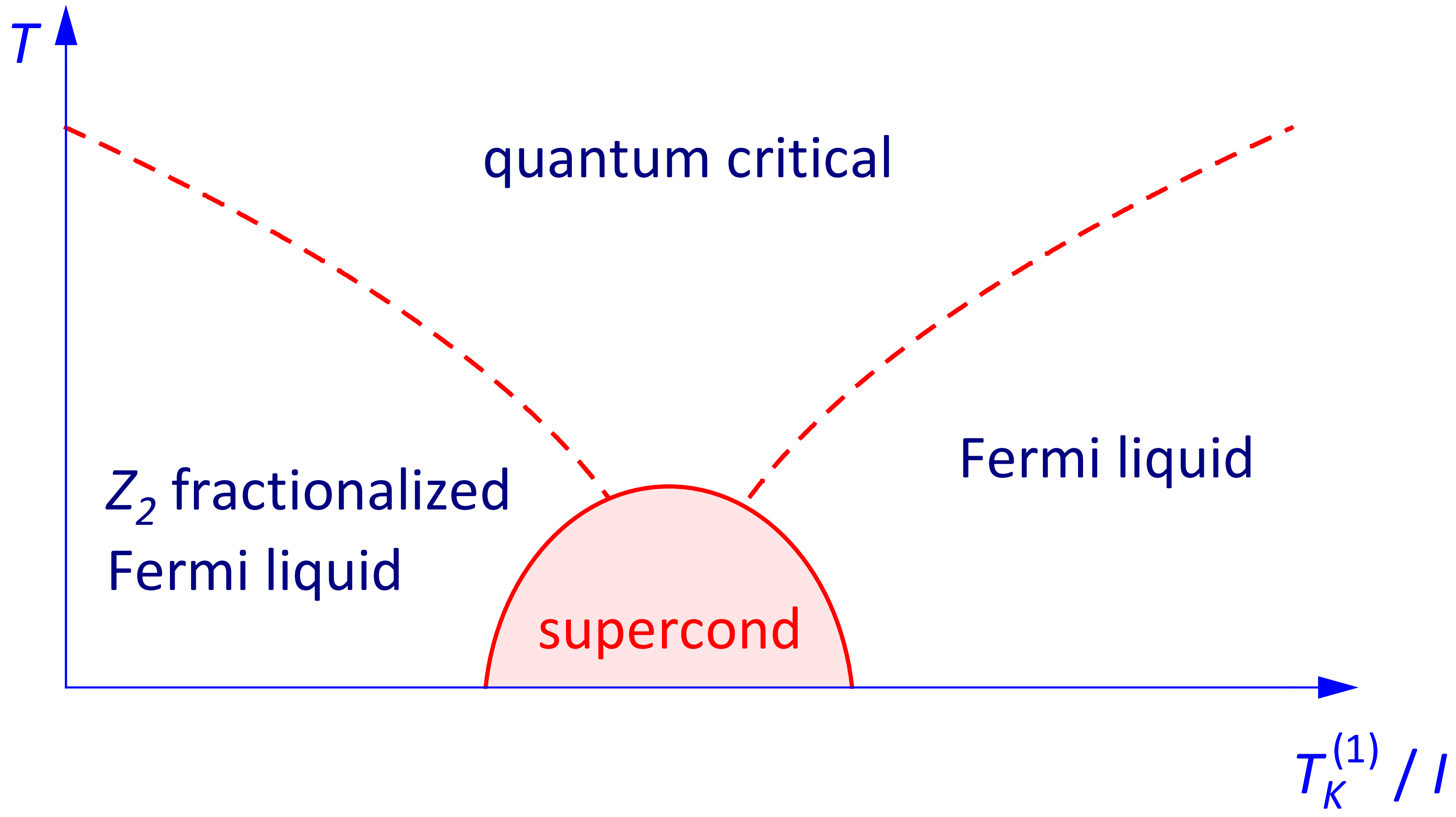
Kondo breakdown transitions: 1st order ...

$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Kondo breakdown transitions: Intermediate phases

$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



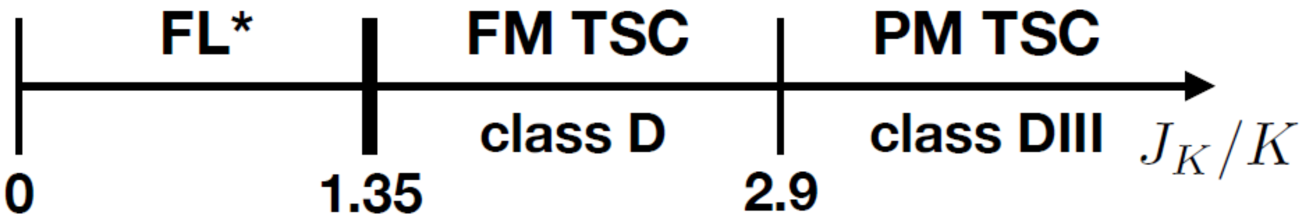
Kitaev Kondo lattice: Mean-field theory

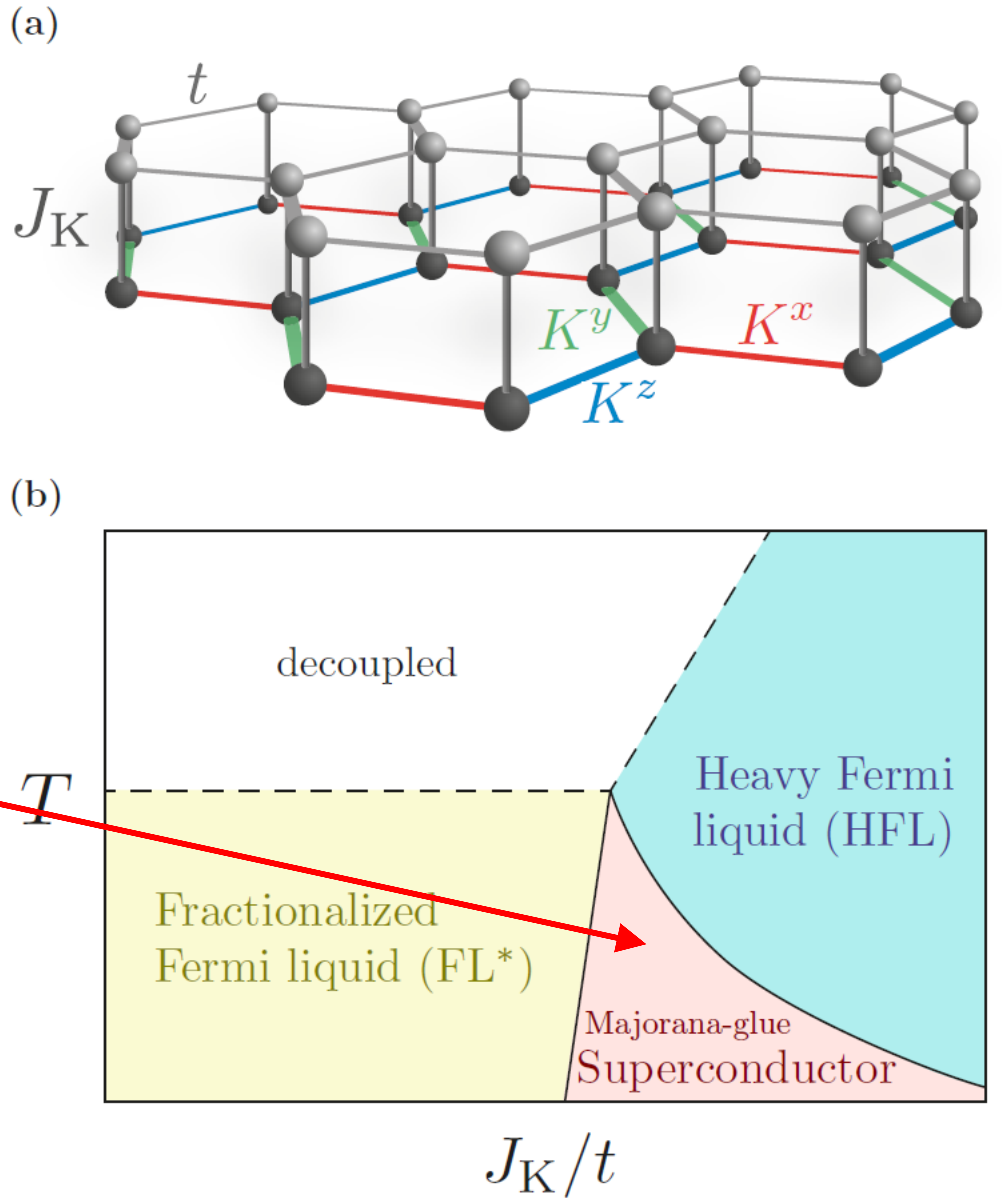
$$\mathcal{H}_t = -t \sum_{\langle ij \rangle \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.),$$

$$\mathcal{H}_K = -\sum_{\langle ij \rangle_\alpha} K^\alpha S_i^\alpha S_j^\alpha,$$

$$\mathcal{H}_J = \frac{1}{2} \sum_{i\sigma\sigma'\alpha} J_{\mathrm{K}}^\alpha c^\dagger_{i\sigma} \tau^\alpha_{\sigma\sigma'} c_{i\sigma'} S_i^\alpha$$

Spin-triplet, nematic s.c.
(of BCS type)



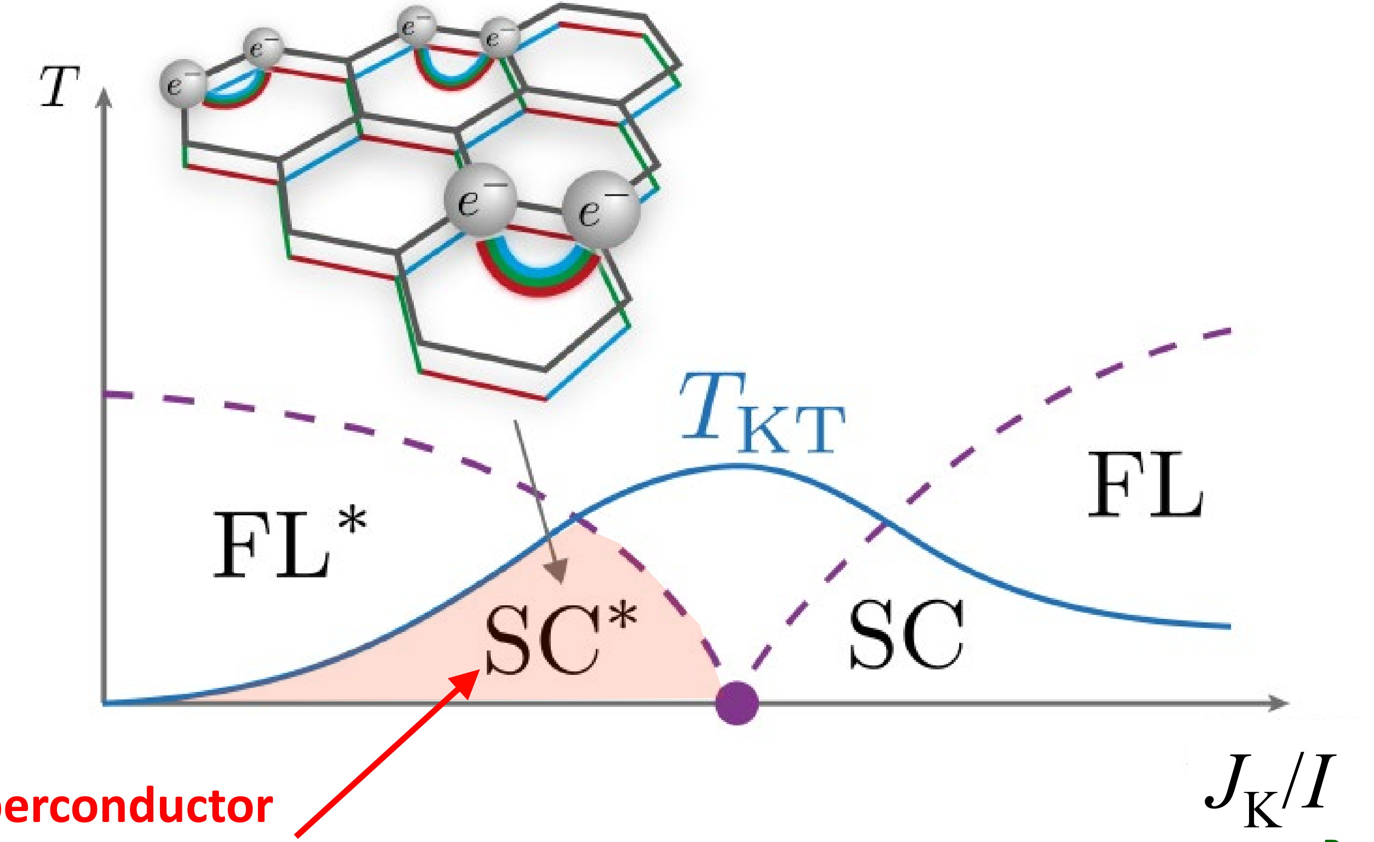


Choi/Kim *et al.*, PRB **98**, 155123 (2018)

Seifert/Meng/Vojta, PRB **97**, 085118 (2018)

Fractionalized superconductivity (SC*)

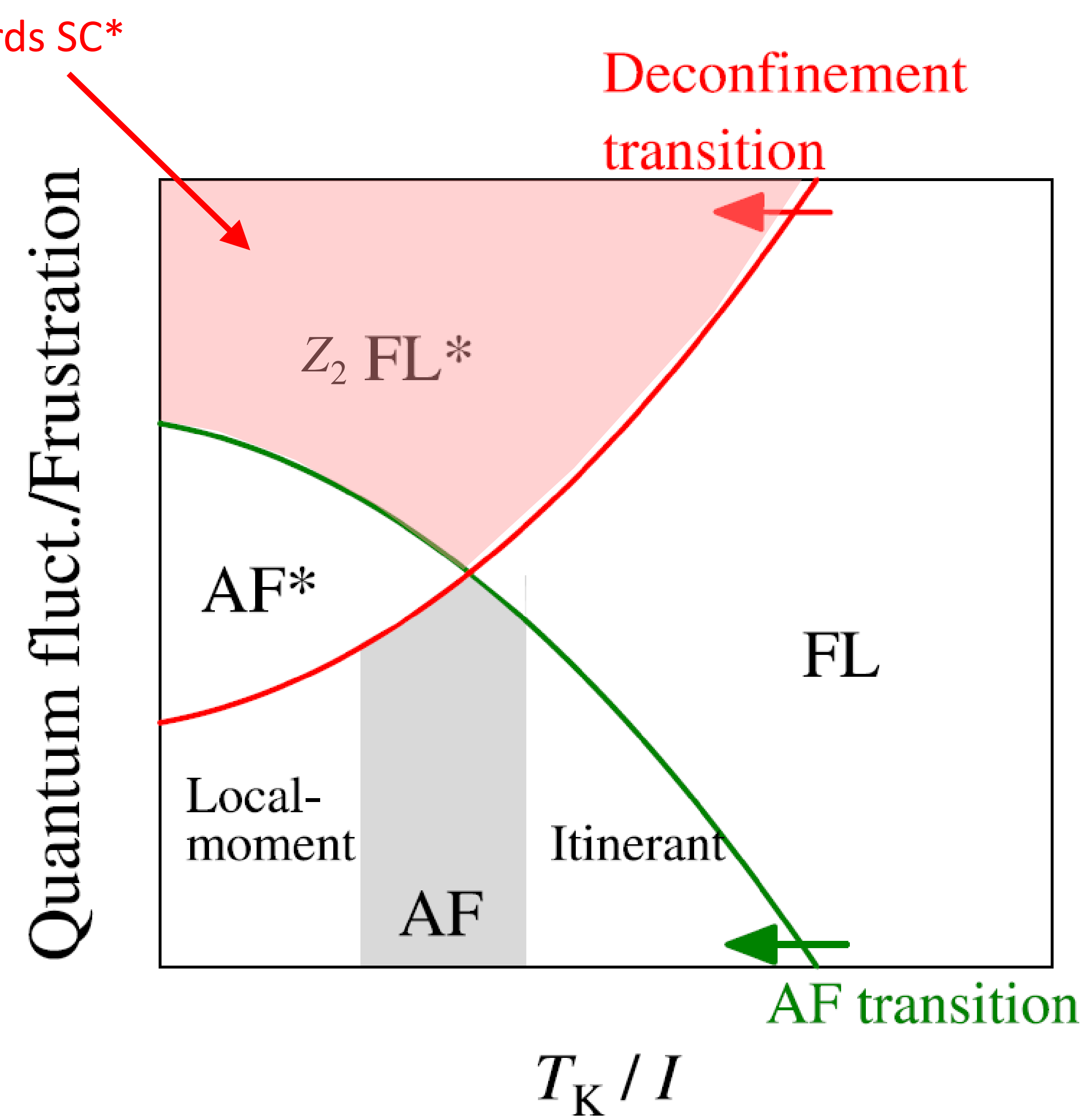
$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_{\mathrm{K}} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Fractionalized superconductor descending from FL*

Bunney *et al.*, PRL **134**, 206602 (2025)
see also Lundemo/Sudbo, PRB **109**, 184508 (2024)

„Global“ heavy-fermion phase diagram …



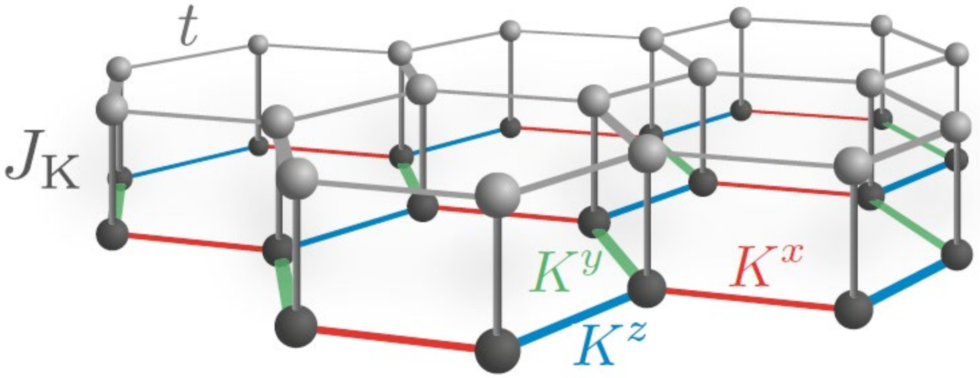
Vojta, JLTP **161**, 203 (2010)
see also Si *et al.*, Coleman *et al.*

Kitaev Kondo lattice → Effective model

$$\mathcal{H}_t = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.),$$

$$\mathcal{H}_K = -\sum_{\langle ij \rangle_\alpha} K^\alpha S_i^\alpha S_j^\alpha,$$

$$\mathcal{H}_J = \frac{1}{2} \sum_{i\sigma\sigma'\alpha} J_{\mathrm{K}}^\alpha c_{i\sigma}^\dagger \tau_{\sigma\sigma'}^\alpha c_{i\sigma'} S_i^\alpha$$



Limit of small J_{K}:
Integrate out local-moment component
(Kitaev spin liquid protected by vison gap)

Generates spin-spin interaction for conduction electrons, dictated by spin correlations of Kitaev spin liquid:

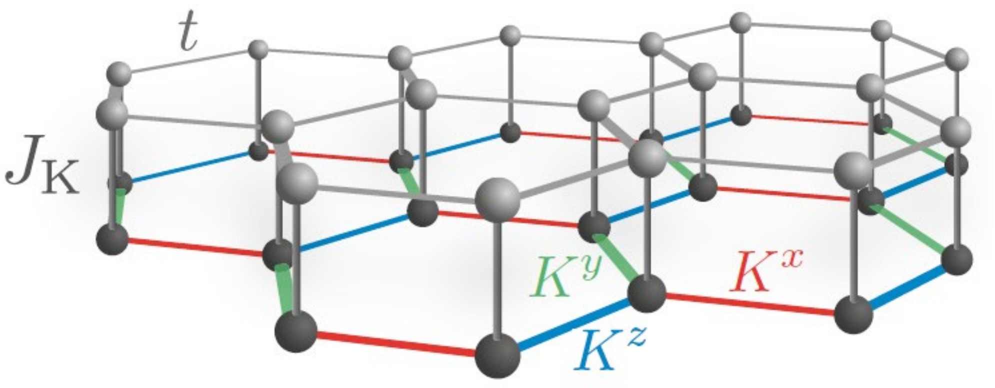
$$\mathcal{S}_{\mathrm{int}} = \sum_{ij} s_i^\alpha(\omega) V^{\alpha\beta}(i,j;\omega) s_j^\beta(\omega) \ ,$$

$$V^{\alpha\beta}(i,j;\omega) = J_{\mathrm{K}}^2 \, \chi_K^{\alpha\beta}(i,j;\omega) \, .$$

Effective Kitaev-Hubbard model

Spin correlations of Kitaev model
only non-zero on-site and for nearest neighbors:

$$\mathcal{H}_{\text{int}} = J_{\text{K}}^2 \left(\chi_0 \sum_i \vec{s}_i^{\,2} + \chi_1 \sum_{\langle ij \rangle_\alpha} s_i^\alpha s_j^\alpha \right)$$



Re-write interaction

$$\boxed{\begin{aligned} \mathcal{H}_t &= -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) \\ \mathcal{H}_{\text{int}} &= U \left[\sum_i n_{i\uparrow} n_{i\downarrow} + \kappa \sum_{\langle ij \rangle_\alpha} s_i^\alpha s_j^\alpha \right] \end{aligned}}$$

$$\begin{aligned} U &= |\chi_0| J_K^2 > 0 \\ \kappa &= \mp 2/3 |\chi_1/\chi_0| = \mp 0.5931 \end{aligned}$$

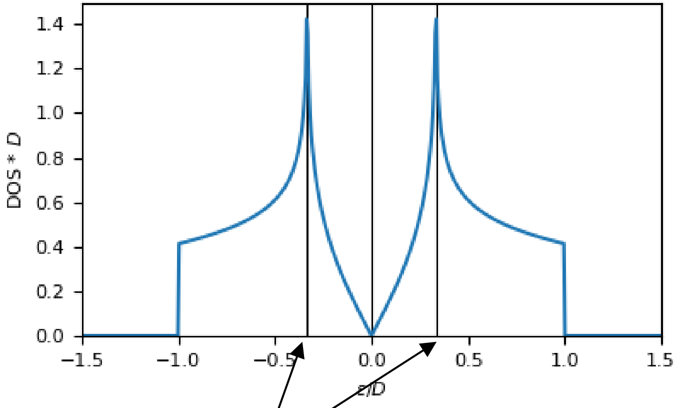
Functional RG analysis of Kitaev-Hubbard model

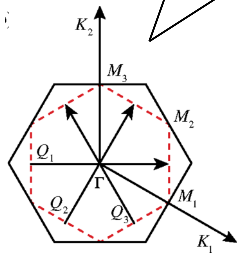
$$\mathcal{H}_t = -t \sum_{\langle ij \rangle \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \text{H.c.})$$

$$\mathcal{H}_{\text{int}} = U \Big[\sum_i n_{i\uparrow} n_{i\downarrow} + \kappa \sum_{\langle ij \rangle_\alpha} s_i^\alpha s_j^\alpha \Big]$$

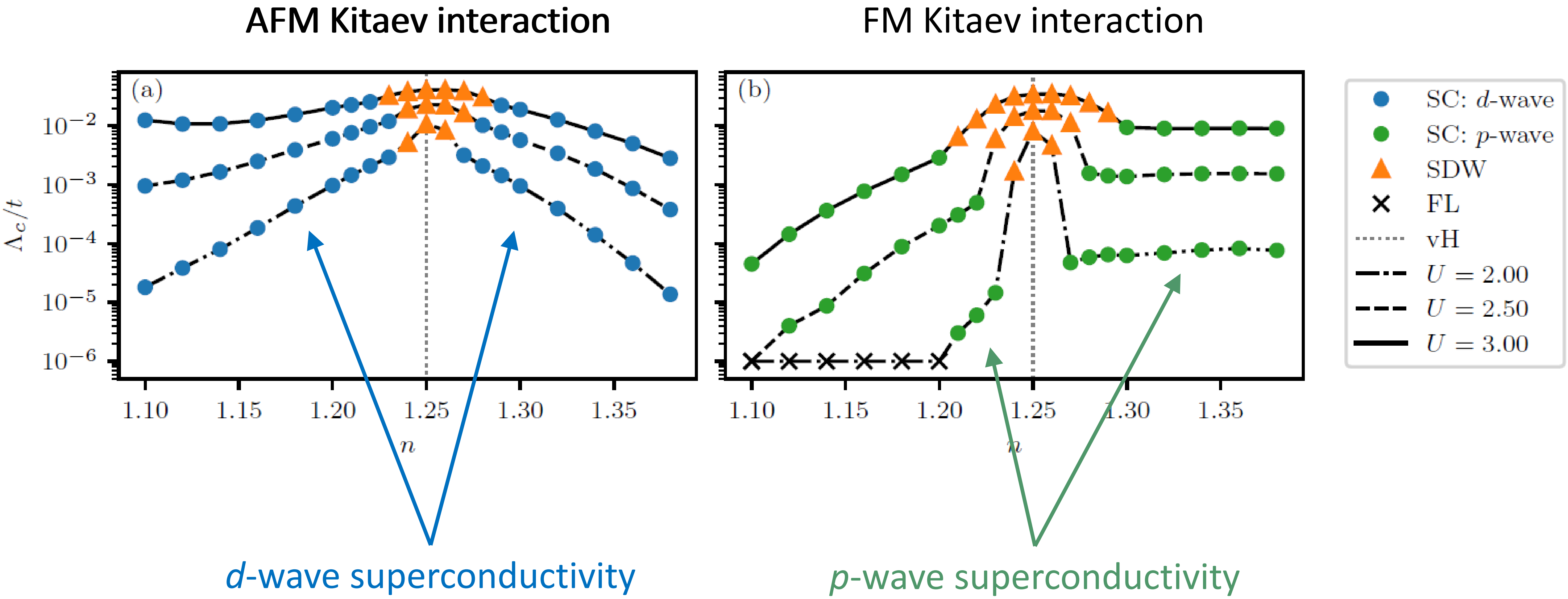
Analyze instabilities perturbatively in *U*
using fRG which keeps track of all two-particle interactions.

In practice:
Moderate interactions *U*=2…3
Band filling near van-Hove filling *n*=5/4





fRG results: Low-temperature instabilities



SC coexists with fractionalized spin-liquid background

Bunney *et al.*, PRL **134**, 206602 (2025)

Pairing in momentum space

Single-orbital case:

$$\hat{\Delta}(\boldsymbol{k}) = (\Psi(\boldsymbol{k})\hat{\sigma}_0 + \boldsymbol{d}(\boldsymbol{k})\cdot\hat{\boldsymbol{\sigma}})i\hat{\sigma}_y$$

Singlet

Triplet

Spin/Nambu space

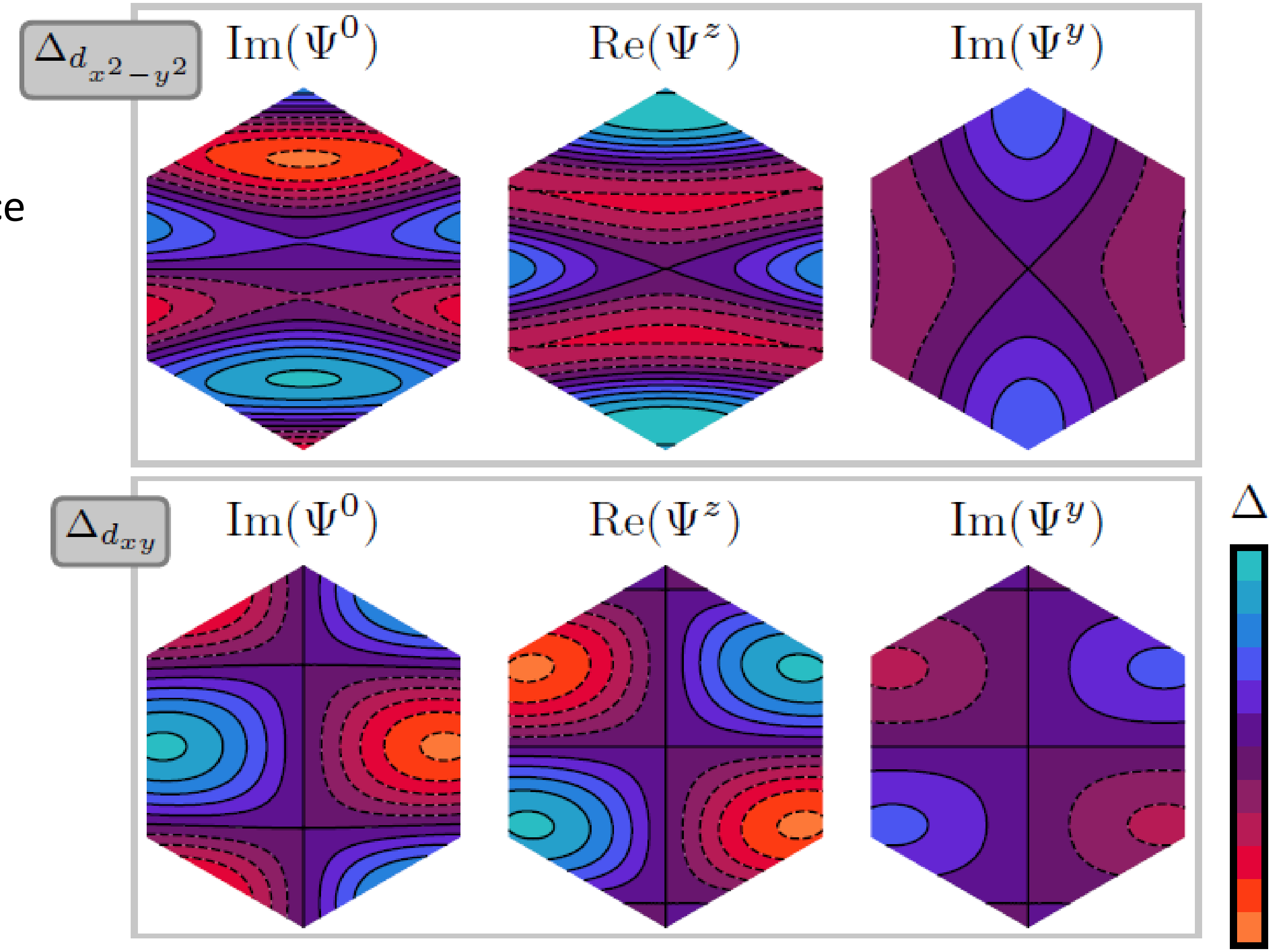
With additional sublattice index:

$$\hat{\Psi} = (\Psi^0\hat{\tau}_0 + \Psi^x\hat{\tau}_x + \Psi^y\hat{\tau}_y + \Psi^z\hat{\tau}_z)i\hat{\tau}_y$$

$$\hat{d}_i = (d_i^0\hat{\tau}_0 + d_i^x\hat{\tau}_x + d_i^y\hat{\tau}_y + d_i^z\hat{\tau}_z)i\hat{\tau}_y$$

Sublattice space

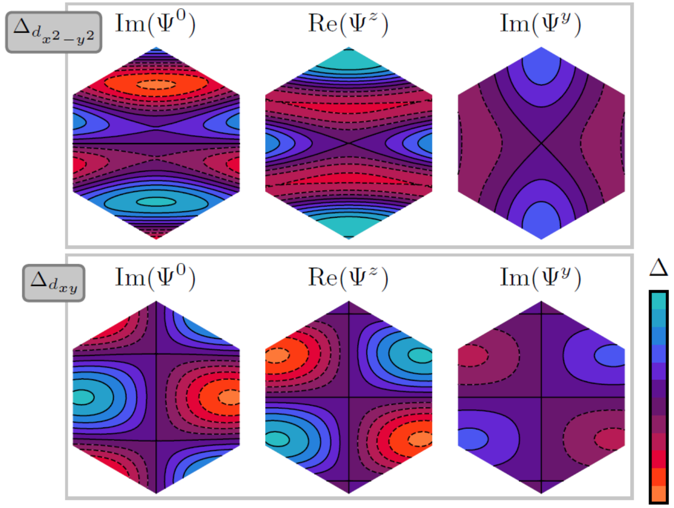
AFM Kitaev interaction → d-wave singlet pairing



Bunney *et al.*, PRL **134**, 206602 (2025)

Stable pairing states (BdG, Ginzburg-Landau)

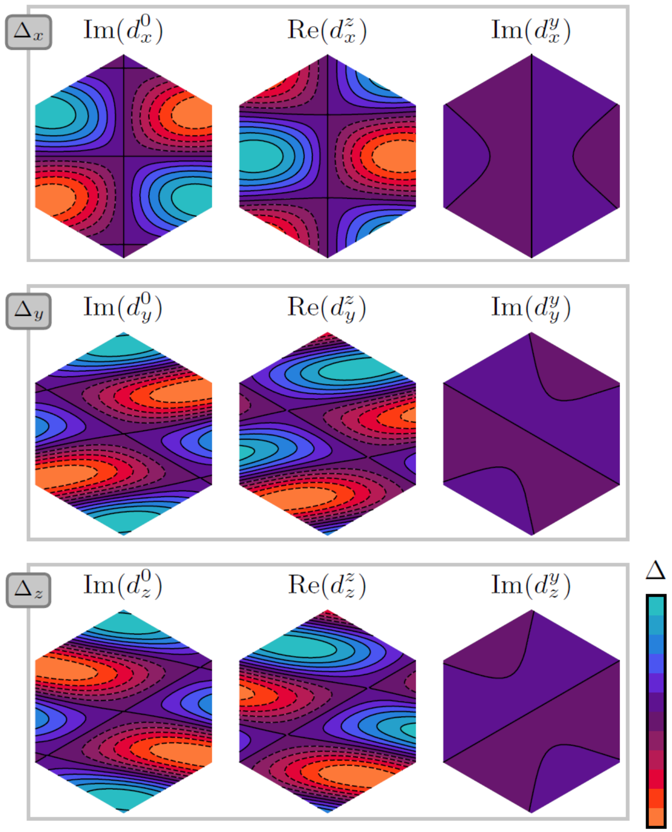
AFM Kitaev interaction



$$\vec{\Delta}_{d\pm id} = \vec{\Delta}_{d_{x^2-y^2}} \pm i\vec{\Delta}_{d_{xy}}$$

„chiral d-wave“

FM Kitaev interaction



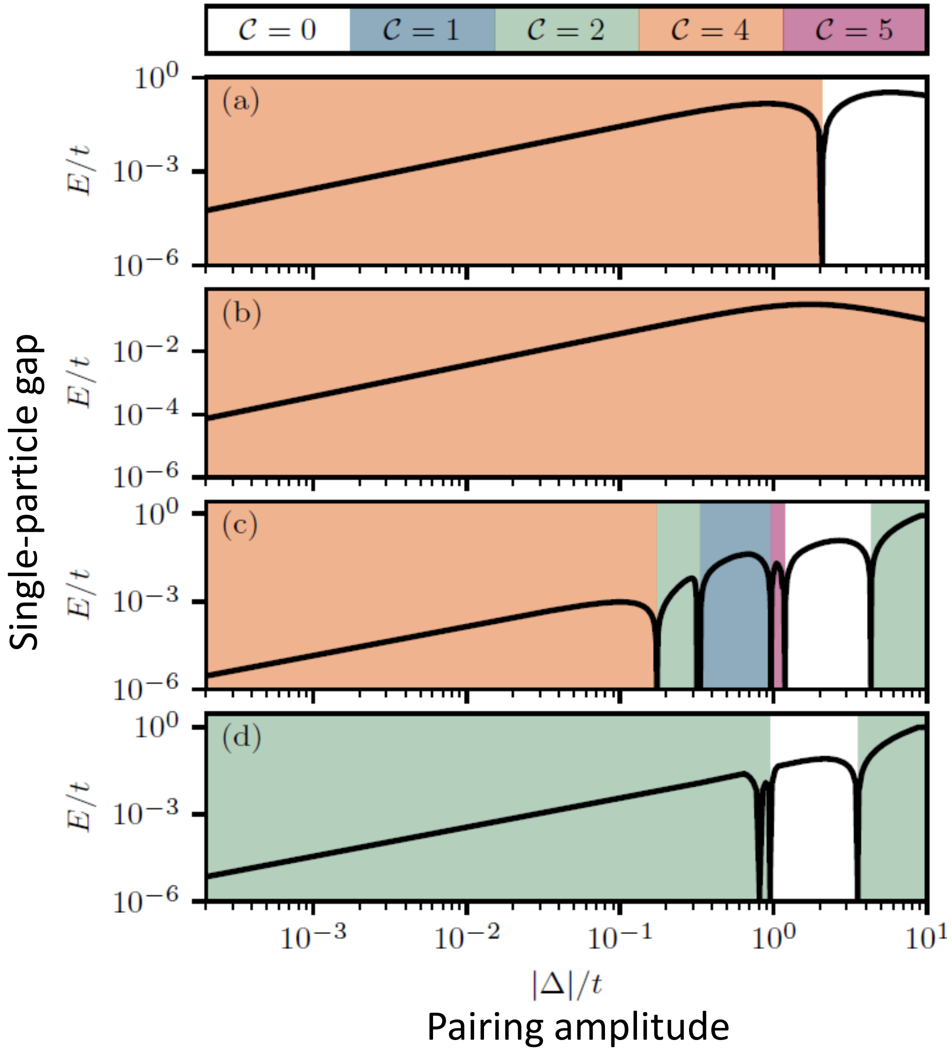
$$\vec{\Delta}_x + \varepsilon\vec{\Delta}_y + \varepsilon^*\vec{\Delta}_z, \text{ with } \varepsilon = e^{i2\pi/3}$$

„chiral p-wave“

As in honeycomb-lattice Hubbard model:
Nandkishore *et al*, Nat Phys. **8**, 158 (2012)
Black-Schaffer *et al*, JPCM **26**, 423201 (2014)

All pairing states are topological (in bandstructure sense)

Chern numbers



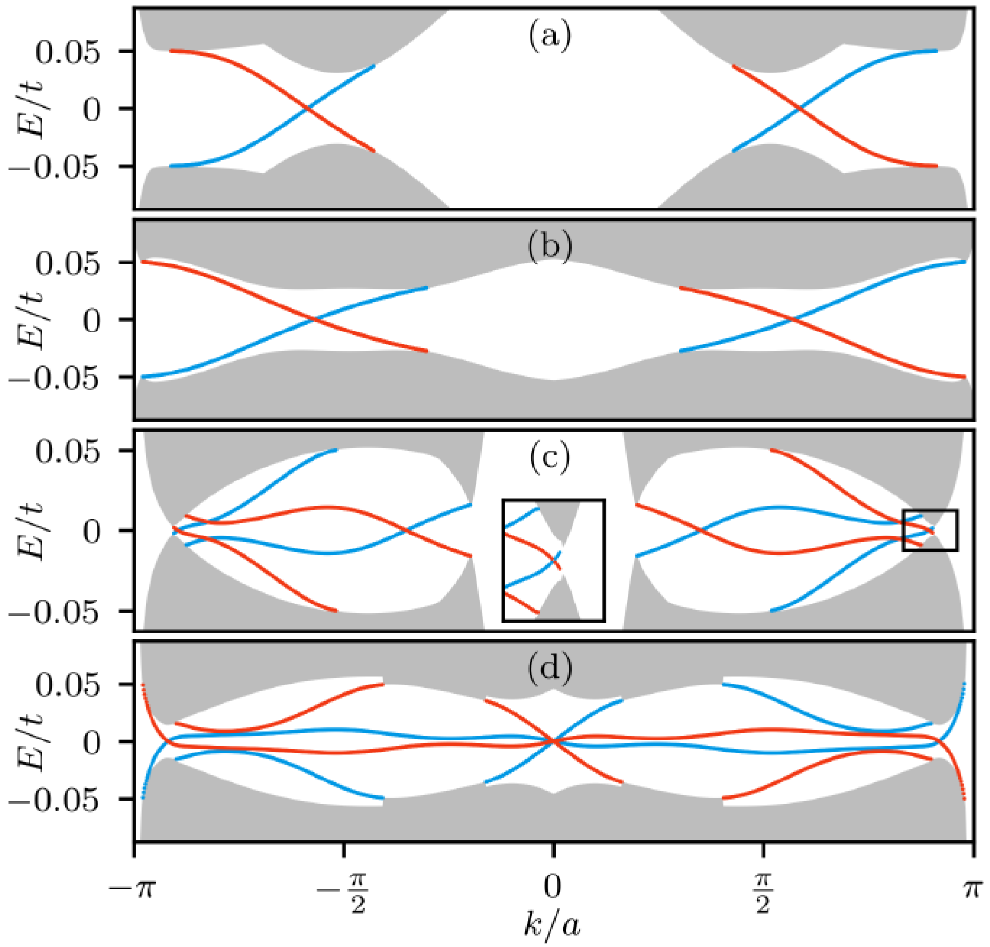
AFM case,
n = 1.18 < n_{vH}

AFM case,
n = 1.30 > n_{vH}

FM case,
n = 1.18 < n_{vH}

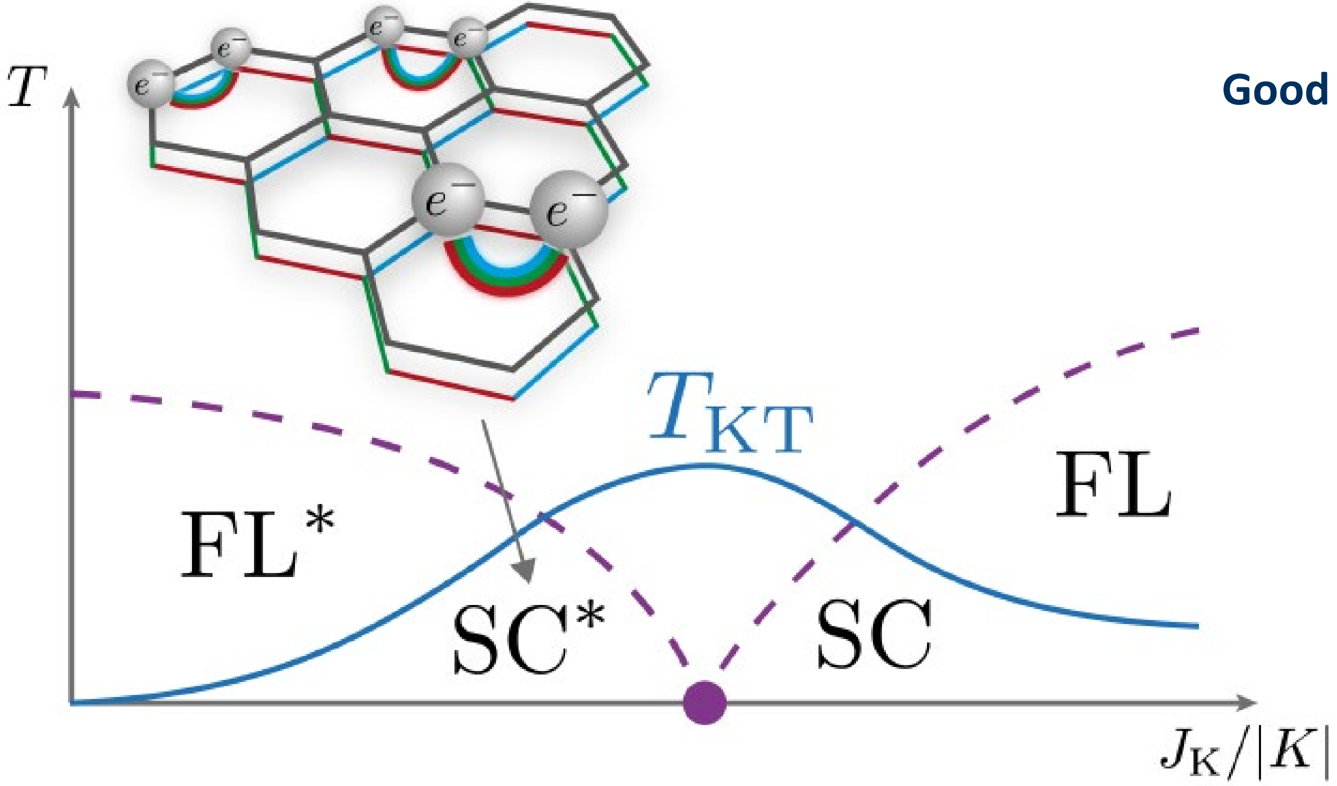
FM case,
n = 1.30 > n_{vH}

Ribbon spectra (for Δ=0.1)



Kondo breakdown transitions: Intermediate phases

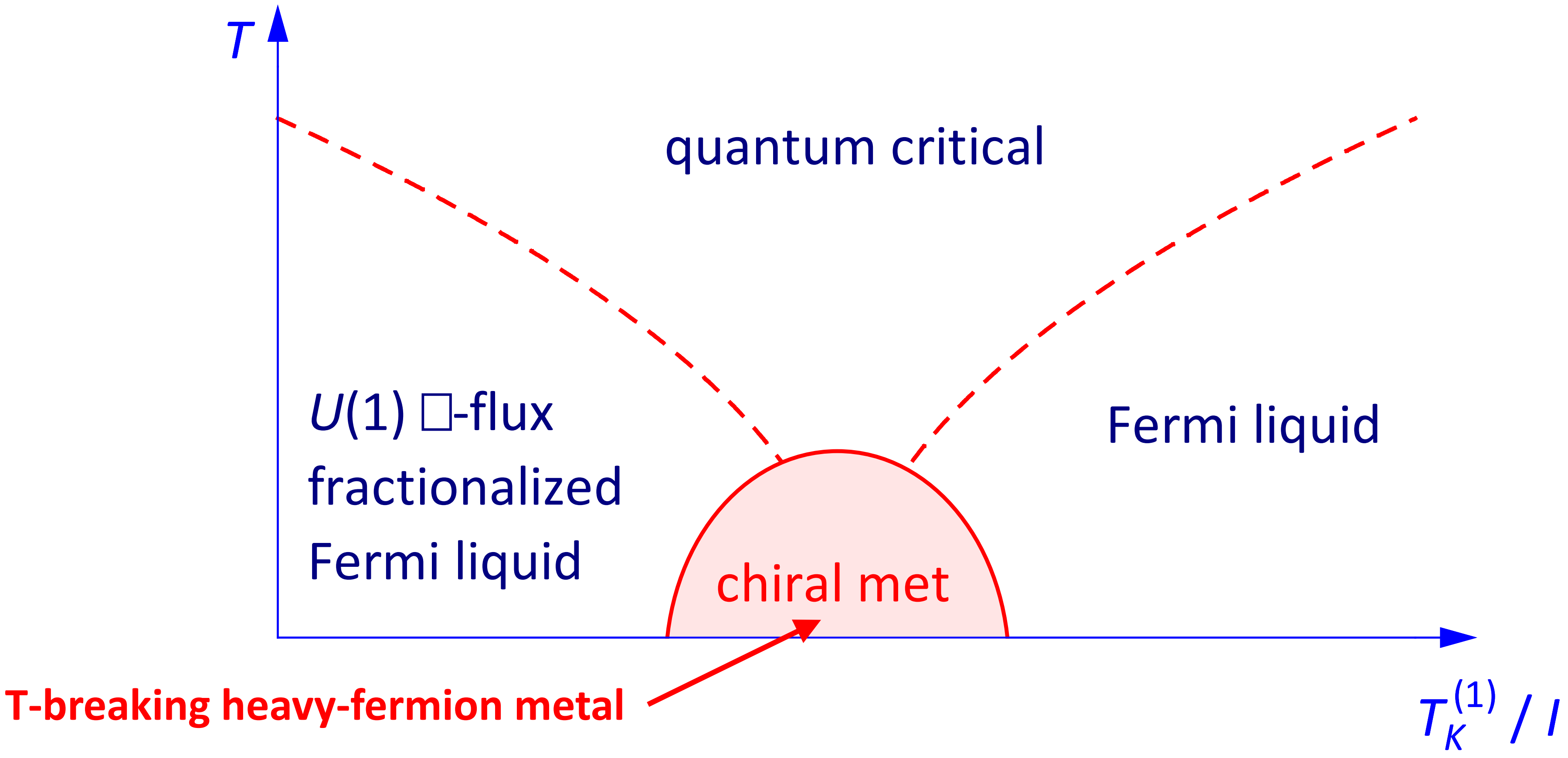
$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^{\dagger}_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_{\mathrm{K}} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^{\dagger}_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Good candidate material lacking (cf. $RuCl_3$/graphene)

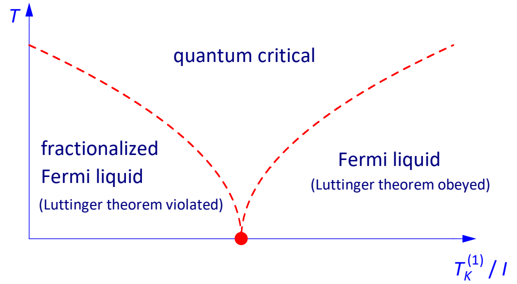
Kondo breakdown transitions: Intermediate phases

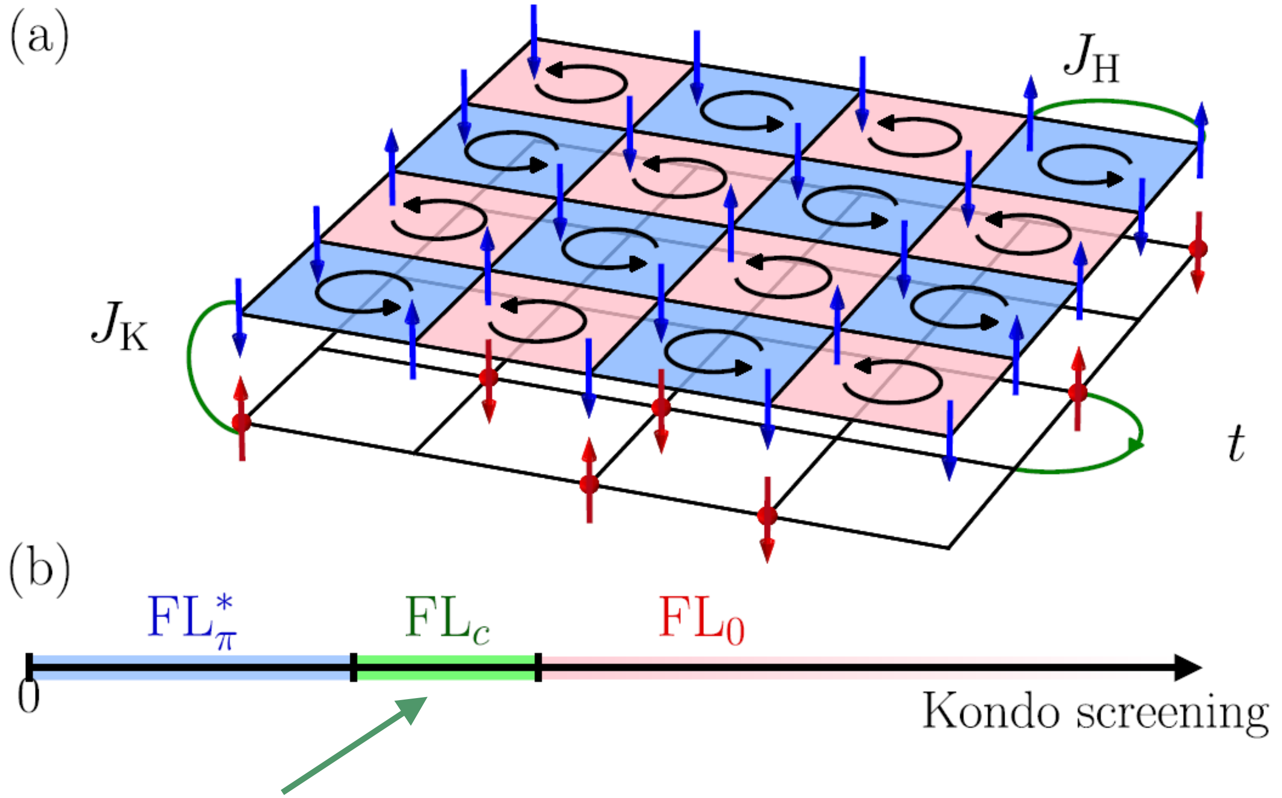
$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



U(1) π-flux FL* → intermediate chiral metal

$$\mathcal{H} = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + J_\mathrm{K} \sum_{i\sigma\sigma'} \vec{S}_i \cdot c^\dagger_{i\sigma} \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} + I \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



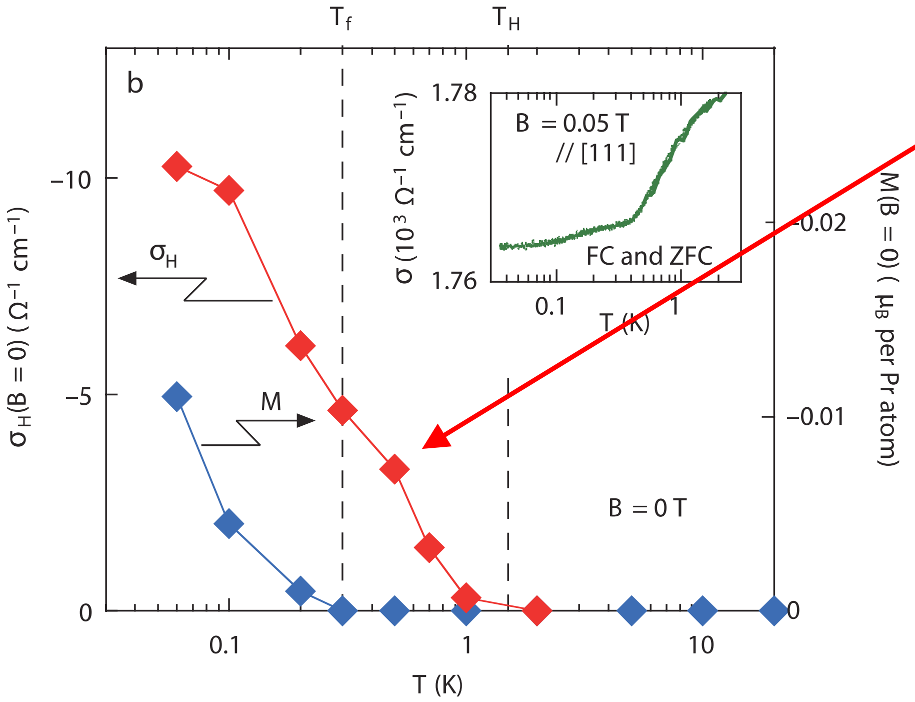


Heavy-fermion metal with spontaneous *T* breaking (orbital antiferromagnet)

Drechsler/Vojta, PRL **134**, 106503 (2025)

$Pr_2Ir_2O_7$ – a candidate material for chiral HF metal?

- Kondo coupling between Ir 5*d* conduction electrons and Pr 4*f* moments
- Small carrier concentration (proximity to quadratic band touching)
- Anomalous Hall effect for $T < 1.5$ K
  → time reversal broken without long-range spin order



Metallic chiral spin liquid (=FL*) at finite J_K?

Machida *et al.*, Nature **463**, 210 (2010)

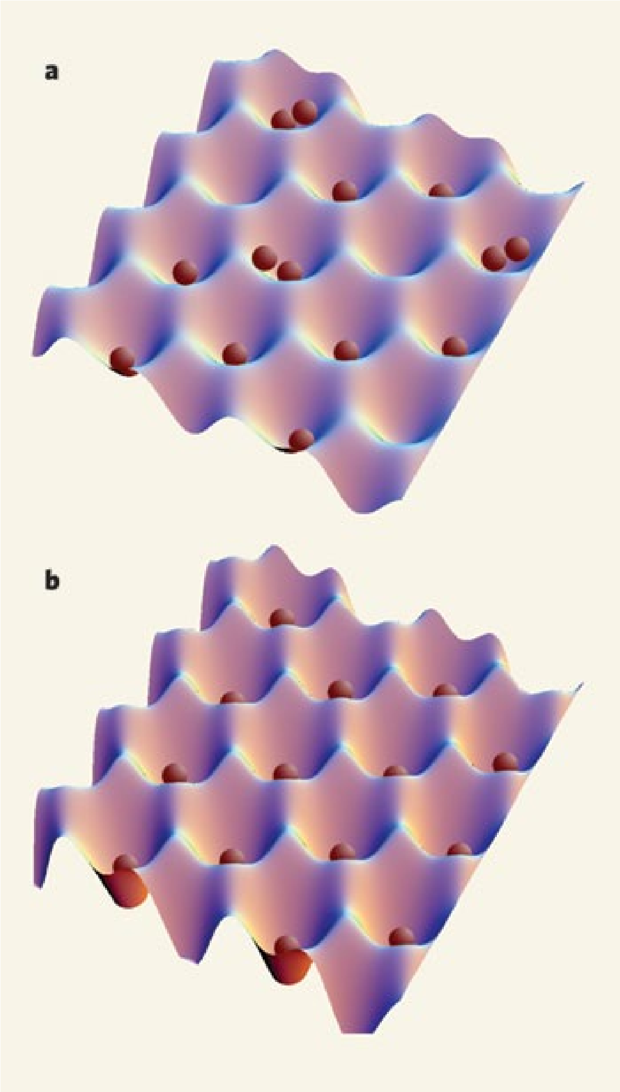
Weakly doped Mott insulators: Cuprates & beyond

What is a Mott insulator?

System of interacting fermions
moving in a lattice with kinetic energy t,
and subject to local repulsion U.

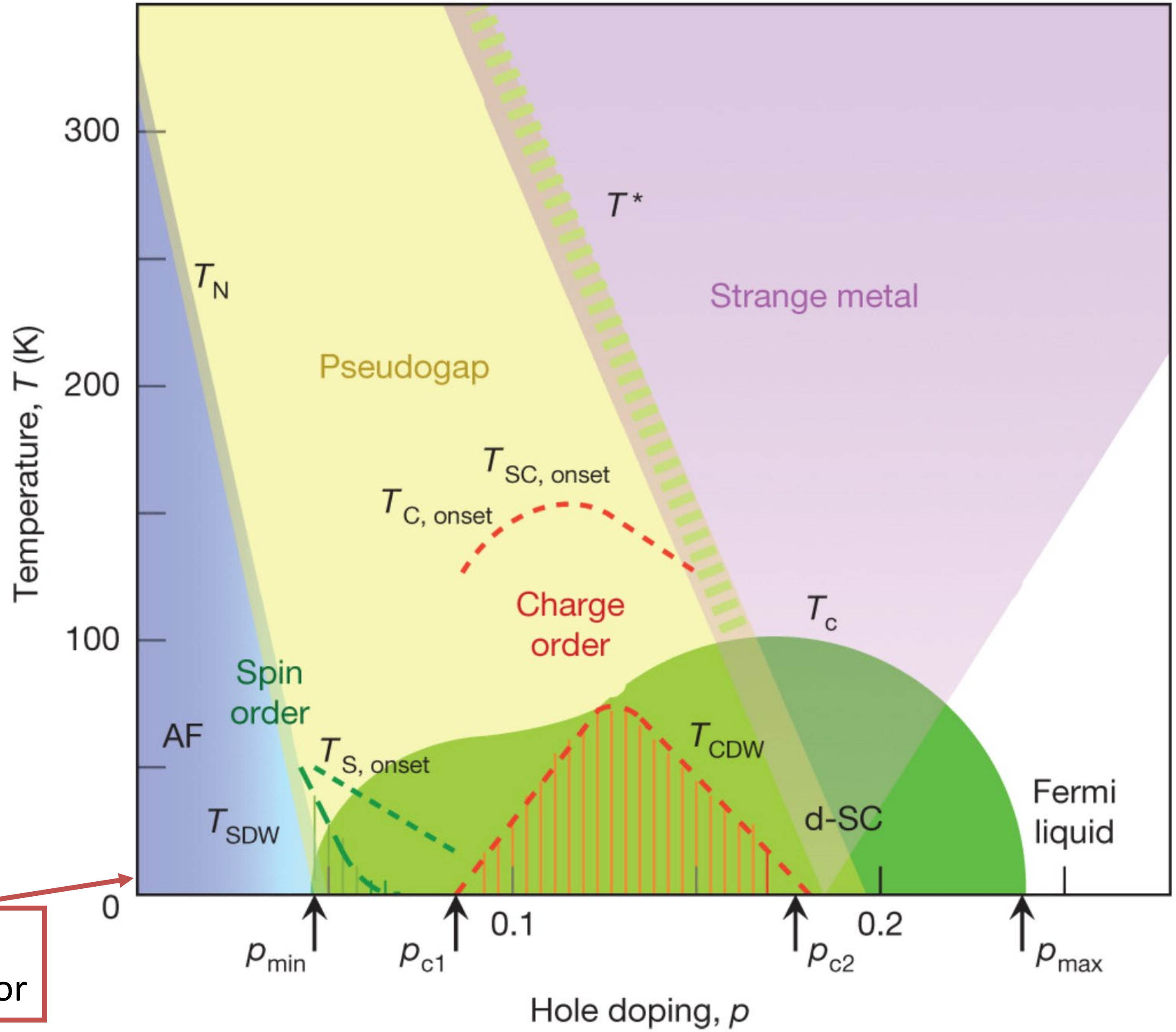
$U \gg t$, half-filling:
($\langle n \rangle$=1 particles per site)

Particles localized due to interactions!
Mott insulator



Characteristics:
- Gap to charge excitations
- No Fermi surface
- Residual spin degrees of freedom → magnetism

Cuprate phase diagram



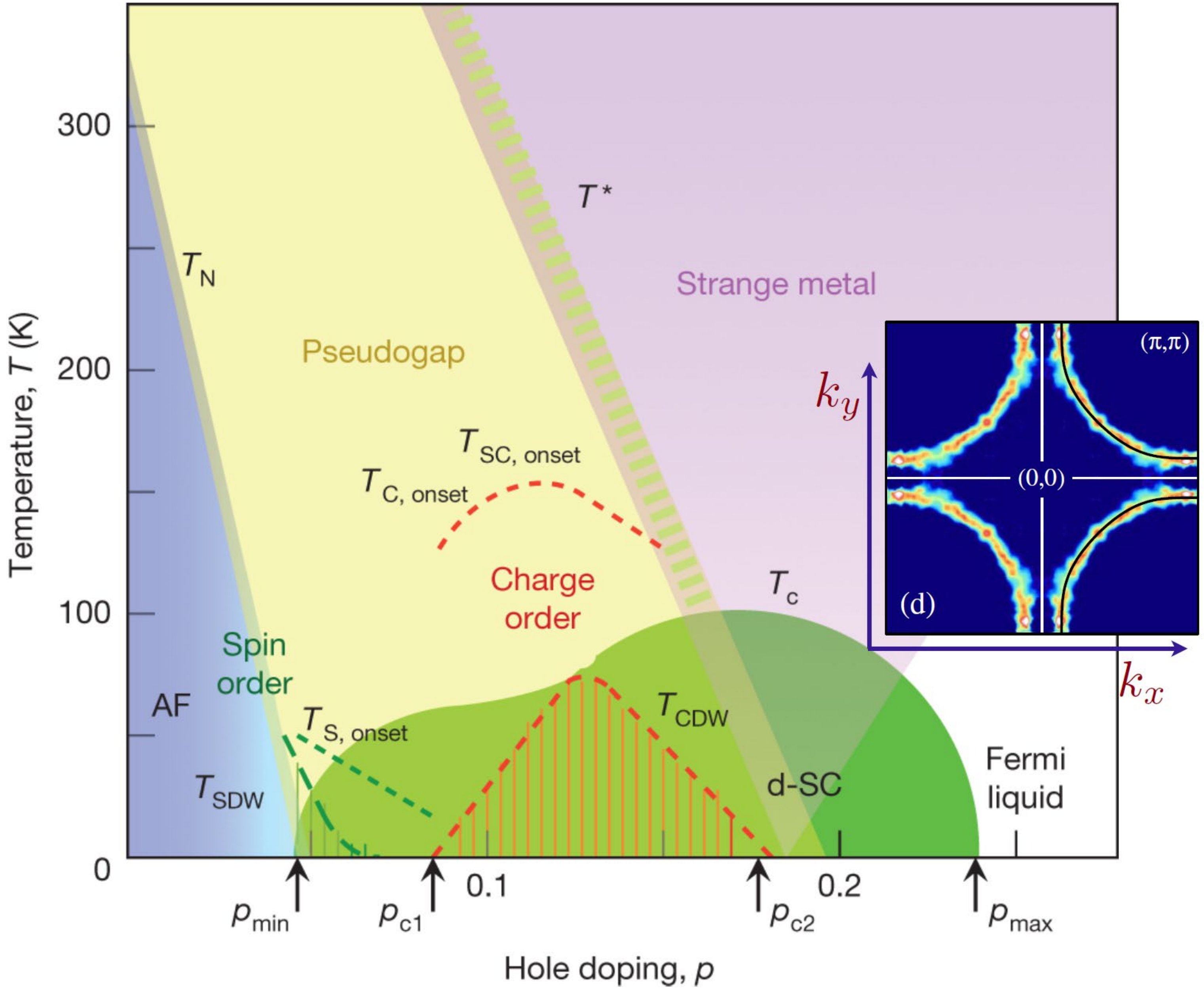
Half-filled Mott insulator

Superconductivity emerges from doping a Mott insulator

Competing orders
(spin & charge density waves)
occur primarily **on top** of pseudogap

Keimer *et al.*, Nature **518**, 179 (2015)

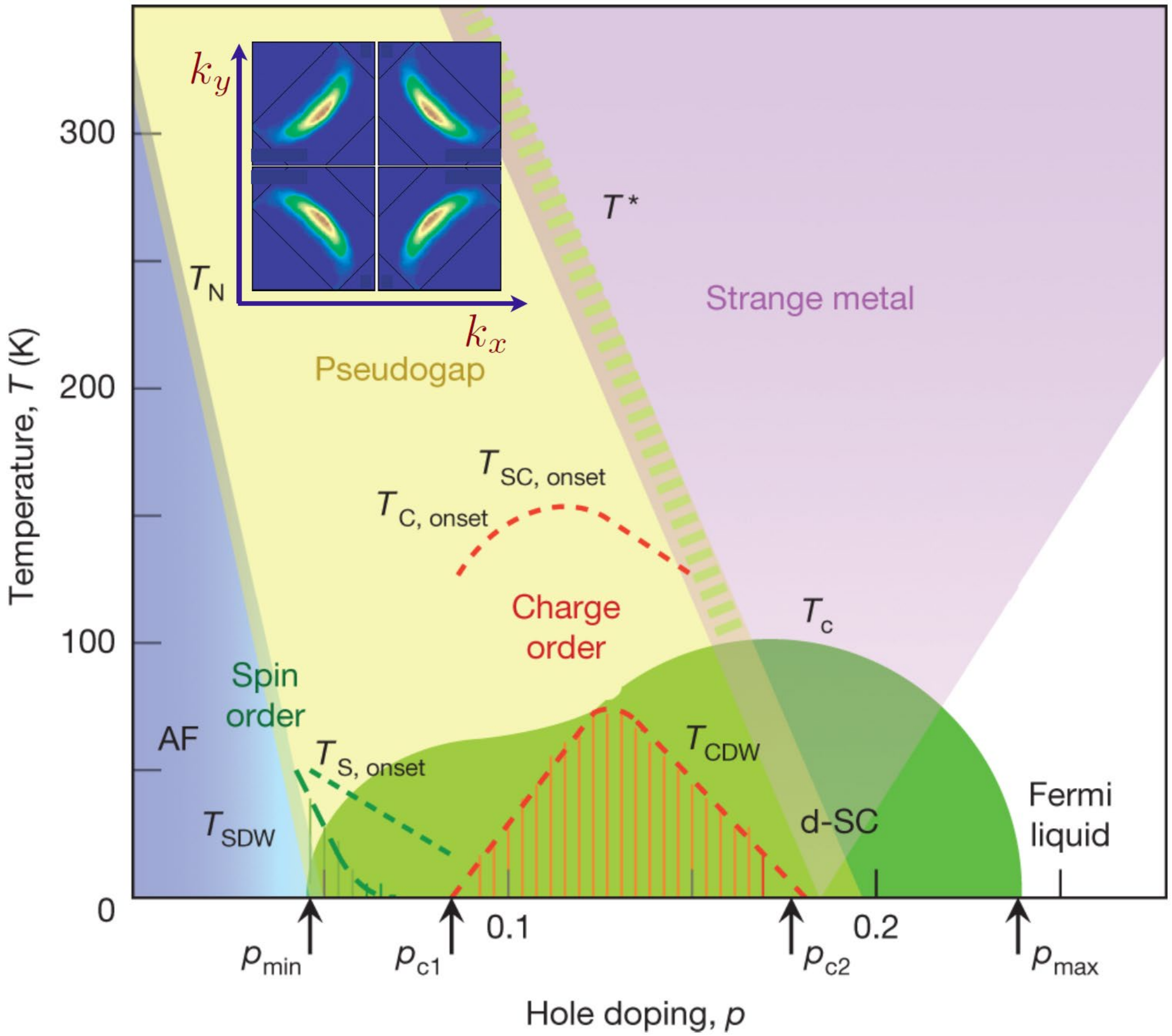
Cuprates: Fermi surface evolution



Overdoped cuprates:

Ordinary metal with „large“ Fermi surface consistent with Luttinger volume (and consistent with DFT)

Cuprates: Fermi surface evolution



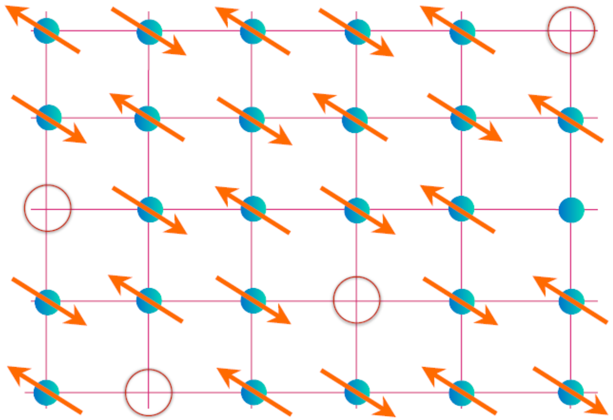
Underdoped cuprates:

Pseudogap metal with
Fermi surface
strongly modified by e-e interactions

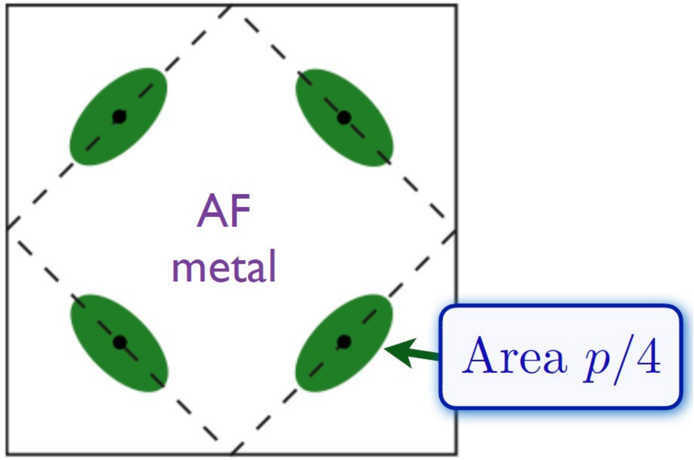
Inconsistent with Luttinger volume

Doping an insulating antiferromagnet

Antiferromagnetic metal

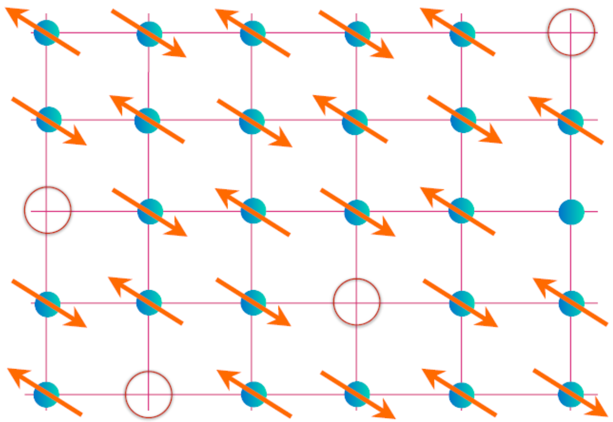


Broken symmetry
Luttinger area



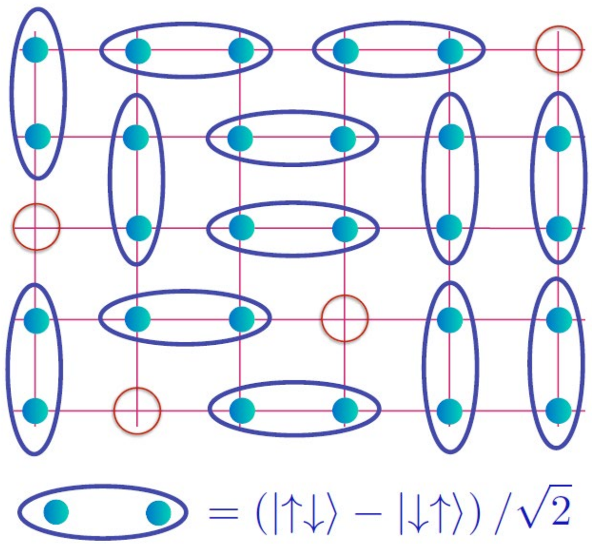
Doping an insulating antiferromagnet

Antiferromagnetic metal



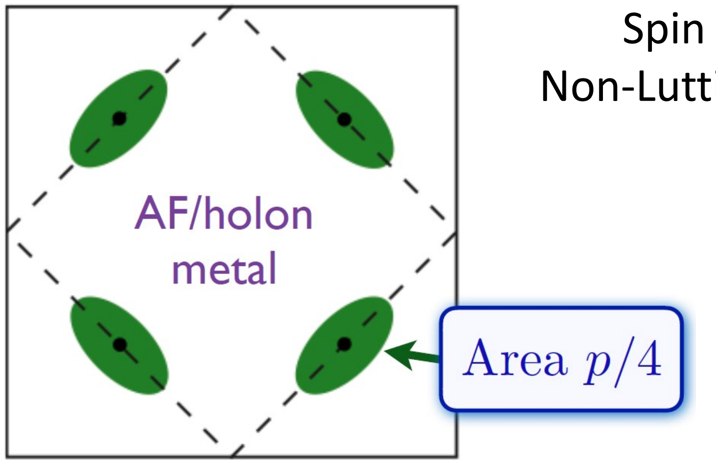
Broken symmetry
Luttinger area

Holon metal



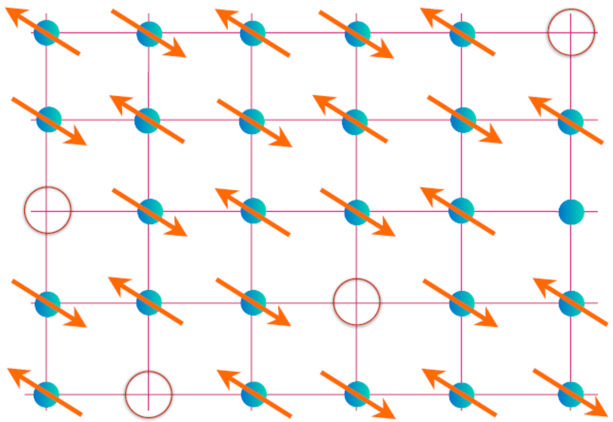
Spin liquid
Non-Luttinger area

Spinless charge-e carriers
in spin-liquid background
(with spinons)

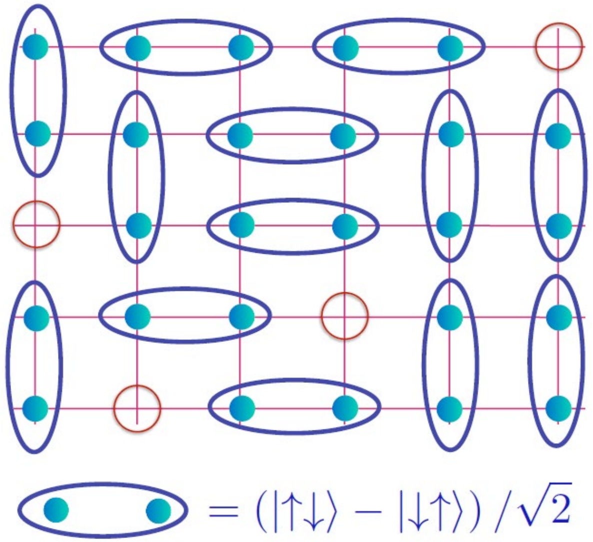


Doping an insulating antiferromagnet

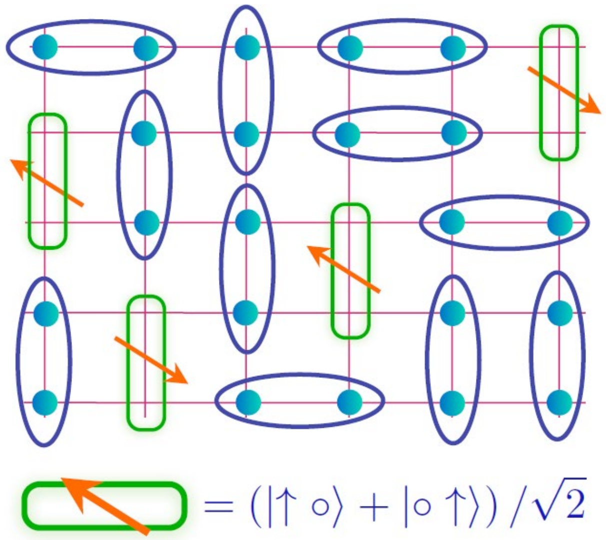
Antiferromagnetic metal



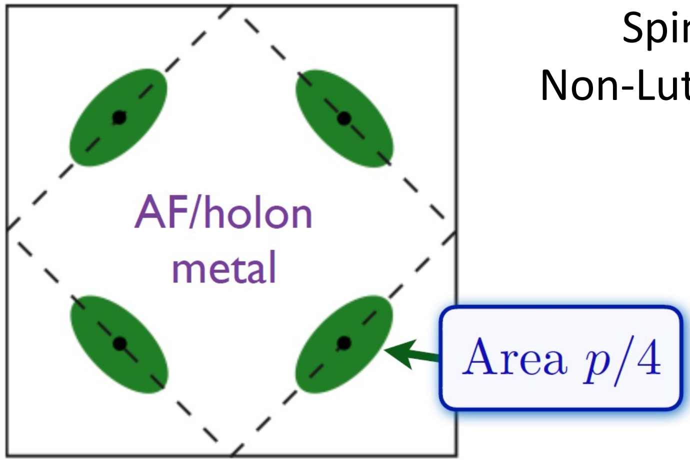
Holon metal



Fractionalized Fermi liquid (FL*)



Broken symmetry
Luttinger area



Spin liquid
Non-Luttinger area

Spin-1/2 charge-e carriers
in spin-liquid background

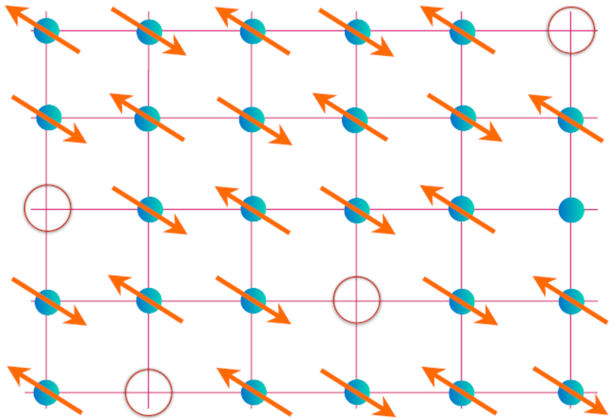
Senthil/Sachdev/Vojta, PRL **90**, 216403 (2003)
Punk/Allais/Sachdev, PNAS **112**, 9552 (2015)

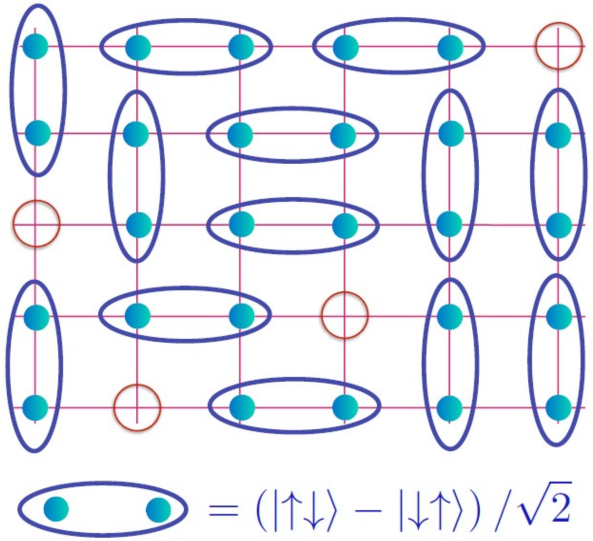
Doping an insulating antiferromagnet

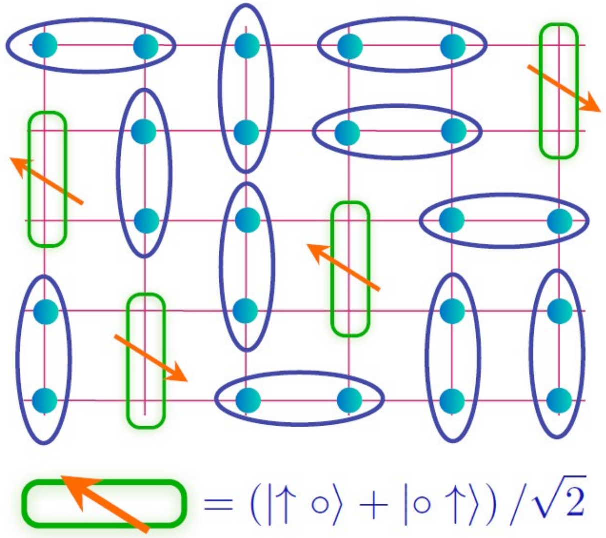
Antiferromagnetic metal

Holon metal

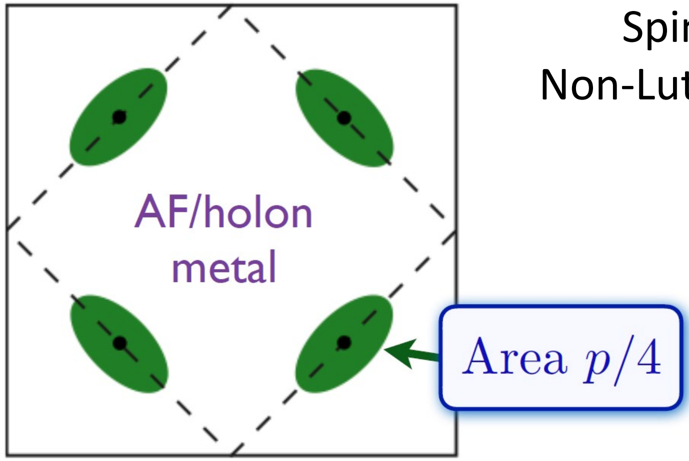
Fractionalized Fermi liquid (FL*)



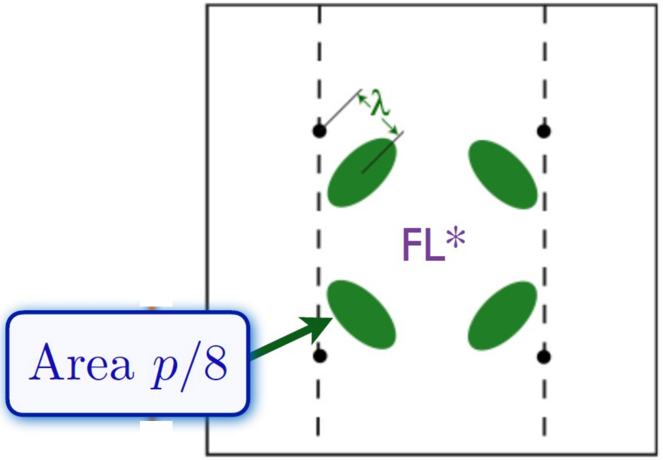




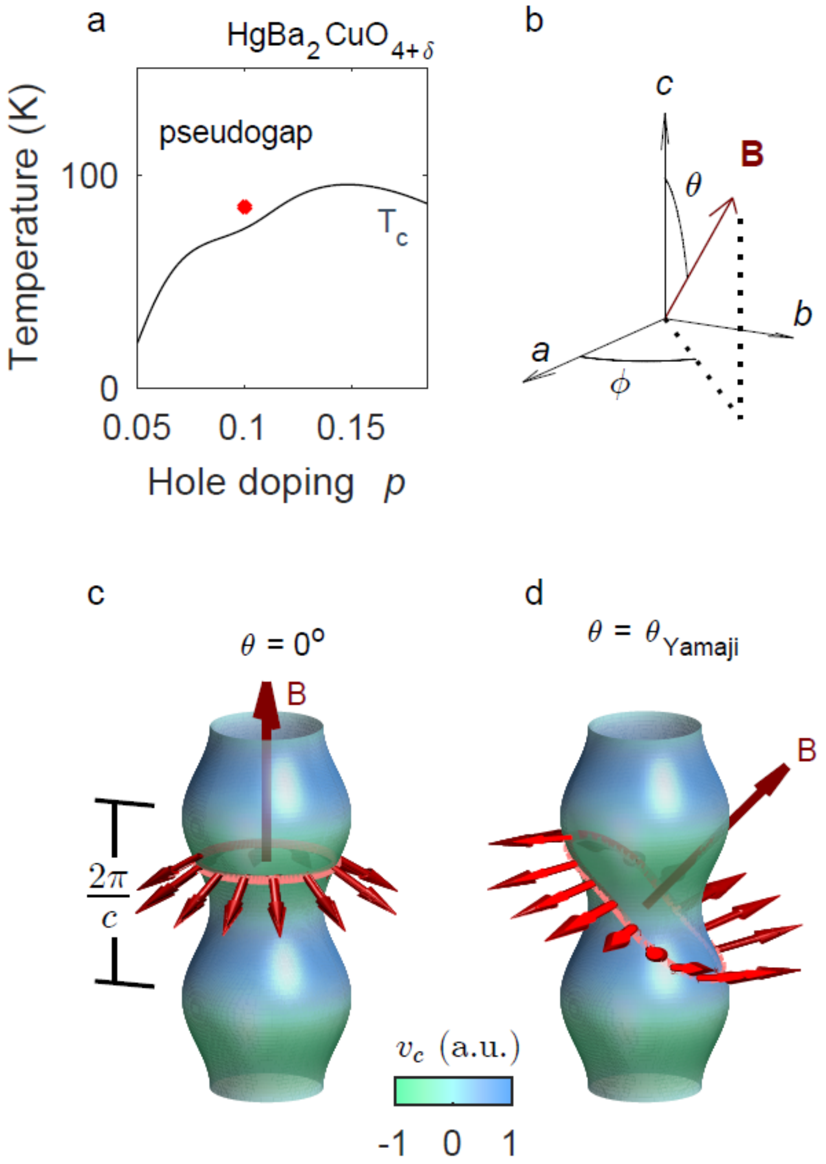
Broken symmetry
Luttinger area



Spin liquid
Non-Luttinger area



Cuprates: Yamaji effect

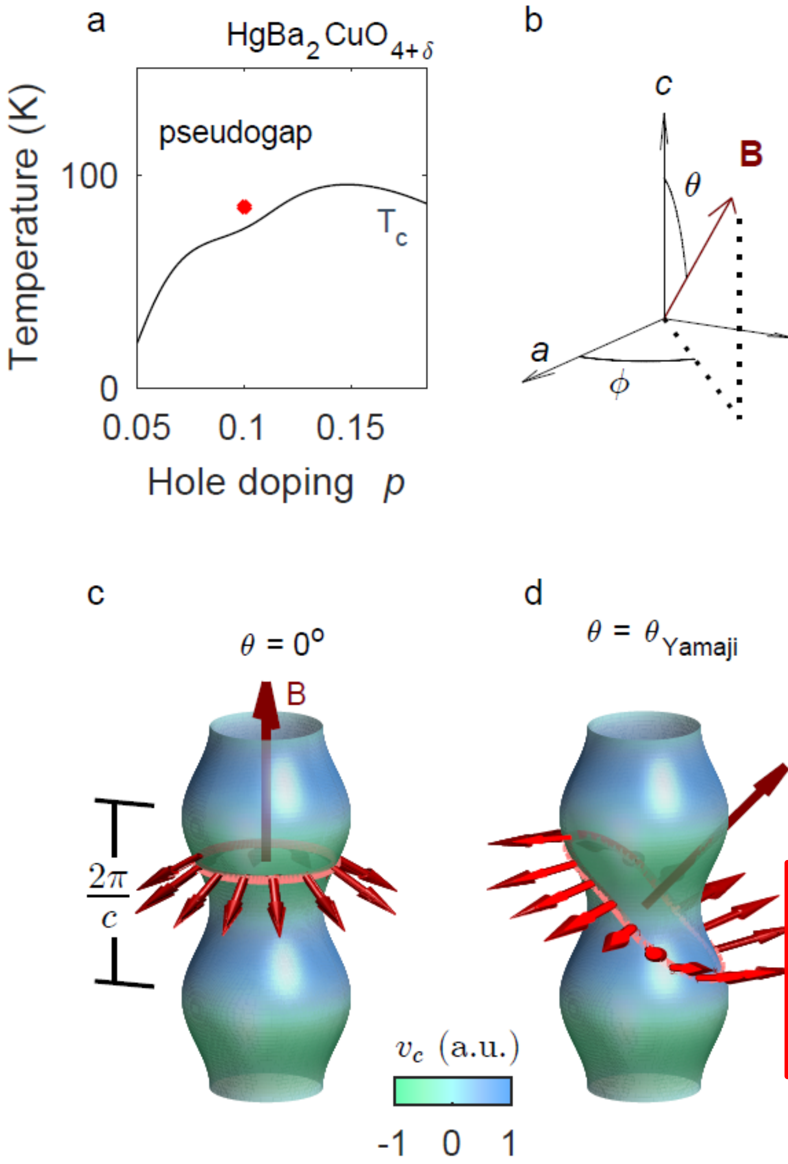


At the Yamaji angle, the orbits in the plane orthogonal to B have an area which is independent of the momentum in c direction, to first order in the hopping along c.

Yamaji, J. Phys. Soc. Jpn. **58**, 1520 (1989)

Chan/Harrison *et al*, Nat. Phys. **21**, 1753 (2025)

Cuprates: Yamaji effect



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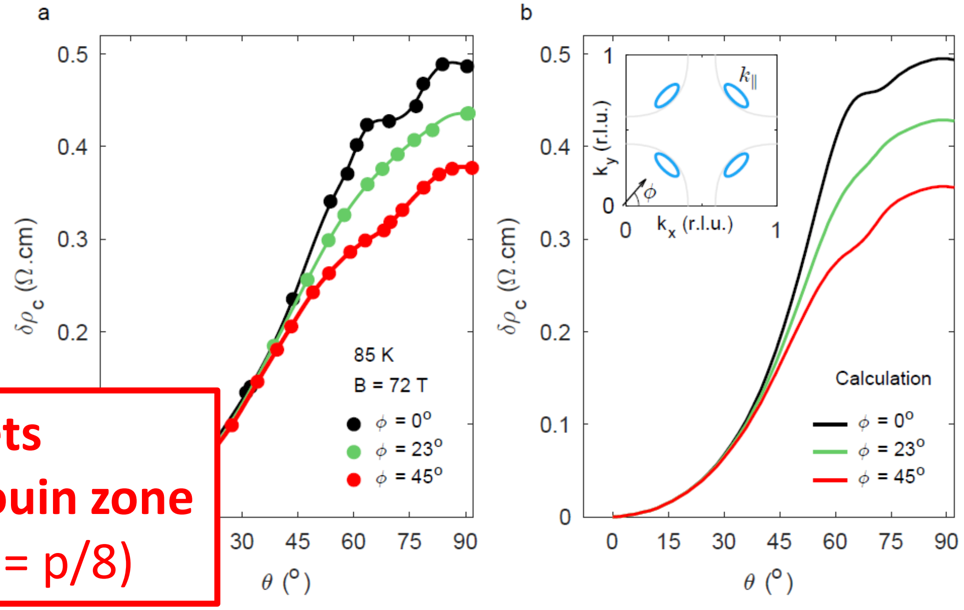
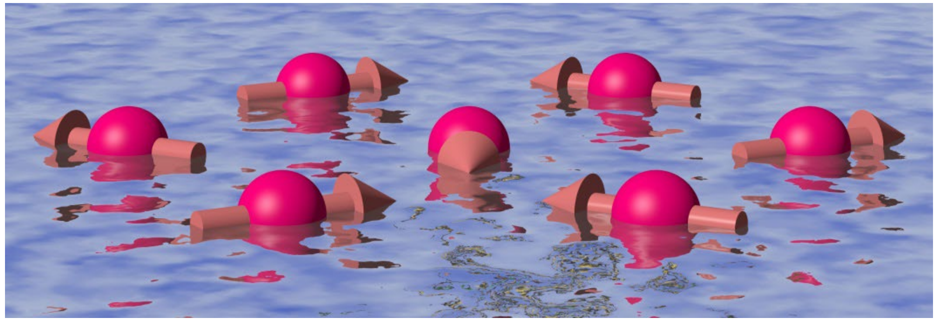
Yamaji, J. Phys. Soc. Jpn. **58**, 1520 (1989)



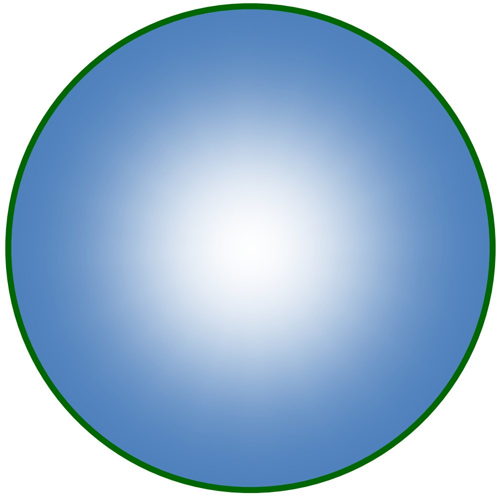
Closed Fermi pockets
with area of 1.3% of Brillouin zone
(doping p=10% → area = p/8)

Chan/Harrison *et al*, Nat. Phys. **21**, 1753 (2025)

Cuprate pseudogap: A fractionalized Fermi liquid



+



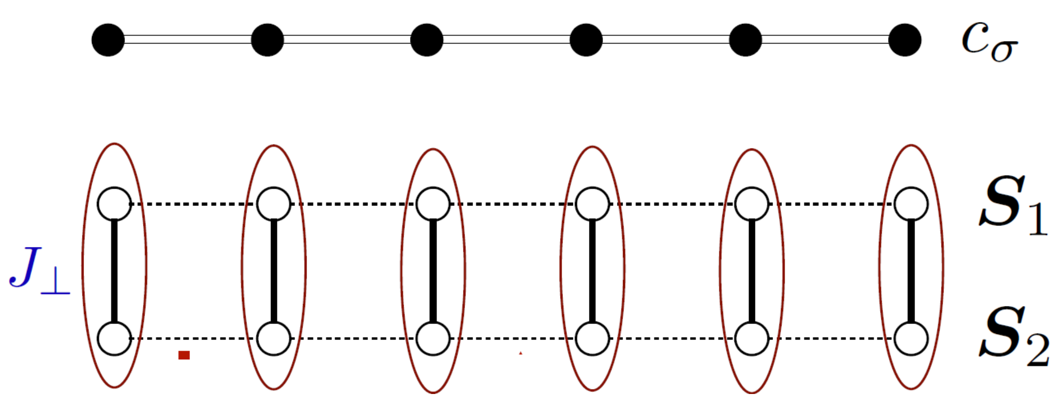
Scenario for low-doping pseudogap state:

„Exotic“ state with topological order,

where local-moment spin liquid co-exists with hole-like quasiparticles

(technically a fractionalized Fermi liquid, FL*)

Ancilla theory of single-band FL*

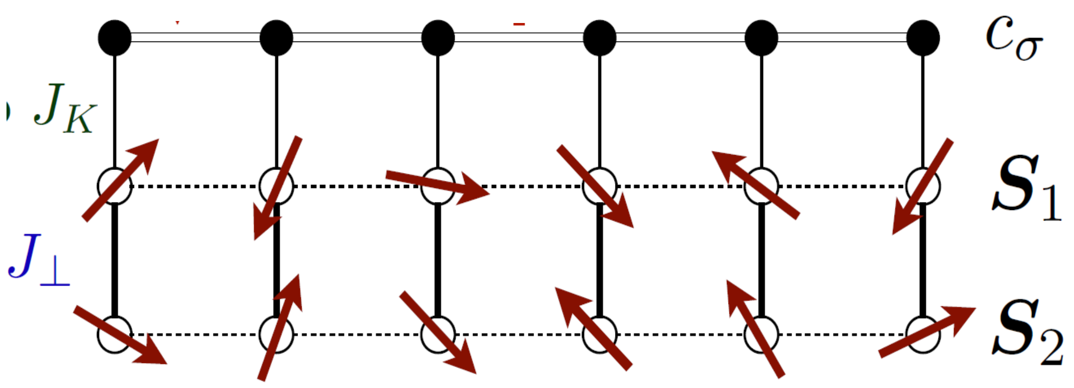


Hubbard band with electron filling (1-*p*)

Two layers of „ancilla spins“ (qubits)

Zhang/Sachdev, PRR **2**, 023172 (2020)
related to Yang/Rice/Zhang, PRB **73**, 174501 (2006)

Ancilla theory of single-band FL*



Free-electron band with electron filling (1-p)

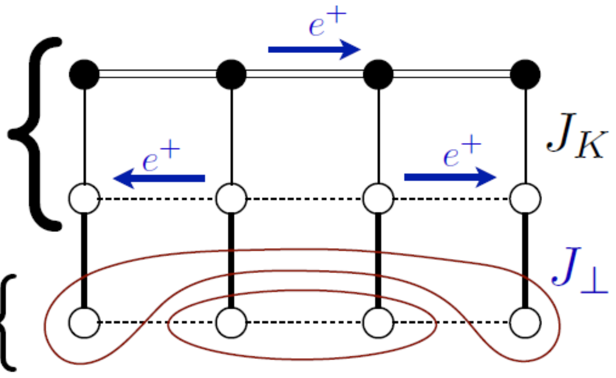
Bilayer antiferromagnet

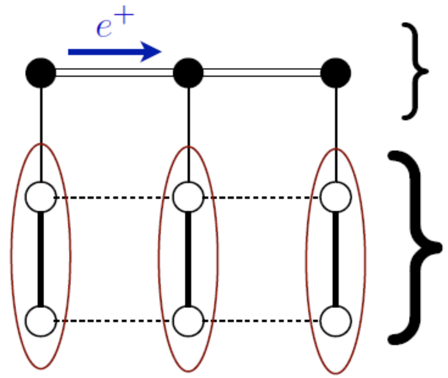
$J_\perp \ll J_K$

$J_\perp \gg J_K$

Kondo lattice with
(2+p) = p (mod 2) holes,
Non-Luttinger volume → FL*

Spin liquid





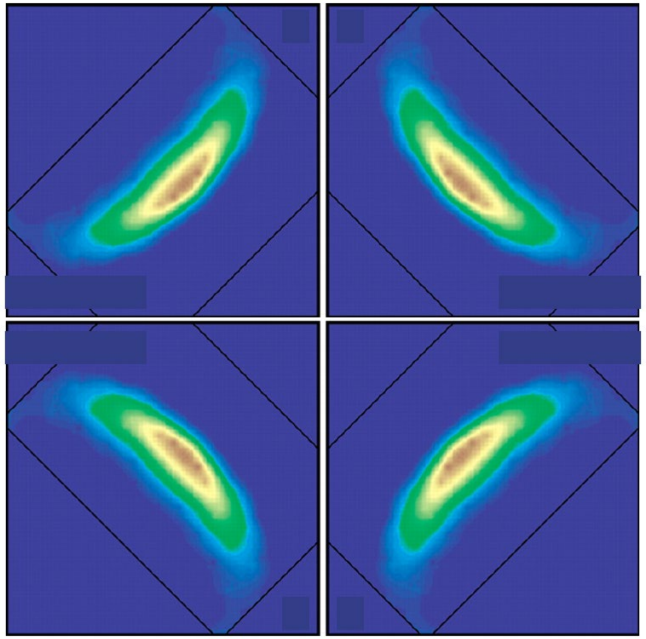
Band with (1+p) holes,
Luttinger volume → FL

Trivial insulator

$J_\perp / J_K$

Pseudogap Fermi surface

Photoemission experiment

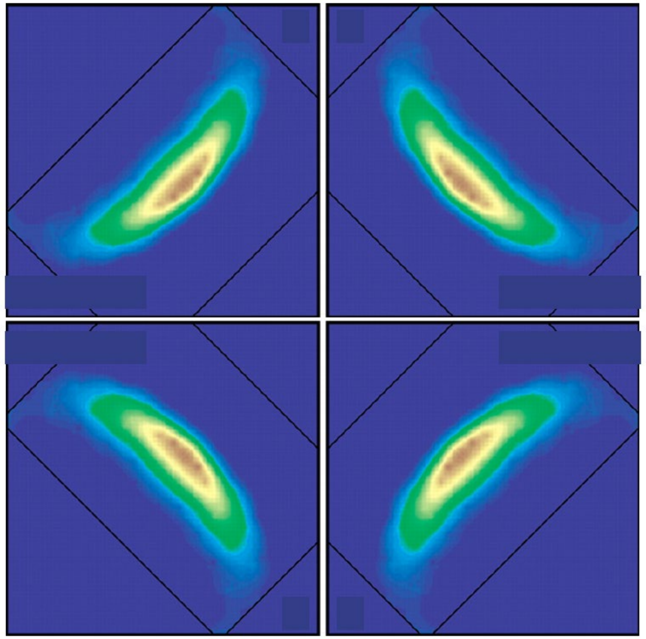


$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$

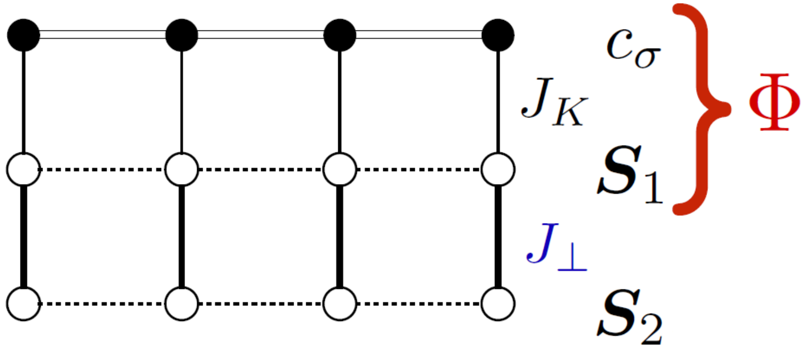
Shen *et al.*, Science **307**, 901 (2007)

Pseudogap Fermi surface

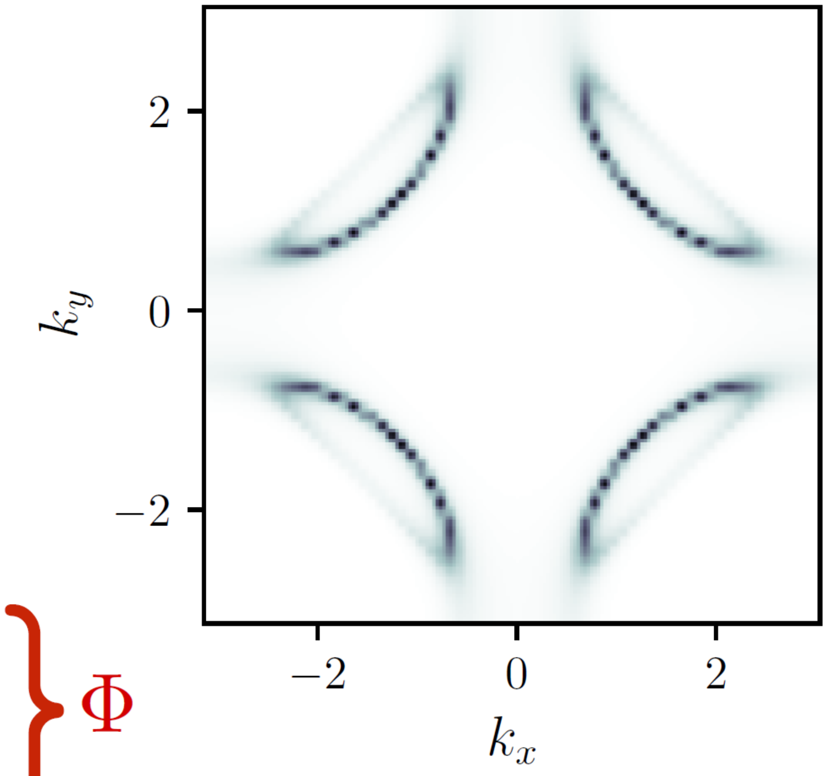
Photoemission experiment



$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$



Ancilla theory for FL*
(mean-field)

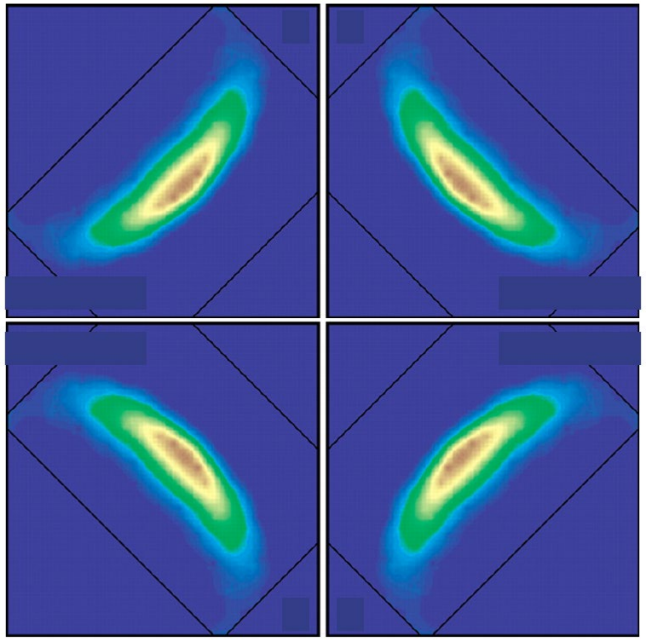


Shen *et al.*, Science **307**, 901 (2007)

Mascot *et al.*, PRB **105**, 075146 (2022)

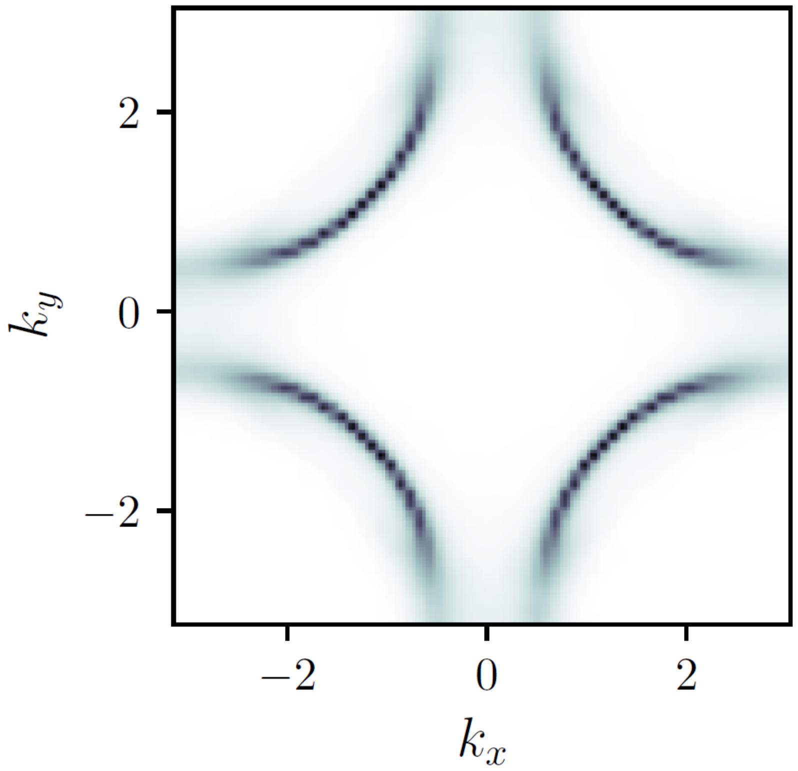
Pseudogap Fermi surface

Photoemission experiment

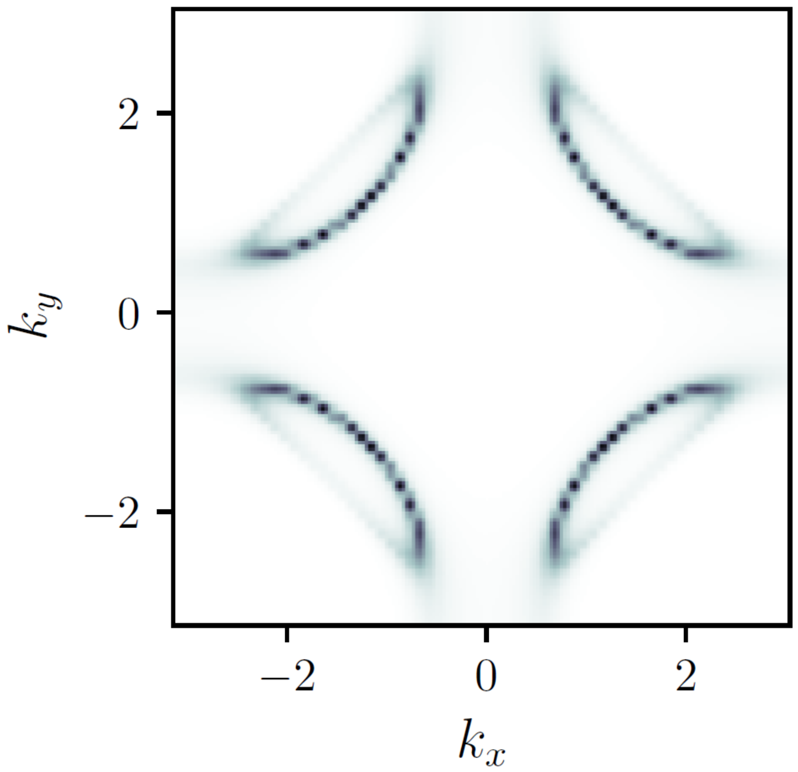


$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$

Ancilla theory for FL* (w/ fluctuations)



Ancilla theory for FL* (mean-field)



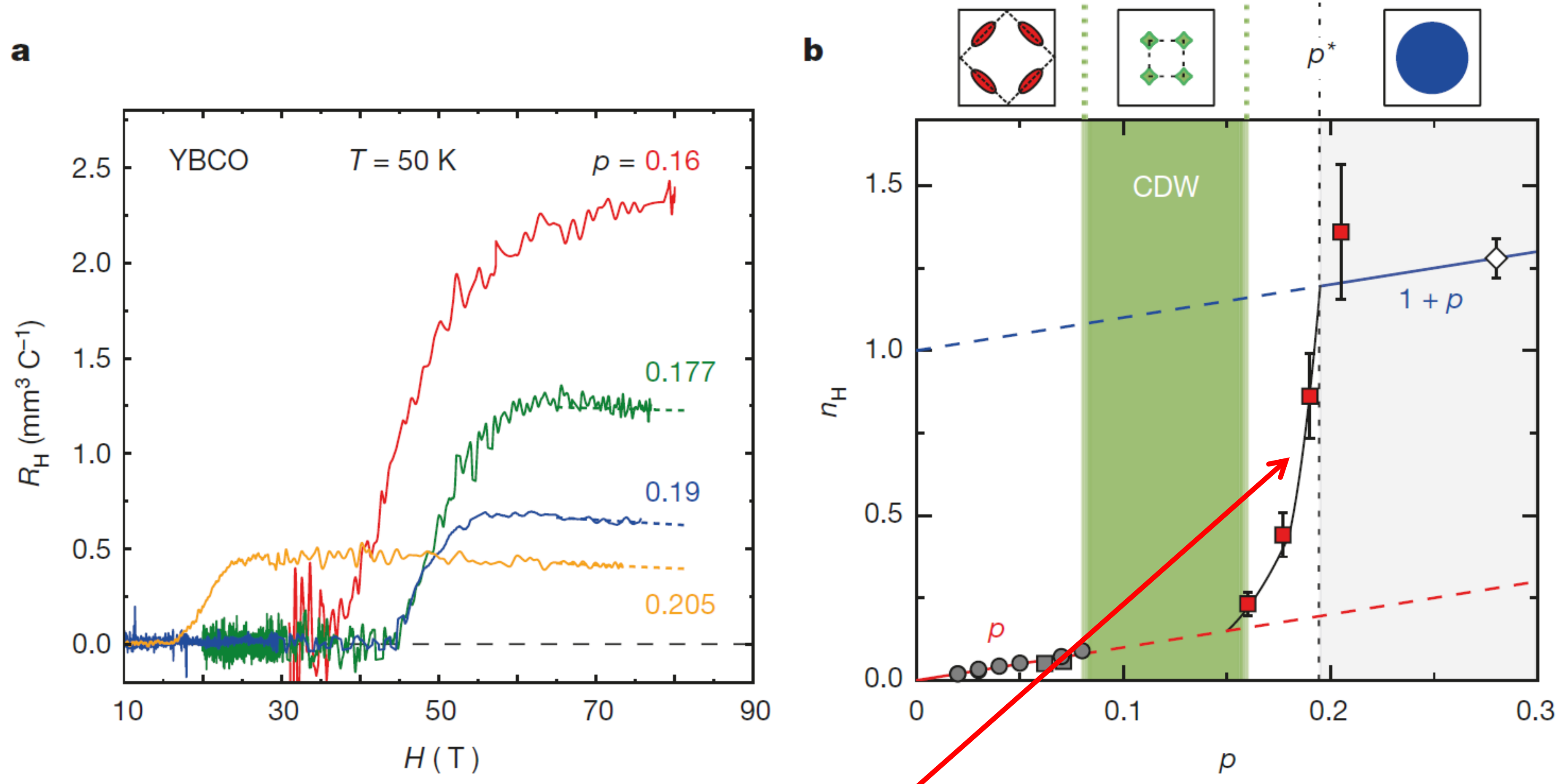
Shen *et al.*, Science **307**, 901 (2007)

Pandey *et al.*, arXiv:2507.05336

Mascot *et al.*, PRB **105**, 075146 (2022)

Fermi-surface reconstruction: More experiments

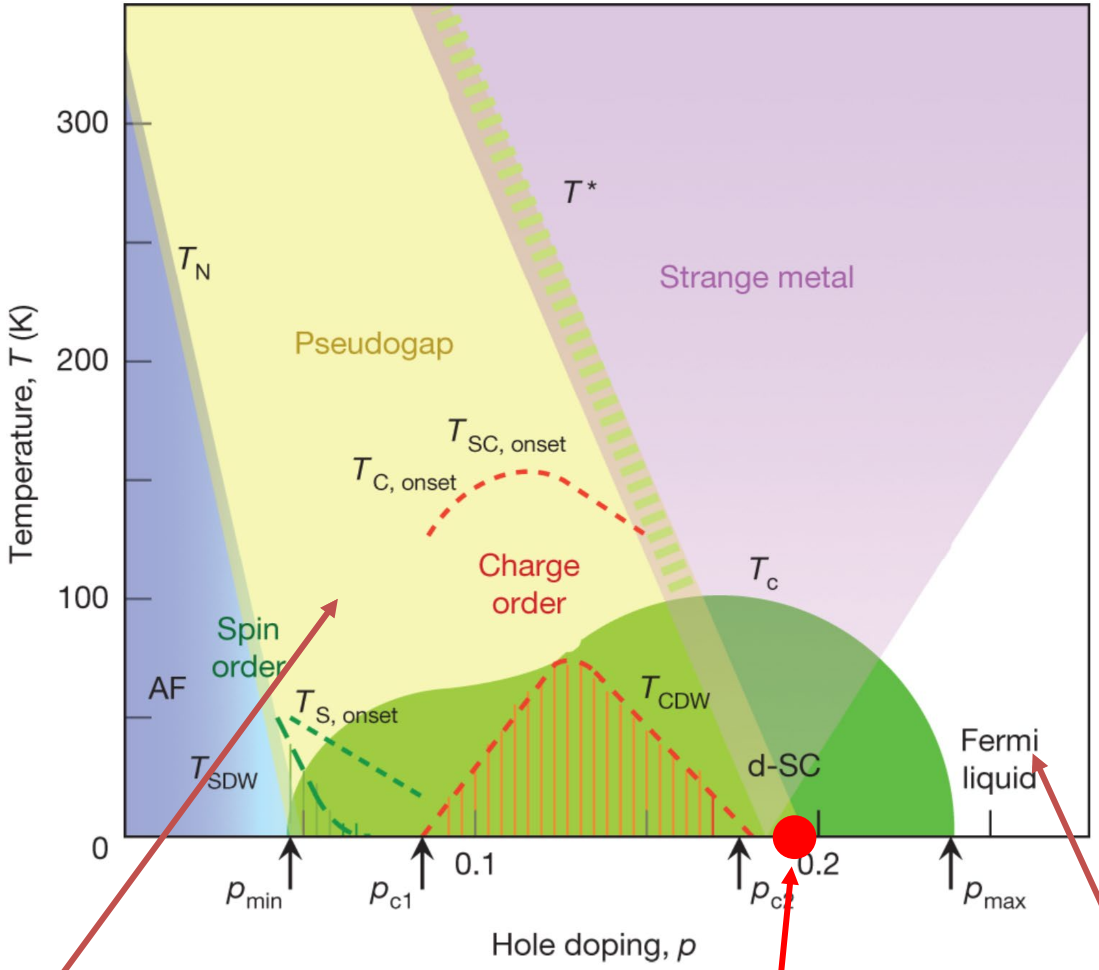
Hall-effect measurements in YBCO in high magnetic field



Normal-state carrier (hole) concentration changes from *p* to (1+*p*)

Badoux / Taillefer *et al.*, Nature **351**, 210 (2016)

Cuprate quantum criticality

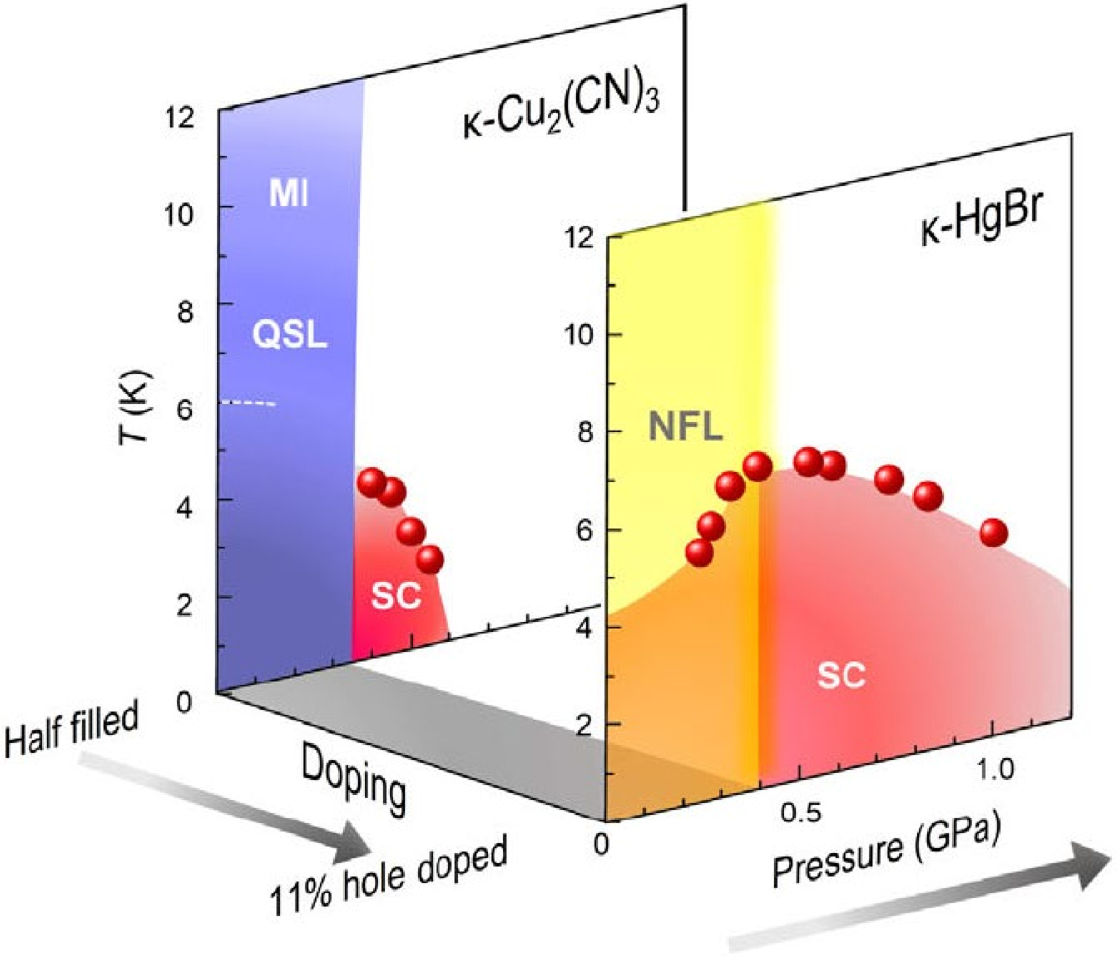


Underdoped normal state is FL*, with Fermi pockets

Quantum phase transition with reconstruction of Fermi surface

Overdoped normal state is FL, with (hole) Fermi volume (1+p)

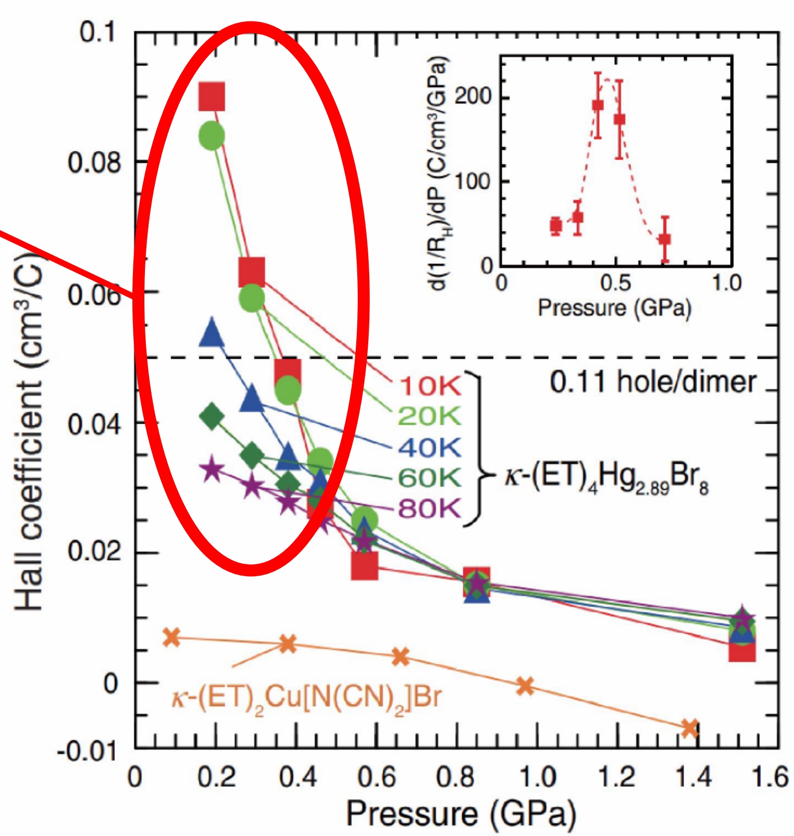
Doped Mott organics



κ-(BEDT-TTF)$_4$Hg$_{2.89}$Br$_8$

Hall effect evolution

Crossover towards FL*



Modelling: Work in progress

Oike *et al*, PRL **114**, 067002 (2015)
Suzuki *et al*, PRX **12**, 011016 (2022)

Mott criticality

Bandwidth-controlled Mott transition: Questions

ct.qmat

Julius-Maximilians-UNIVERSITÄT WÜRZBURG

TECHNISCHE UNIVERSITÄT DRESDEN

- Conductivity always finite at $T>0$ → transition only well defined at ***$T=0$***?
- What is fate of **Fermi surface** at transition?
  What happens to low-energy quasiparticles?
- Is **magnetism** relevant for transition?

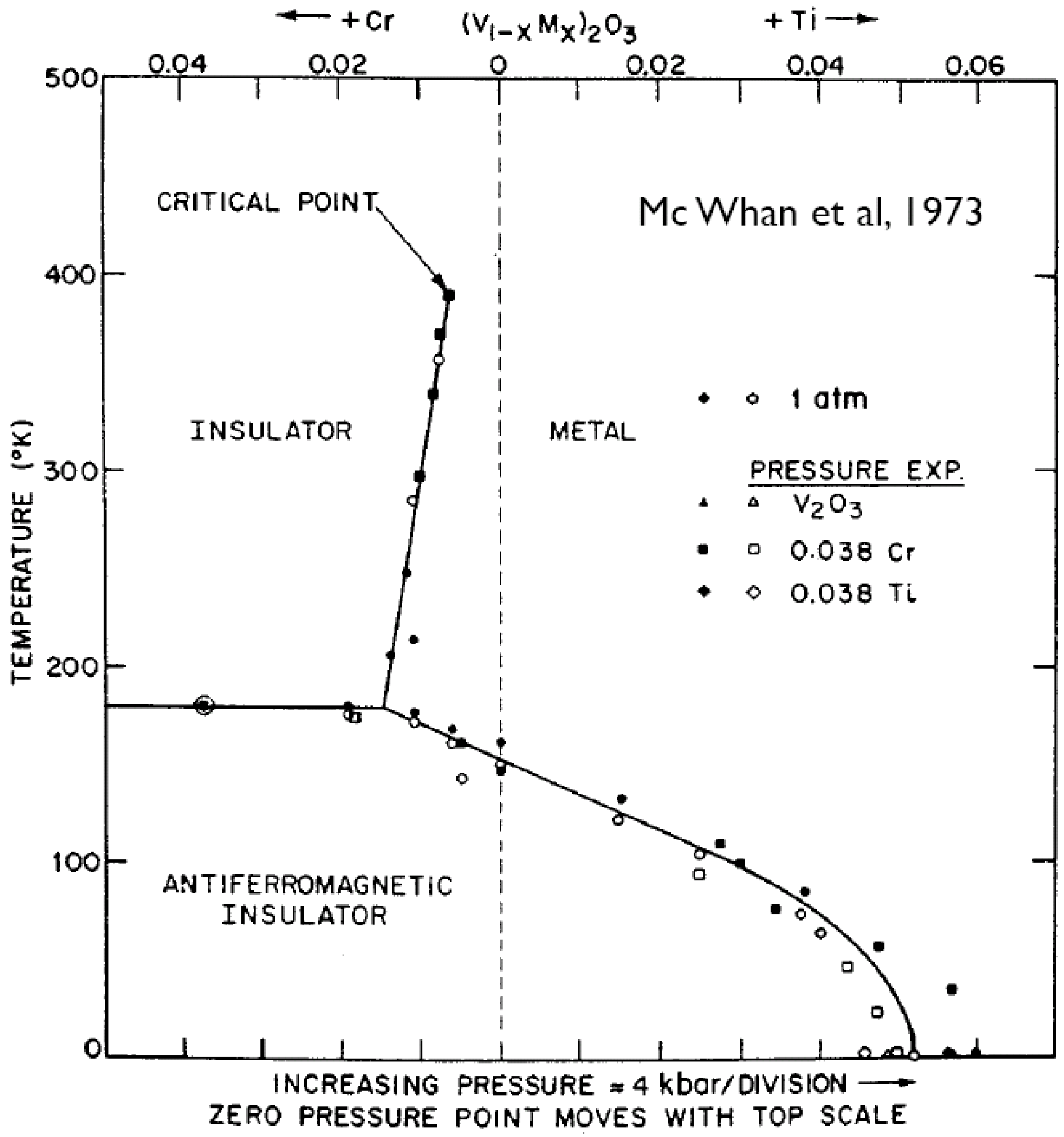
…

- Does genuine **Mott quantum criticality** exist?
  (metal ←→ topological spin liquid)

Senthil, PRB **78**, 045109 (2008)
Qi / Sachdev, PRB **77**, 165112 (2008)
Mishmash *et al*, PRB **91**, 235140 (2015)
+ many others

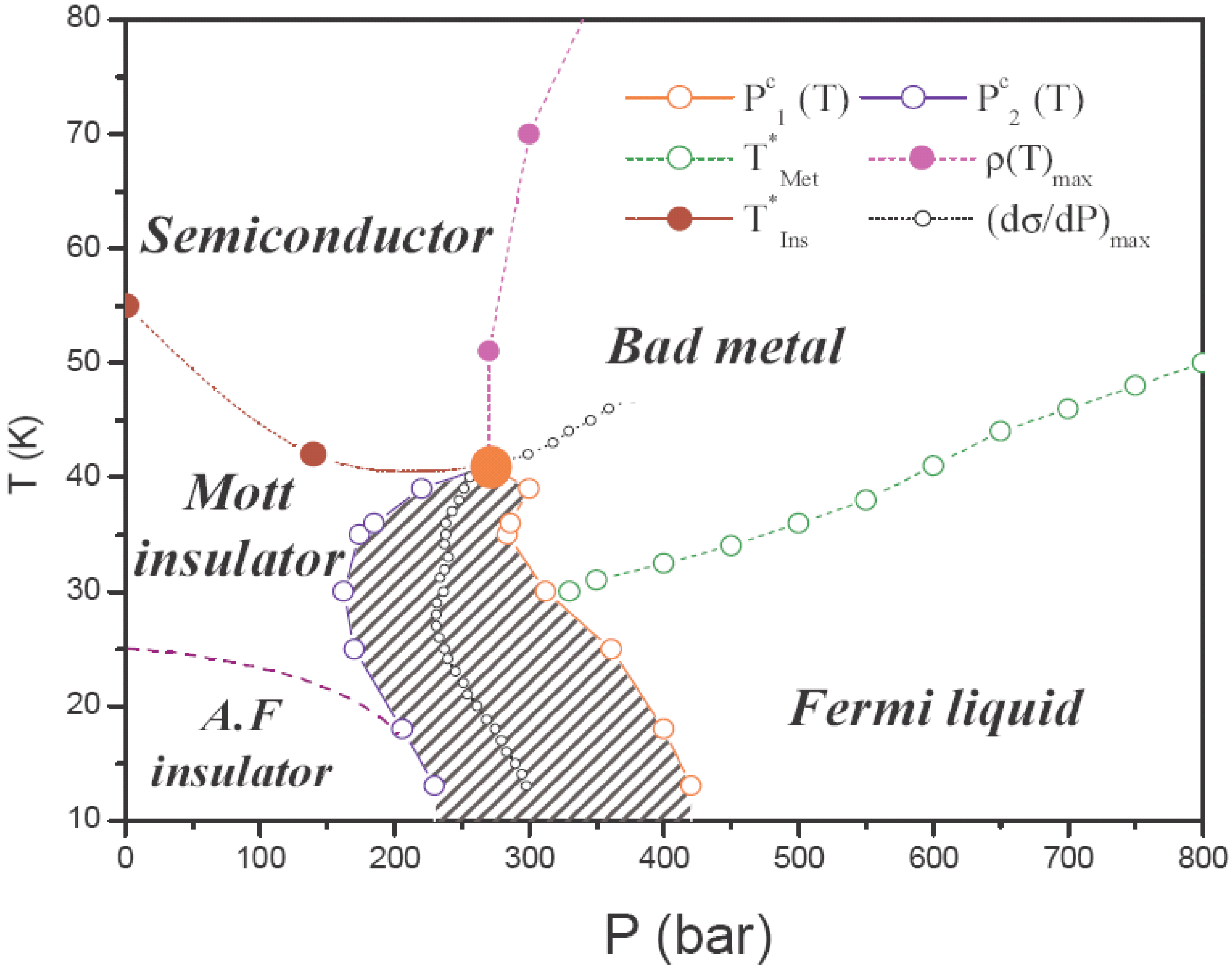
Mott transition: Experiments

V_2O_3



Mott transition: Experiments

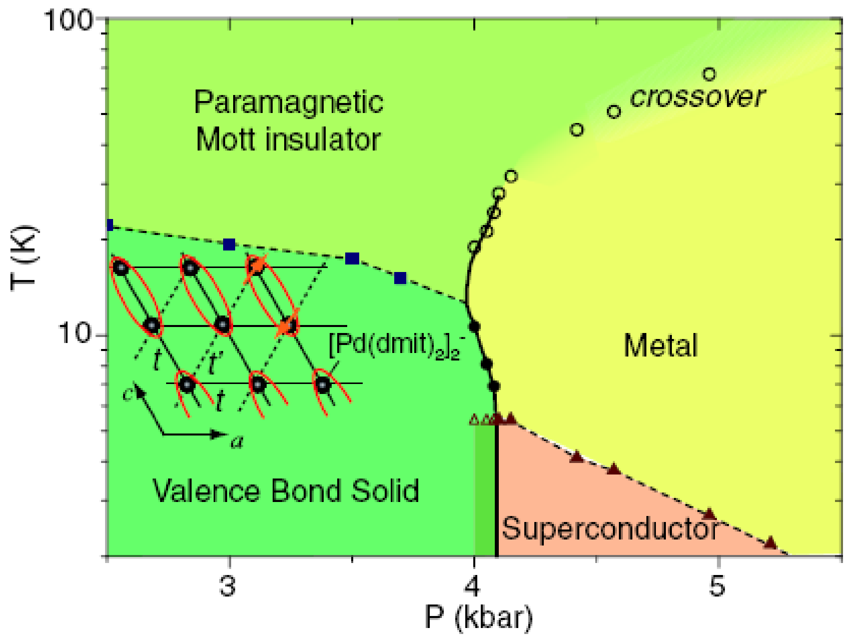
κ-$(BEDT\text{-}TTF)_2Cu[N(CN)_2]Cl$



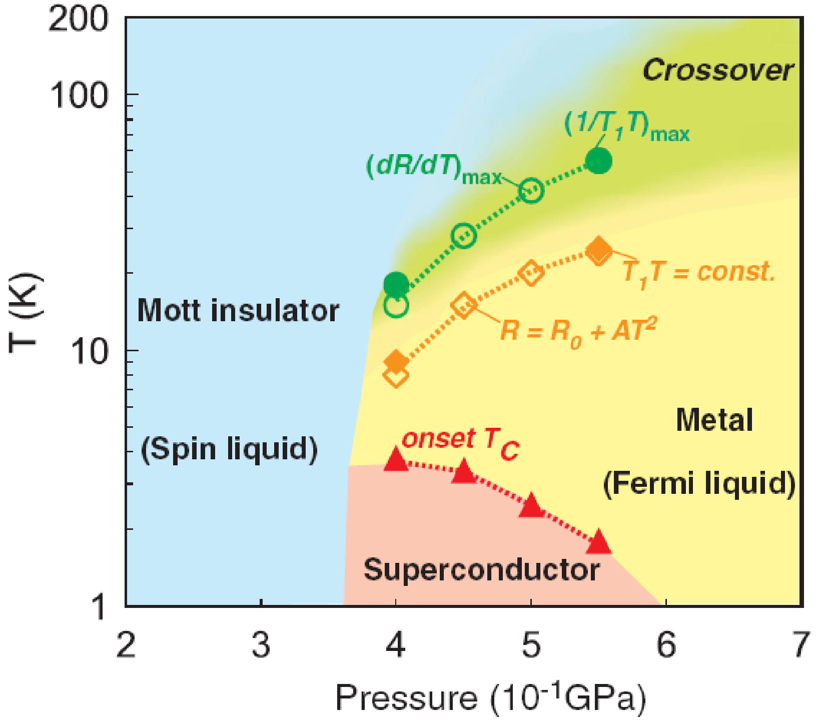
Limelette *et al*, PRL **91**, 016401 (2003)

Mott transition: Experiments

$[Pd(dmit)_2]_2$



κ-$(ET)_2Cu_2(CN)_3$



Shimizu *et al.*, PRL (2007)

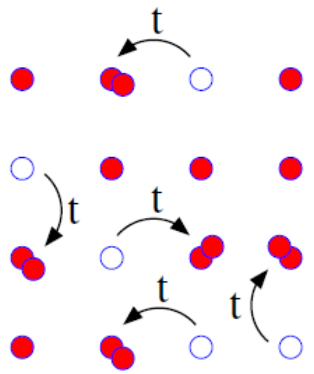
Kurosaki *et al.*, PRL (2005)

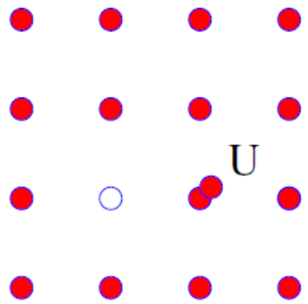
Hubbard model

$$H = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Kinetic energy
(bandwidth $W \sim t$)

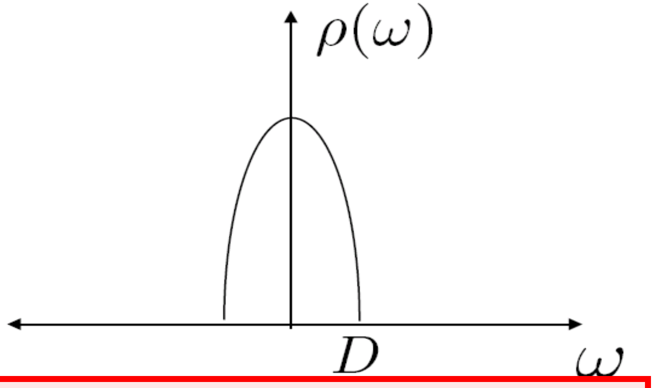
Coulomb repulsion
(local strength U)





Mott transition: Single-particle spectrum @ half-filling

Uncorrelated metal, U=0

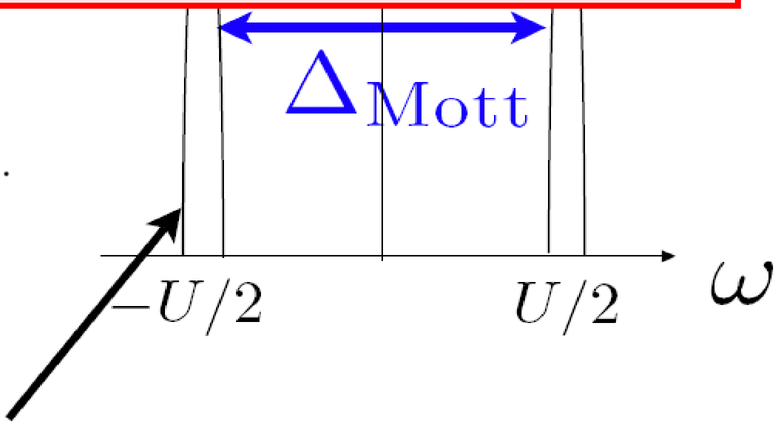


What happens at low energies at $U = U_c$?

- **Quasiparticle weight Z → 0 ?**
- **Effective mass m*→ ∞ ?**
- **Mott gap Δ → 0 ?**

Mott insulator, U large
(close to atomic limit!)

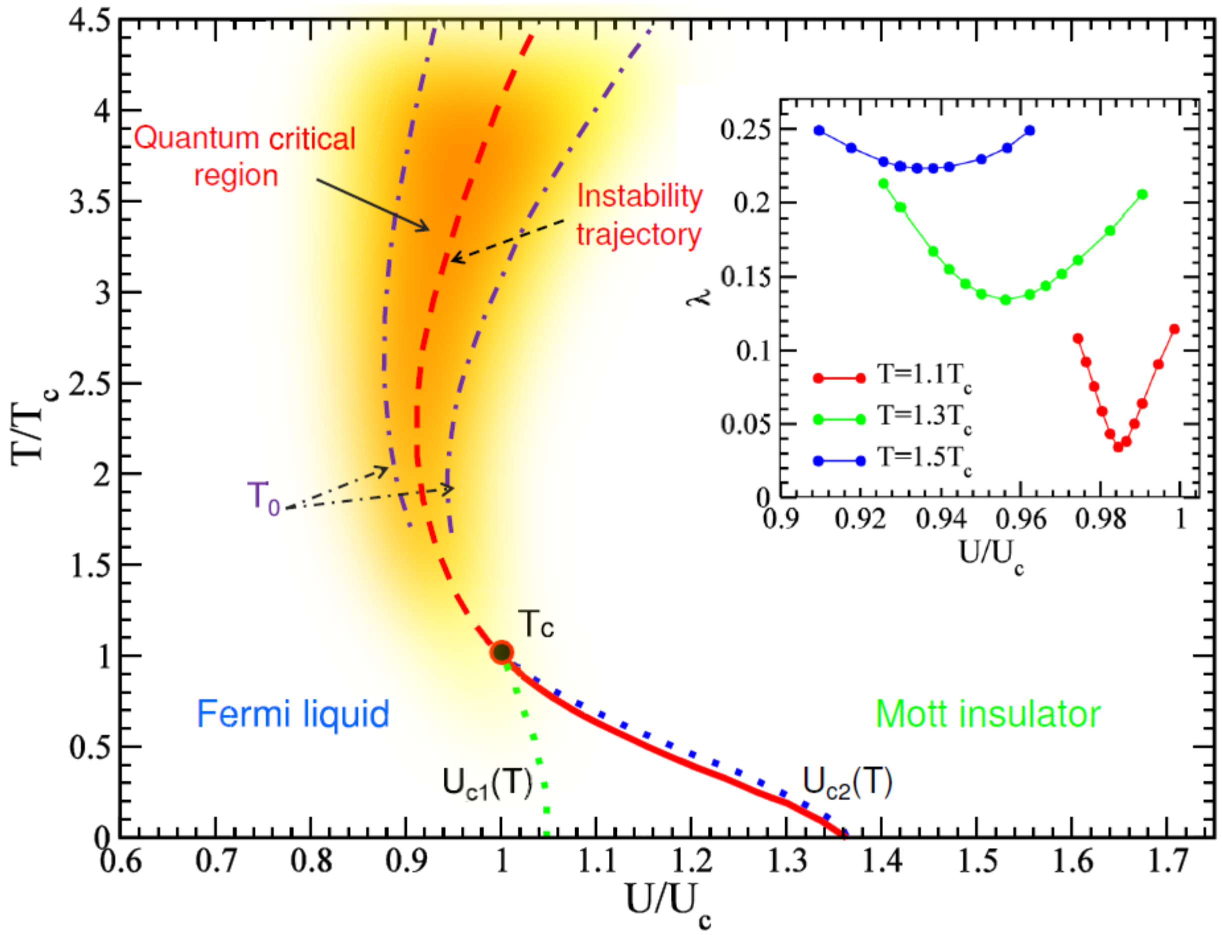
$$G(i\omega_n)_{\mathrm{at}} = \frac{1/2}{i\omega_n + U/2} + \frac{1/2}{i\omega_n - U/2}.$$



Hubbard bands
(reflect local moments)

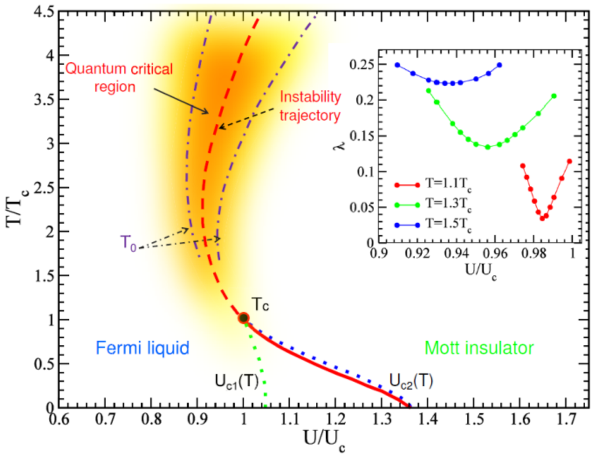
Mott transitions & quantum criticality?

Signs of **quantum** criticality in DMFT for single-band Hubbard model

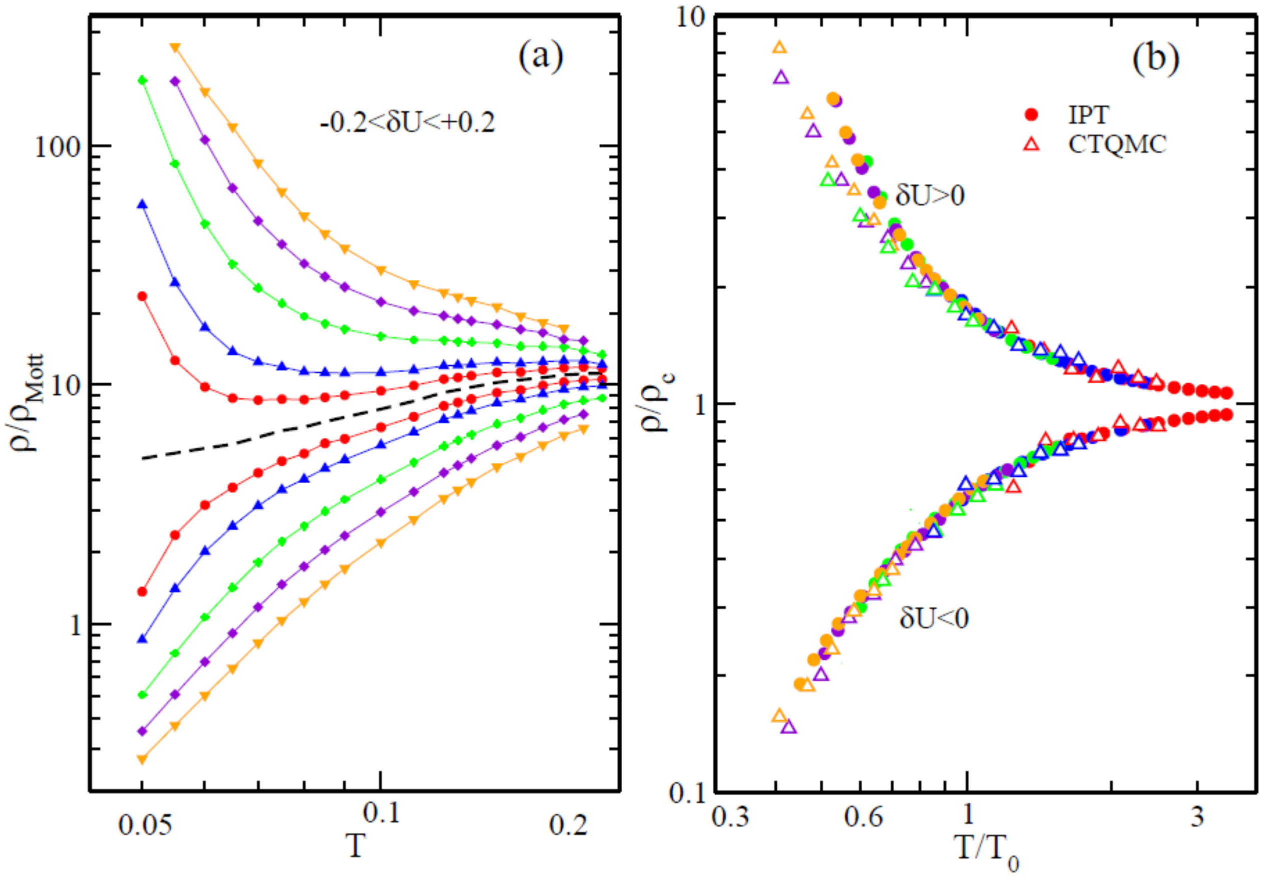


Terletska / Dobrosavljevic *et al*, PRL **107**, 026401 (2011)

Mott transitions & quantum criticality?



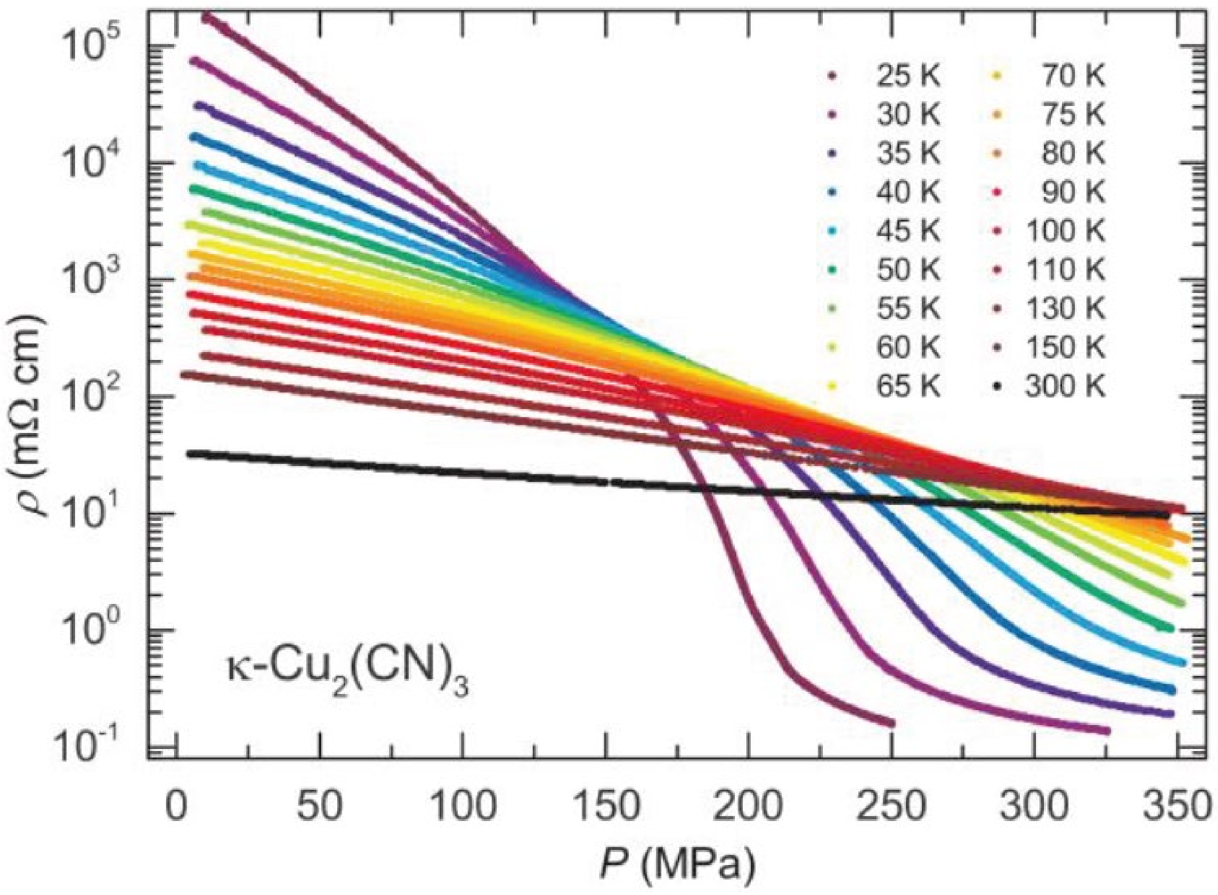
Resistivity near Mott endpoint



Terletska / Dobrosavljevic *et al*, PRL **107**, 026401 (2011)

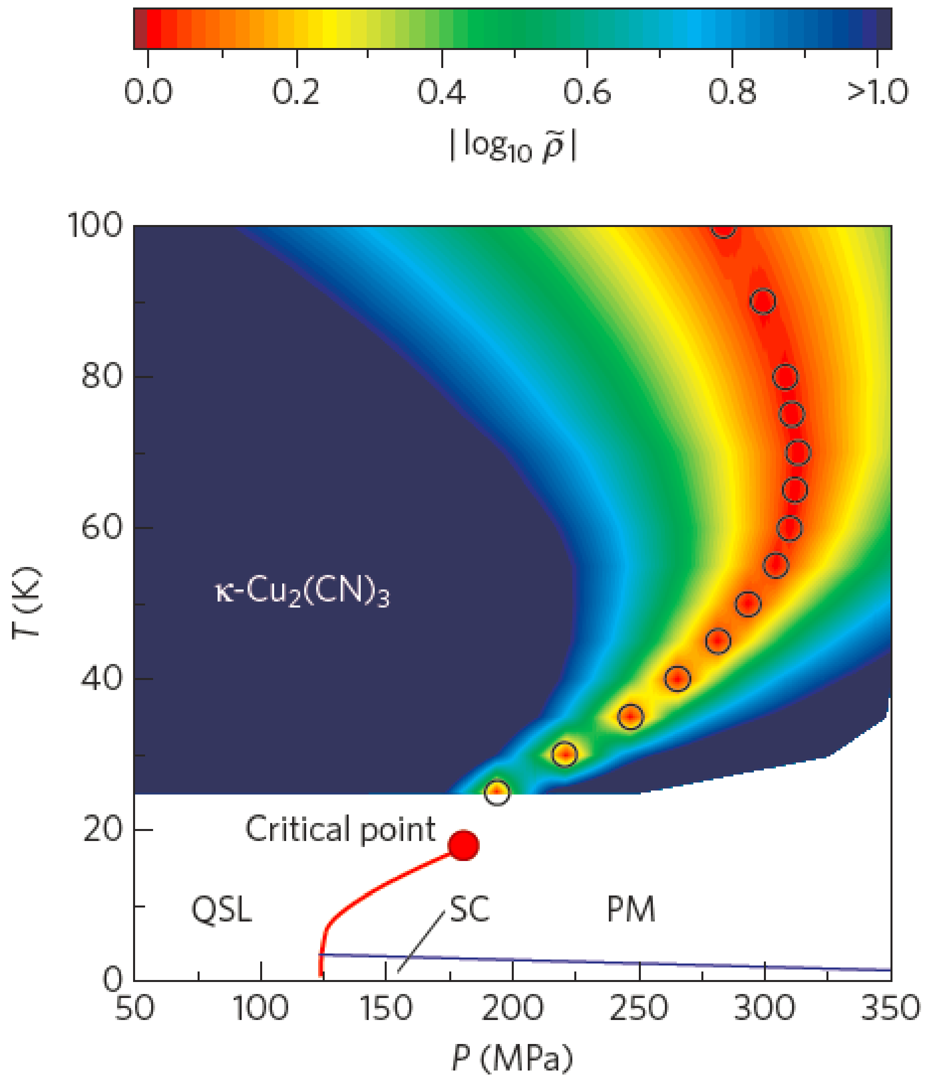
Mott transitions & quantum criticality!

κ-$(ET)_2Cu_2(CN)_3$
κ-$(ET)_2Cu[N(CN)_2]Cl$
$EtMe_3Sb[Pd(dmit)_2]_2$



Furukawa / Kanoda *et al.,* Nat. Phys. **11**, 221 (2015)

Mott transitions & quantum criticality!



κ-$(ET)_2Cu_2(CN)_3$

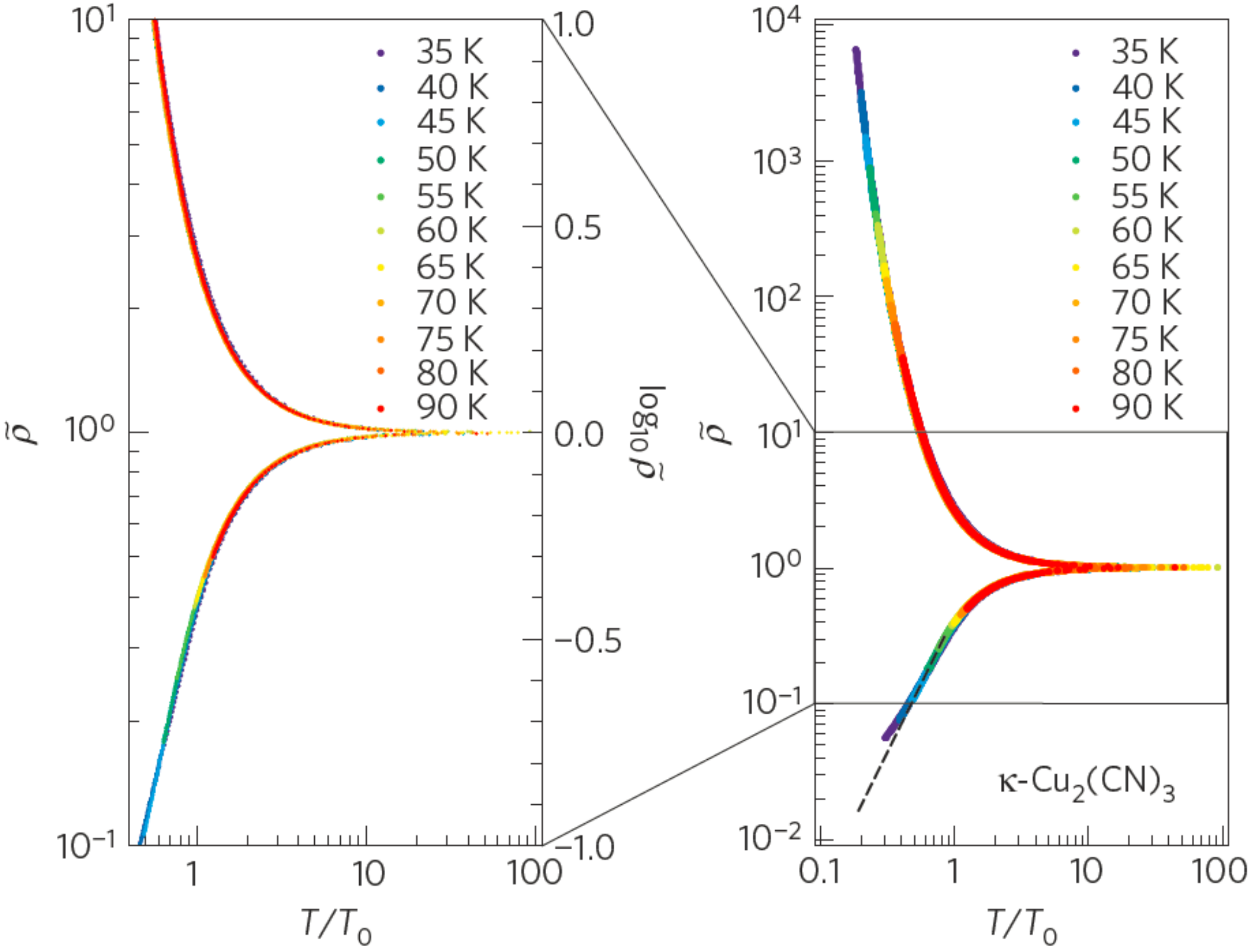
κ-$(ET)_2Cu[N(CN)_2]Cl$

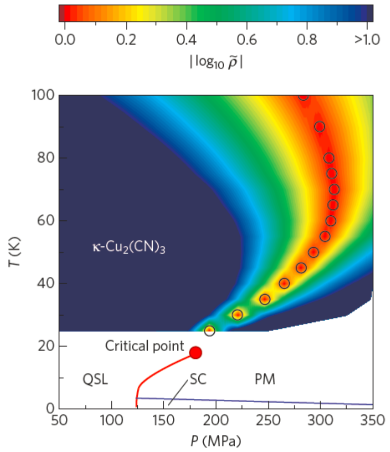
$EtMe_3Sb[Pd(dmit)_2]_2$

$$\tilde{\rho}(\delta P, T) \equiv \rho(\delta P, T)/\rho_c(T)$$
$$\rho_c(T) \equiv \rho(\delta P = 0,\ T)$$

Furukawa / Kanoda *et al.,* Nat. Phys. **11**, 221 (2015)

Mott transitions & quantum criticality!



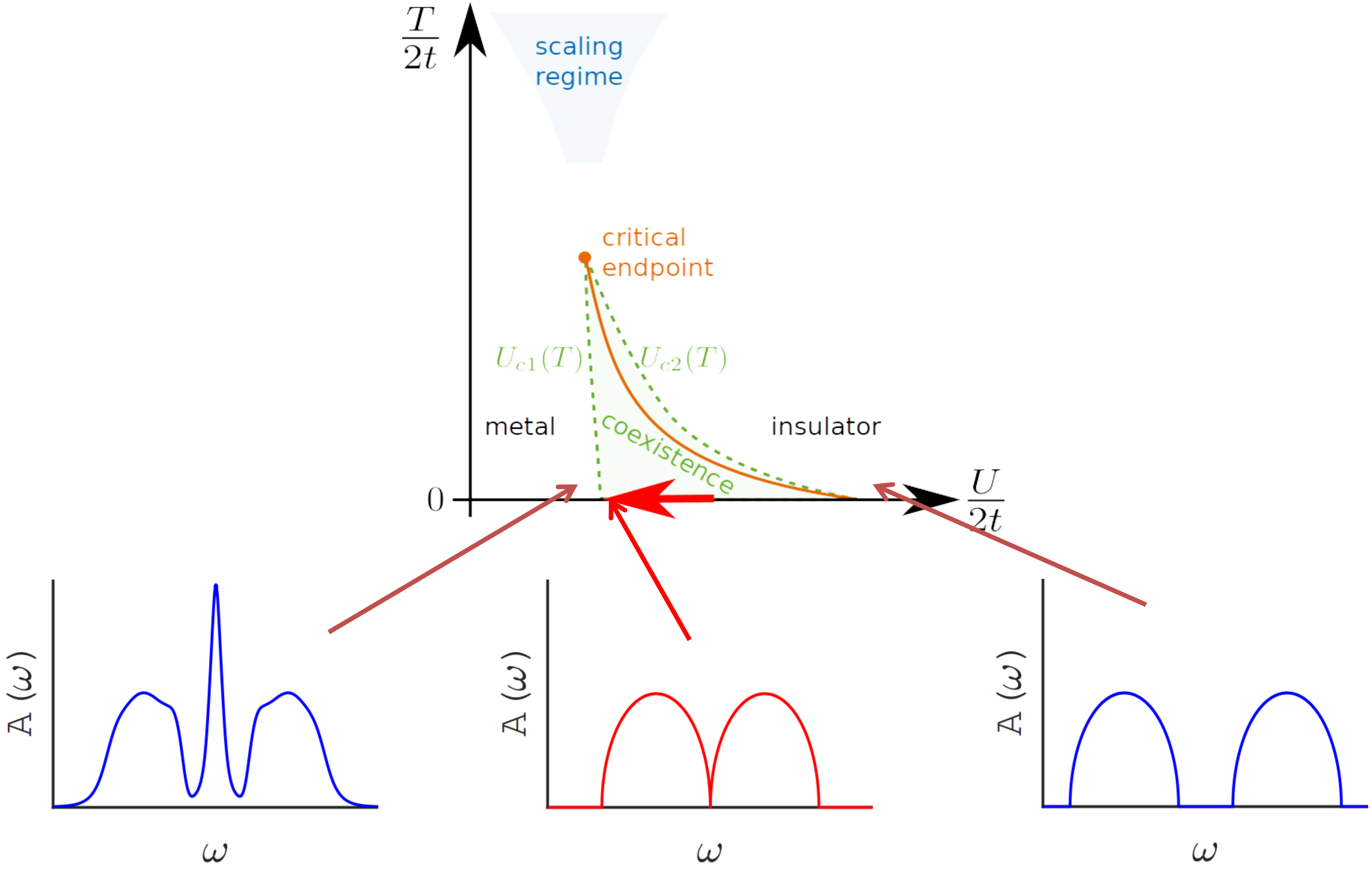


$$\tilde{\rho}(\delta P, T) \equiv \rho(\delta P, T)/\rho_c(T)$$
$$\rho_c(T) \equiv \rho(\delta P = 0,\ T)$$

$$T/T_0 = T/|c\delta P|^{z\nu}$$
$$z\nu = 0.62$$

Furukawa / Kanoda *et al.,* Nat. Phys. **11**, 221 (2015)

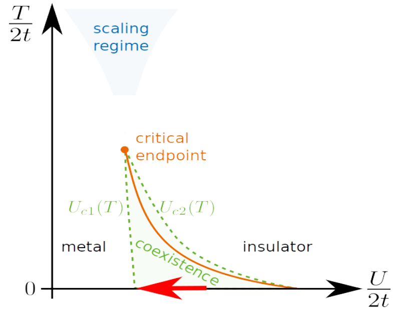
Scale-invariant single-particle spectra? (frequency power laws)



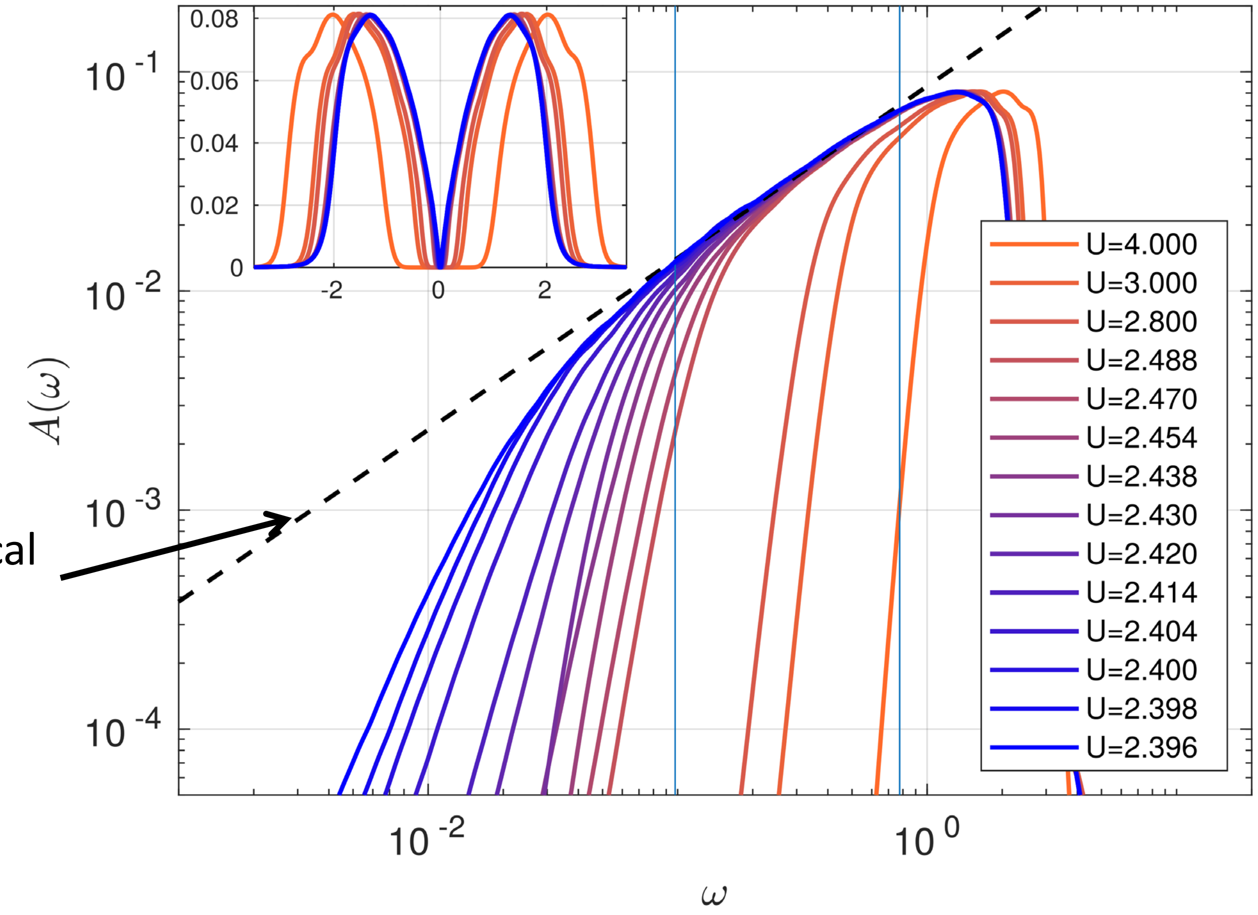
Metastable insulator at $U \rightarrow U_{c1}^{+}$
could develop frequency power laws (!)

Metastable insulator: DMFT results

Single-particle spectrum of insulating solution at T=0 upon approaching U_{c1}



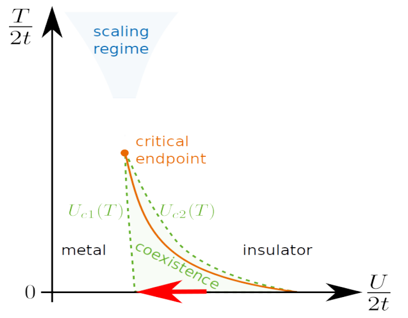
quantum critical power law

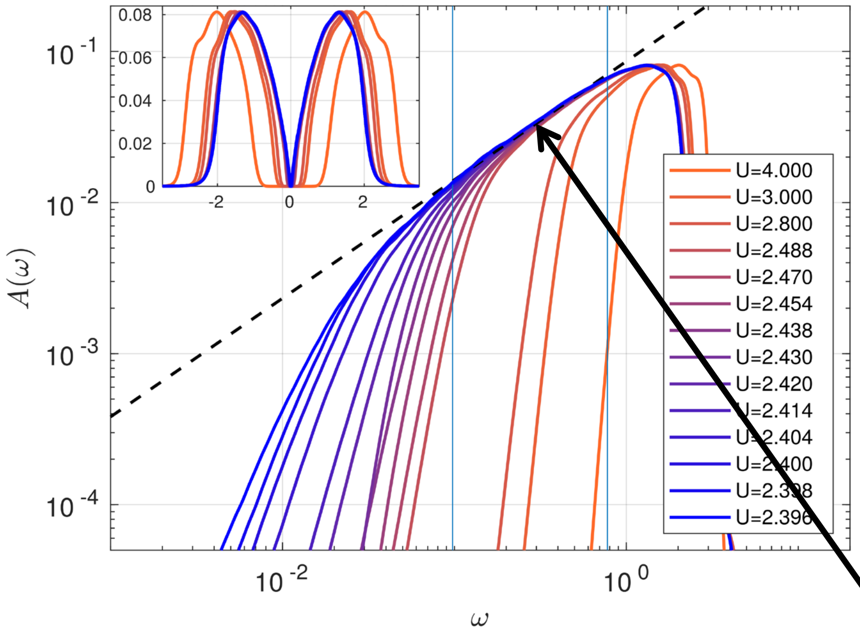


Eisenlohr / Lee / Vojta, PRB **100**, 155152 (2019)

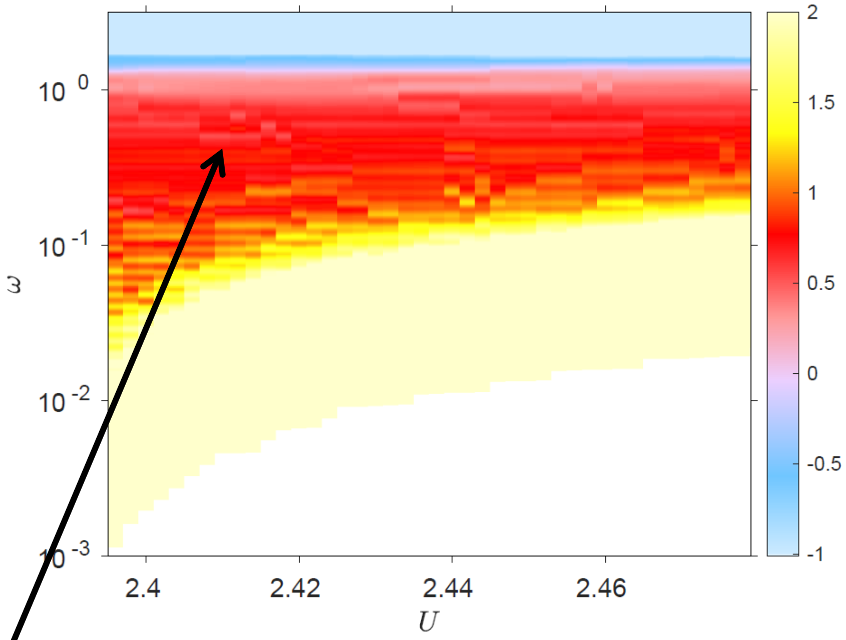
Metastable insulator: DMFT results

Single-particle spectrum of insulating solution at T=0 upon approaching U_{c1}





$$\frac{d\log A(\omega)}{d\log\omega}$$



quantum critical power law

Resistivity from DMFT

$$\sigma(0) = \frac{1}{\rho} = 2\pi \frac{e^2}{\hbar a} \int_{-\infty}^{\infty} d\nu \, \boxed{n'_F(\nu)} \int_{-\infty}^{\infty} d\epsilon \, \boxed{DOS(\epsilon)\,(v(\epsilon))^2} \boxed{\left[-\frac{1}{\pi}\Im G(\epsilon,\nu)\right]^2}$$

derivative of Fermi function

lattice

single-particle spectrum

Results for resistivity (of insulator) at U_{c1}: $\rho_c(T)$

Expect

$$\rho(U_c(T), T) \sim T^{\alpha}$$

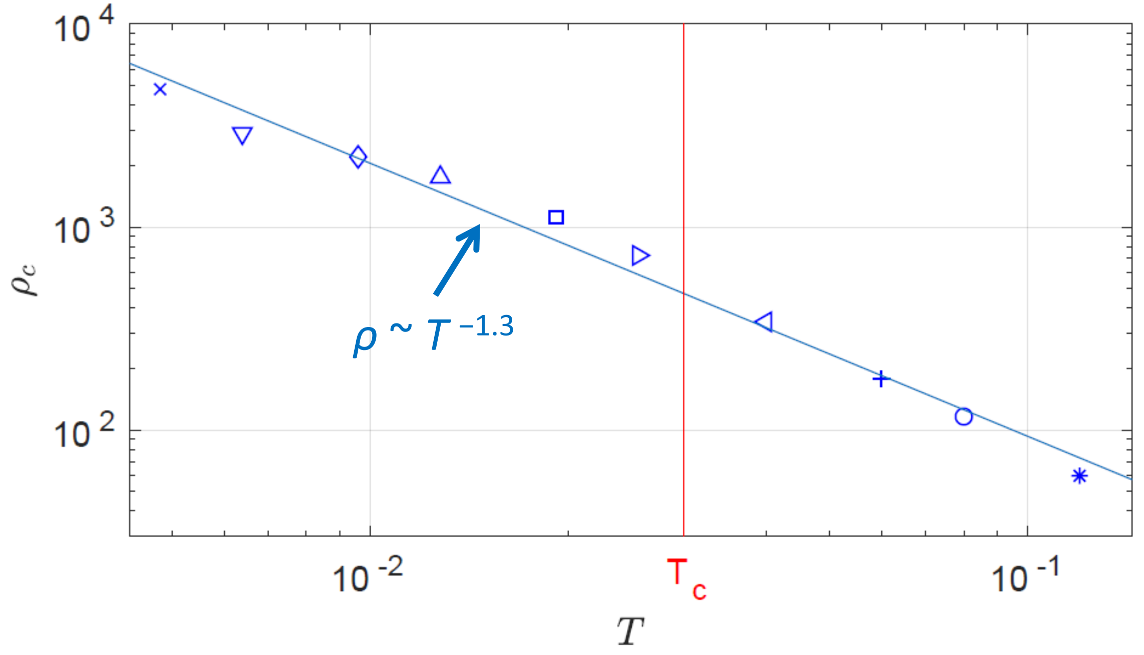
with $\alpha \approx -2r_{hyb}$

Fit:

$$\alpha = -1.3(1)$$

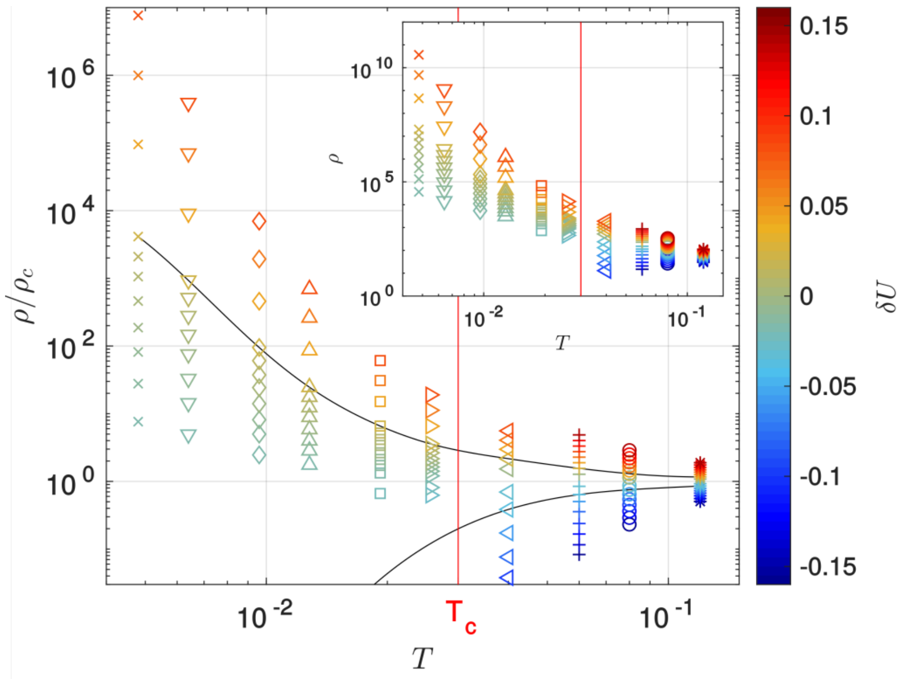
Recall:

$$r_{hyb} = 0.79(3)$$



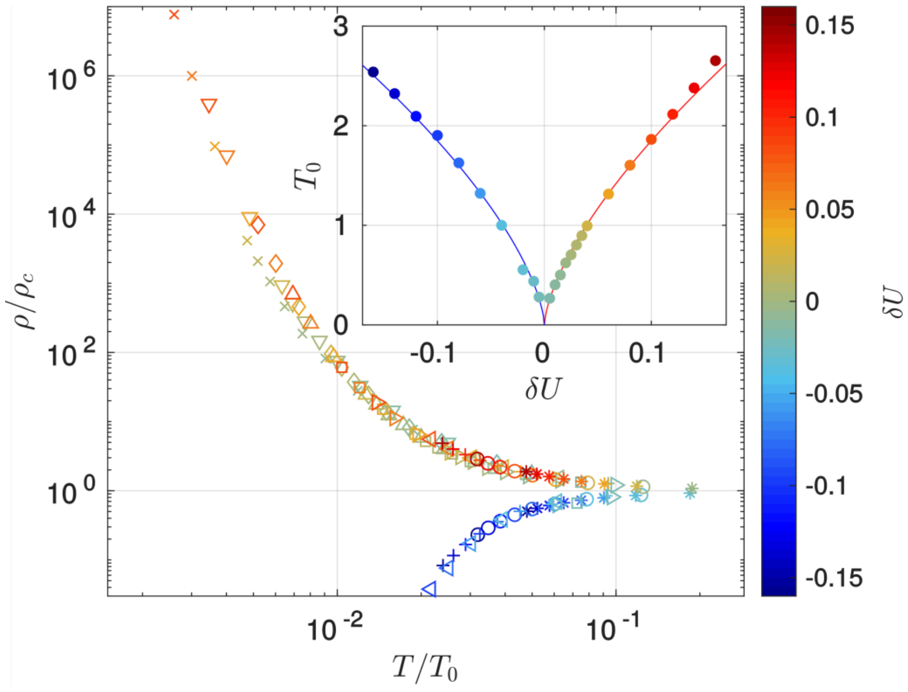
DMFT: Resistivity scaling

Resistivity from DMFT,
plotted as $\rho(T)$ / ρ_c(T)
for different $\delta U = U - U_c$



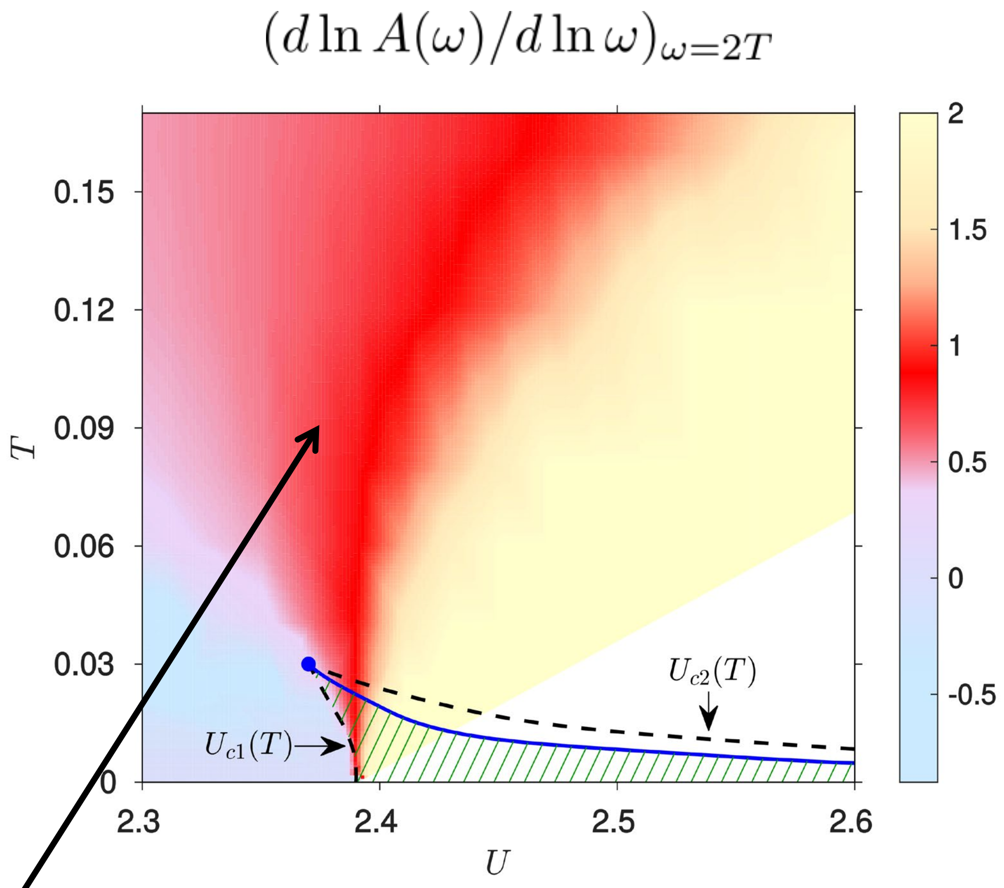
Rescaled resistivity

$$\rho(U,T) = \rho_c(U_c(T),T) f_\pm(T/T_0)$$
$$T_0 \sim |\delta U|^{z\nu} = |U - U_c(T)|^{z\nu}$$



Eisenlohr/Lee/Vojta, PRB **100**, 155152 (2019)

Finite-T phase diagram from DMFT



Local Mott quantum criticality (!)

Agreement between expt and theory suggests that (incoherent!) intermediate-T regime is dominated by local physics!

Eisenlohr/Lee/Vojta, PRB **100**, 155152 (2019)

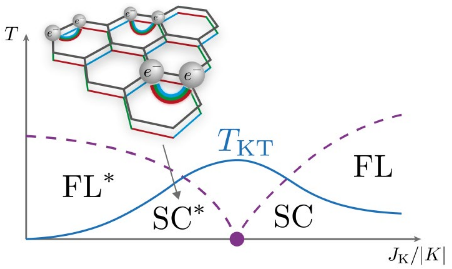
Summary & outlook

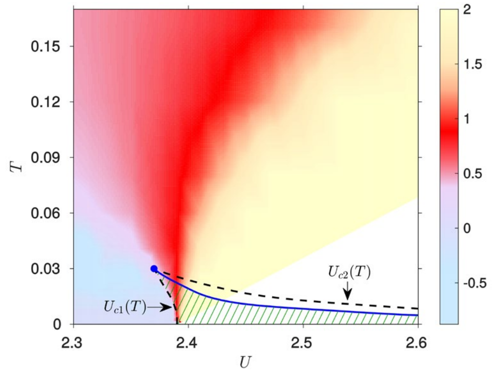
Summary

Proximity to (frustrated) Mott insulators → novel physics
(not covered here: Moire systems!)

Fractionalization → Non-Fermi liquid metals and exotic superconductors
(including cuprates)

Finite-T crossovers often dominated by strongly inelastic scattering
(and spatially very local physics)





What's next?

Improve effective theories for fractionalized metals (predictive power!)

Interrogate numerics (e.g. cluster DMFT) to characterize non-FL states

Find unambiguous signatures for fractionalization (theory + experiment!)

Develop realistic theories of strange metals (linear-in-*T* resistivity)