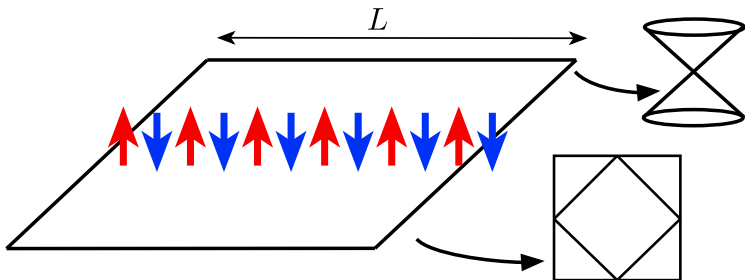


Fakher F. Assaad, Hong-Kong, November 19th 2025

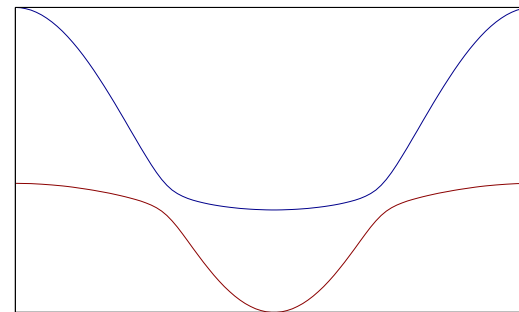
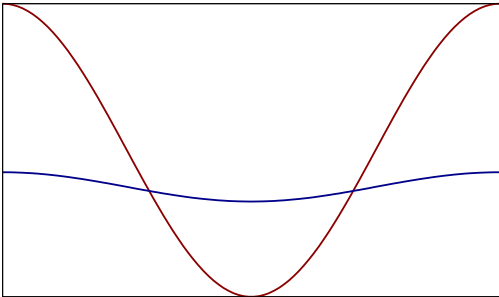
Dimensional mismatch Kondo system:



model systems for heavy fermion quantum criticality.

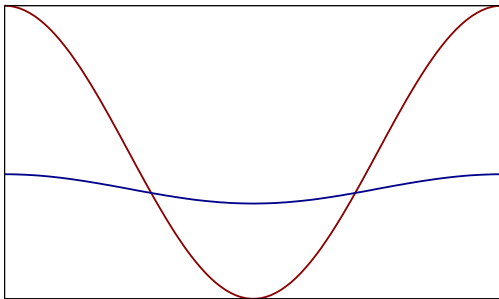
Fakher F. Assaad, Hong-Kong, November 19th 2025

Heavy fermions: two types of electrons, fast and slow, that can hybridize.



Fakher F. Assaad, Hong-Kong, November 19th 2025

Heavy fermions: two types of electrons, fast and slow, that can hybridize.



VOLUME 90, NUMBER 11

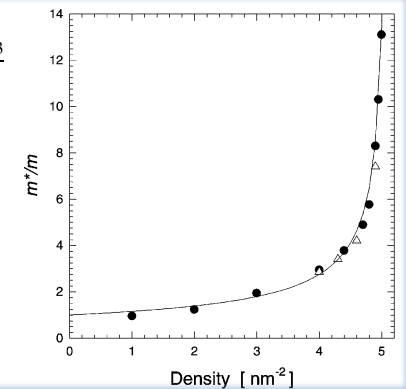
PHYSICAL REVIEW LETTERS

week ending
21 MARCH 2003

Evidence for a Mott-Hubbard Transition in a Two-Dimensional ³He Fluid Monolayer

A. Casey, H. Patel, J. Nyéki, B. P. Cowan, and J. Saunders

Mott transition triggered by diverging mass:
fermions become slow.



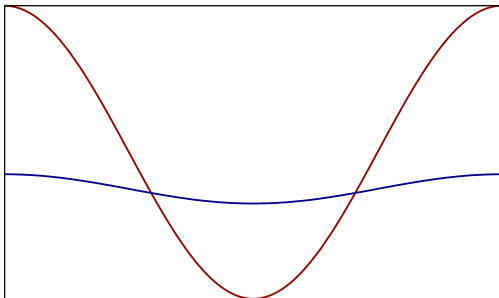
$$C_v = \gamma T \propto k_B^2 g(\epsilon_F) T$$

$$\chi = \left. \frac{\partial M}{\partial H} \right|_{H=0} \propto g \mu_B g(\epsilon_F)$$

$$g(\epsilon_F) \propto m^{d/2}$$

Fakher F. Assaad, Hong-Kong, November 19th 2025

Heavy fermions: two types of electrons, fast and slow, that can hybridize.



VOLUME 90, NUMBER 11

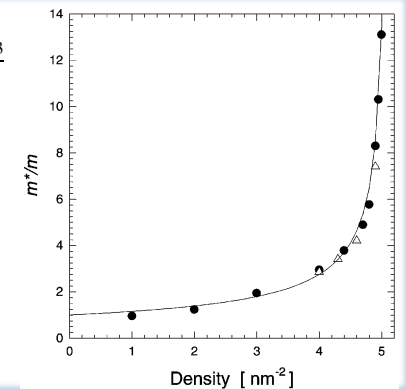
PHYSICAL REVIEW LETTERS

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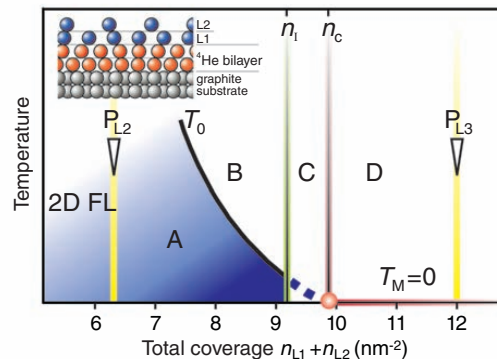
Mott transition triggered by diverging mass:
fermions become slow.



Bilayer ^3He : A Simple Two-Dimensional Heavy-Fermion System with Quantum Criticality

Michael Neumann, Ján Nyéki, Brian Cowan, John Saunders*

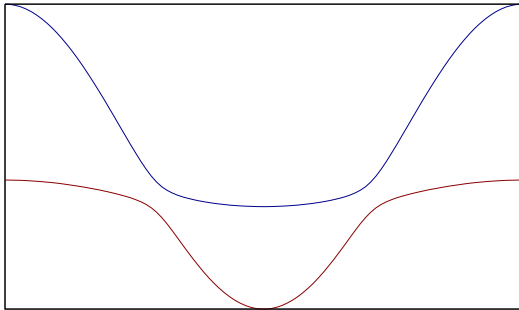
7 SEPTEMBER 2007 VOL 317 SCIENCE www.sciencemag.org



The first layer is close to the Mott transition (slow) fermions. The second layer is far from half filling (fast)

Fakher F. Assaad, Hong-Kong, November 19th 2025

Heavy fermions: two types of electrons, fast and slow, that can hybridize.



Topological Kondo insulator. $n_{\text{slow}} + n_{\text{fast}}$ is even

Resulting band structure can have topological properties

PRL **104**, 106408 (2010)

PHYSICAL REVIEW LETTERS

week ending
12 MARCH 2010

Topological Kondo Insulators

Maxim Dzero,¹ Kai Sun,¹ Victor Galitski,¹ and Piers Coleman²

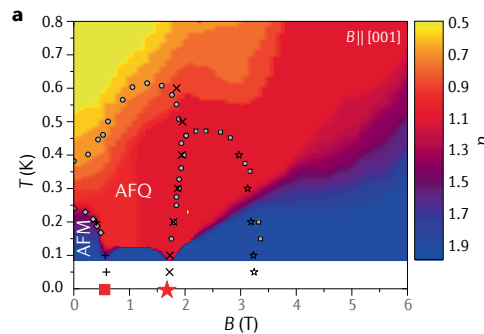
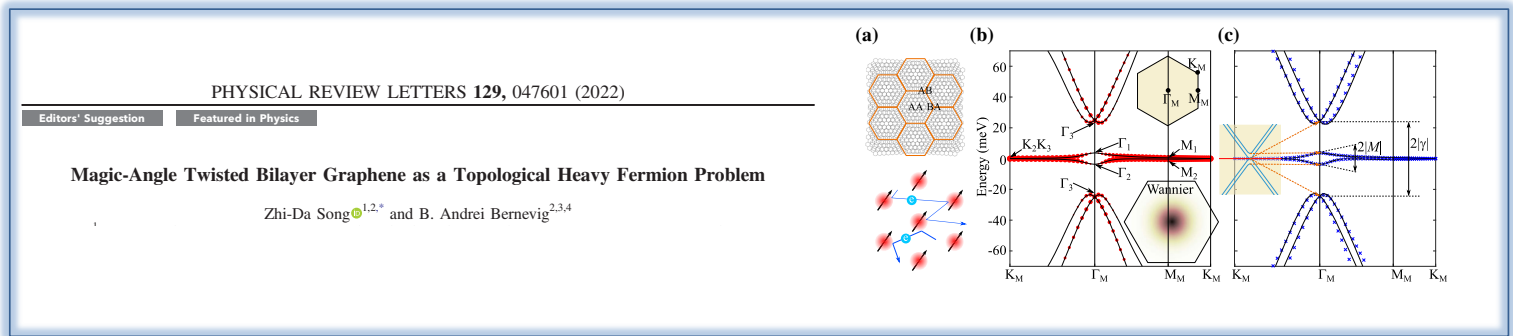
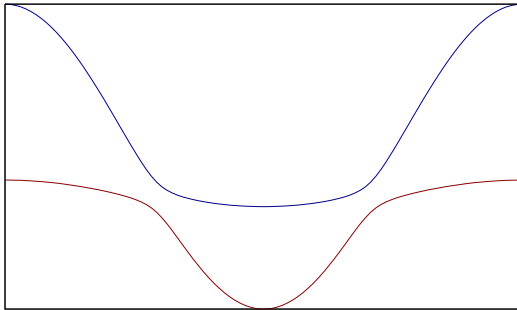
Giant spontaneous Hall effect in a nonmagnetic Weyl–Kondo semimetal

Sami Dzsaber^a, Xinlin Yan^a, Mathieu Taupin^a, Gaku Eguchi^a, Andrey Prokofiev^a, Toni Shiroka^{b,c}, Peter Blaha^d, Oleg Rubel^e, Sarah E. Grefe^f, Hsin-Hua Lai^f, Qimiao Si^f, and Silke Paschen^{a,f,1}

PNAS 2021 Vol. 118 No. 8 e2013386118

Fakher F. Assaad, Hong-Kong, November 19th 2025

Heavy fermions: two types of electrons, fast and slow, that can hybridize.



Exotic quantum criticality and strange metallicity.

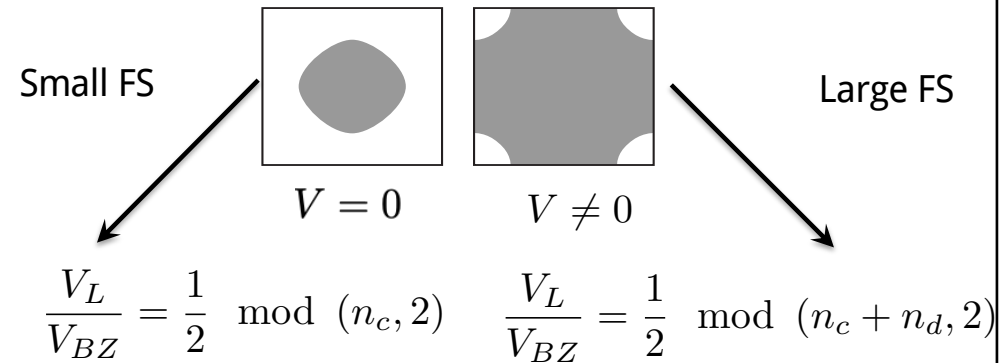
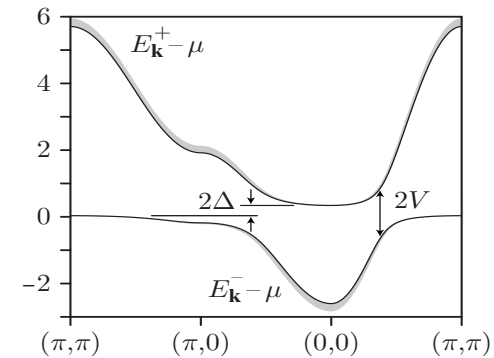
Sequential localization of a complex electron fluid

Valentina Martelli^{a,1,2}, Ang Cai^{b,c,1}, Emilian M. Nica^{b,c,3}, Mathieu Taupin^a, Andrey Prokofiev^a, Chia-Chuan Liu^{b,c}, Hsin-Hua Lai^{b,c}, Rong Yu^{b,c,d}, Kevin Ingersent^e, Robert K  chler^f, Andr   M. Strydom^g, Diana Geiger^a, Jonathan Haenel^a, Julio Larrea^{a,2}, Qimiao Si^{b,c,4}, and Silke Paschen^{a,b,c,4}

PNAS | September 3, 2019 | vol. 116 | no. 36 | 17701–17706

Periodic Anderson model

$$\hat{H} = \sum_{i,j,\sigma} \hat{c}_{i,\sigma}^\dagger T_{i,j} \hat{c}_{j,\sigma} + V \sum_{i,\sigma} \left(\hat{c}_{i,\sigma}^\dagger \hat{d}_{i,\sigma} + \hat{d}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} \right) + \epsilon_d \sum_{i,\sigma} \hat{d}_{i,\sigma}^\dagger \hat{d}_{i,\sigma} + U \sum_i \hat{d}_{i,\uparrow}^\dagger \hat{d}_{i,\uparrow} \hat{d}_{i,\downarrow}^\dagger \hat{d}_{i,\downarrow}$$



PHYSICAL REVIEW B 77, 205123 (2008)

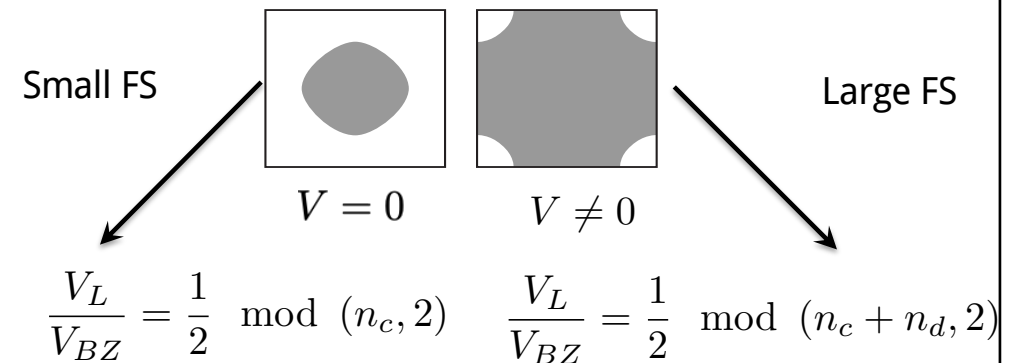
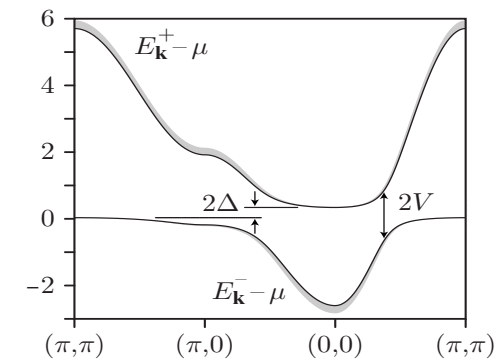
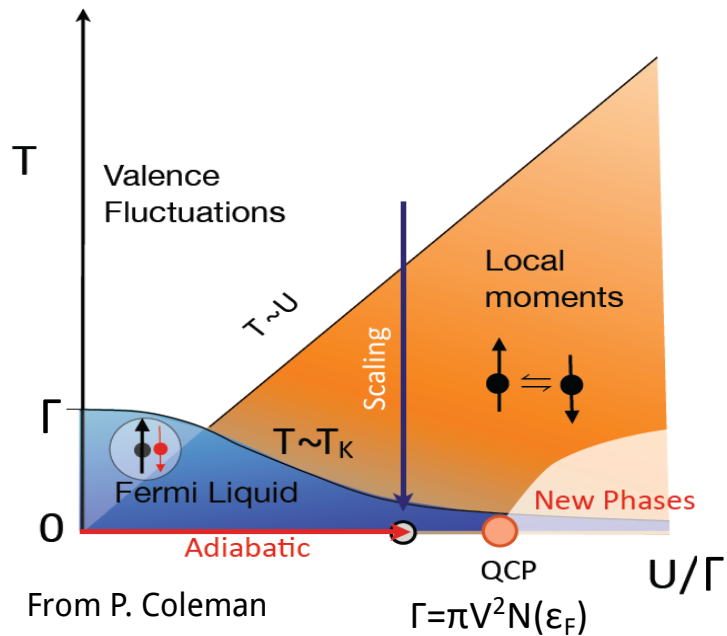
Coherence and metamagnetism in the two-dimensional Kondo lattice model

K. S. D. Beach^{1,2} and F. F. Assaad¹

Periodic Anderson model

$$\hat{H} = \sum_{i,j,\sigma} \hat{c}_{i,\sigma}^\dagger T_{i,j} \hat{c}_{j,\sigma} + V \sum_{i,\sigma} \left(\hat{c}_{i,\sigma}^\dagger \hat{d}_{i,\sigma} + \hat{d}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} \right) + \epsilon_d \sum_{i,\sigma} \hat{d}_{i,\sigma}^\dagger \hat{d}_{i,\sigma} + U \sum_i \hat{d}_{i,\uparrow}^\dagger \hat{d}_{i,\uparrow} \hat{d}_{i,\downarrow}^\dagger \hat{d}_{i,\downarrow}$$

Heavy fermion phase paramagnetic ground state is adiabatically connected to the $U=0$ limit!



The Kondo limit

$$\hat{H} = \sum_{i,j,\sigma} \hat{c}_{i,\sigma}^\dagger T_{i,j} \hat{c}_{j,\sigma} + V \sum_{i,\sigma} \left(\hat{c}_{i,\sigma}^\dagger \hat{d}_{i,\sigma} + \hat{d}_{i,\sigma}^\dagger \hat{c}_{i,\sigma} \right) + \epsilon_d \sum_{i,\sigma} \hat{d}_{i,\sigma}^\dagger \hat{d}_{i,\sigma} + U \sum_i \hat{d}_{i,\uparrow}^\dagger \hat{d}_{i,\uparrow} \hat{d}_{i,\downarrow}^\dagger \hat{d}_{i,\downarrow}$$

Freeze out charge fluctuations on the d-orbitals. Schrieffer- Wolff, Phys. Rev. 149, 491, (1966).

$$\hat{H}_{KLM} = e^S \hat{H} e^{-S} \simeq \sum_{i,j,\sigma} \hat{c}_{i,\sigma}^\dagger T_{i,j} \hat{c}_{j,\sigma} + J_K \sum_i \hat{c}_i^\dagger \frac{\boldsymbol{\sigma}}{2} \hat{c}_i \cdot \hat{\mathbf{S}}_i$$

$$\hat{d}^\dagger \hat{d} = 1, \quad J_K \simeq \frac{V^2}{U}$$

What happens to the Luttinger count?

Count the number of conduction and number of composite fermions

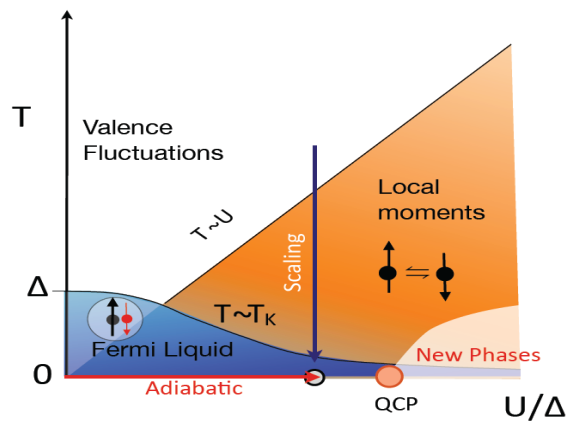
$$e^S \hat{d}_i^\dagger e^{-S} \simeq \hat{c}_i^\dagger \boldsymbol{\sigma} \cdot \hat{\mathbf{S}}_i \equiv \hat{\Psi}_i^\dagger$$

M. Raczkowski and FFA, Phys. Rev. Lett. 122, 097203, (2019).

T. A. Costi, Phys. Rev. Lett. 85 (2000), 1504–1507.

The Kondo model

Competing interactions, origin of quantum criticality.

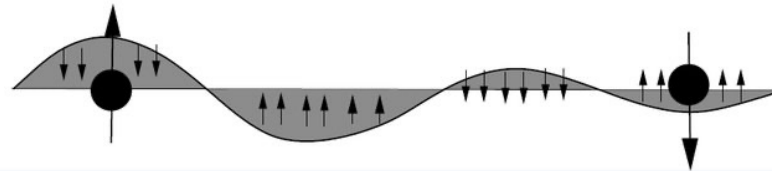


From P. Coleman

Heavy fermion: d-electrons (composite-fermions) are delocalized

As U increases (J_k -decreases) d-electrons localize and favor magnetic states.

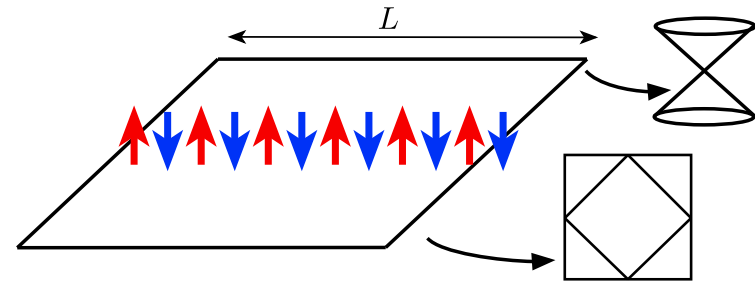
$$\mathcal{S}_{\text{RKKY}}(\mathbf{n}) = \frac{J_k^2}{8} \int d\tau d\tau' \sum_{\mathbf{r}, \mathbf{r}'} \mathbf{n}_{\mathbf{r}}(\tau) \chi^0(\mathbf{r} - \mathbf{r}', \tau - \tau') \mathbf{n}_{\mathbf{r}'}(\tau')$$



Fakher F. Assaad, Hong-Kong, November 19th 2025

Dimensional mismatch Kondo system

Provide a realization of:



Heavy fermion phase (metal, impurities spins participate in Luttinger volume)

Kondo breakdown phase (metal, impurities spins do **not** participate in Luttinger volume)

Magnetic ordering in metallic environment

And can be solved without the encountering the negative sign problem in quantum Monte Carlo simulations

Fakher F. Assaad, Hong-Kong, November 19th 2025

Many thanks to...



B. Danu
(Wü)



Z. Liu
(Shanghai)



B. Frank
(Former TUD)



M. Raczkowski
(Wü)



T. Grover (UCSD)



M. Vojta (TUD)



G. Pan (Wü)



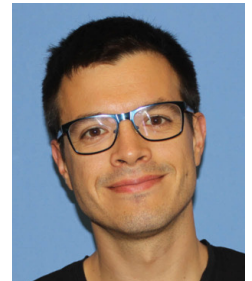
M. Weber
(TUD)



S. Biswas
(ICTS Bangalore)



L. Janssen (TUD)



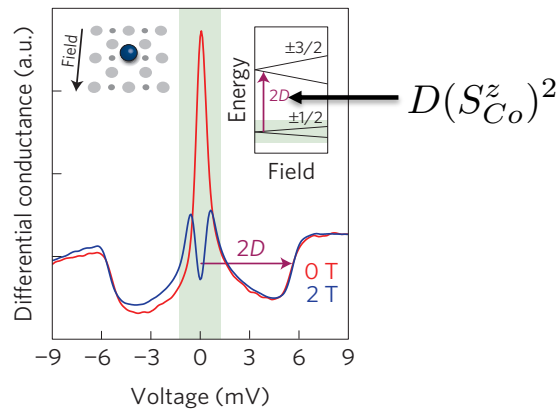
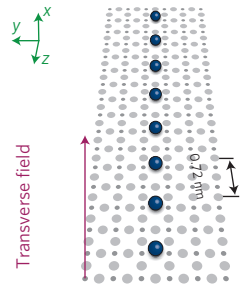
D. Luitz (Bonn)

Experimental realization of dimensional mismatch Kondo systems.

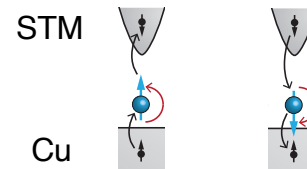
Co atoms on Cu₂N/Cu(100) in magnetic field.

R. Toskovic, R. van den Berg, A. Spinelli, I. S. Eliens, B. van den Toorn, B. Bryant, J. S. Caux, and A. F. Otte, Nature Physics 12 (2016), 656

Single impurity limit



Cu₂N decoupling layer, co-tunneling through Co-d orbital.



$$T_k \sim 0.2 \text{ meV}$$

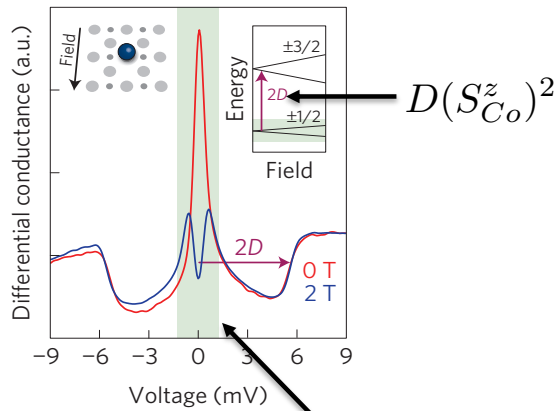
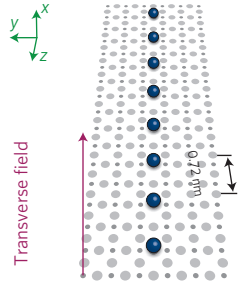
$$\mu_B \sim 0.06 \text{ meV/T}$$

Dimensional mismatch Kondo: Experiments & Models

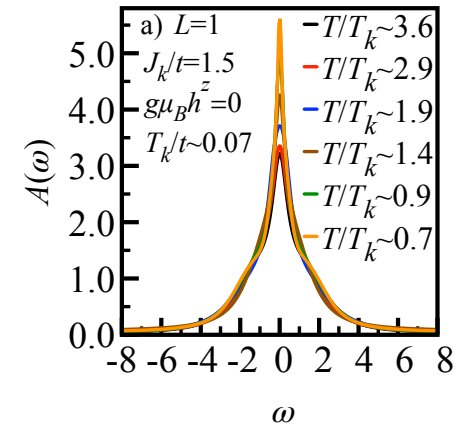
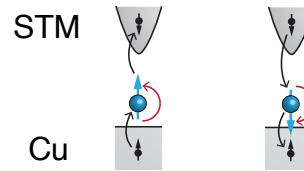
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Single impurity limit



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$T_k \sim 0.2$ meV

$\mu_B \sim 0.06$ meV/T

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \hat{c}_{r=0}^\dagger \boldsymbol{\sigma} \hat{c}_{r=0} \cdot \hat{\mathbf{S}} - g\mu_B h^z \hat{S}^z$$

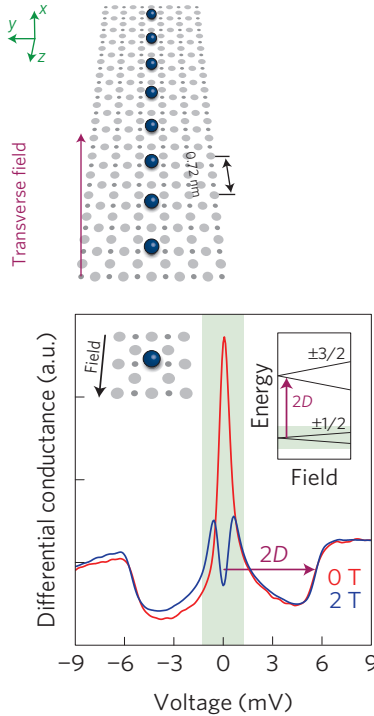
$$\hat{c}_i^\dagger = (\hat{c}_{i,\uparrow}^\dagger, \hat{c}_{i,\downarrow}^\dagger)$$

$$A(\omega) = -\text{Im}G^{\text{ret}}(\omega), \quad G^{\text{ret}}(\omega) = -i \int_0^\infty dt e^{i\omega t} \sum_\sigma \langle \{ \hat{\psi}_\sigma(t), \hat{\psi}_\sigma^\dagger(0) \} \rangle$$

$$\hat{\psi}^\dagger = e^{-S} \hat{d}^\dagger e^S = \hat{c}_{r=0}^\dagger \boldsymbol{\sigma} \cdot \hat{\mathbf{S}}$$

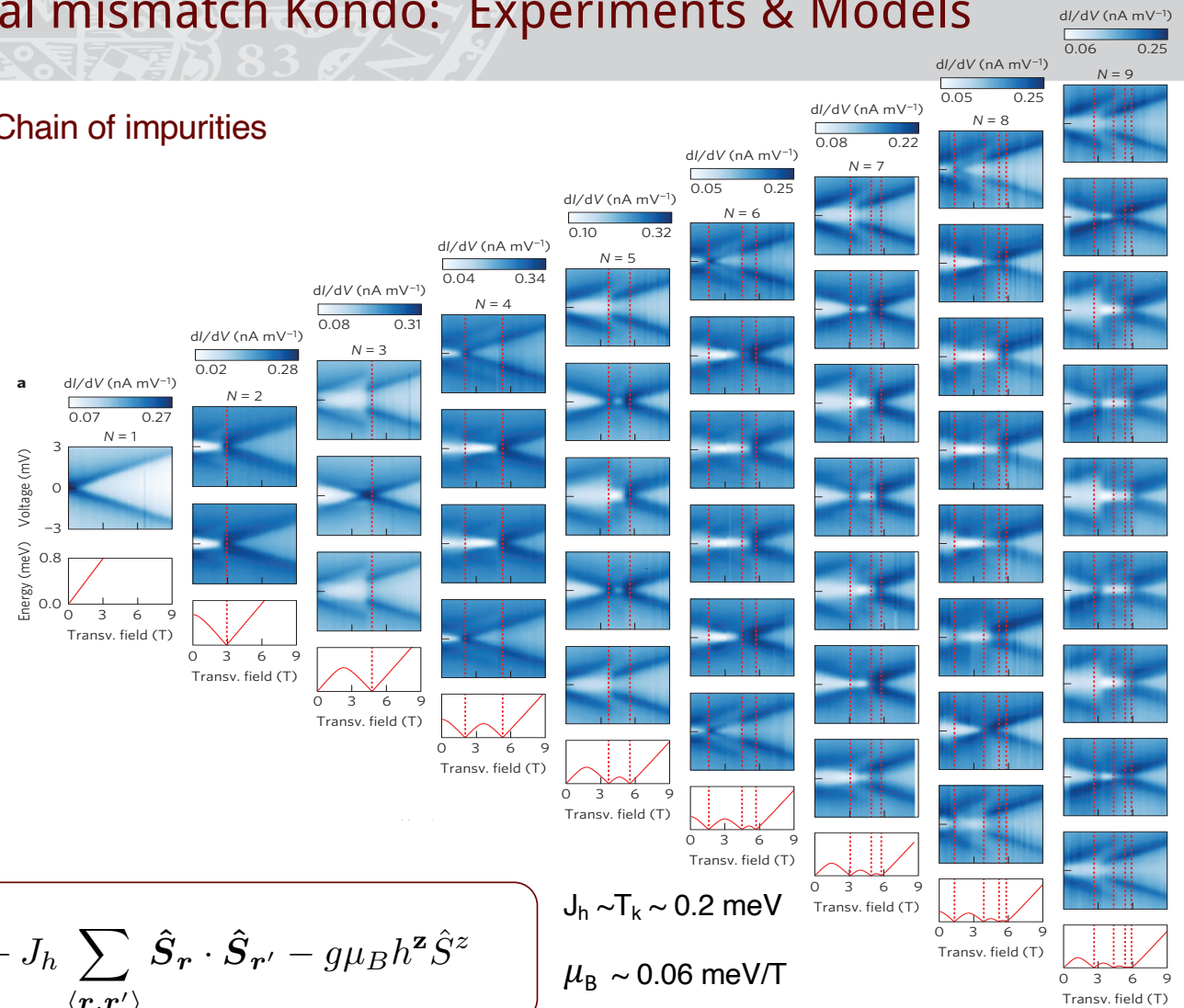
T.A. Costi, Phys. Rev. Lett. 85, 1504 (2000).

Dimensional mismatch Kondo: Experiments & Models



Site dependent
field induced
“Kondo resonance”

Chain of impurities



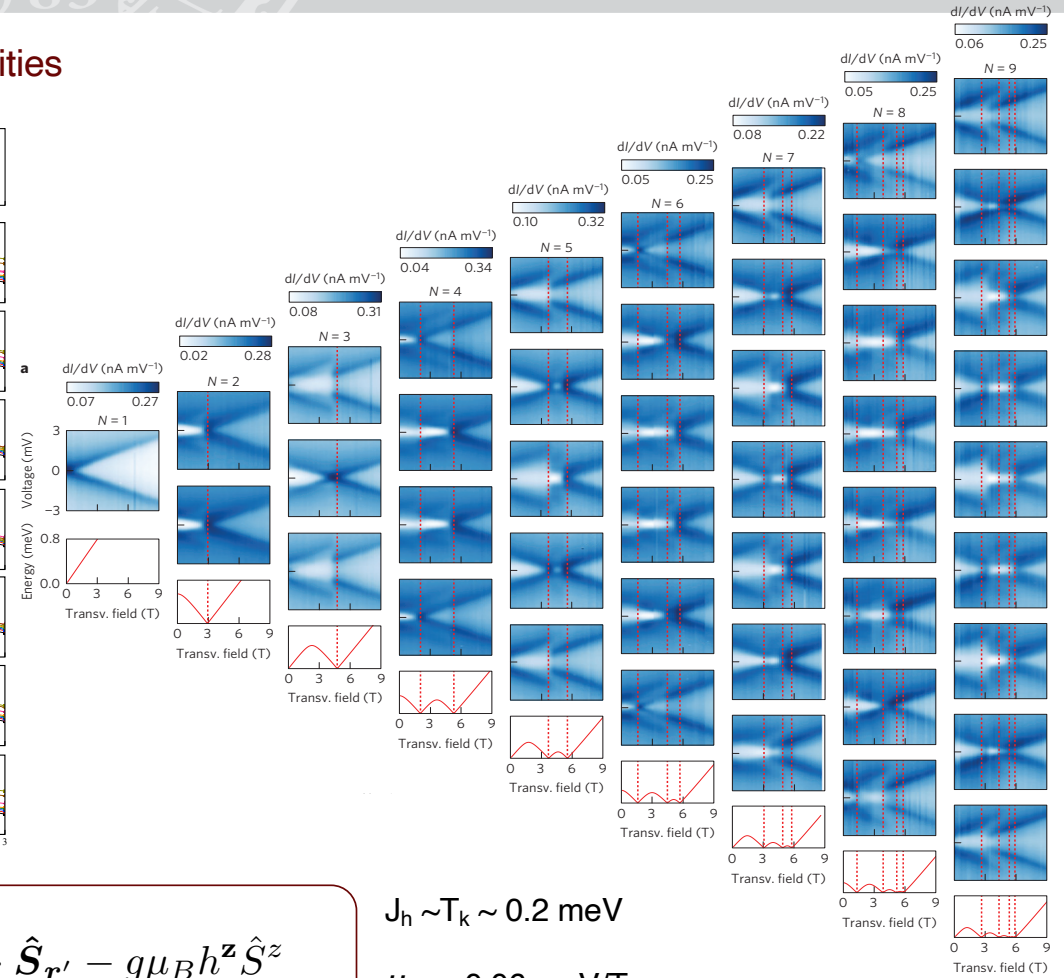
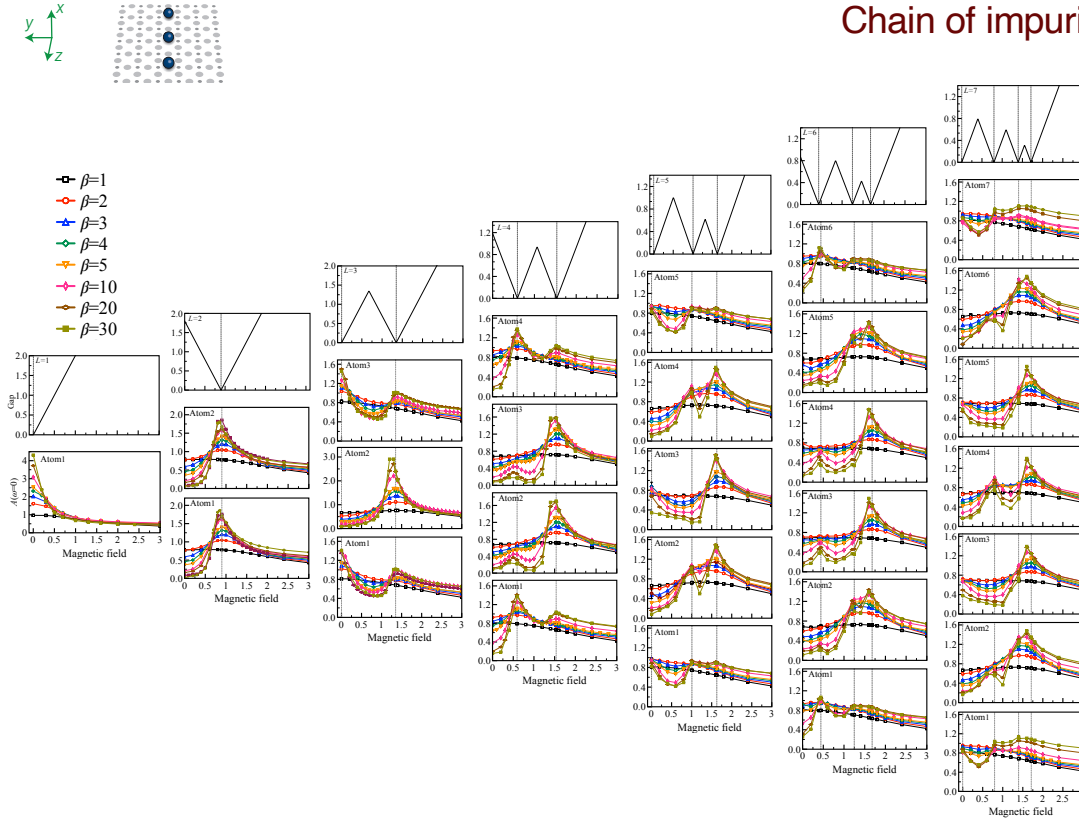
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'} - g\mu_B h^z \hat{S}^z$$

$$J_h \sim T_K \sim 0.2 \text{ meV}$$

$$\mu_B \sim 0.06 \text{ meV/T}$$

Dimensional mismatch Kondo: Experiments & Models

Chain of impurities



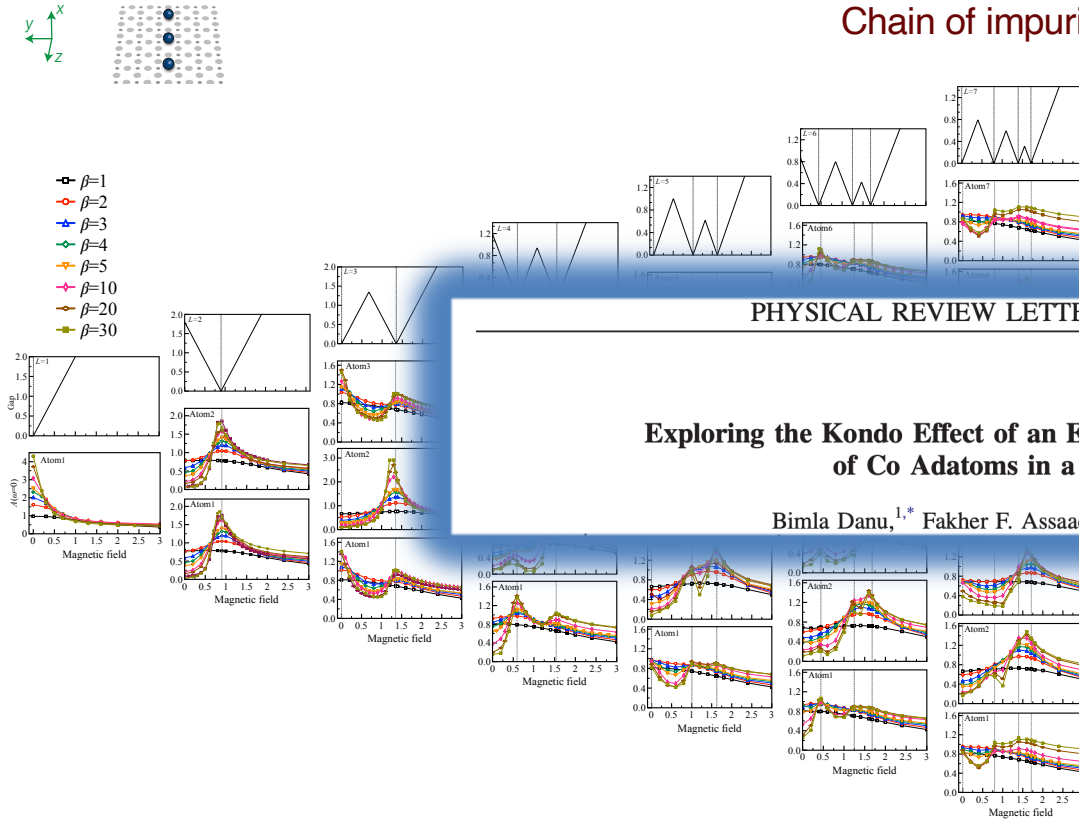
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'} - g\mu_B h^z \hat{S}^z$$

$$J_h \sim T_K \sim 0.2 \text{ meV}$$

$$\mu_B \sim 0.06 \text{ meV/T}$$

Dimensional mismatch Kondo: Experiments & Models

Chain of impurities



PHYSICAL REVIEW LETTERS **123**, 176601 (2019)

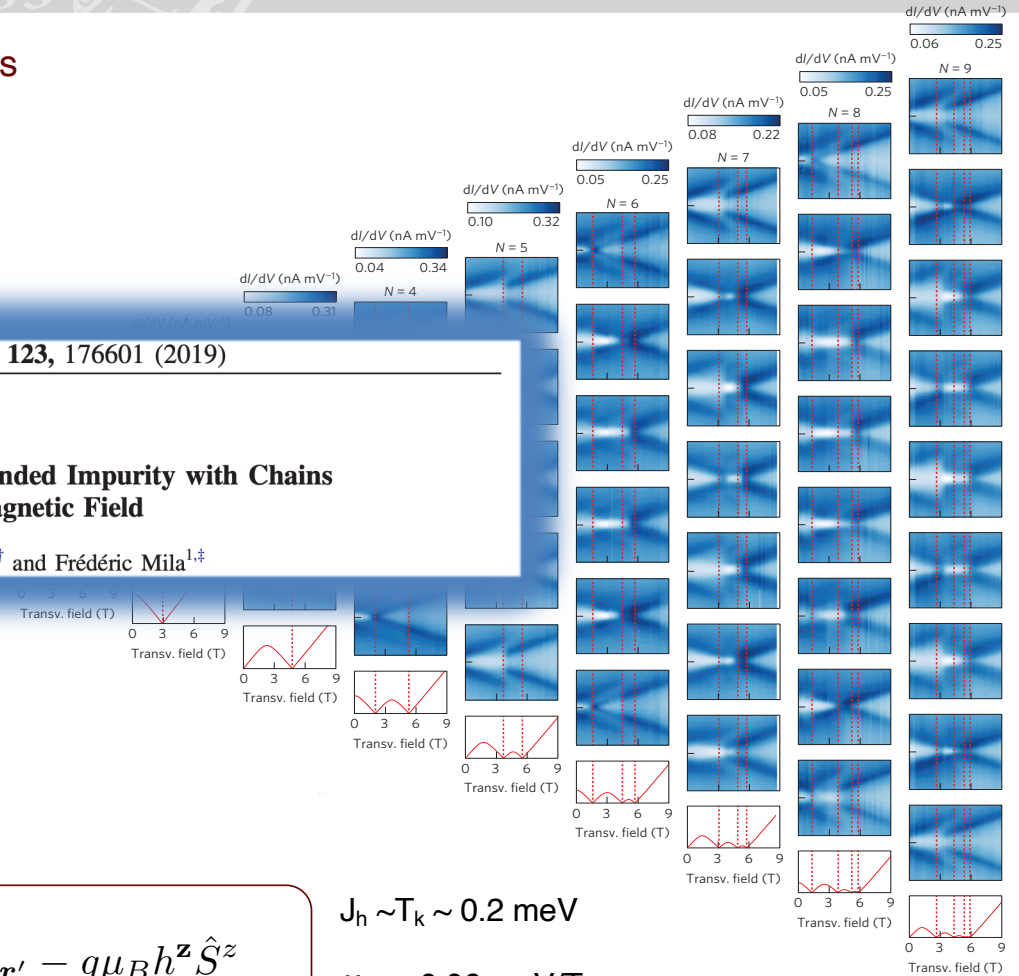
Exploring the Kondo Effect of an Extended Impurity with Chains of Co Adatoms in a Magnetic Field

Bimla Danu,^{1,*} Fakher F. Assaad,^{2,3,†} and Frédéric Mila^{1,‡}

$$J_h \sim T_K \sim 0.2 \text{ meV}$$

$$\mu_B \sim 0.06 \text{ meV/T}$$

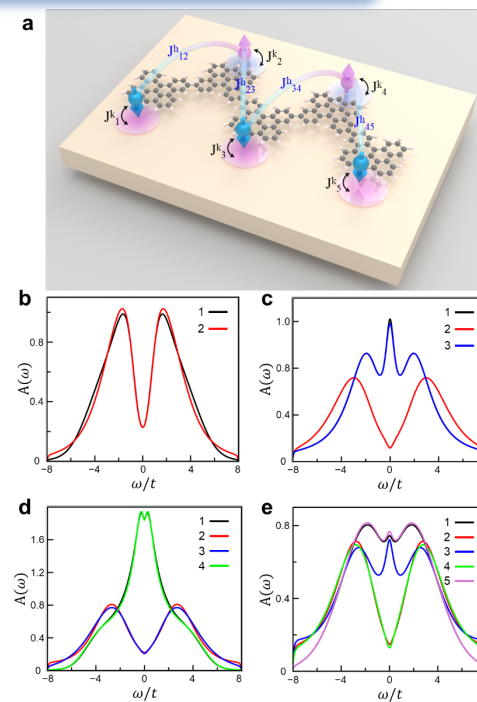
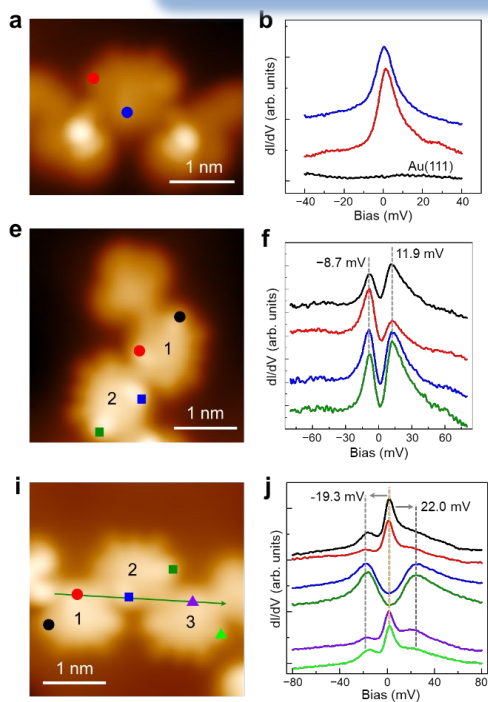
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \sigma \hat{c}_r \cdot \hat{S}_r + J_h \sum_{\langle r,r' \rangle} \hat{S}_r \cdot \hat{S}_{r'} - g\mu_B h^z \hat{S}^z$$



Dimensional mismatch Kondo: Experiments & Models

**Artificially built Kondo chains with organic radicals on metallic
surfaces: new model system of heavy fermion quantum criticality**

En Li¹, Bimla Danu², Yufeng Liu³, Huilin Xie⁴, Jacky Wing Yip Lam⁴, Ben Zhong
Tang^{4, 5}, Shiyong Wang^{3, 6, 7*}, Fakher F. Assaad^{2*}, Nian Lin^{1*}

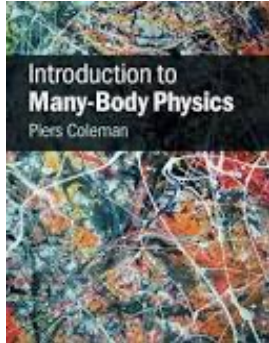


Accepted in JACS

<https://arxiv.org/pdf/2408.01236>

Rings / Infinite chains

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$



PHYSICAL REVIEW B **75**, 165110 (2007)

Quantum critical point in the Kondo-Heisenberg model on the honeycomb lattice

Saeed Saremi and Patrick A. Lee

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 2 November 2006; revised manuscript received 2 February 2007; published 17 April 2007)

Goal: Mapping onto problem of quantum electrodynamics [U(1) gauge theory]. Allows for saddle point approximations and bias free quantum Monte Carlo simulations.

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \boldsymbol{\sigma} \hat{c}_r \cdot \hat{\mathbf{S}}_r + J_h \sum_{\langle r,r' \rangle} \hat{\mathbf{S}}_r \cdot \hat{\mathbf{S}}_{r'}$$

Abrikosov fermions

$$\hat{\mathbf{S}}_r = \frac{1}{2} \hat{\mathbf{f}}_r^\dagger \boldsymbol{\sigma} \hat{\mathbf{f}}_r \quad \text{with constraint} \quad \hat{\mathbf{f}}_r^\dagger \hat{\mathbf{f}}_r = 1$$

$$\text{Def} \quad \hat{\mathbf{f}}_r^\dagger = (\hat{f}_{r,\uparrow}^\dagger, \hat{f}_{r,\downarrow}^\dagger)$$

$$\begin{aligned} \hat{H} = & -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) - \frac{J_k}{8} \sum_r \left[(\hat{V}_r + \hat{V}_r^\dagger)^2 + (i\hat{V}_r - i\hat{V}_r^\dagger)^2 \right] \\ & - \frac{J_h}{8} \sum_{b=\langle r,r' \rangle} \left[(\hat{D}_b + \hat{D}_b^\dagger)^2 + (i\hat{D}_b - i\hat{D}_b^\dagger)^2 \right] + \underbrace{U \sum_r (\hat{\mathbf{f}}_r^\dagger \hat{\mathbf{f}}_r - 1)^2}_{= \hat{H}_U} \end{aligned}$$

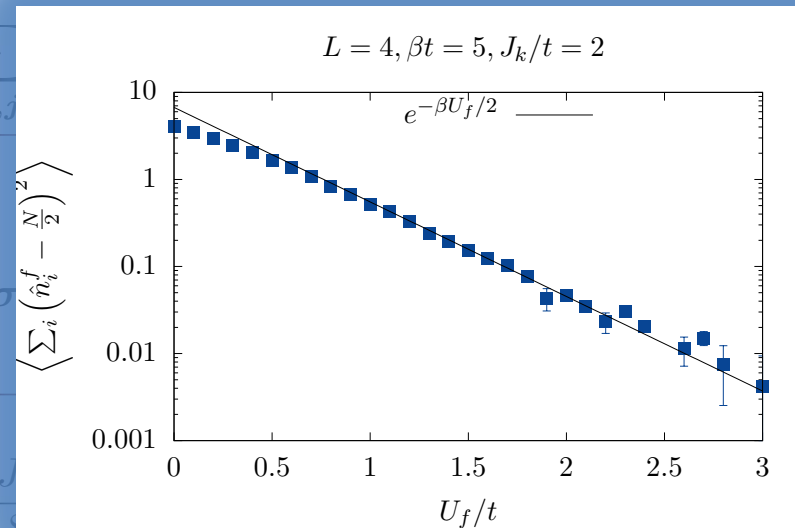
$$\hat{D}_b = \hat{\mathbf{f}}_r^\dagger \hat{\mathbf{f}}_{r'}, \quad \hat{V}_r = \hat{\mathbf{f}}_r^\dagger \hat{c}_r$$

Exact in the limit $U \rightarrow \infty$. But since $[\hat{H}_U, \hat{H}] = 0$ the constraint is imposed very efficiently.

$$\hat{H} = -t \sum_{\langle i,j \rangle}$$

Abrikosov fermions

$$\hat{S}_r = \frac{1}{2} \hat{f}_r^\dagger \sigma \hat{f}_r$$



Def $\hat{f}_r^\dagger = \left(\hat{f}_{r,\uparrow}^\dagger, \hat{f}_{r,\downarrow}^\dagger \right)$

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.)$$

$$- \frac{J_h}{8} \sum_{b=\langle r,r' \rangle} \left[\left(\hat{D}_b + \hat{D}_b^\dagger \right)^2 + \left(i\hat{D}_b - i\hat{D}_b^\dagger \right)^2 \right] + \underbrace{U \sum_r \left(\hat{f}_r^\dagger \hat{f}_r - 1 \right)^2}_{= \hat{H}_U}$$

$$\hat{D}_b = \hat{f}_r^\dagger \hat{f}_{r'}, \quad \hat{V}_r = \hat{f}_r^\dagger \hat{c}_r$$

Exact in the limit $U \rightarrow \infty$. But since $[\hat{H}_U, \hat{H}] = 0$ the constraint is imposed very efficiently.

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) - \frac{J_k}{8} \sum_{\mathbf{r}} \left[\left(\hat{V}_{\mathbf{r}} + \hat{V}_{\mathbf{r}}^\dagger \right)^2 + \left(i\hat{V}_{\mathbf{r}} - i\hat{V}_{\mathbf{r}}^\dagger \right)^2 \right]$$

$$- \frac{J_h}{8} \sum_{b=\langle \mathbf{r}, \mathbf{r}' \rangle} \left[\left(\hat{D}_b + \hat{D}_b^\dagger \right)^2 + \left(i\hat{D}_b - i\hat{D}_b^\dagger \right)^2 \right] + U \sum_{\mathbf{r}} \left(\hat{f}_{\mathbf{r}}^\dagger \hat{f}_{\mathbf{r}} - 1 \right)^2$$

$$\hat{D}_b = \hat{f}_{\mathbf{r}}^\dagger \hat{f}_{\mathbf{r}'}, \quad \hat{V}_{\mathbf{r}} = \hat{f}_{\mathbf{r}}^\dagger \hat{c}_{\mathbf{r}}$$

$$b_{\mathbf{r}} = |b_{\mathbf{r}}| e^{i\varphi_{\mathbf{r}}} \quad \chi_b = |\chi_b| e^{i \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{a} \cdot d\mathbf{l}} \quad a_0$$

Partition function, $Z = \text{Tr} e^{-\beta \hat{H}} = \int D\{f^\dagger f\} D\{c^\dagger c\} D\{\chi_b\} D\{b_{\mathbf{r}}\} D\{a_0\} e^{-S}$ with, for $U \rightarrow \infty$

$$S = \int_0^\beta d\tau \left\{ \frac{2}{J_h} \sum_b |\chi_b(\tau)|^2 + \frac{2}{J_k} \sum_{\mathbf{r}} |b_{\mathbf{r}}(\tau)|^2 + \sum_{i,j} c_i^\dagger(\tau) [\partial_\tau \delta_{i,j} - T_{i,j}] c_j(\tau) \right.$$

$$+ \sum_{\mathbf{r}} |b_{\mathbf{r}}(\tau)| \left[e^{i\varphi_{\mathbf{r}}(\tau)} \mathbf{f}_{\mathbf{r}}^\dagger(\tau) \mathbf{c}_{\mathbf{r}}(\tau) + h.c. \right]$$

$$+ \sum_{\mathbf{r}} \mathbf{f}_{\mathbf{r}}^\dagger(\tau) [\partial_\tau - ia_{0,\mathbf{r}}(\tau)] \mathbf{f}_{\mathbf{r}}(\tau) + ia_{0,\mathbf{r}}(\tau) + \sum_{b=\langle \mathbf{r}, \mathbf{r}' \rangle} |\chi_b(\tau)| \left[\mathbf{f}_{\mathbf{r}}^\dagger(\tau) e^{-i \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{a}(l,\tau) d\mathbf{l}} \mathbf{f}_{\mathbf{r}'}(\tau) + h.c. \right]$$

$$\begin{aligned}
 S = \int_0^\beta d\tau \quad & \left\{ \frac{2}{J_h} \sum_b |\chi_b(\tau)|^2 + \frac{2}{J_k} \sum_{\mathbf{r}} |b_{\mathbf{r}}(\tau)|^2 + \sum_{i,j} c_i^\dagger(\tau) [\partial_\tau \delta_{i,j} - T_{i,j}] c_j(\tau) \right. \\
 & + \sum_{\mathbf{r}} |b_{\mathbf{r}}(\tau)| \left[e^{i\varphi_{\mathbf{r}}(\tau)} \mathbf{f}_{\mathbf{r}}^\dagger(\tau) \mathbf{c}_{\mathbf{r}}(\tau) + h.c. \right] \\
 & + \sum_{\mathbf{r}} \mathbf{f}_{\mathbf{r}}^\dagger(\tau) [\partial_\tau - i a_{0,\mathbf{r}}(\tau)] \mathbf{f}_{\mathbf{r}}(\tau) + i a_{0,\mathbf{r}}(\tau) + \sum_{b=\langle \mathbf{r}, \mathbf{r}' \rangle} |\chi_b(\tau)| \left[\mathbf{f}_{\mathbf{r}}^\dagger(\tau) e^{-i \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{a}(l,\tau) dl} \mathbf{f}_{\mathbf{r}'}(\tau) + h.c. \right]
 \end{aligned}$$

Numerical simulations

$$Z = \text{Tr} e^{-\beta \hat{H}} = \int D\{f^\dagger f\} D\{c^\dagger c\} D\{\chi_b\} D\{b_{\mathbf{r}}\} D\{a_0\} e^{-S} = \int D\{\chi_b\} D\{b_{\mathbf{r}}\} D\{a_0\} e^{-(S_0 - \log \det M)}$$

For our specific case, the action is real ! → No sign problem.

(T. Sato, FFA, and T. Grover, Phys. Rev. Lett. 120 (2018), 107201)

The integration over the fields is carried out with Monte Carlo importance sampling.

R. Blankenbecler, D. J. Scalapino, and R. L. Sugar, Phys. Rev. D 24 (1981), 2278–2286.

Kinetic

$$\hat{H} = \sum_{k=1}^{M_T} \sum_{\sigma=1}^{N_{\text{col}}} \sum_{s=1}^{N_{\text{fl}}} \sum_{x,y}^{N_{\text{dim}}} \hat{c}_{x\sigma s}^\dagger T_{xy}^{(ks)} \hat{c}_{y\sigma s} + \sum_{k=1}^{M_V} U_k \left\{ \sum_{\sigma=1}^{N_{\text{col}}} \sum_{s=1}^{N_{\text{fl}}} \left[\left(\sum_{x,y}^{N_{\text{dim}}} \hat{c}_{x\sigma s}^\dagger V_{xy}^{(ks)} \hat{c}_{y\sigma s} \right) + \alpha_{ks} \right] \right\}^2$$

Potential (sum of perfect squares)

Coupling of fermions to bosonic fields with predefined dynamics

$$+ \sum_{k=1}^{M_I} \hat{Z}_k \left(\sum_{\sigma=1}^{N_{\text{col}}} \sum_{s=1}^{N_{\text{fl}}} \sum_{x,y}^{N_{\text{dim}}} \hat{c}_{x\sigma s}^\dagger I_{xy}^{(ks)} \hat{c}_{y\sigma s} \right) + \hat{H}_{\text{Ising}}$$

- Block diagonal in flavors, N_{fl}
- $SU(N_{\text{col}})$ symmetric in colors N_{col}
- Arbitrary Bravais lattice for $d=1,2$
- Model can be specified at minimal programming cost
- Fortran 2008 standard
- MPI implementation
- Global and local moves, Parallel tempering, Langevin, HMC
- Projective and finite T approaches
- pyALF: easy access python interface
- Predefined models



F. Goth



M. Bercx



J. Hoffmann



J. S.E. Portela



J. Schwab



Z. Liu



E. Huffman



A. Götz



F. Parisen Toldin



Wissenschaftliche
Literaturversorgungs-
und Informationssysteme (LIS)



$$S = N \int_0^\beta d\tau \left\{ \frac{2}{J_h} \sum_{b=\langle \mathbf{r}, \mathbf{r}' \rangle} |\chi_b(\tau)|^2 + \frac{2}{J_k} \sum_{\mathbf{r}} |b_{\mathbf{r}}(\tau)|^2 + \sum_{i,j} c_i^\dagger(\tau) [\partial_\tau \delta_{i,j} - T_{i,j}] c_j(\tau) \right. \\ \left. + \sum_{\mathbf{r}} |b_{\mathbf{r}}(\tau)| \left[e^{i\varphi_{\mathbf{r}}(\tau)} \mathbf{f}_{\mathbf{r}}^\dagger(\tau) \mathbf{c}_{\mathbf{r}}(\tau) + h.c. \right] \right. \\ \left. + \sum_{\mathbf{r}} \mathbf{f}_{\mathbf{r}}^\dagger(\tau) [\partial_\tau - i a_{0,\mathbf{r}}(\tau)] \mathbf{f}_{\mathbf{r}}(\tau) + i a_{0,\mathbf{r}}(\tau) + \sum_b |\chi_b(\tau)| \left[\mathbf{f}_{\mathbf{r}}^\dagger(\tau) e^{-i \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{a}(\mathbf{l}, \tau) d\mathbf{l}} \mathbf{f}_{\mathbf{r}'}(\tau) + h.c. \right] \right\}$$

PHYSICAL REVIEW RESEARCH 2, 013276 (2020)

Phase diagram and dynamics of the SU(N) symmetric Kondo lattice model

Marcin Raczkowski 

Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

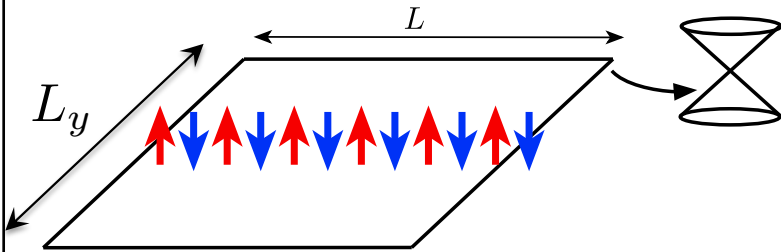
Fakher F. Assaad

Institut für Theoretische Physik und Astrophysik and Würzburg-Dresden Cluster of Excellence ct.qmat, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

The large-N Kondo insulating state is
adiabatically connected to N=2 !

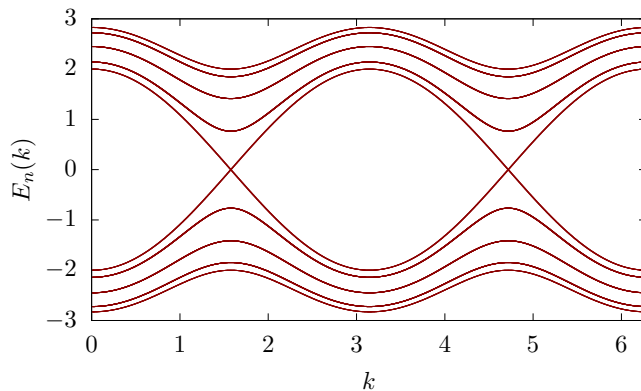
$$|b_{\mathbf{r}}(\tau)| e^{i\varphi_{\mathbf{r}}(\tau)} \rightarrow b$$

Kondo and Kondo breakdown phases (mean-field)



$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \boldsymbol{\sigma} \hat{c}_r \cdot \hat{\mathbf{S}}_r + J_h \sum_{\langle r,r' \rangle} \hat{\mathbf{S}}_r \cdot \hat{\mathbf{S}}_{r'}$$

No impurities

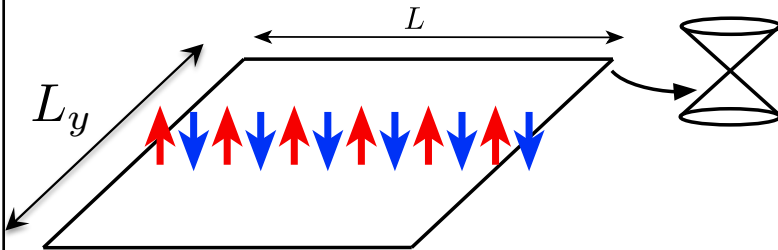


Unit cell has L_y orbitals and one impurity spin.

Half-filling: $n_c = L_y$

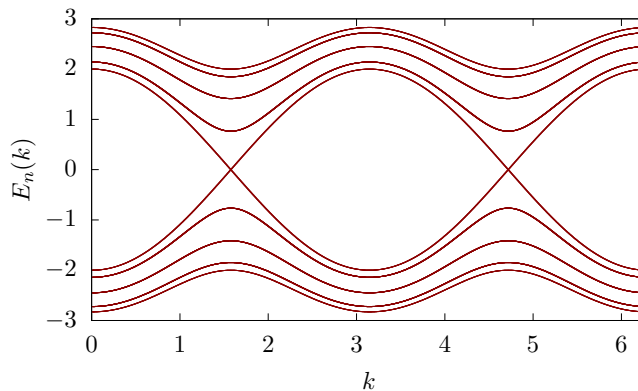
$$\frac{V}{V_{BZ}} = \frac{1}{2} \bmod (n_c, 2)$$

Kondo and Kondo breakdown phases (mean-field)



$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \sigma \hat{c}_r \cdot \hat{S}_r + J_h \sum_{\langle r,r' \rangle} \hat{S}_r \cdot \hat{S}_{r'}$$

No impurities



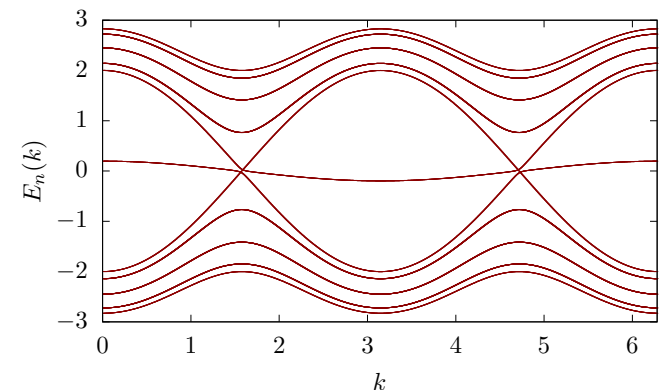
$$\frac{V}{V_{BZ}} = \frac{1}{2} \bmod (n_c, 2)$$

Unit cell has L_y orbitals and one impurity spin.

Half-filling: $n_c = L_y$

Spin-spin correlations of spin chain inherit those of the conduction electrons.

Impurities participate in Luttinger volume

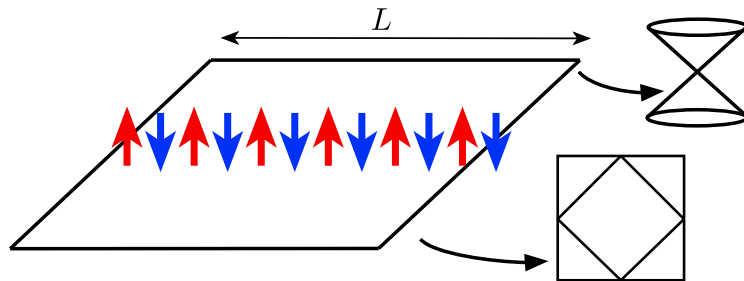


$$\frac{V}{V_{BZ}} = \frac{1}{2} \bmod (n_c + 1, 2)$$

Fakher F. Assaad, Hong-Kong, November 19th 2025

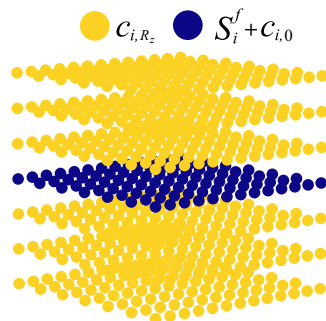
Co atoms
on $\text{Cu}_2\text{N}/\text{Cu}(100)$

R. Toskovic
et al. Nat. Phys.
2016



$\text{LaIn}_3/\text{CeIn}_3$

H. Shishido et al.
Science 2010



Kondo breakdown transitions and phases

B. Danu, M. Vojta, FFA, and T. Grover, Phys. Rev. Lett. 125 (2020), 206602.

Dissipation induced magnetic order-disorder transitions

B. Danu, M. Vojta, T. Grover FFA, Phys. Rev. B 106 (2022), L161103.

M. Weber, D. J. Luitz, and FFA, Phys. Rev. Lett. 129 (2022), 056402.

Spin 3/2 chains on metals and semi-metals

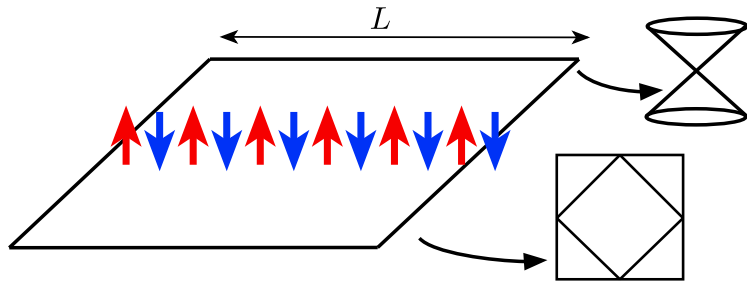
B. Danu, FFA arXiv:2509.11392

Marginal Fermi liquid at magnetic quantum criticality from dimensional confinement

Zi Hong Liu, B. Frank, L. Janssen, M. Vojta, FFA, Phys. Rev. B 107, 165104 (2023)

B. Frank, Zi Hong Liu, FFA, M. Vojta, and L. Janssen, Phys. Rev. B 108 (2023), L100405.

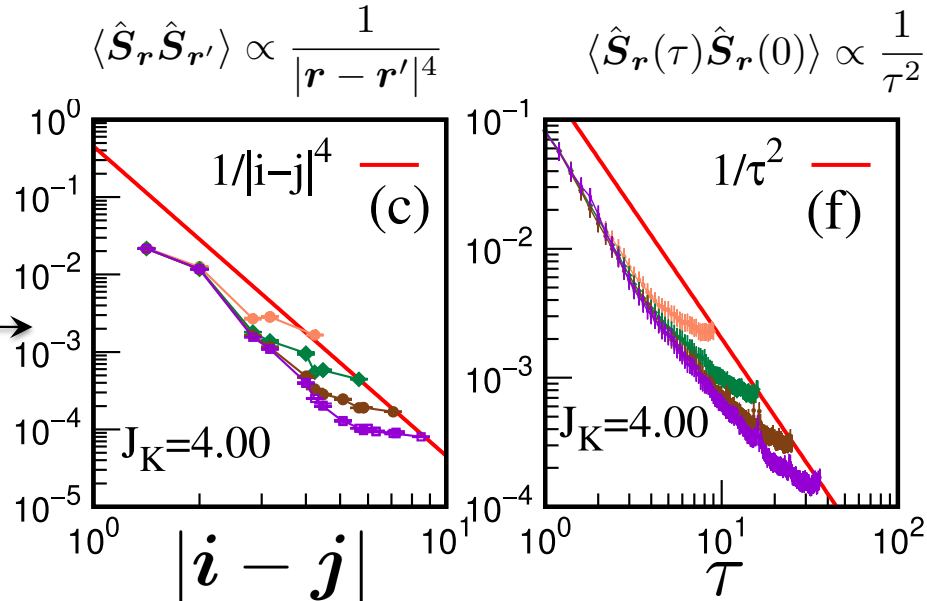
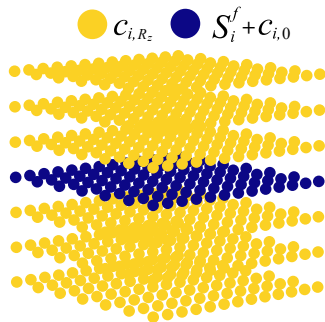
Kondo phase $J_K \gg t$



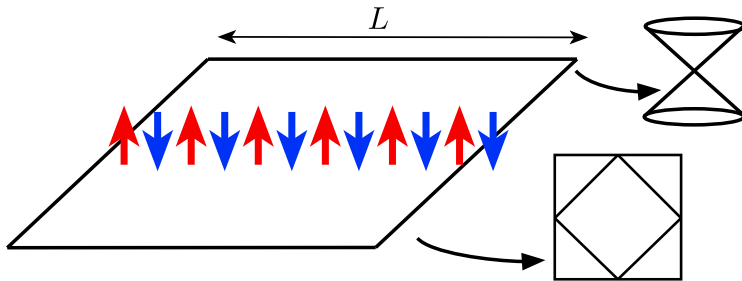
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \boldsymbol{\sigma} \hat{c}_r \cdot \hat{\mathbf{S}}_r + J_h \sum_{\langle r,r' \rangle} \hat{\mathbf{S}}_r \cdot \hat{\mathbf{S}}_{r'}$$

$\hat{\Psi}_i = \hat{\mathbf{S}}_i \cdot \boldsymbol{\sigma} \hat{c}_i$ Emergent composite fermion that participates in Luttinger count

Spin-spin correlations inherit power-law of conduction electrons.



Weak-coupling $J_k \ll t$



$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

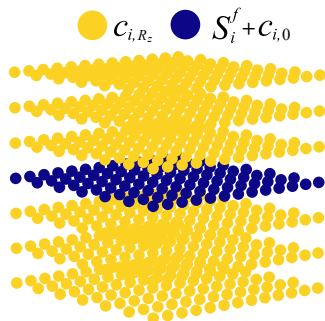
Integrate out electrons à la Hertz-Millis

$$\mathcal{S}(\mathbf{n}) = \mathcal{S}_{\text{spin}}(\mathbf{n}) + \mathcal{S}_{\text{diss}}(\mathbf{n}) + \dots$$

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \frac{J_k^2}{8} \int d\tau d\tau' \sum_{\mathbf{r}, \mathbf{r}'} \mathbf{n}_{\mathbf{r}}(\tau) \chi^0(\mathbf{r} - \mathbf{r}', \tau - \tau') \mathbf{n}_{\mathbf{r}'}(\tau').$$

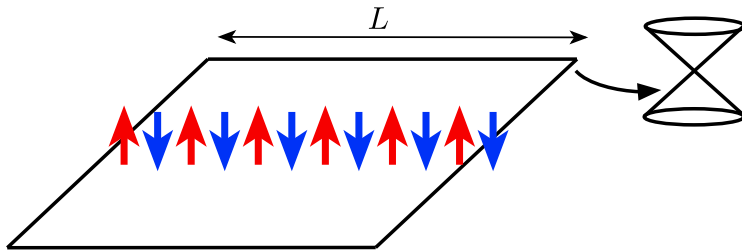


Spin susceptibility of the host metal

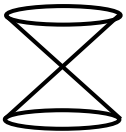


Spin chain on semi-metal

B. Danu, M. Vojta, FFA, and T. Grover, Phys. Rev. Lett. 125 (2020), 206602.



$$\Delta_n = \frac{1}{2}$$



$$\chi^0(\mathbf{0}, \tau - \tau') \propto \frac{1}{v_F^2 (\tau - \tau')^4}$$

$$\chi^0(r\mathbf{e}_x, 0) \propto \frac{1}{r^4}$$

Kondo is irrelevant

For: $\mathbf{r} \rightarrow \lambda \mathbf{r}, \tau \rightarrow \lambda \tau,$

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \frac{J_k^2}{8} \int d\tau d\tau' d\mathbf{r} \mathbf{n}_{\mathbf{r}}(\tau) \chi^0(0, \tau - \tau') \mathbf{n}_{\mathbf{r}}(\tau') \rightarrow \lambda^{-2} \mathcal{S}_{\text{diss}}(\mathbf{n})$$

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

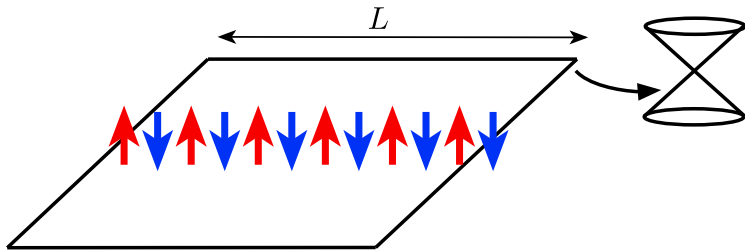
Integrate out electrons à la Hertz-Millis

$$\mathcal{S}(\mathbf{n}) = \mathcal{S}_{\text{spin}}(\mathbf{n}) + \mathcal{S}_{\text{diss}}(\mathbf{n}) + \dots$$

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \frac{J_k^2}{8} \int d\tau d\tau' \sum_{\mathbf{r}, \mathbf{r}'} \mathbf{n}_{\mathbf{r}}(\tau) \chi^0(\mathbf{r} - \mathbf{r}', \tau - \tau') \mathbf{n}_{\mathbf{r}'}(\tau').$$

Spin chain on semi-metal

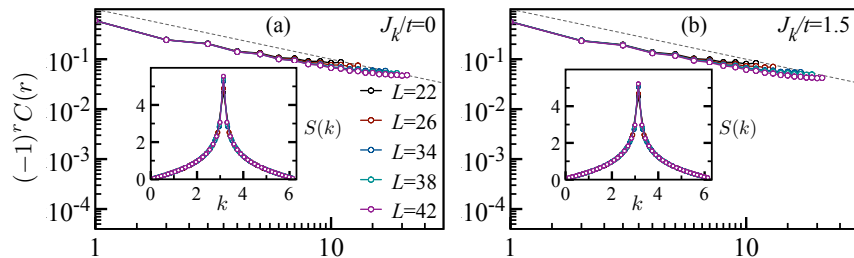
B. Danu, M. Vojta, FFA, and T. Grover, Phys. Rev. Lett. 125 (2020), 206602.



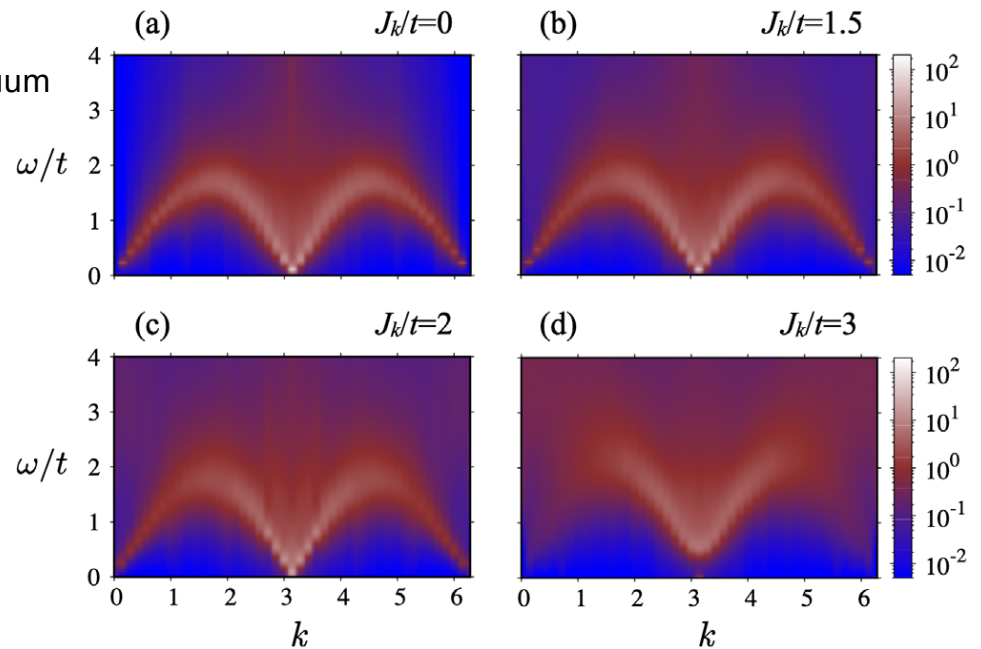
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \boldsymbol{\sigma} \hat{c}_r \cdot \hat{\mathbf{S}}_r + J_h \sum_{\langle r,r' \rangle} \hat{\mathbf{S}}_r \cdot \hat{\mathbf{S}}_{r'}$$

$$S(\mathbf{q}, \omega) = \frac{\chi''(\mathbf{q}, \omega)}{1 - e^{-\beta\omega}}$$

$S(\mathbf{q})$



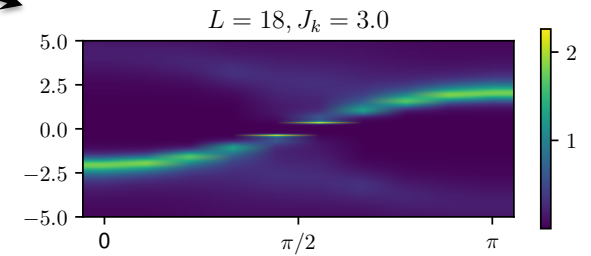
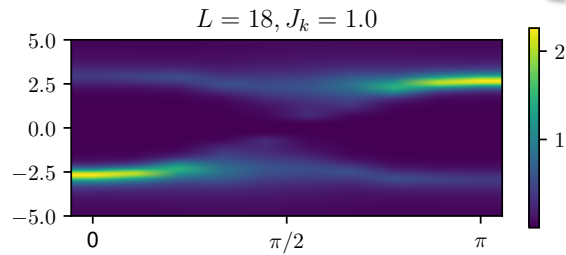
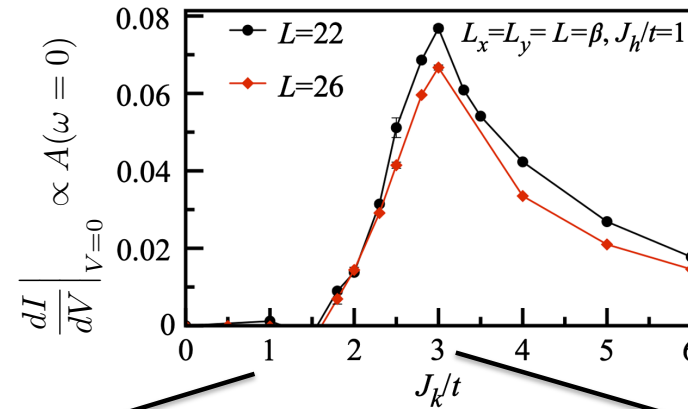
Two
spinon
continuum



Spin chain on semi-metal

B. Danu, M. Vojta, FFA, and T. Grover, Phys. Rev. Lett. 125 (2020), 206602.

Composite fermion spectral function



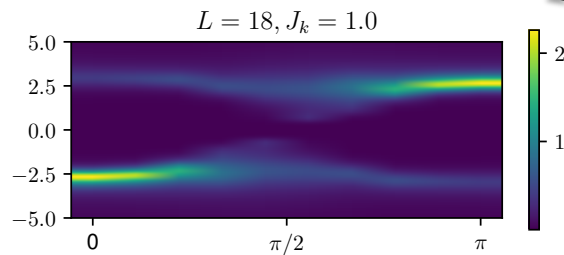
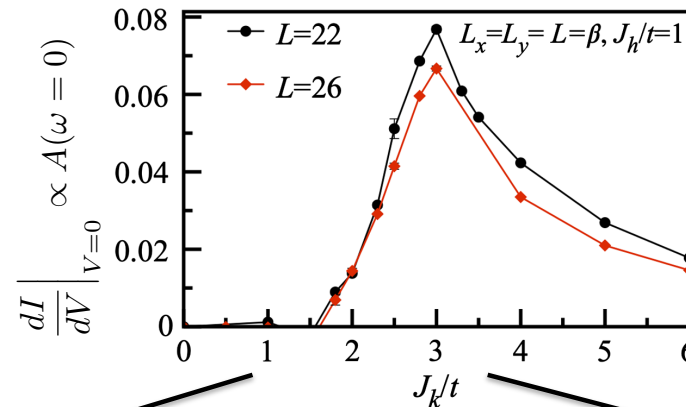
$$A(k, \omega) = -\text{Im}G^{\text{ret}}(\omega), \quad G^{\text{ret}}(\omega) = -i \int_0^\infty dt e^{i\omega t} \sum_{\sigma} \langle \{ \hat{\psi}_{k,\sigma}(t), \hat{\psi}_{k,\sigma}^\dagger(0) \} \rangle \quad \hat{\psi}_r^\dagger = \hat{c}_r^\dagger \boldsymbol{\sigma} \cdot \hat{\mathbf{S}}_r$$

F. Mazza, S. Biswas, X. Yan, A. Prokofiev, P. Steffens, Q. Si, FFA, and S. Paschen, arXiv:2403.12779 (2024).

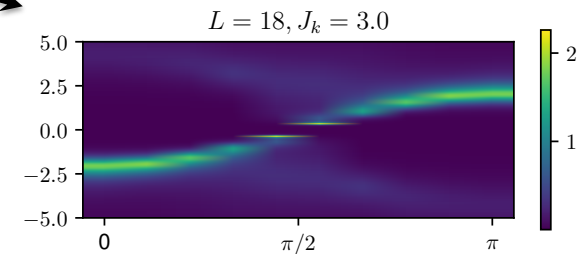
Spin chain on semi-metal

B. Danu, M. Vojta, FFA, and T. Grover, Phys. Rev. Lett. 125 (2020), 206602.

Composite fermion spectral function



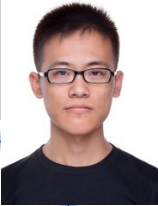
Example of an orbital selective Mott transition in which the composite fermion does not contribute to transport.



Question: can we compute the conductivity?

$$\sigma'(\omega) \propto \frac{1}{\omega} \text{Im} \int_0^\infty dt e^{i\omega t} \langle [\hat{J}, \hat{J}(-t)] \rangle$$

$$\hat{J} = \sum_{\mathbf{r}, \sigma} i \hat{\Psi}_{\mathbf{r}, \sigma}^\dagger \hat{\Psi}_{\mathbf{r} + \mathbf{a}_x, \sigma} + \text{H.c.}$$

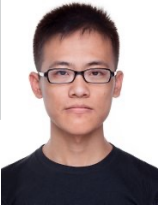


$$\hat{H} = \sum_{\langle i,j \rangle, \sigma} t_{i,j} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + J_h \sum_{\mathbf{r}, \sigma} \left(\hat{f}_{\mathbf{r},\sigma}^\dagger \hat{U}_{\mathbf{r},\mathbf{r}+\mathbf{a}_x} \hat{f}_{\mathbf{r}+\mathbf{a}_x,\sigma} + h.c. \right) + h \sum_b \left(\hat{E}_b \right)^2$$

$$+ h_m \sum_{\mathbf{r}} (\hat{e}_{\mathbf{r}})^2 + V \sum_{\mathbf{r}, \sigma} \left(\hat{c}_{\mathbf{r},\sigma}^\dagger \hat{z}_{\mathbf{r}}^\dagger \hat{f}_{\mathbf{r},\sigma} + h.c. \right)$$

$$\begin{aligned} [\hat{E}_b, \hat{U}_b] &= \hat{U}_b \\ [\hat{e}_{\mathbf{r}}, \hat{z}_{\mathbf{r}}] &= \hat{z}_{\mathbf{r}} \\ \hat{\Psi}_{\mathbf{r},\sigma} &= \hat{z}_{\mathbf{r}}^\dagger \hat{f}_{\mathbf{r},\sigma} \end{aligned}$$

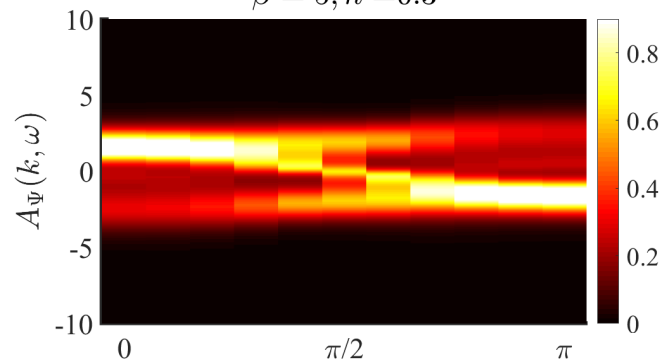
- Limiting cases:
1. $h_m \rightarrow 0$ $\hat{z}_{\mathbf{r}}^\dagger$ is slow so that it can be replaced by a constant. $\hat{\Psi}_{\mathbf{r},\sigma}^\dagger$ is a well defined electronic mode.
 2. $h_m \rightarrow \infty$ $\hat{z}_{\mathbf{r}}^\dagger$ fluctuates quickly such that it averages out to zero on the time scale of the f-fermion. $\hat{\Psi}_{\mathbf{r},\sigma}^\dagger$ is not well defined.
 3. $h \rightarrow \infty$ $J_h \sum_{\mathbf{r}} \left(\hat{f}_{\mathbf{r},\sigma}^\dagger \hat{U}_{\mathbf{r},\mathbf{r}+\mathbf{a}_x} \hat{f}_{\mathbf{r}+\mathbf{a}_x,\sigma} + h.c. \right) + h \sum_{\mathbf{r}} \left(\hat{E}_{\mathbf{r},\mathbf{r}+\mathbf{a}_x} \right)^2 \rightarrow \frac{J_h^2}{h} \hat{S}_{\mathbf{r}} \hat{S}_{\mathbf{r}+\mathbf{a}_x}$ $\hat{S}_{\mathbf{r}} = \frac{1}{2} \hat{f}^\dagger \sigma \hat{f}$



$h = h_m$

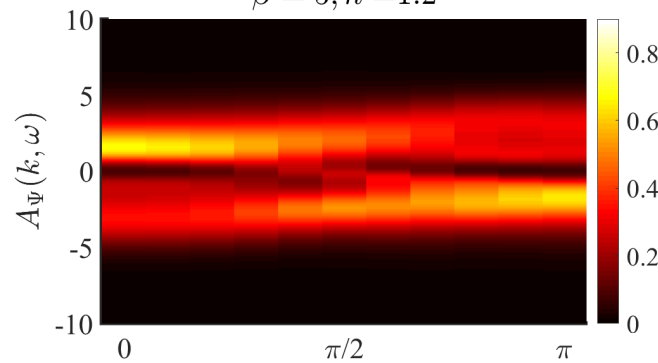
Kondo

$\beta = 5, h = 0.3$



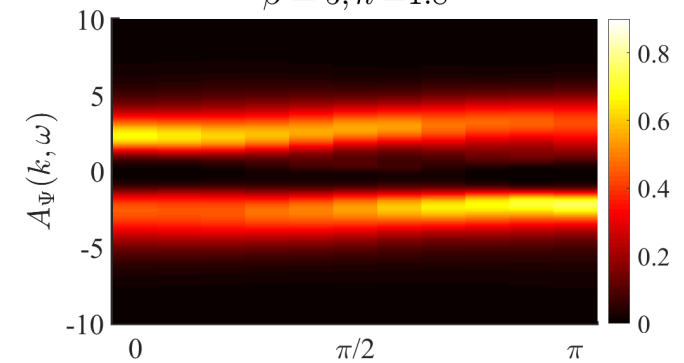
Composite fermion
Spectral function

$\beta = 5, h = 1.2$



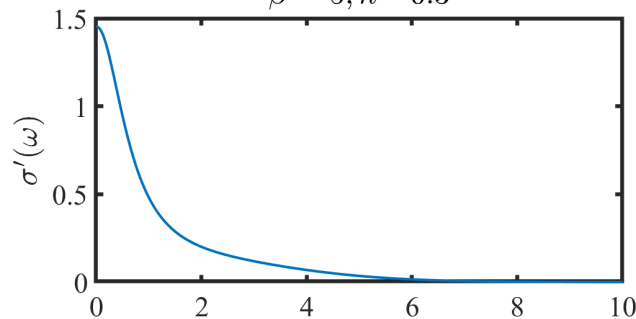
Kondo breakdown

$\beta = 5, h = 1.8$

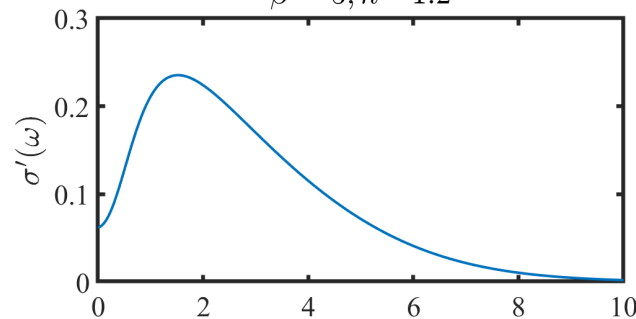


Optical conductivity

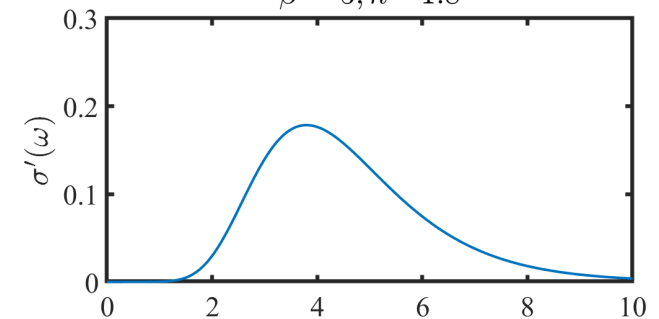
$\beta = 5, h = 0.3$



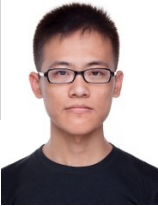
$\beta = 5, h = 1.2$



$\beta = 5, h = 1.8$



Orbital selective Mott transition. (Conduction electrons are metallic)

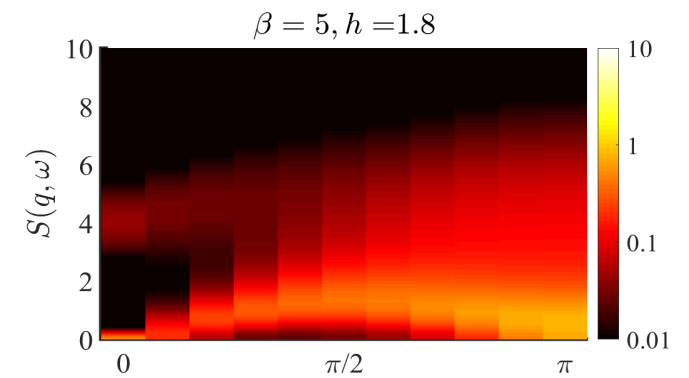
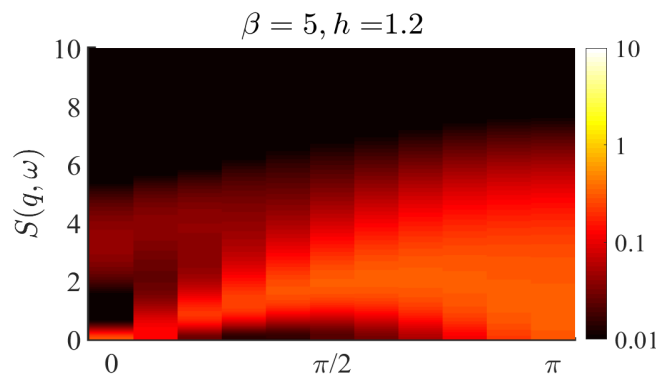
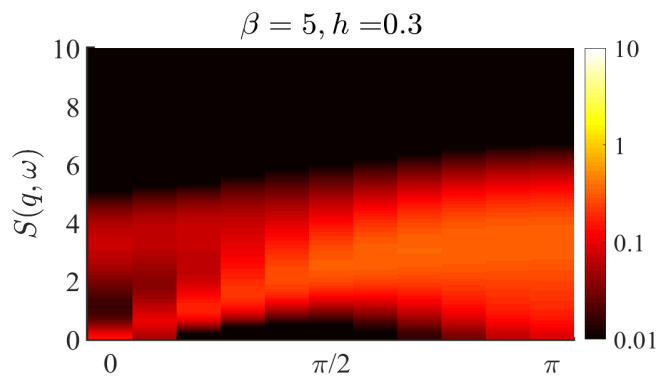


$h = h_m$

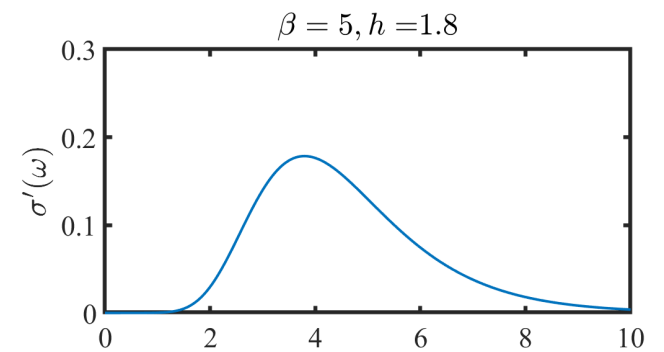
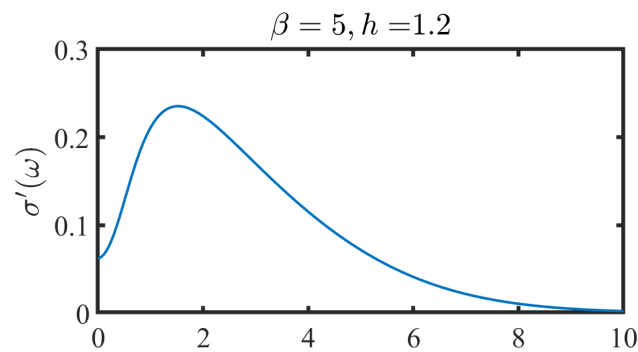
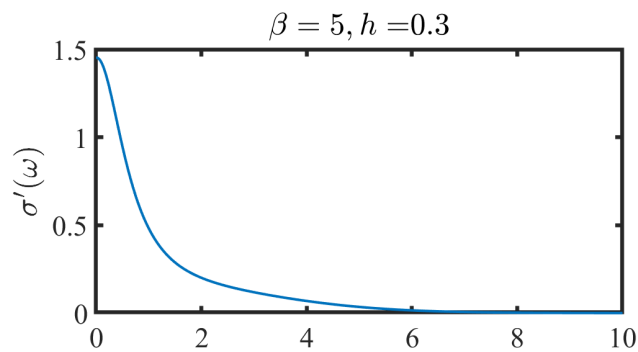
Kondo

Composite fermion
Spectral function

Kondo breakdown



Optical conductivity



Orbital selective Mott transition. (Conduction electrons are metallic)

Fakher F. Assaad, Hong-Kong, November 19th 2025

Goal:

Experimentally relevant, simple (sign free) model systems that can capture the physics of heavy fermion systems.

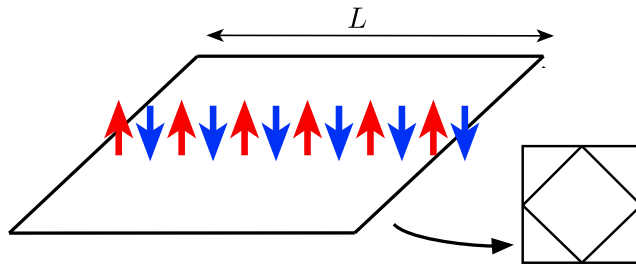
Heavy fermion phase (metal, impurities spin do participate in Luttinger volume) ✓

Kondo breakdown phase (metal, impurities spin do **not** participate in Luttinger volume) ✓

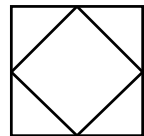
Magnetic ordering in metallic environment

Spin chain on metallic surface

B. Danu, M. Vojta, T. Grover, and F.F. Assaad, Phys. Rev. B 106 (2022), L161103.



$$\Delta_n = \frac{1}{2}$$



$$\chi^0(\mathbf{0}, \tau - \tau') \propto \frac{1}{v_F^2 (\tau - \tau')^2}$$

$$\chi^0(r\mathbf{e}_x, 0) \propto \frac{1}{r^4}$$

Kondo is marginal $\rightarrow$ Dissipation leads to long ranged magnetic order.

M. Weber, D. Luitz, FFA PRL 2022

For: $\mathbf{r} \rightarrow \lambda \mathbf{r}, \tau \rightarrow \lambda \tau,$ $\mathcal{S}_{\text{diss}}(\mathbf{n}) = \frac{J_k^2}{8} \int d\tau d\tau' d\mathbf{r} \mathbf{n}_{\mathbf{r}}(\tau) \chi^0(0, \tau - \tau') \mathbf{n}_{\mathbf{r}}(\tau') \rightarrow \mathcal{S}_{\text{diss}}(\mathbf{n})$

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

Integrate out electrons à la Hertz-Millis

$$\mathcal{S}(\mathbf{n}) = \mathcal{S}_{\text{spin}}(\mathbf{n}) + \mathcal{S}_{\text{diss}}(\mathbf{n}) + \dots$$

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \frac{J_k^2}{8} \int d\tau d\tau' \sum_{\mathbf{r}, \mathbf{r}'} \mathbf{n}_{\mathbf{r}}(\tau) \chi^0(\mathbf{r} - \mathbf{r}', \tau - \tau') \mathbf{n}_{\mathbf{r}'}(\tau').$$



$$\mathcal{S}(\mathbf{n}) = \mathcal{S}_{\text{chain}}(\mathbf{n}) + \mathcal{S}_{\text{diss}}(\mathbf{n})$$

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \alpha \int d\tau d\tau' \sum_{\mathbf{r}} \frac{\mathbf{n}_{\mathbf{r}}(\tau) \mathbf{n}_{\mathbf{r}}(\tau')}{(\tau - \tau')^2}.$$

PHYSICAL REVIEW LETTERS **129**, 056402 (2022)

Dissipation-Induced Order: The $S = 1/2$ Quantum Spin Chain Coupled to an Ohmic Bath

Manuel Weber¹, David J. Luitz^{2,1} and Fakher F. Assaad^{3,4}

Hamiltonian

$$\hat{H} = J \sum_i \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_{i+1} + \sum_{i,q} \omega_q \hat{\mathbf{a}}_{i,q}^\dagger \hat{\mathbf{a}}_{i,q} + \sum_{i,q} \lambda_q \left(\hat{\mathbf{a}}_{i,q}^\dagger + \hat{\mathbf{a}}_{i,q} \right) \cdot \hat{\mathbf{S}}_i$$

$$J(\omega) = \pi \sum_q \lambda_q \delta(\omega - \omega_q) = 2\pi\alpha\omega$$

Conservation law

$$\hat{\mathbf{J}}_{\text{tot}} = \sum_{iq} \hat{\mathbf{Q}}_{iq} \times \hat{\mathbf{P}}_{iq} + \sum_i \hat{\mathbf{S}}_i$$

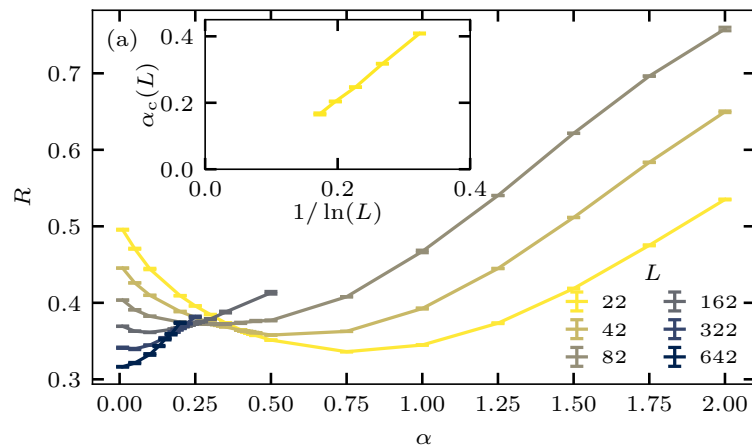
$$\hat{\mathbf{P}}_{iq} = \frac{1}{\sqrt{2}} \left(\hat{\mathbf{a}}_{iq}^\dagger - \hat{\mathbf{a}}_{iq} \right)$$

$$\hat{\mathbf{Q}}_{iq} = \frac{1}{\sqrt{2}} \left(\hat{\mathbf{a}}_{iq}^\dagger + \hat{\mathbf{a}}_{iq} \right)$$

A bath cannot screen

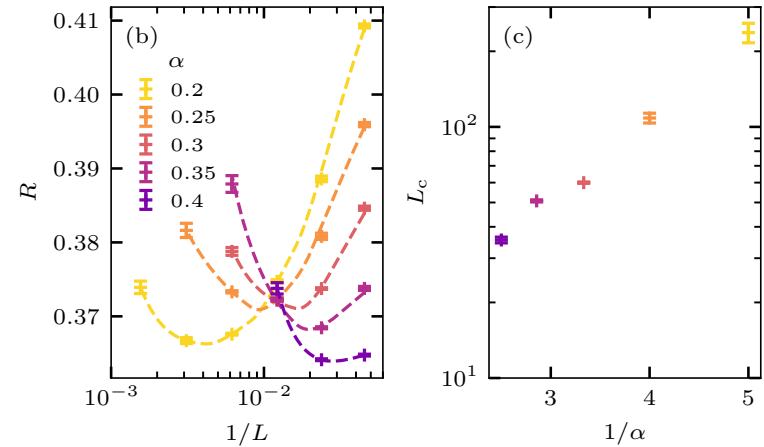
$$\mathcal{S}(\mathbf{n}) = \mathcal{S}_{\text{chain}}(\mathbf{n}) + \mathcal{S}_{\text{diss}}(\mathbf{n})$$

$$\mathcal{S}_{\text{diss}}(\mathbf{n}) = \alpha \int d\tau d\tau' \sum_{\mathbf{r}} \frac{\mathbf{n}_{\mathbf{r}}(\tau) \mathbf{n}_{\mathbf{r}}(\tau')}{(\tau - \tau')^2}.$$



$$S(\mathbf{q}) = \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}} \langle \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_0 \rangle$$

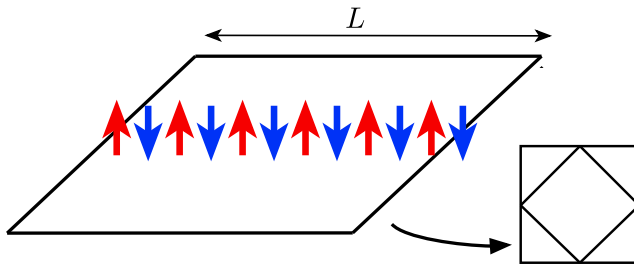
$$R = 1 - \frac{S(\mathbf{Q} + \delta\mathbf{q})}{S(\mathbf{Q})}$$



Long ranged interaction along the imaginary time leads to breakdown of Mermin-Wagner theorem and triggers long-ranged order (Dissipation is marginally relevant)

Spin chain on metallic surface

B. Danu, M. Vojta, T. Grover, and F.F. Assaad, Phys. Rev. B 106 (2022), L161103.

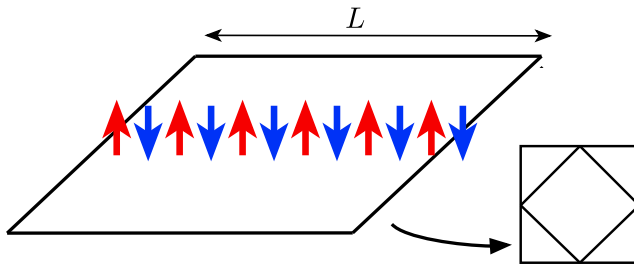


$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

Dissipation and Kondo singlet formation compete and trigger an order-disorder transition.

Spin chain on metallic surface

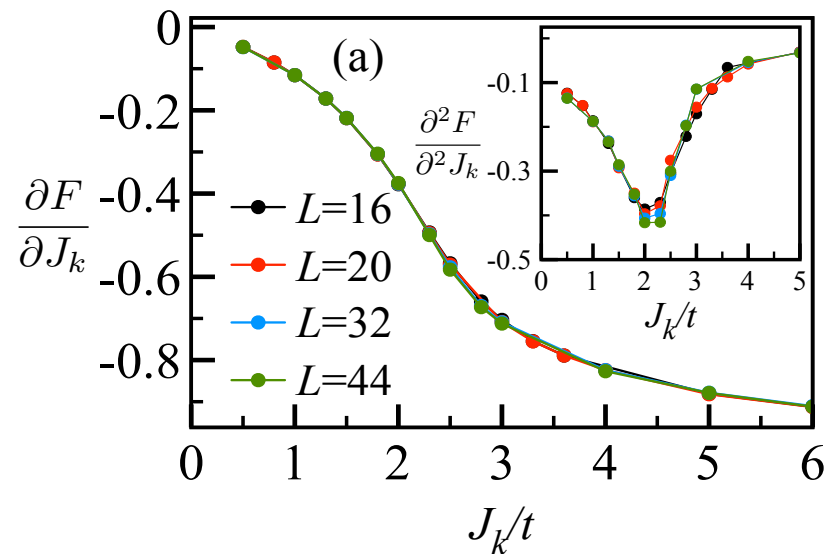
B. Danu, M. Vojta, T. Grover, and F.F. Assaad, Phys. Rev. B 106 (2022), L161103.



$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

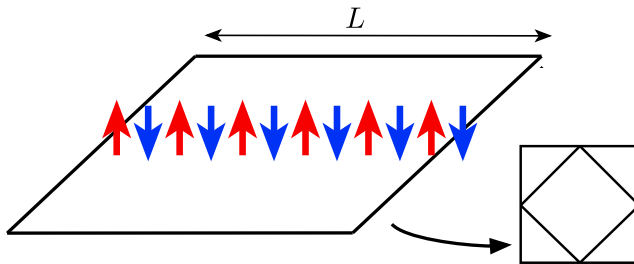
Transition

$$J_k^c/t \simeq 2.1$$



Spin chain on metallic surface

B. Danu, M. Vojta, T. Grover, and F.F. Assaad, Phys. Rev. B 106 (2022), L161103.

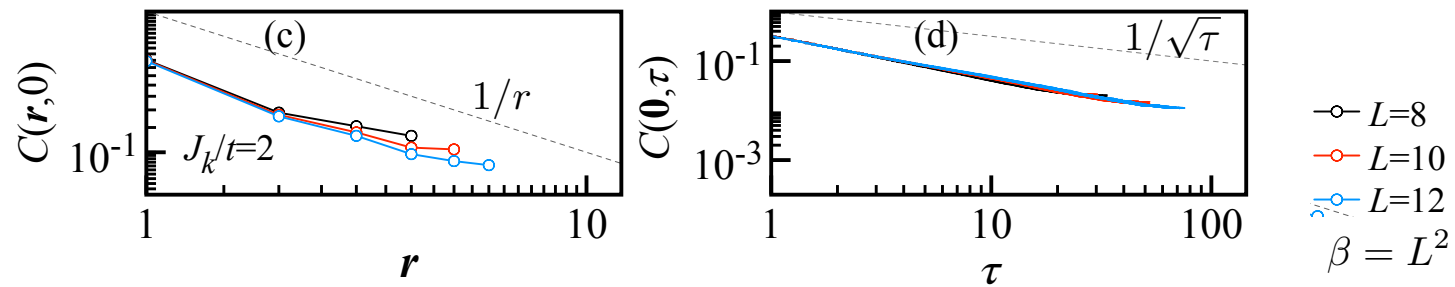


$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

Dynamical exponent

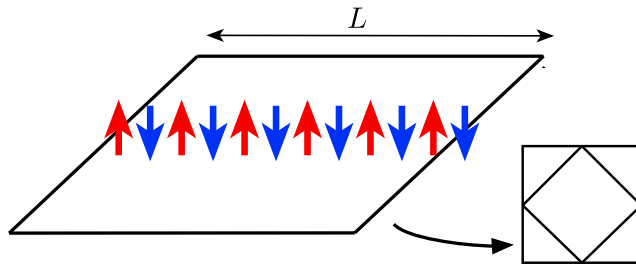
$$J_k^c/t \simeq 2.1$$

$$z \simeq 2$$



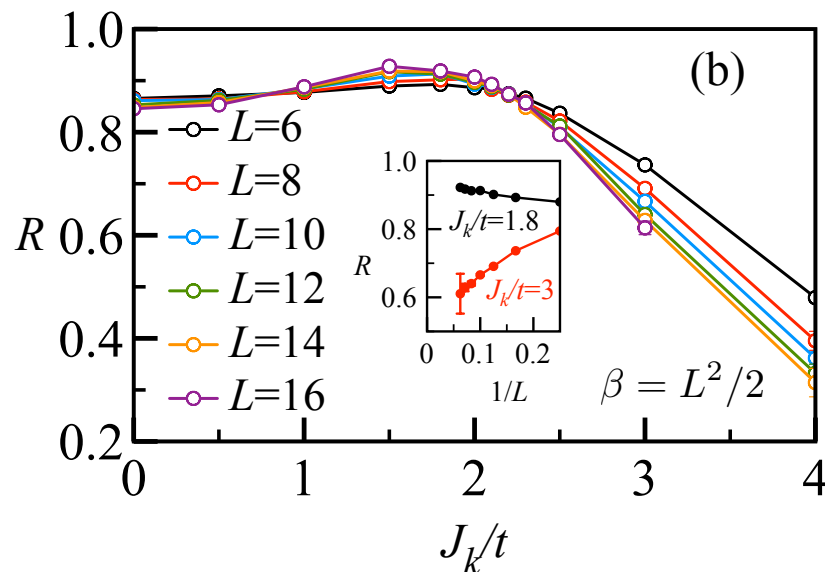
Spin chain on metallic surface

B. Danu, M. Vojta, T. Grover, and F.F. Assaad, Phys. Rev. B 106 (2022), L161103.



$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

Correlation ratio



$$J_k^c/t \simeq 2.1$$

$$z \simeq 2$$

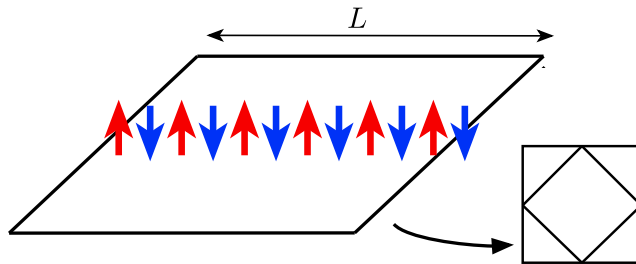
$$\chi(\mathbf{k}, i\Omega_m) = \int_0^\beta d\tau \sum_{\mathbf{r}} e^{i(\Omega_m \tau - \mathbf{k} \cdot \mathbf{r})} \langle \hat{S}_{\mathbf{r}}^z(\tau) \hat{S}_{\mathbf{0}}^z(0) \rangle$$

$$R = 1 - \frac{\chi(\mathbf{Q} - \delta \mathbf{k}, 0)}{\chi(\mathbf{Q}, 0)}$$

$$R = f([J_k - J_k^c] L^{1/\nu}, L^z/\beta, L^{-\omega})$$

Spin chain on metallic surface

B. Danu, M. Vojta, T. Grover, and F.F. Assaad, Phys. Rev. B 106 (2022), L161103.

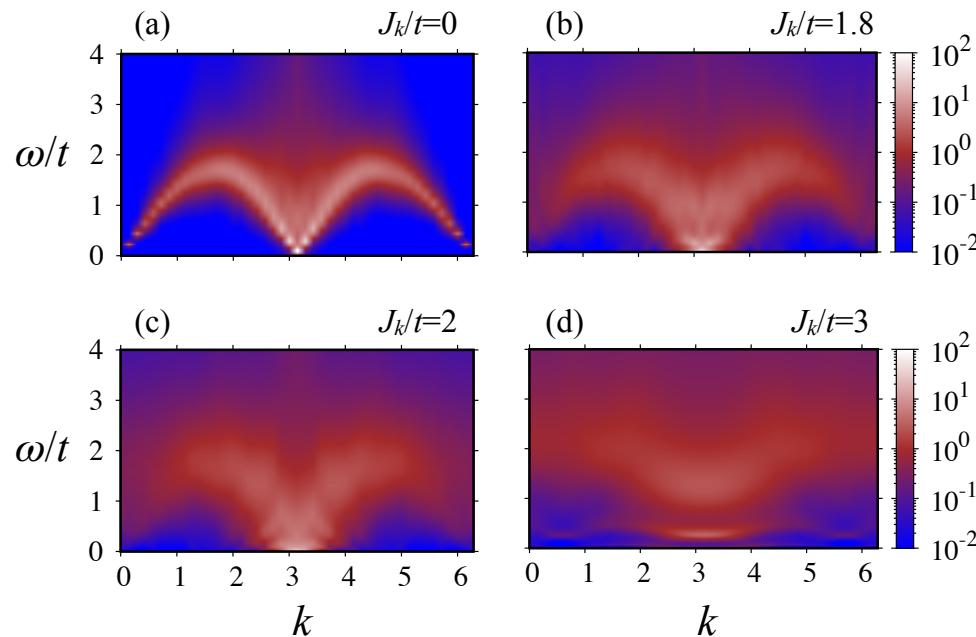


$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

Dynamical spin structure factor

Two spinon continuum

$$S(\mathbf{k}, \omega) = \frac{\text{Im} \chi(\mathbf{k}, \omega + i0^+)}{1 - e^{-\beta \omega}}$$

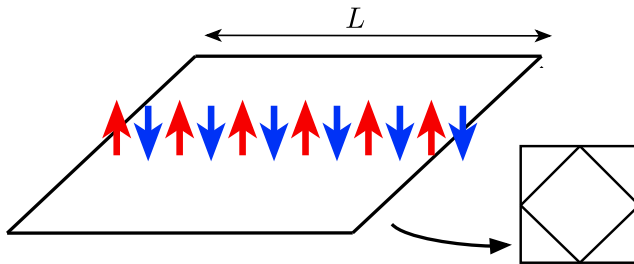


Landau damped Goldstone mode

Spin inherit properties of metal

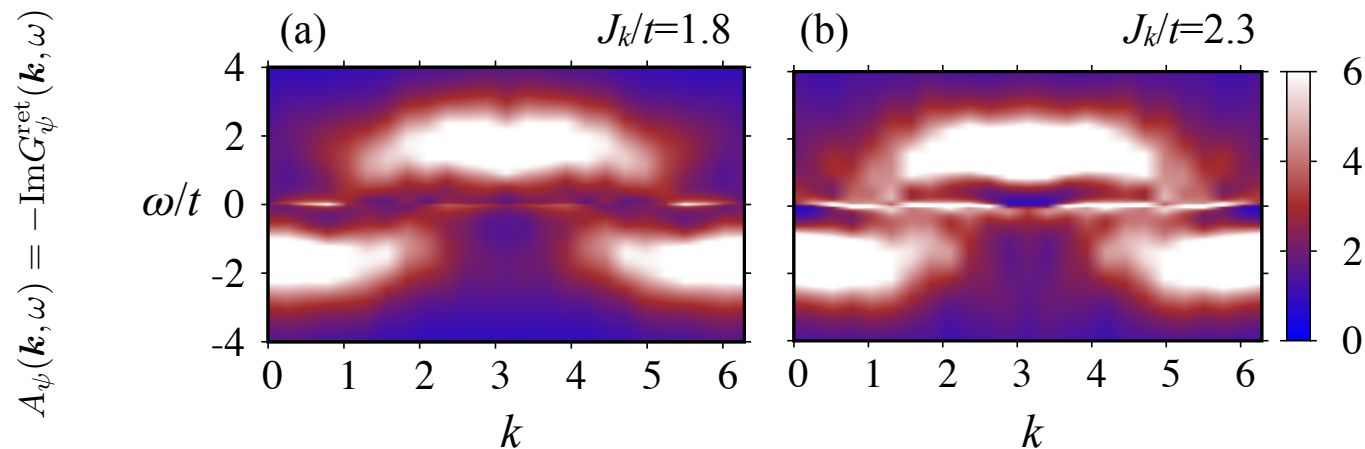
Spin chain on metallic surface

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$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'}$$

Single particle spectral function shows heavy bands in both phases.



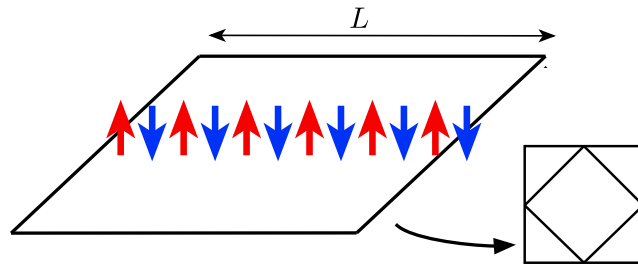
Wanted: U(1) gauge
theories calculations
for this setup.

$$G_{\psi}^{\text{ret}}(\mathbf{k}, \omega) = -i \int_0^{\infty} dt e^{i\omega t} \sum_{\sigma} \langle \{ \hat{\psi}_{\mathbf{k},\sigma}(t), \hat{\psi}_{\mathbf{k},\sigma}^{\dagger}(0) \} \rangle$$

$$\hat{\psi}_{\mathbf{r},\sigma}^{\dagger} = 2 \sum_{\sigma'} \hat{c}_{\mathbf{r},\sigma'}^{\dagger} \boldsymbol{\sigma}_{\sigma',\sigma} \cdot \hat{\mathbf{S}}_{\mathbf{r}}$$

Spin chain on metallic surface

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$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \boldsymbol{\sigma} \hat{c}_r \cdot \hat{\mathbf{S}}_r + J_h \sum_{\langle r,r' \rangle} \hat{\mathbf{S}}_r \cdot \hat{\mathbf{S}}_{r'}$$

$$S(\mathbf{q}, \omega) = \frac{\chi''(\mathbf{q}, \omega)}{1 - e^{-\beta\omega}}$$

Two
spinon
continuum ω/t

$$J_k^c \propto 2.1t$$

AFM
metal

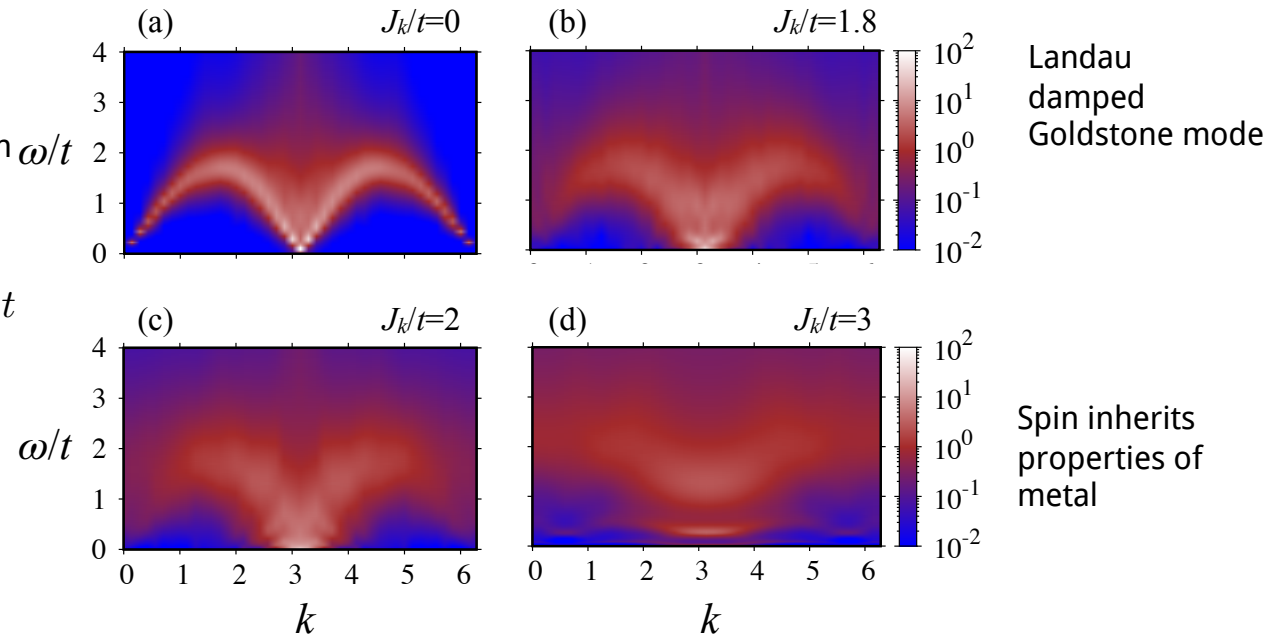
$$\langle \mathbf{n} \rangle \neq 0, \langle \mathbf{b} \rangle \neq 0$$



Heavy fermion
metal

$$\langle \mathbf{n} \rangle = 0, \langle \mathbf{b} \rangle \neq 0$$

$$J_k/t$$



Fakher F. Assaad, Hong-Kong, November 19th 2025

Goal:

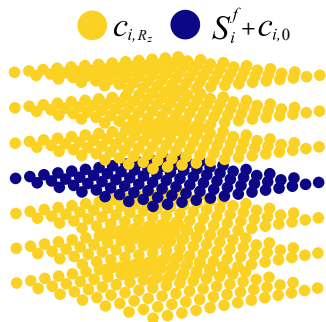
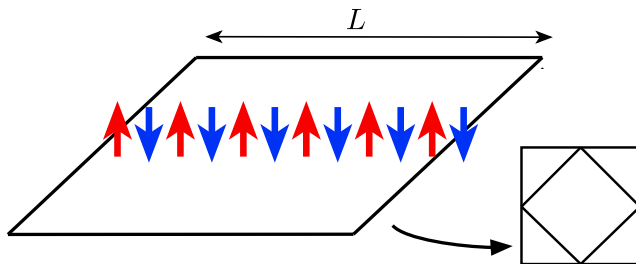
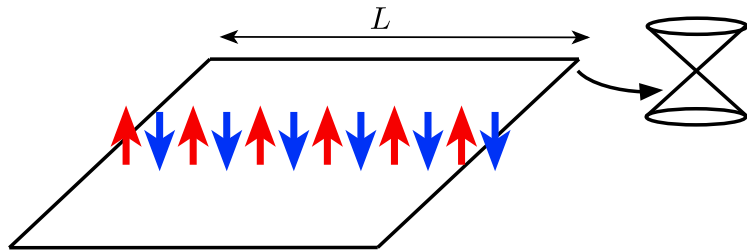
Experimentally relevant, simple (sign free) model systems that can capture the physics of heavy fermion systems.

Heavy fermion phase (metal, impurities spin do participate in Luttinger volume) ✓

Kondo breakdown phase (metal, impurities spin do **not** participate in Luttinger volume) ✓

Magnetic ordering in metallic environment ✓

Summary



$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_r \hat{c}_r^\dagger \boldsymbol{\sigma} \hat{c}_r \cdot \hat{\mathbf{S}}_r + J_h \sum_{\langle r,r' \rangle} \hat{\mathbf{S}}_r \cdot \hat{\mathbf{S}}_{r'}$$

Kondo breakdown transition

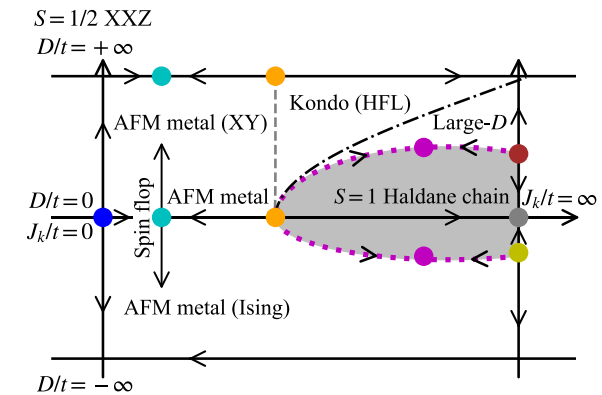
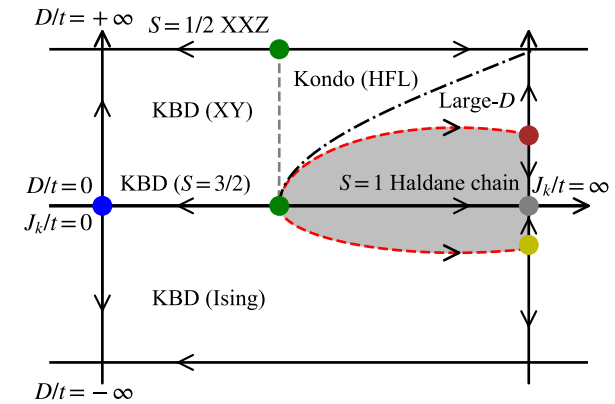
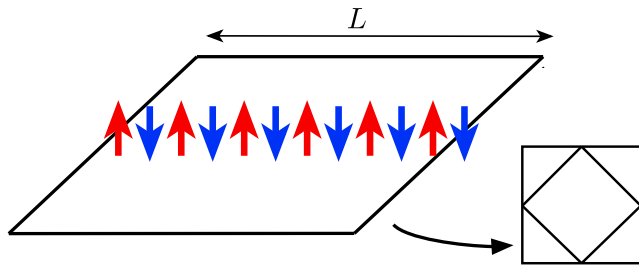
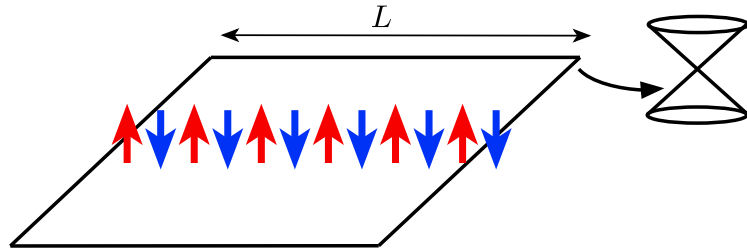
Dissipation induced long range order

QCP between antiferromagnetic heavy fermion metal and heavy fermion metal

Realization of marginal Fermi liquid at QCP

between antiferromagnetic heavy fermion metal and heavy fermion metal

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \frac{J_k}{2} \sum_{\mathbf{r}} \hat{c}_{\mathbf{r}}^\dagger \boldsymbol{\sigma} \hat{c}_{\mathbf{r}=0} \cdot \hat{\mathbf{S}}_{\mathbf{r}} + J_h \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \hat{\mathbf{S}}_{\mathbf{r}} \cdot \hat{\mathbf{S}}_{\mathbf{r}'} + D \sum_{\mathbf{r}} \left(\hat{S}_{\mathbf{r}}^z \right)^2$$



Phases and phase transitions of an $S = 3/2$ chain on metallic and semi-metallic surfaces

Bimla Danu ¹ and Fakher F. Assaad ² [arXiv:2509.11392](https://arxiv.org/abs/2509.11392)