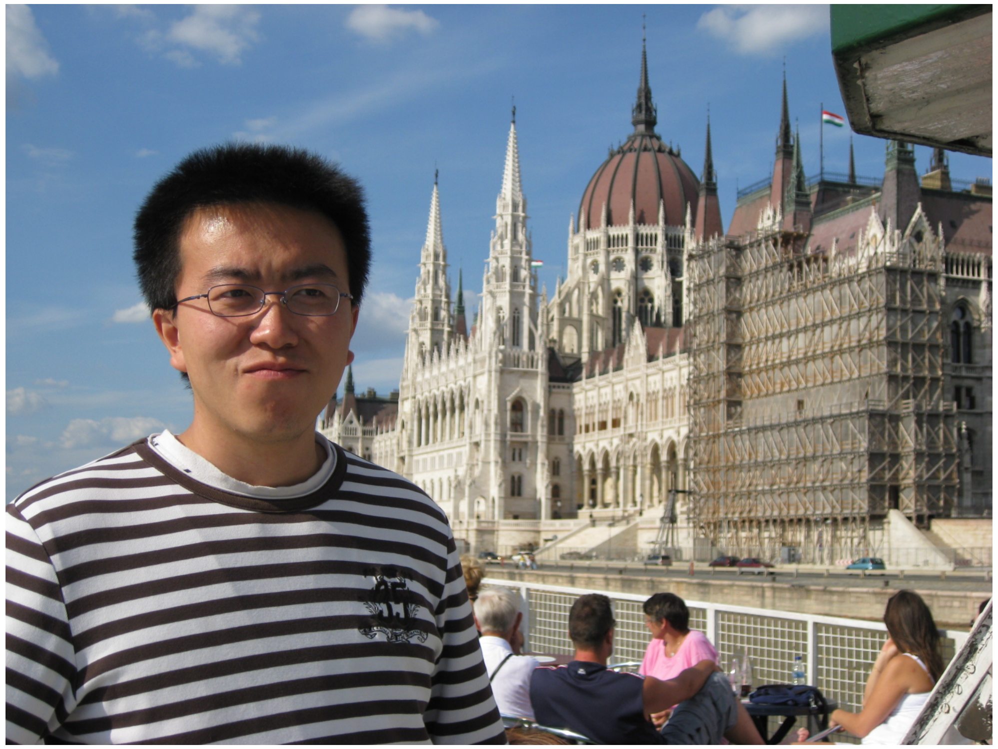
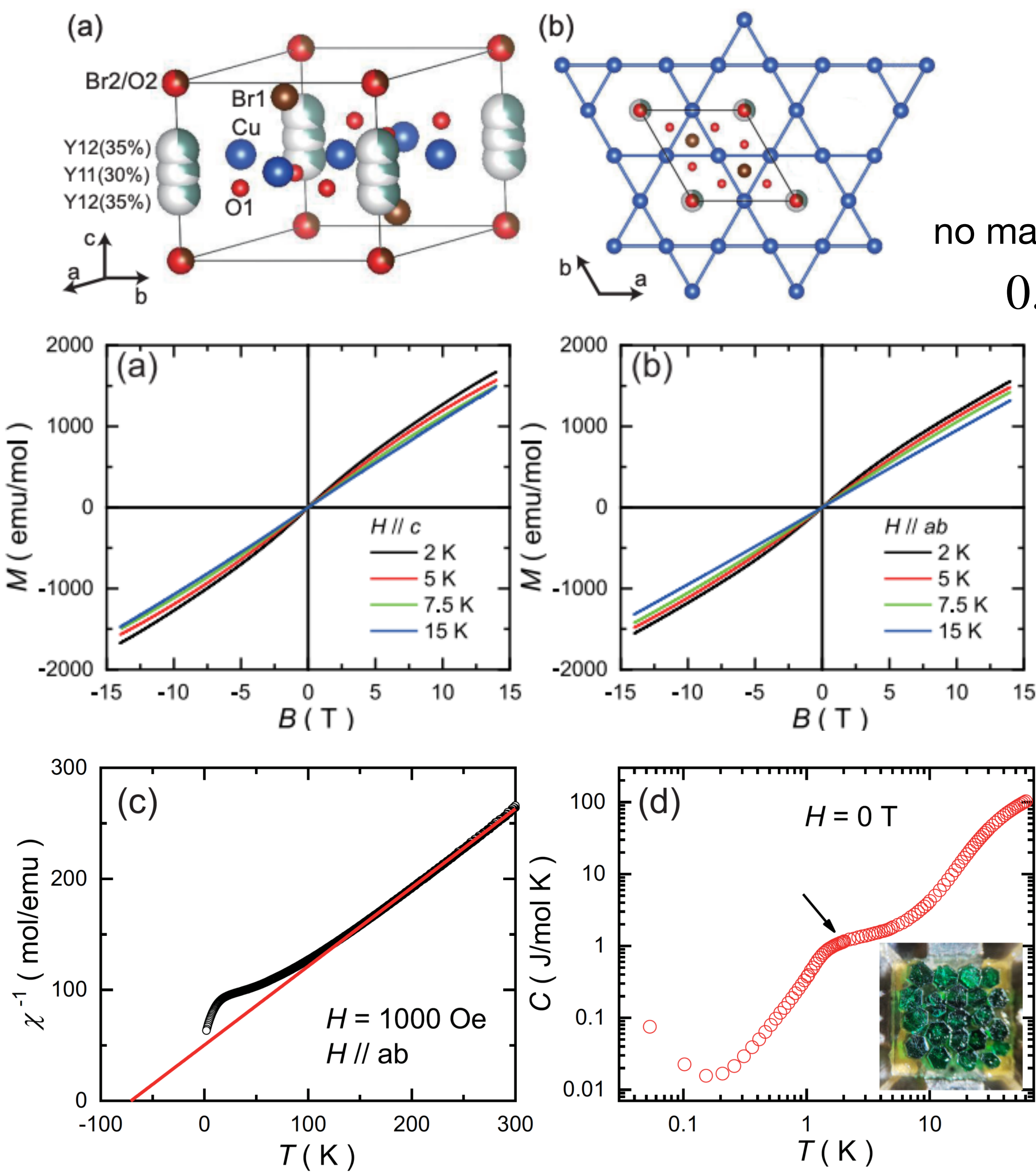
U(1) Dirac spin liquid, QED3, Monte Carlo, GPU and all that

ZI YANG MENG
孟子楊
https://quantummc.xyz/

2009/07, (probably not) thinking about QSL

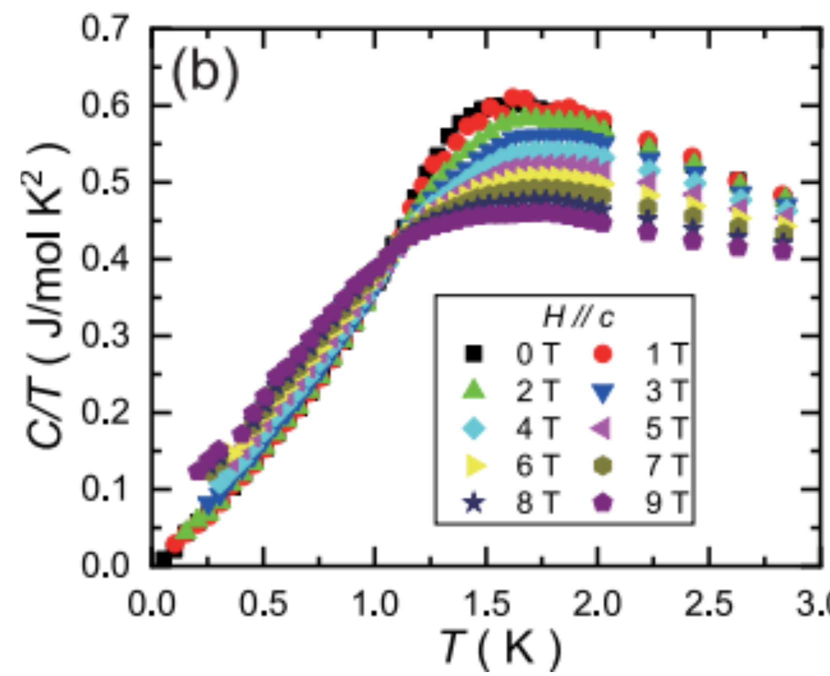


$YCu_3(OH)_6Br_2[Br_x(OH)_{1-x}]$ (YCu_3-Br)



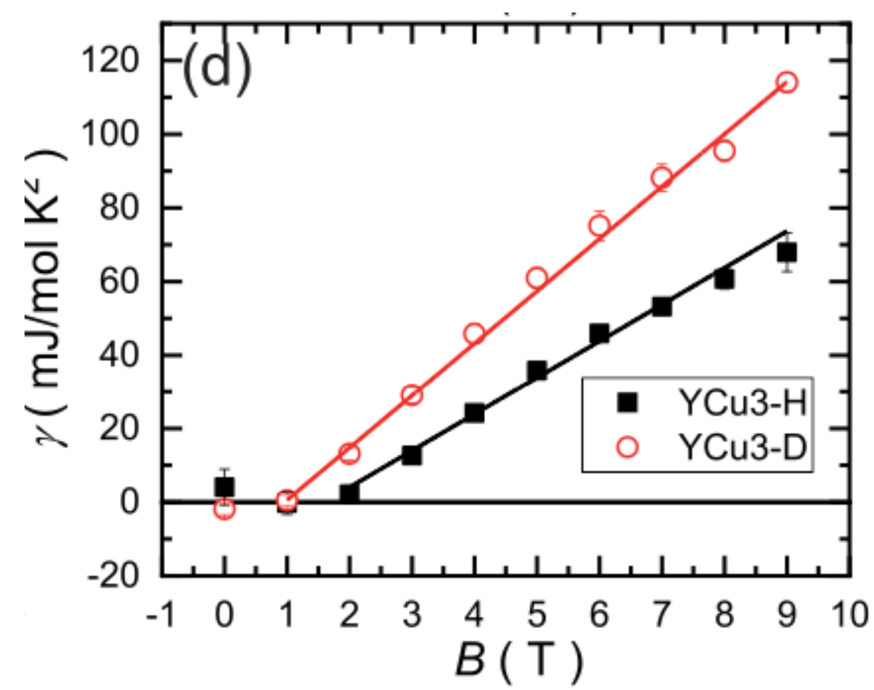
no mag. order

$0.05K$



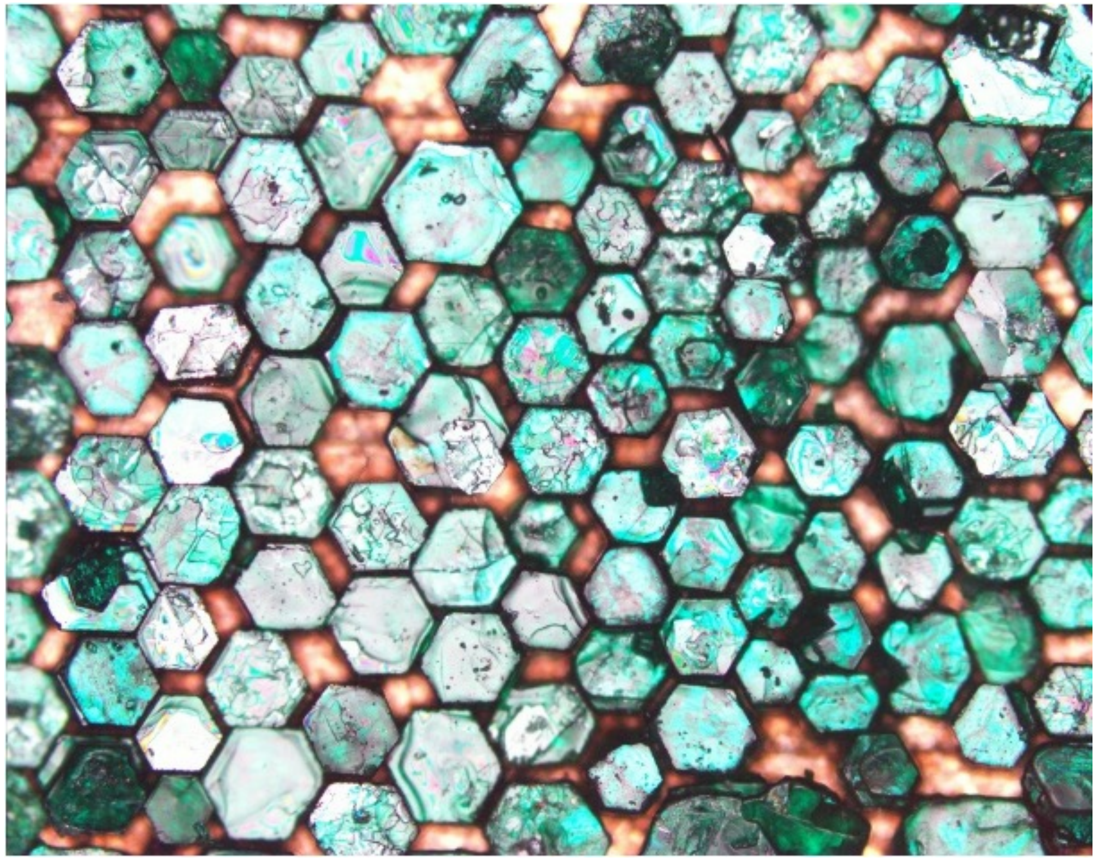
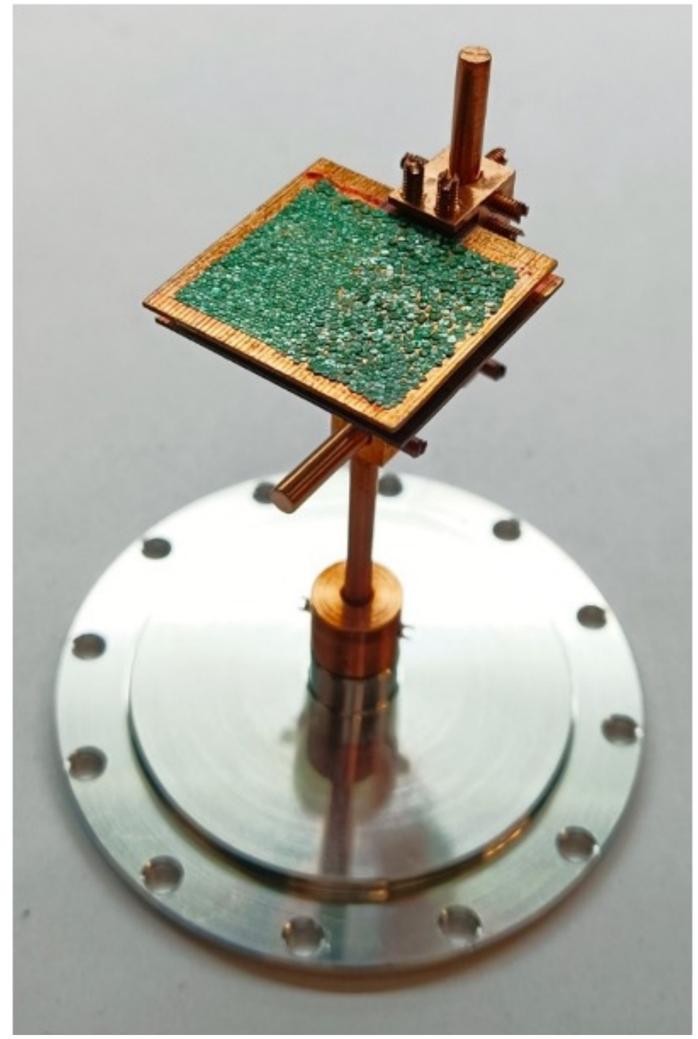
$C/T = \gamma + \alpha T$

$k_B T \ll \mu_B B$



Possible Dirac quantum spin liquid in a kagome quantum antiferromagnet YCu3(OH)6Br2[Brx(OH)1−x], Zeng, Ma, …, ZYM, Shiliang Li, [Phys. Rev. B 105, L121109 (2022)]

$YCu_3(OH)_6Br_2[Br_x(OH)_{1-x}]$ (YCu_3-Br)

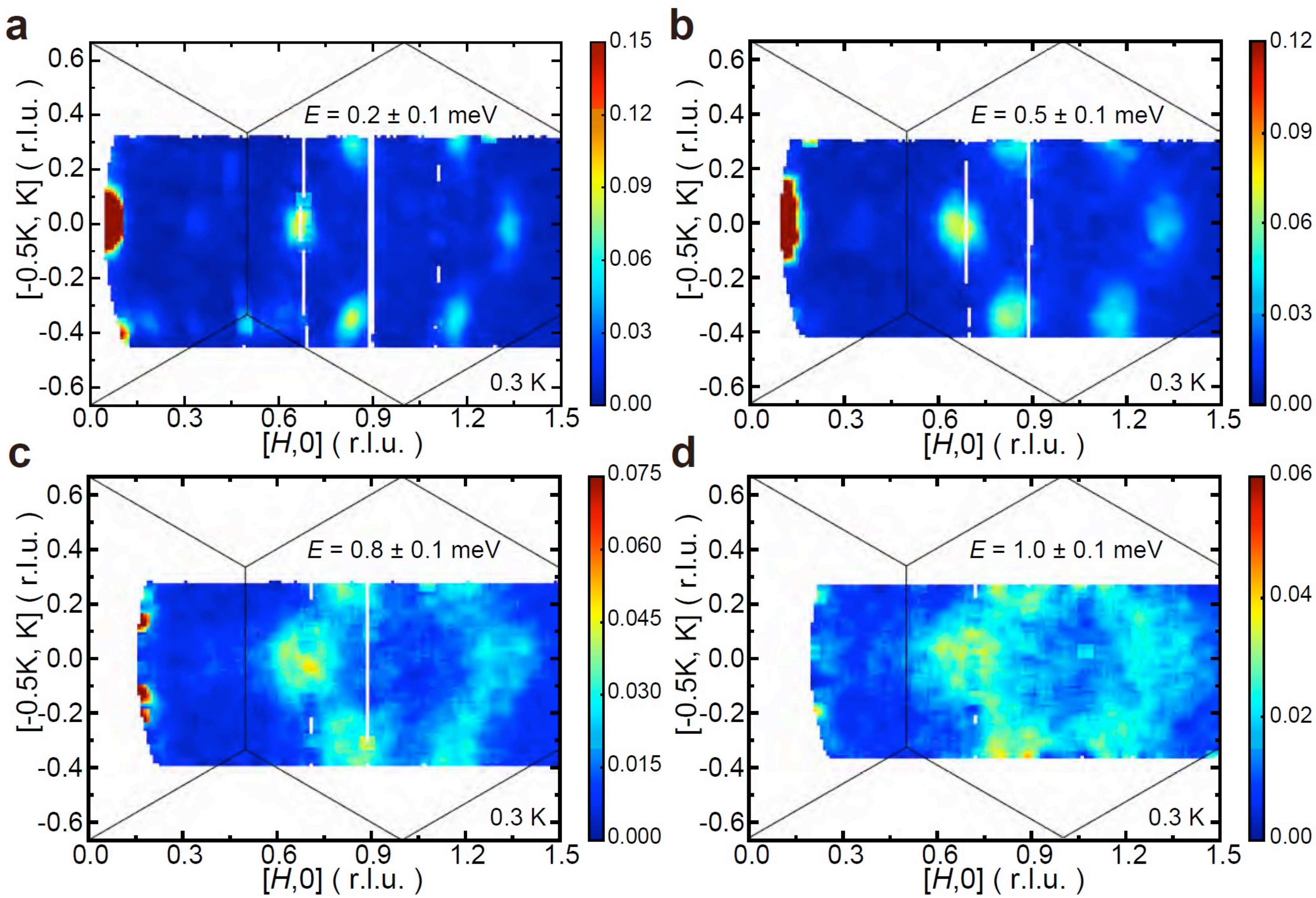



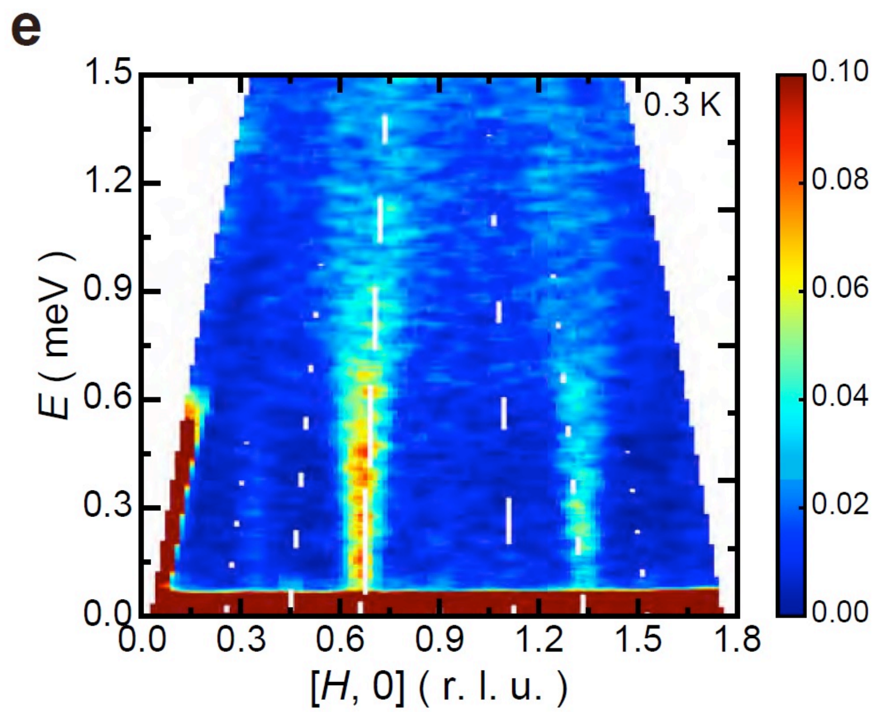

Sample #1: ~ 5000 single crystals, ~ 0.5 g @ AMATERAS, J-Parc

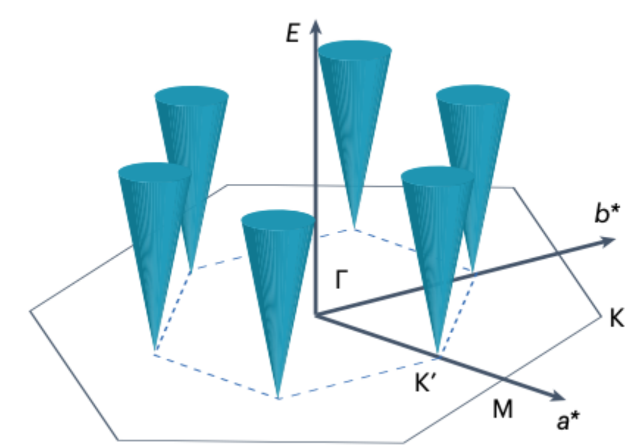
Sample #2: ~ 800 single crystals, ~ 0.5 g @ AMATERAS, J-Parc & THALES, ILL

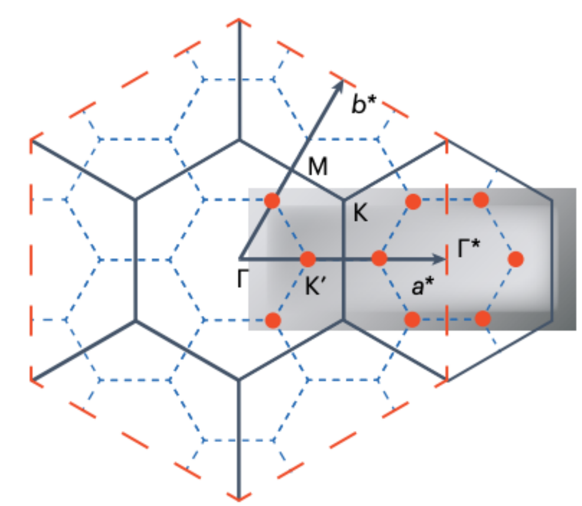
📍 Spectral evidence for Dirac spinons in a kagome lattice antiferromagnet,
Zeng, Zhou, …, ZYM, Shiliang Li,
[Nat. Phys. 20, 1097 (2024)]

$YCu_3(OH)_6Br_2[Br_x(OH)_{1-x}]$ (YCu_3-Br or YCOB)









Spectral evidence for Dirac spinons in a kagome lattice antiferromagnet,
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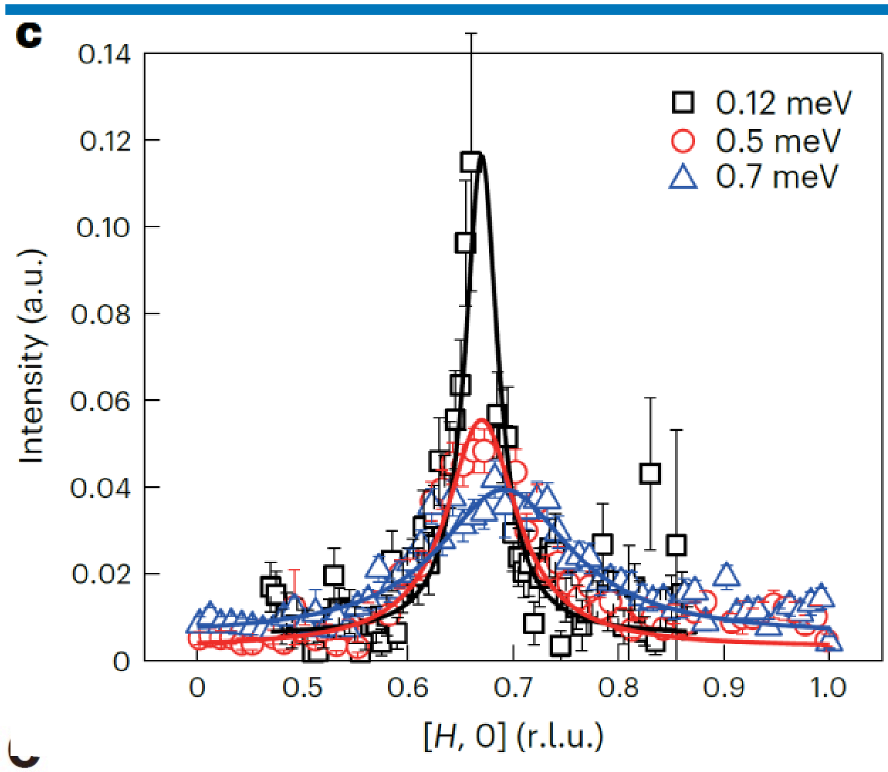
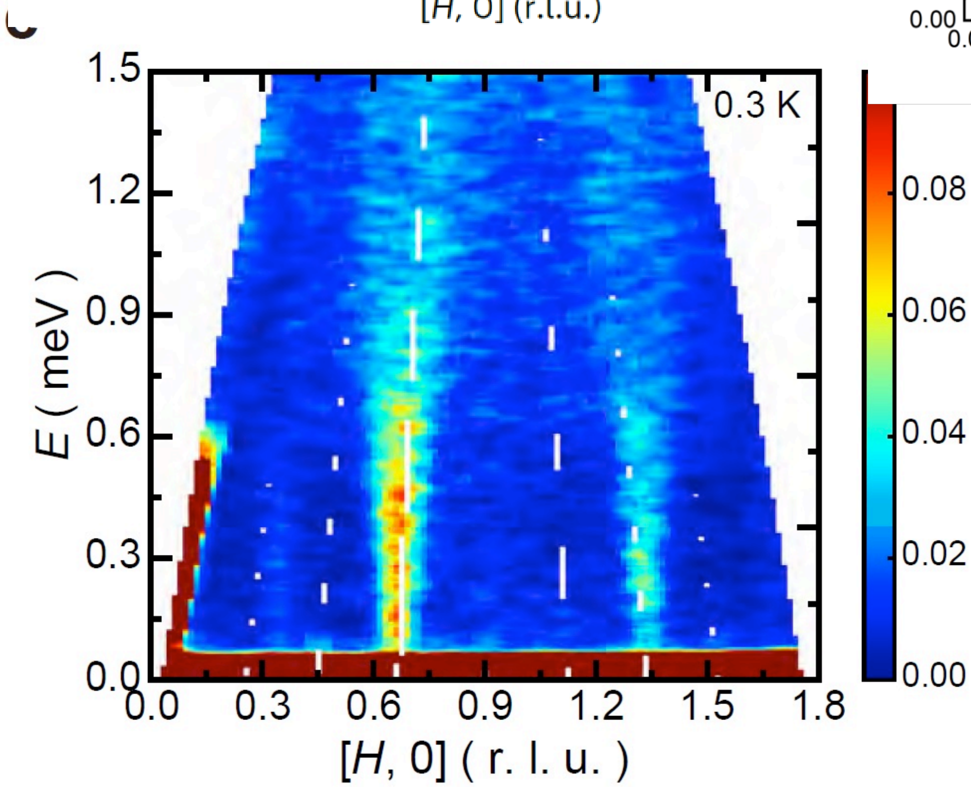
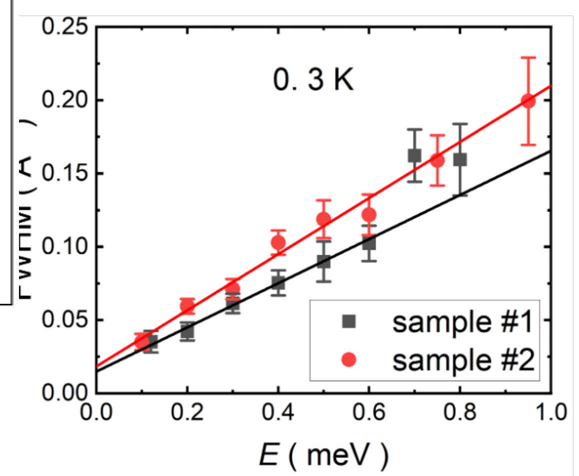
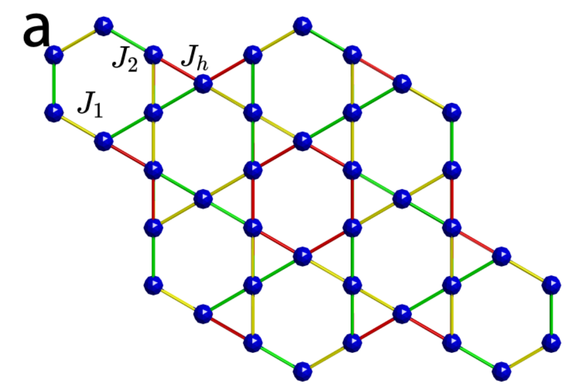
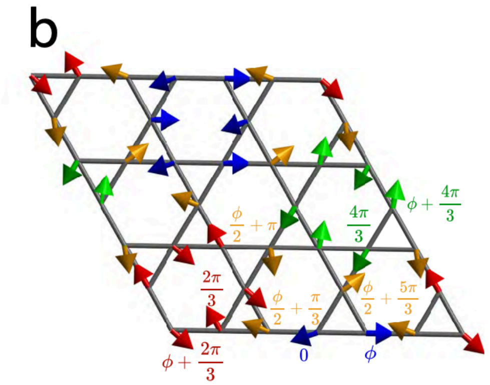


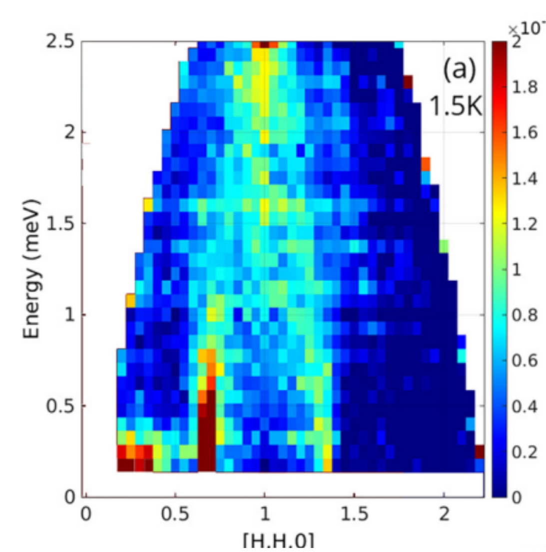


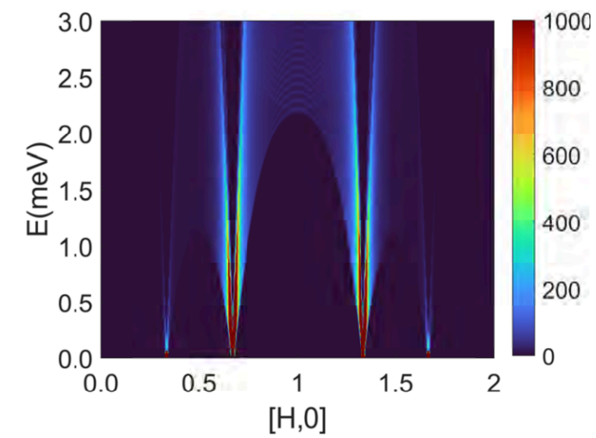


• $Y_3Cu_9(OH)_{19}Cl_8$, Y-kapellasite, Phys. Rev. B 107, 125156 (2023)





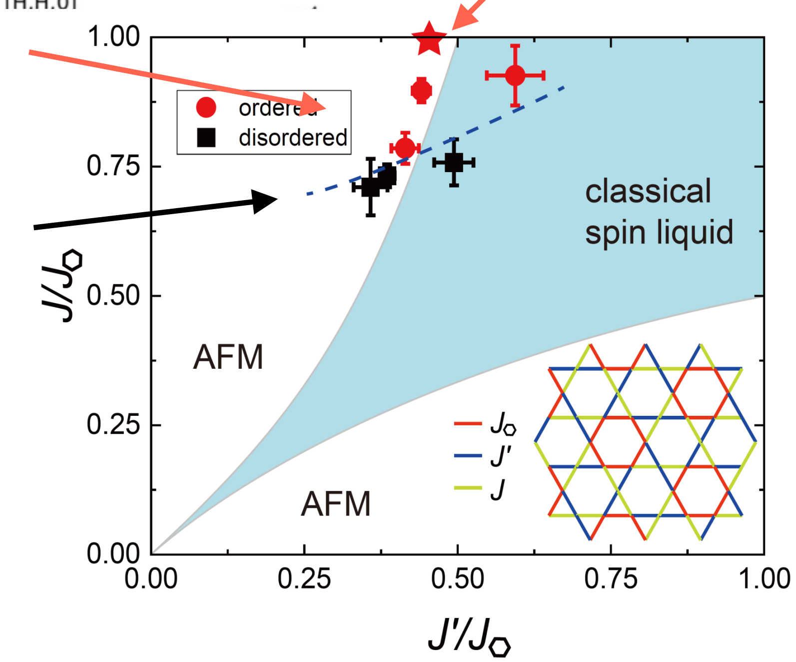




$Y_3Cu_9(OH)_{19}Cl_8$

$LuCu_3(OH)_6Br_2[Br_{0.65}(OH)_{0.35}]$,

$LuCu_3(OH)_6Br_2[Br_{0.33}(OH)_{0.67}]$,
$LuCu_3(OH)_6Br_2[Br_{0.29}(OH)_{0.71}]$,
$YCu_3(OH)_6Br_2[Br_{0.33}(OH)_{0.67}]$



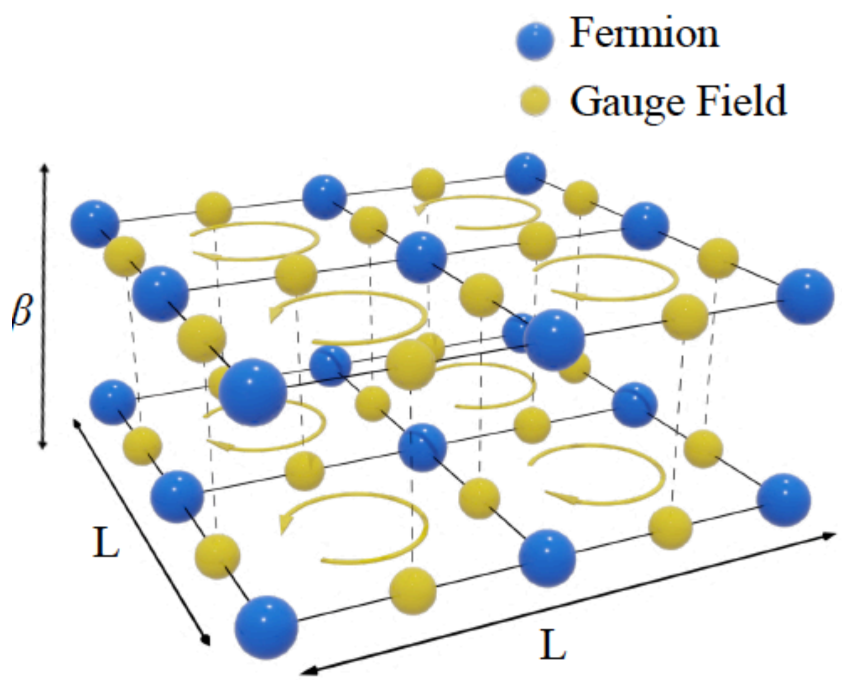
Spectral evidence for Dirac spinons in a kagome lattice antiferromagnet, Zeng, Zhou, …, ZYM, Shiliang Li, [Nat. Phys. 20, 1097 (2024)]

Antiferromagnetic Order and Possible Quantum Spin Liquid in $LuCu_3(OH)_6Br_2[Br_x(OH)_{1-x}]$, Z. Li, …, Shiliang Li, [Chin. Phys. Lett. 42, 027504 (2025)]

Monte Carlo Study of Lattice Compact Quantum Electrodynamics with Fermionic Matter: The Parent State of Quantum Phases

Xiao Yan Xu,[1,*] Yang Qi,[2–4,†] Long Zhang,[5] Fakher F. Assaad,[6] Cenke Xu,[7] and Zi Yang Meng[8,9,10,11,‡]

Phys. Rev. X 9, 021022 (2019)



$$S = S_B + S_F = \int_0^\beta d\tau\ (L_B + L_F)$$

$$i, j \in L \times L$$

$$L = 16{,}20$$

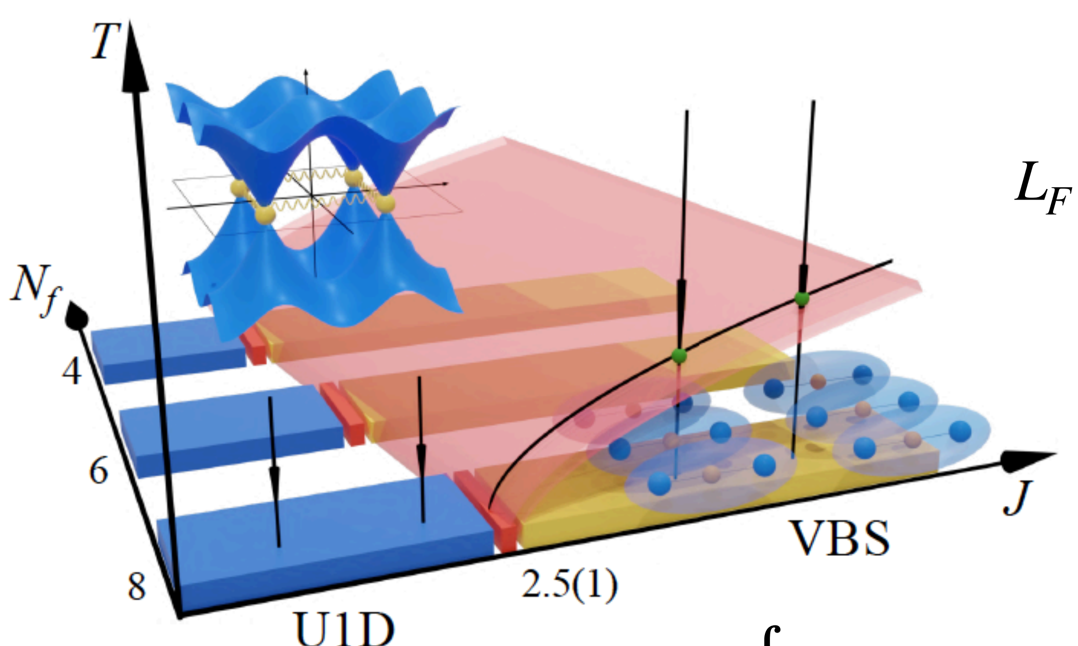
$$L_B^{nc} = \frac{1}{JN_f} \sum_{\langle i,j \rangle} \frac{(\phi_{ij,\tau+1} - \phi_{ij,\tau})^2}{\Delta\tau^2} + K \sum_{\square} \cos\Big(\sum_{\langle i,j \rangle \in \square} \phi_{ij,\tau}\Big)$$

$$\tau \in [0,\ \beta = \frac{1}{T}]$$

$$L_B^{c} = \frac{1}{JN_f} \sum_{\langle i,j \rangle} \frac{1 - \cos(\phi_{ij,\tau+1} - \phi_{ij,\tau})}{\Delta\tau^2} + K \sum_{\square} \cos\Big(\sum_{\langle i,j \rangle \in \square} \phi_{ij,\tau}\Big)$$

$$\beta = 2L, 4L$$

$$\Delta\tau = 0.1$$



$$L_F = \sum_{\langle i,j \rangle, \alpha} \psi^\dagger_{i,\tau,\alpha} (\partial_\tau \delta_{ij} - t e^{i\phi_{ij,\tau}}) \psi_{j,\tau,\alpha} + h.c.$$

$$\alpha \in 2, 4, \cdots, N_f$$

$$N_f = 2, \cdots, 8, 10$$

$$O(\beta \times (L \times L)^3) \sim O(L^7)$$

$$Z = \int D(\phi, \psi^\dagger, \psi)\ e^{-(S_B + S_F)} = \int D\phi\ e^{-S_B}\ \mathbf{Tr}[e^{-S_F}] = \int D\phi\ e^{-S_B}\ [\det(\mathbf{1} + \prod_{\tau=1}^{\beta} B_\tau(\phi_\tau))]^{N_f}$$

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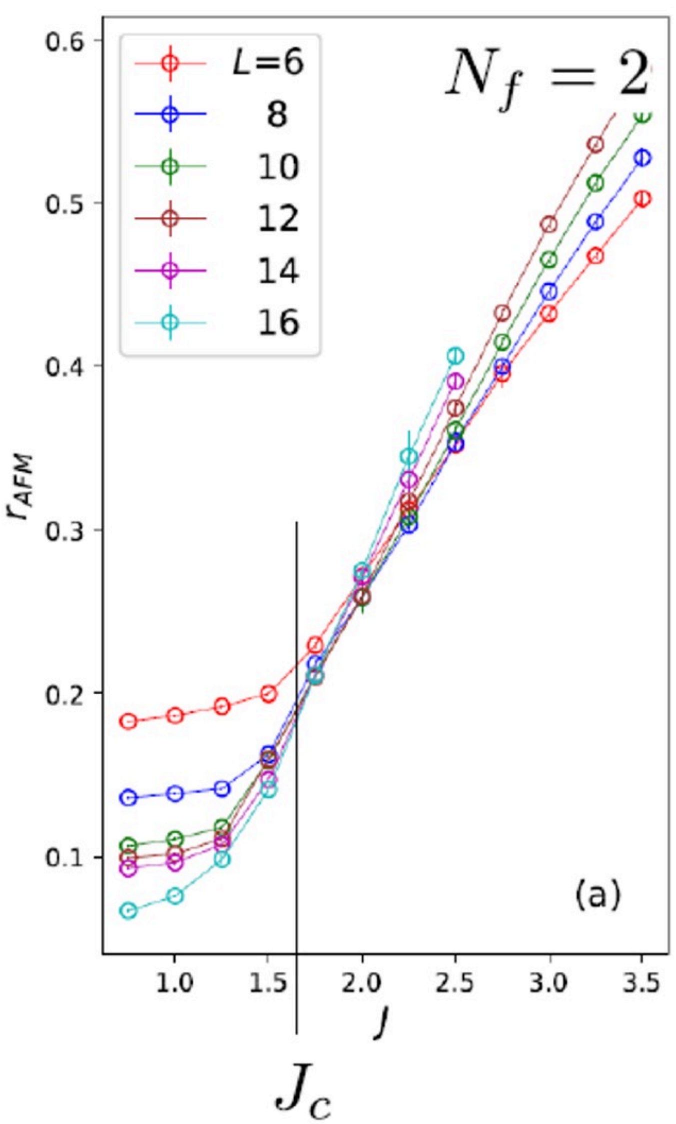
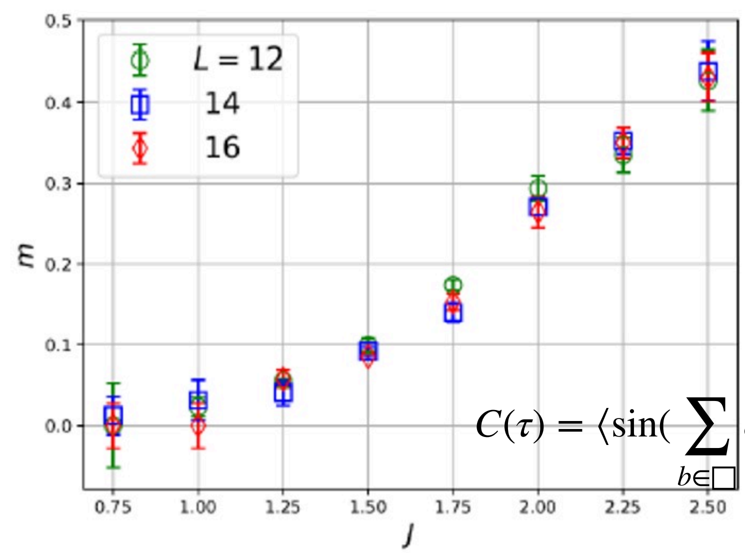
$$S^{\alpha}_{\beta}(i) = \psi^{\dagger}_{i\alpha}\psi_{i\beta} - \tfrac{1}{N_{\mathrm{f}}}\delta_{\alpha\beta}\textstyle\sum_{\gamma}\psi^{\dagger}_{i\gamma}\psi_{i\gamma}$$

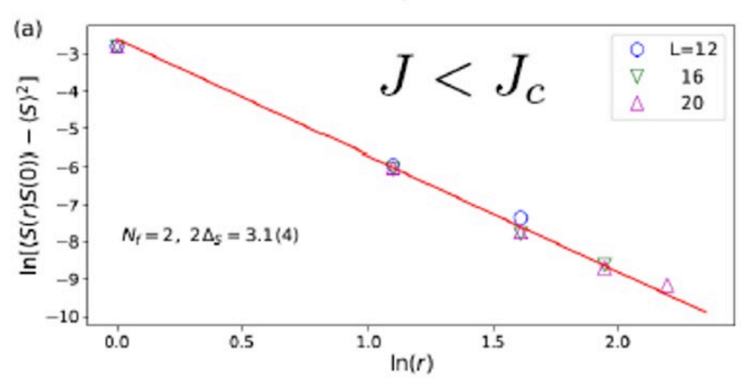
$$\chi_S(\mathbf{k}) = \frac{1}{L^4}\sum_{ij}\sum_{\alpha\beta}\langle S^{\alpha}_{\beta}(i)S^{\beta}_{\alpha}(j)\rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)}$$

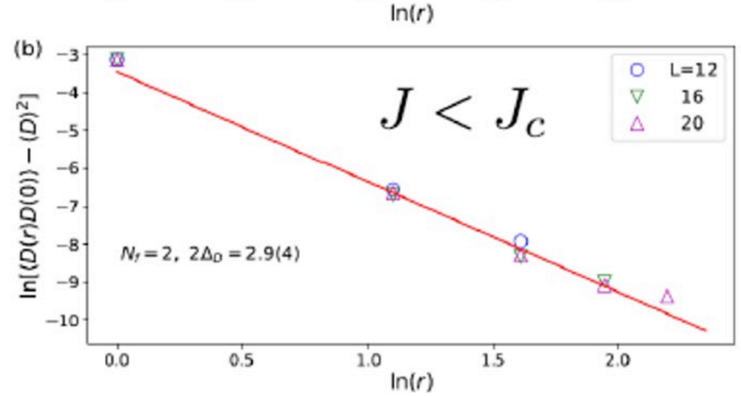
$$r_{\mathrm{AFM}} = 1 - \frac{\chi_S(\mathbf{X}+\delta\mathbf{q})}{\chi_S(\mathbf{X})}$$

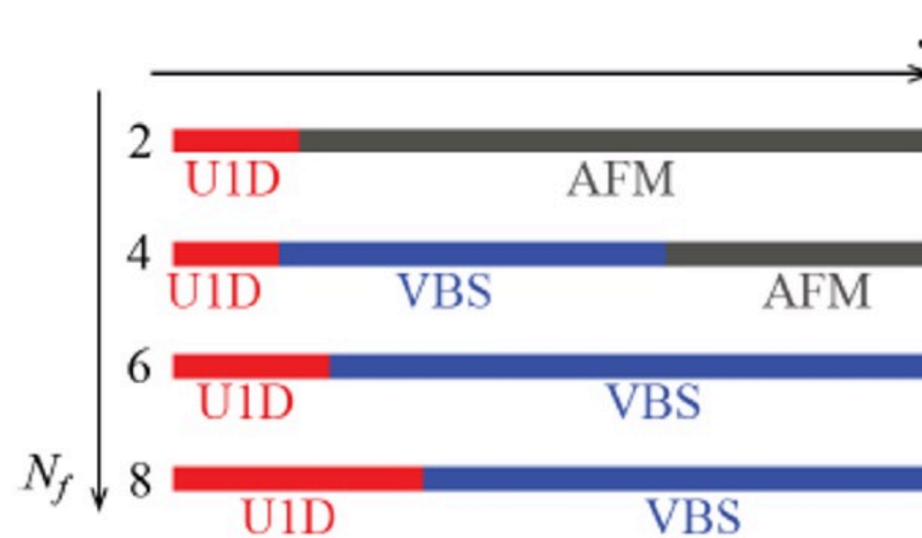


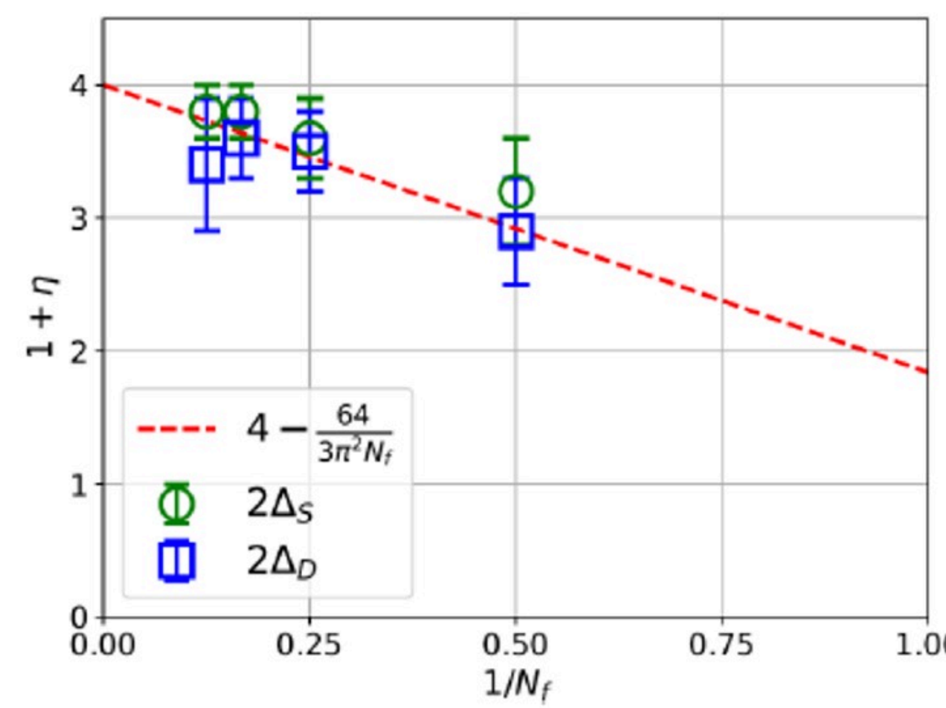


$$C(\tau) = \langle \sin(\sum_{b\in\square}\phi_b(\tau)) \cdot \sin(\sum_{b\in\square}\phi_b(0))\rangle \sim \exp(-m\tau)$$









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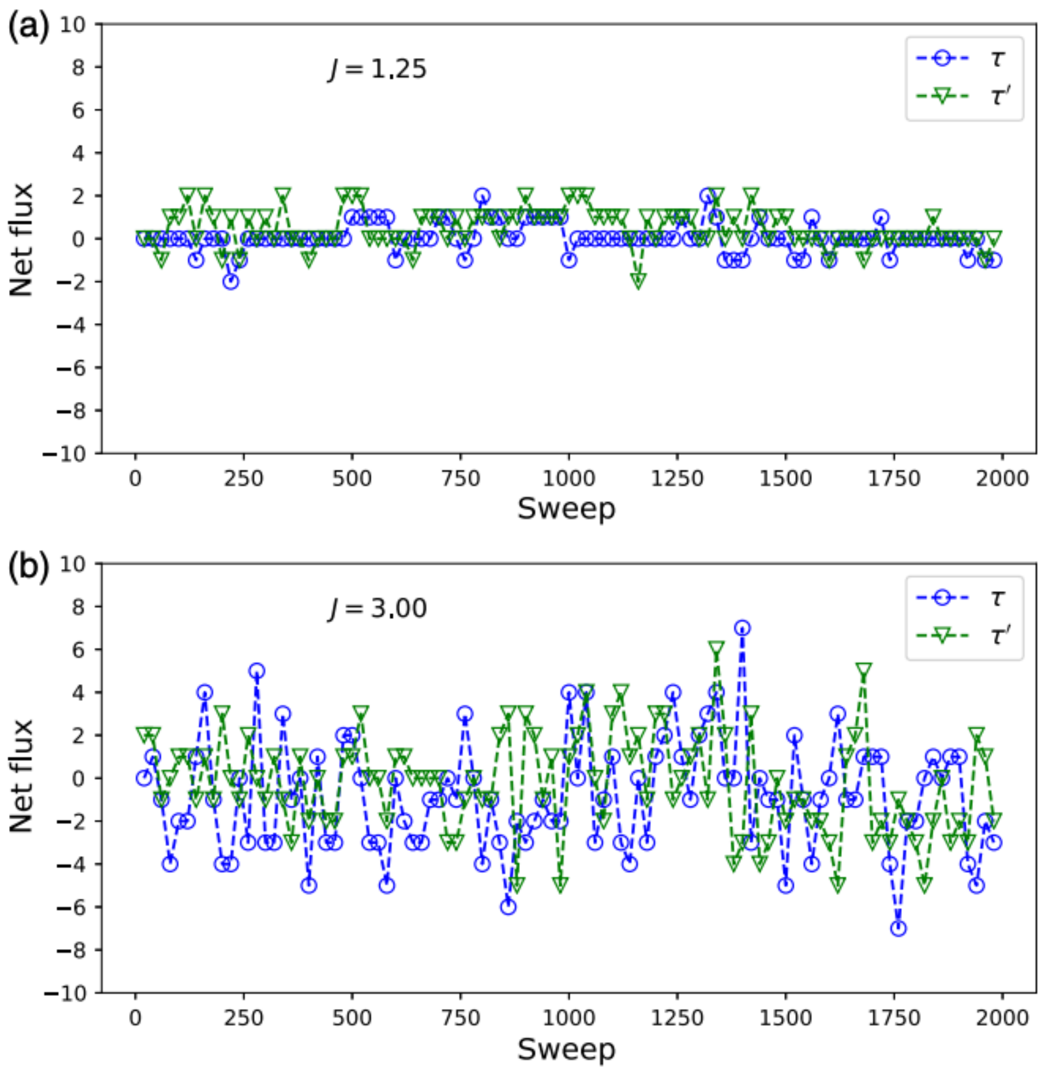
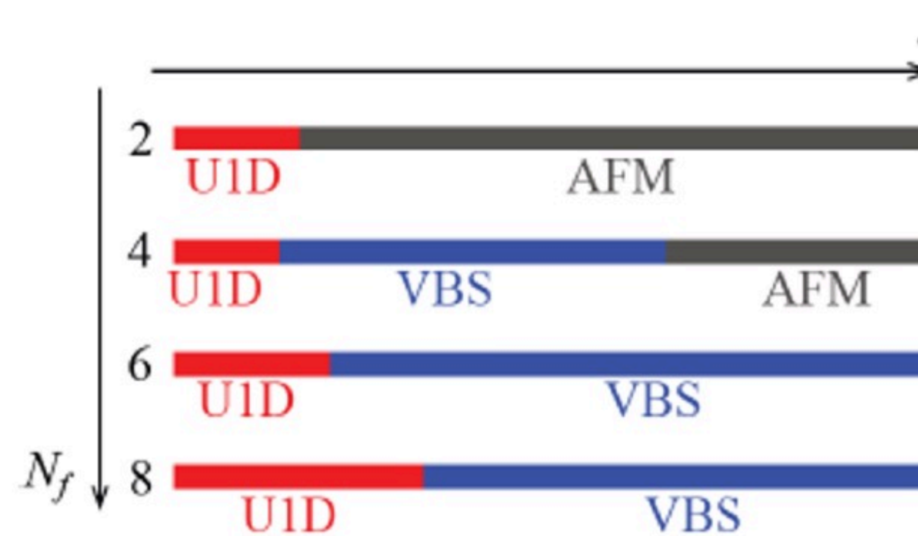
$$\sum_{b\in\square} \phi_b(\tau) = \Phi_\square(\tau) + 2\pi m_\square(\tau) \qquad \Phi_\square(\tau) \in [0,2\pi)$$

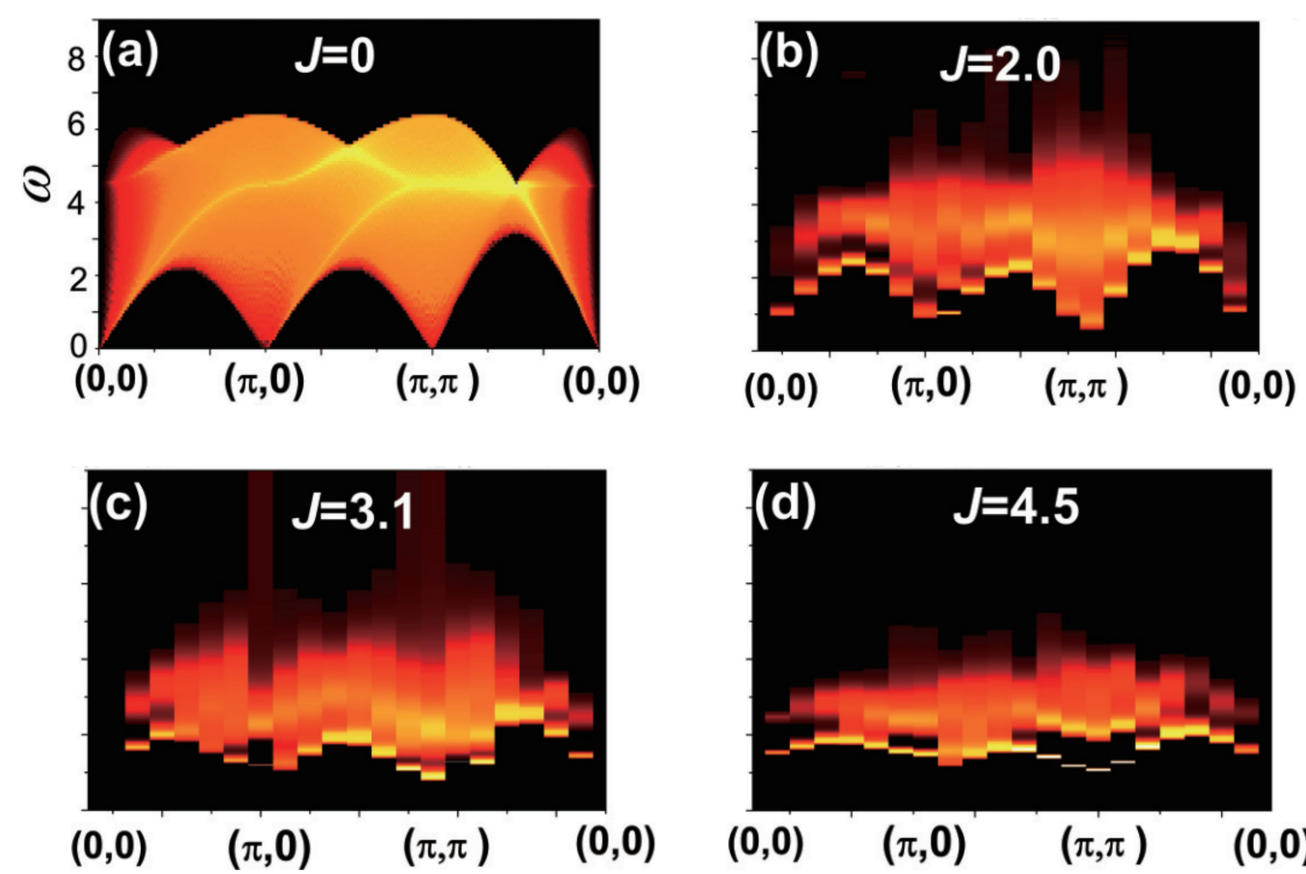
Net flux $$M(\tau) = \sum_{\square} m_\square(\tau)$$

$$N_f = 2, \quad J_c \sim 1.6(2), \quad L = 12, \quad \tau' - \tau = 8\Delta\tau$$





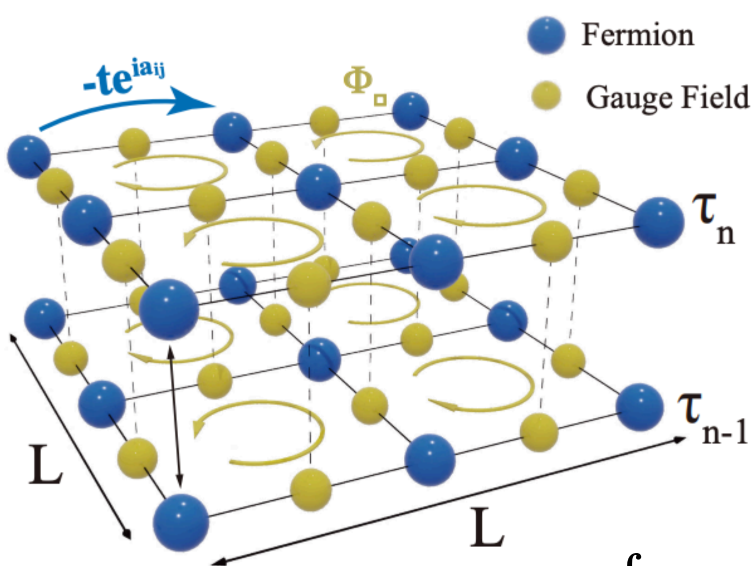
$$N_f = 8, \quad J_c \sim 2.5(1), \quad L = 14$$



Emergent gauge flux in QED$_3$ with flavor chemical potential: application to magnetized U(1) Dirac spin liquids

Chuang Chen,[1,2] Urban F. P. Seifert,[3] Kexin Feng,[1] Oleg A. Starykh,[4] Leon Balents,[5,6,7] and Zi Yang Meng[1]

arXiv: 2508.08528



$$S = \sum_{i,n}\left[\psi_i^\dagger(\tau_n)(\psi_i(\tau_n)-\psi_i(\tau_{n-1})) - \frac{1}{2}B\psi_i^\dagger(\tau_n)\sigma^z\psi_i(\tau_n)\right]$$
$$-t\sum_{\langle ij\rangle,n}\left[e^{ia_{ij}(\tau_n)}\psi_i^\dagger(\tau_n)\psi_j(\tau_n)+\text{h.c.}\right]$$
$$+\frac{1}{J}\sum_{\langle ij\rangle,n}\left[a_{ij}(\tau_n)-a_{ij}(\tau_{n-1})\right]^2$$

$i,j \in L\times L$

$K=0$

$L=16{,}20$

$\tau\in[0,\beta=1/T]$

$$Z=\int D(\phi,\psi^\dagger,\psi)\ e^{-(S_B+S_F)} = \int D\phi\ e^{-S_B}\ \mathbf{Tr}[e^{-S_F}] = \int D\phi\ e^{-S_B}\ [\det(\mathbf{1}+\prod_{\tau=1}^{\beta}B_\tau(\phi_\tau,B))]^{N_f}$$

$\beta = 2L$

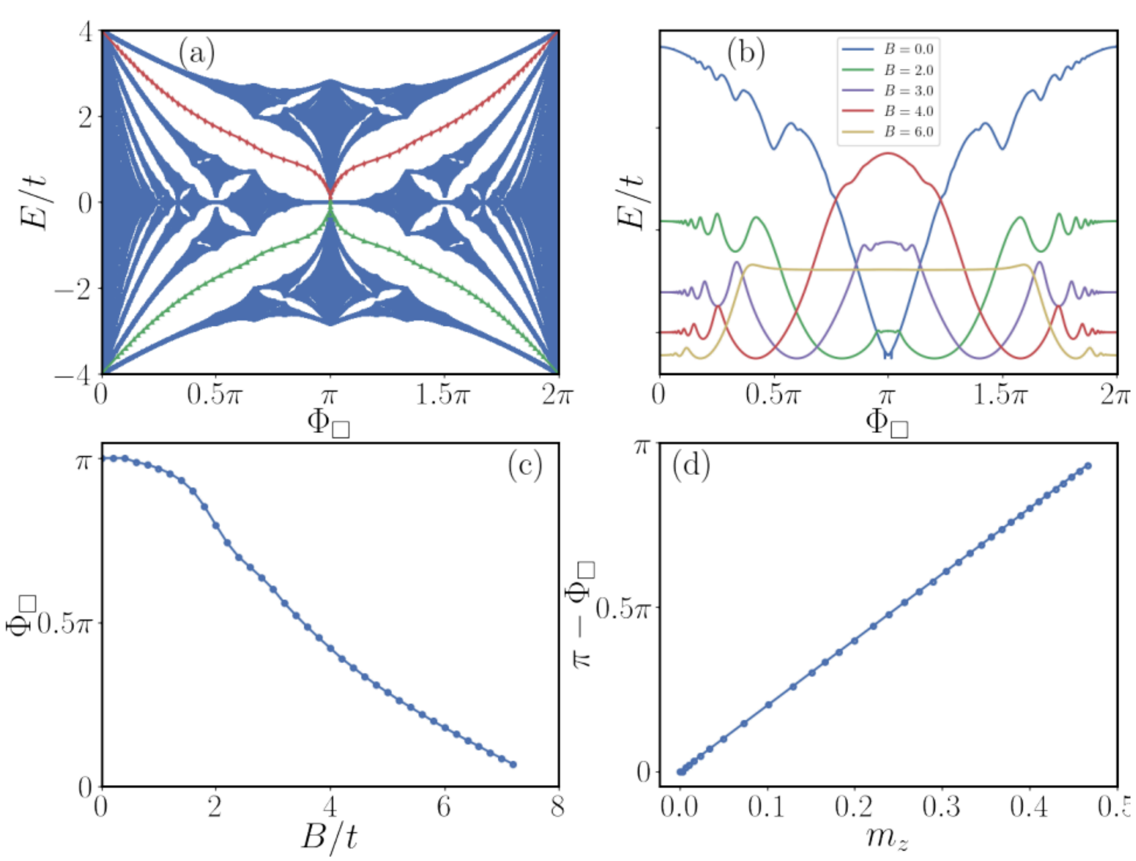
$J\to 0$: mean-field limit, temporal gauge fluctuations frozen

$\Delta\tau=0.1$

$N_f=2$

$O(\beta\times(L\times L)^3)\sim O(L^7)$

$L=32$



$$\Phi_{\square}(\tau)=\nabla\times a_{\square}(\tau)=\sum_{b\in\square}\phi_b(\tau)$$

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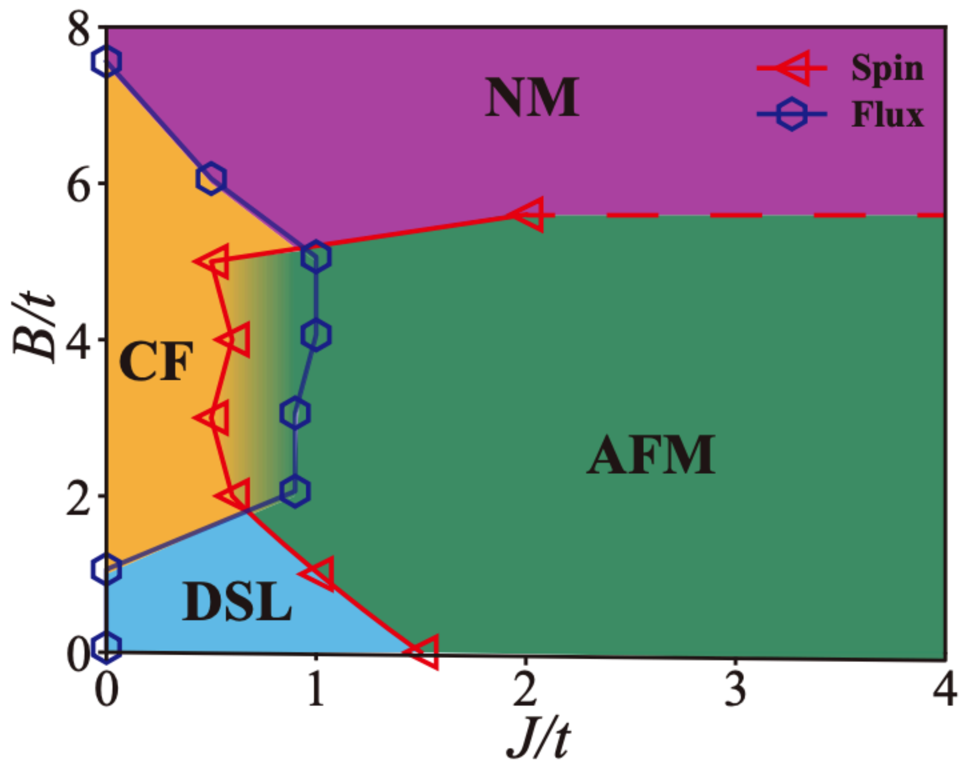


CF: $$\chi = \langle \sin(\sum_{b \in \square} \phi_b) \rangle = \langle \sin(\nabla \times a) \rangle \neq 0$$

emergent gauge flux

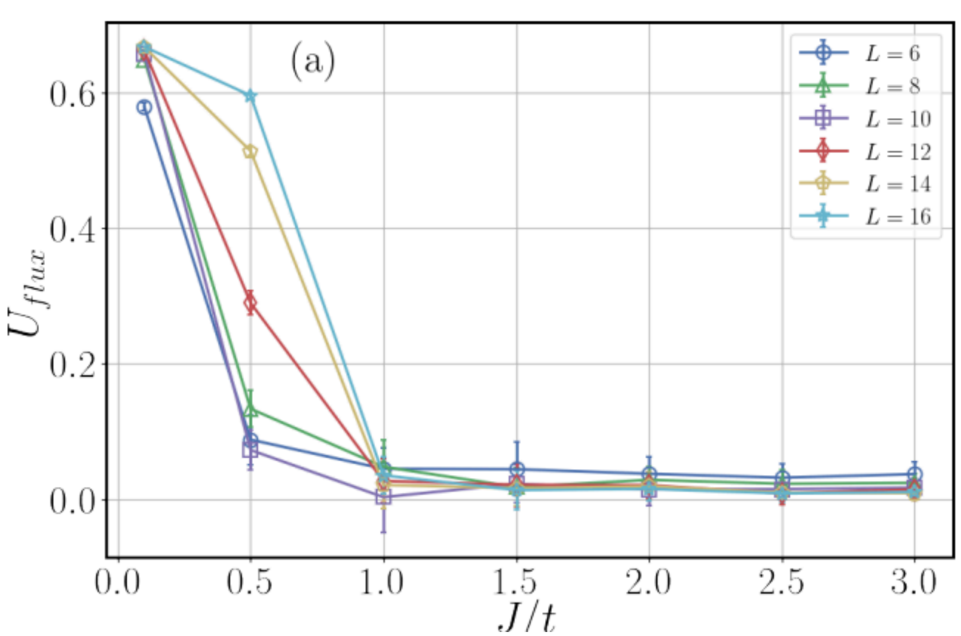
$$S^{\pm}(\mathbf{q}) = \frac{1}{N} \sum_{ij} \langle \sin(\Phi_{\square,i}) \ \sin(\Phi_{\square,j}) \rangle \ e^{-i\mathbf{q}\cdot\mathbf{r}_{ij}} \qquad \mathbf{q} = \Gamma$$

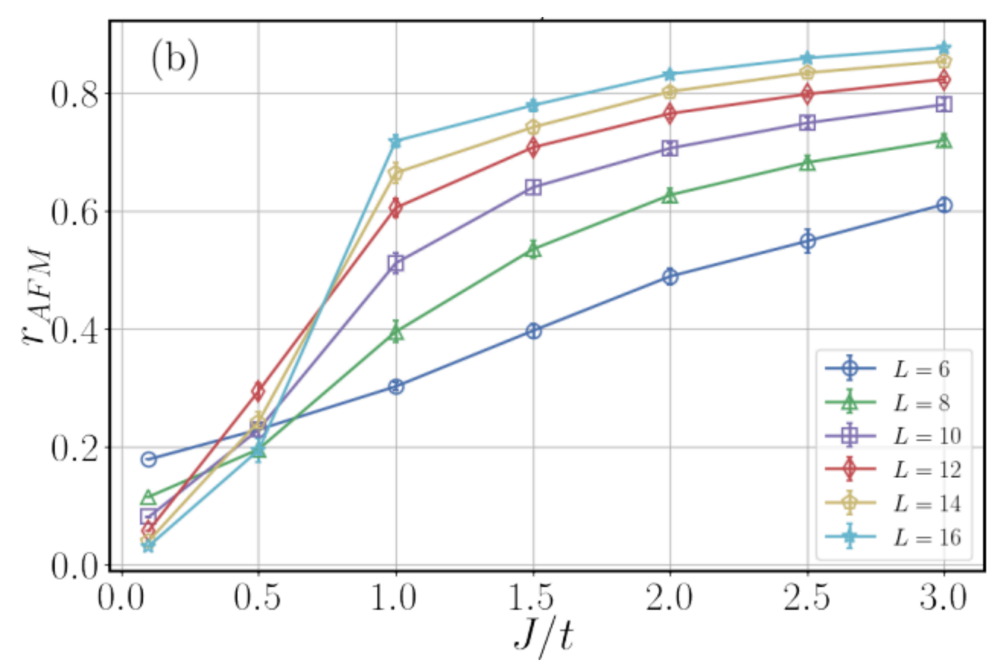
AFM: $N^{+} = (-1)^{i} \langle \psi_i^{\dagger} \sigma^{+} \psi_i \rangle \neq 0$ in-plane AFM order

$$S^{\pm}(\mathbf{q}) = \frac{1}{N} \sum_{ij} \langle S_i^{+} S_j^{-} + h.c. \rangle \ e^{-i\mathbf{q}\cdot\mathbf{r}_{ij}} \qquad \mathbf{q} = M = (\pi, \pi)$$

$$N^{+} = \lim_{N \to \infty} \sqrt{\frac{1}{N} S^{\pm}(M)}$$

$B/t = 2$





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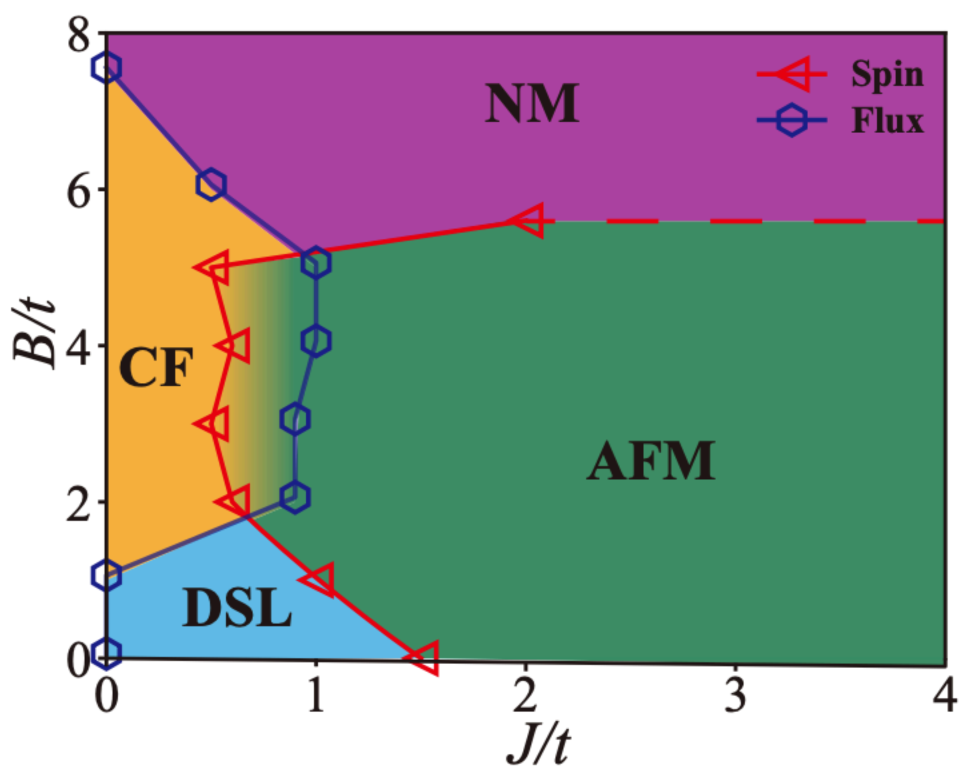


CF: $\chi = \langle \sin(\sum_{b\in\square} \phi_b)\rangle = \langle \sin(\nabla \times a)\rangle \neq 0$

emergent gauge flux

AFM: $N^+ = (-1)^i \langle \psi_i^\dagger \sigma^+ \psi_i \rangle \neq 0$

in-plane AFM order

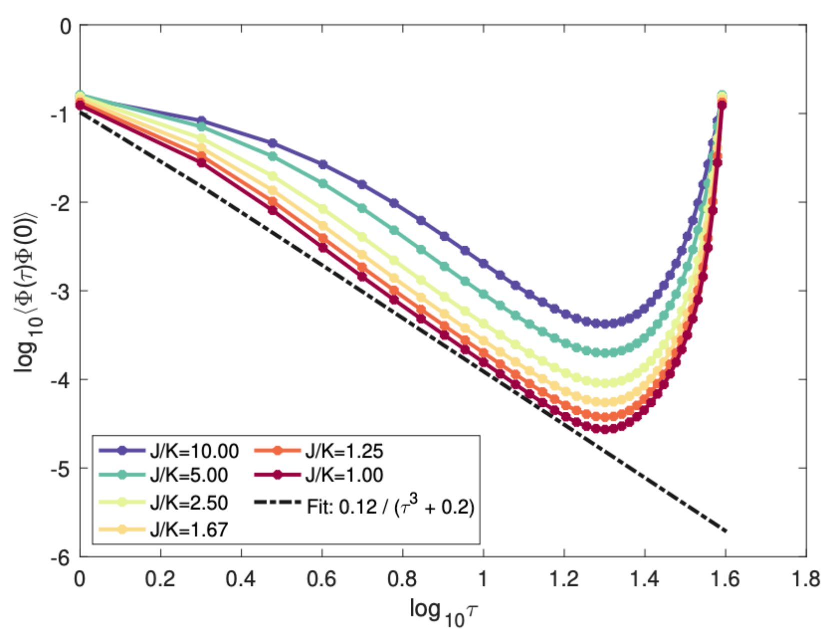
$$C_\chi(\tau) = \frac{1}{N_\square} \sum_{\square} \langle \sin(\sum_{b\in\square} \phi_b(\tau)) \cdot \sin(\sum_{b\in\square} \phi_b(0))\rangle$$

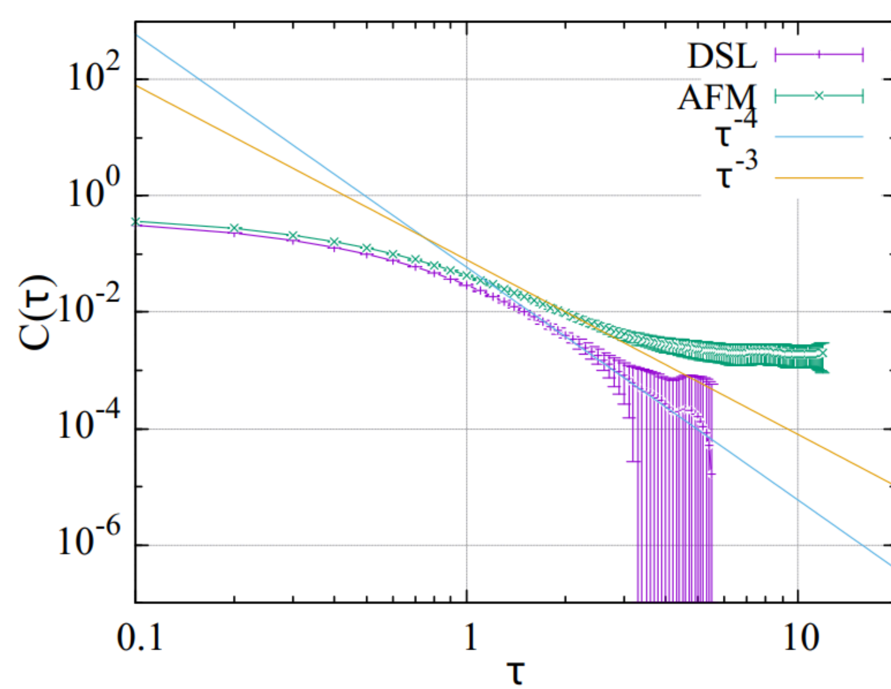
$C_\chi(\tau) \sim \tau^3$

in free photon theory and in AFM

$C_\chi(\tau) \sim \tau^4$

in DSL, conserved currents





$B=0, J=3, K=1, L=12$

$B=3, J=3, K=0, L=12$

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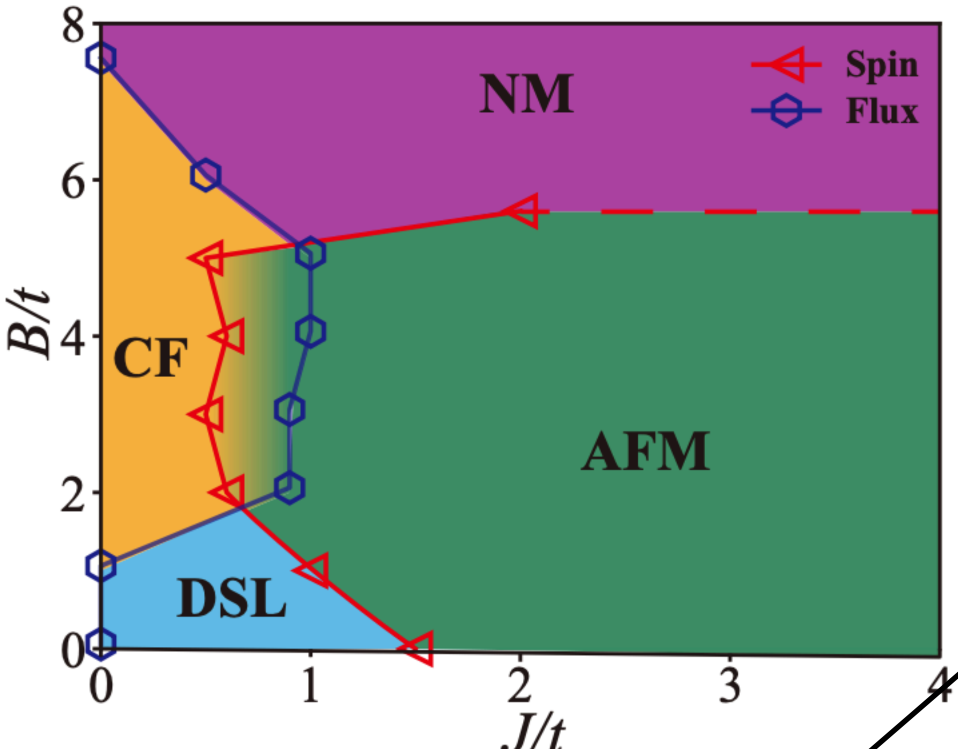
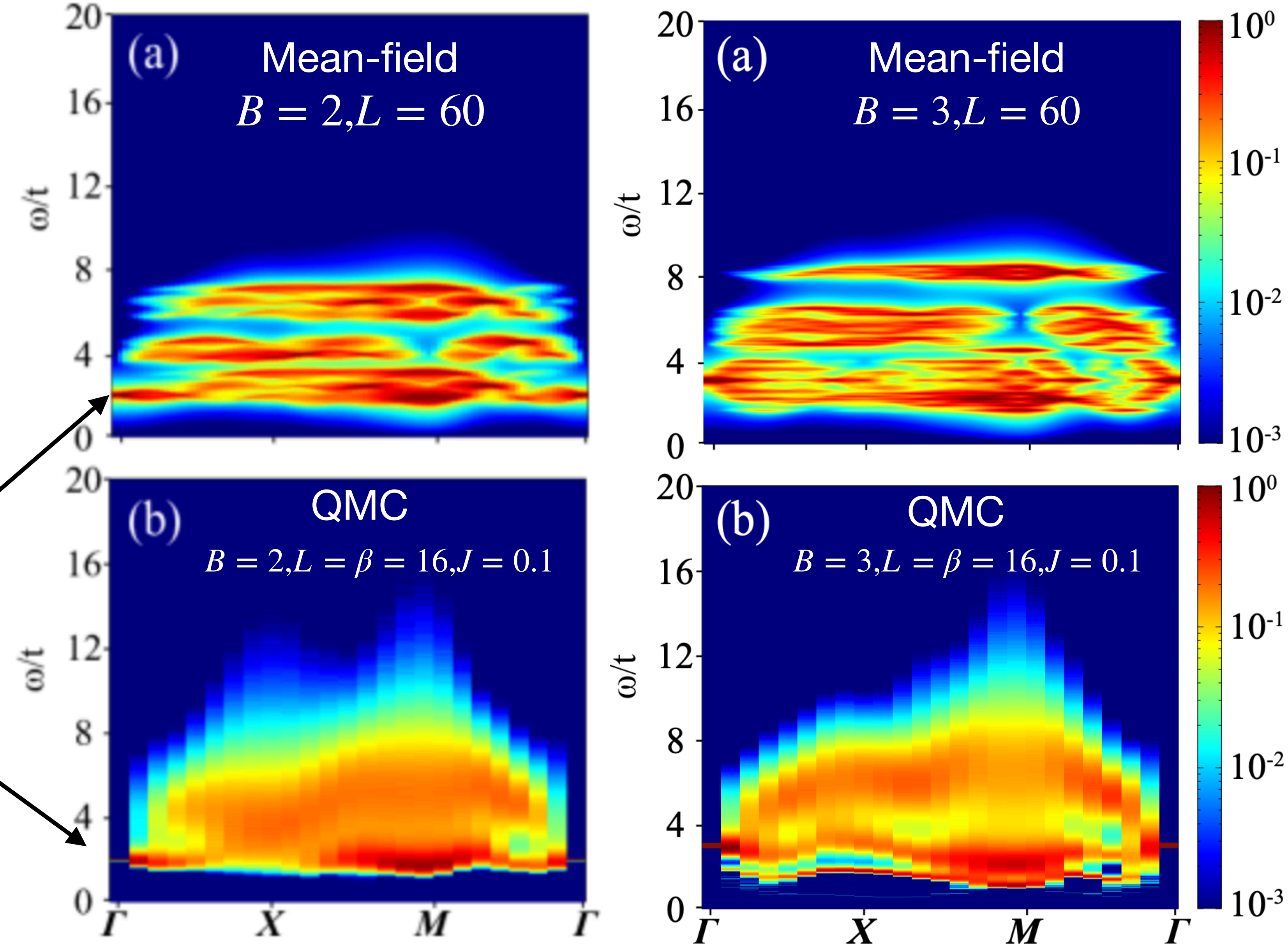
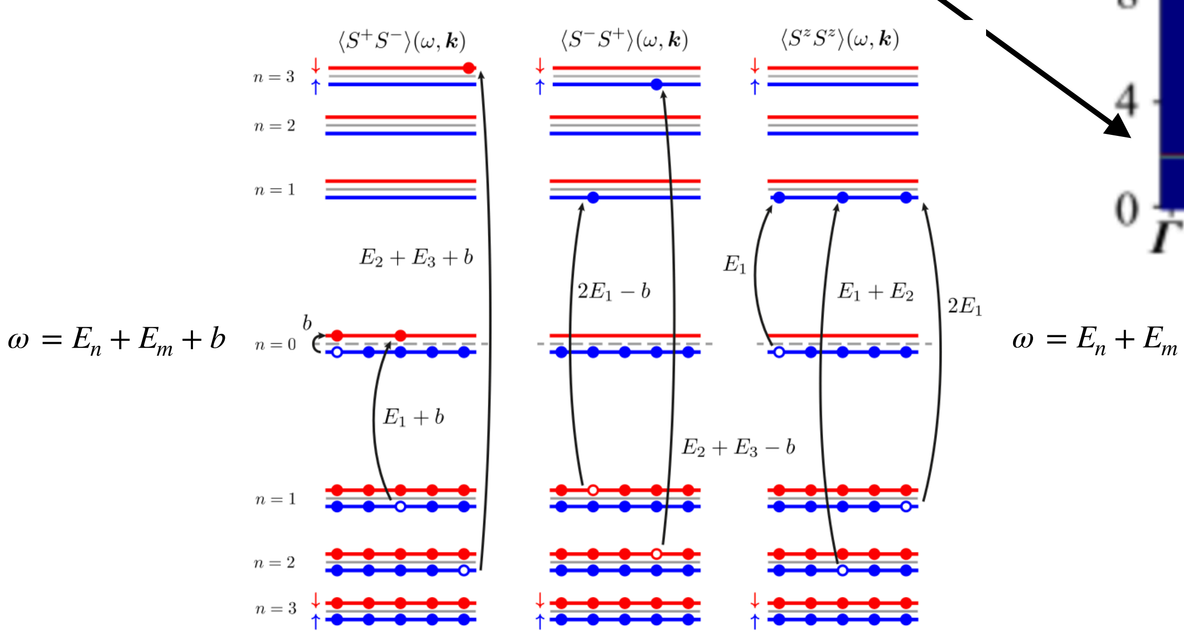


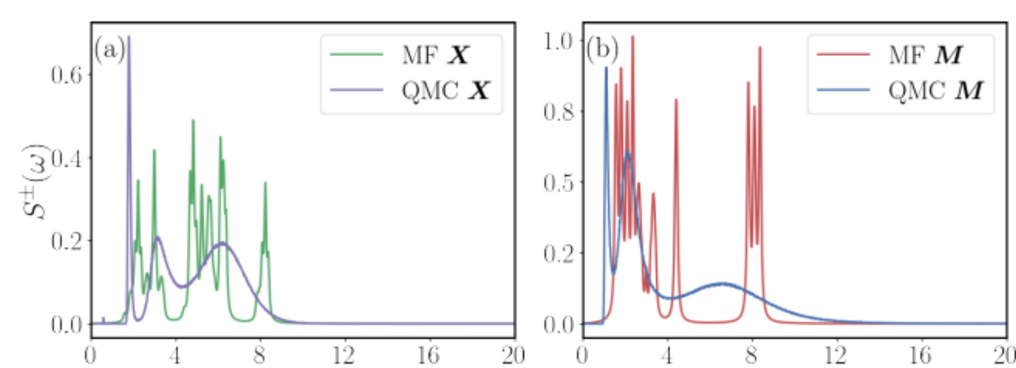
$S^{\pm}(\mathbf{q}, \omega)$

Larmor pole at

$\omega = B$







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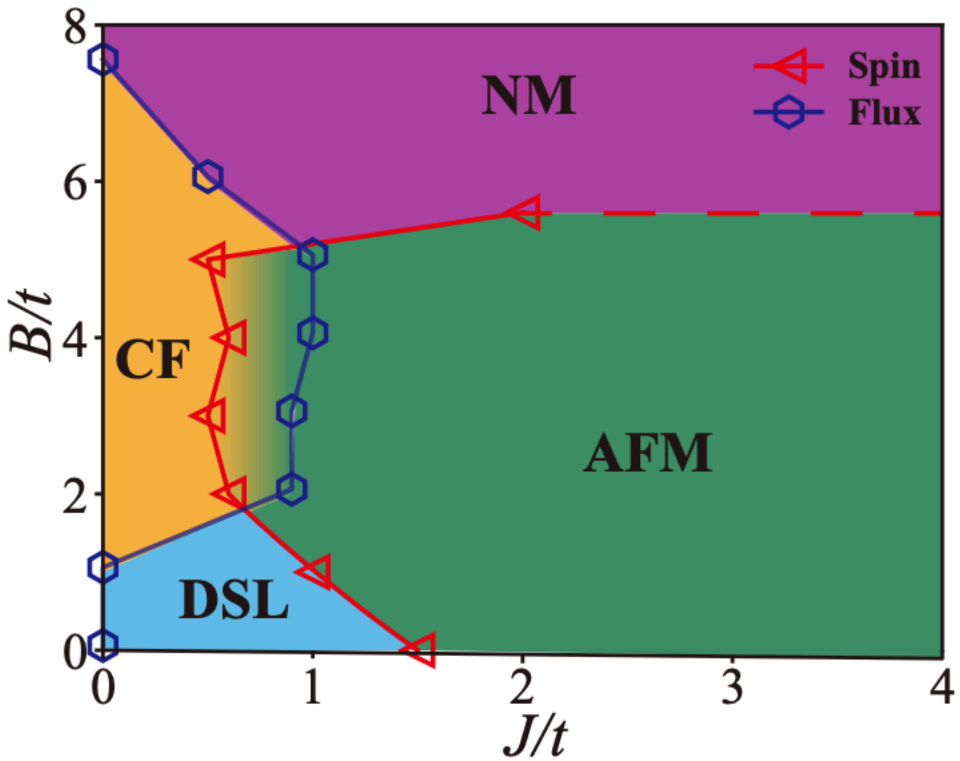


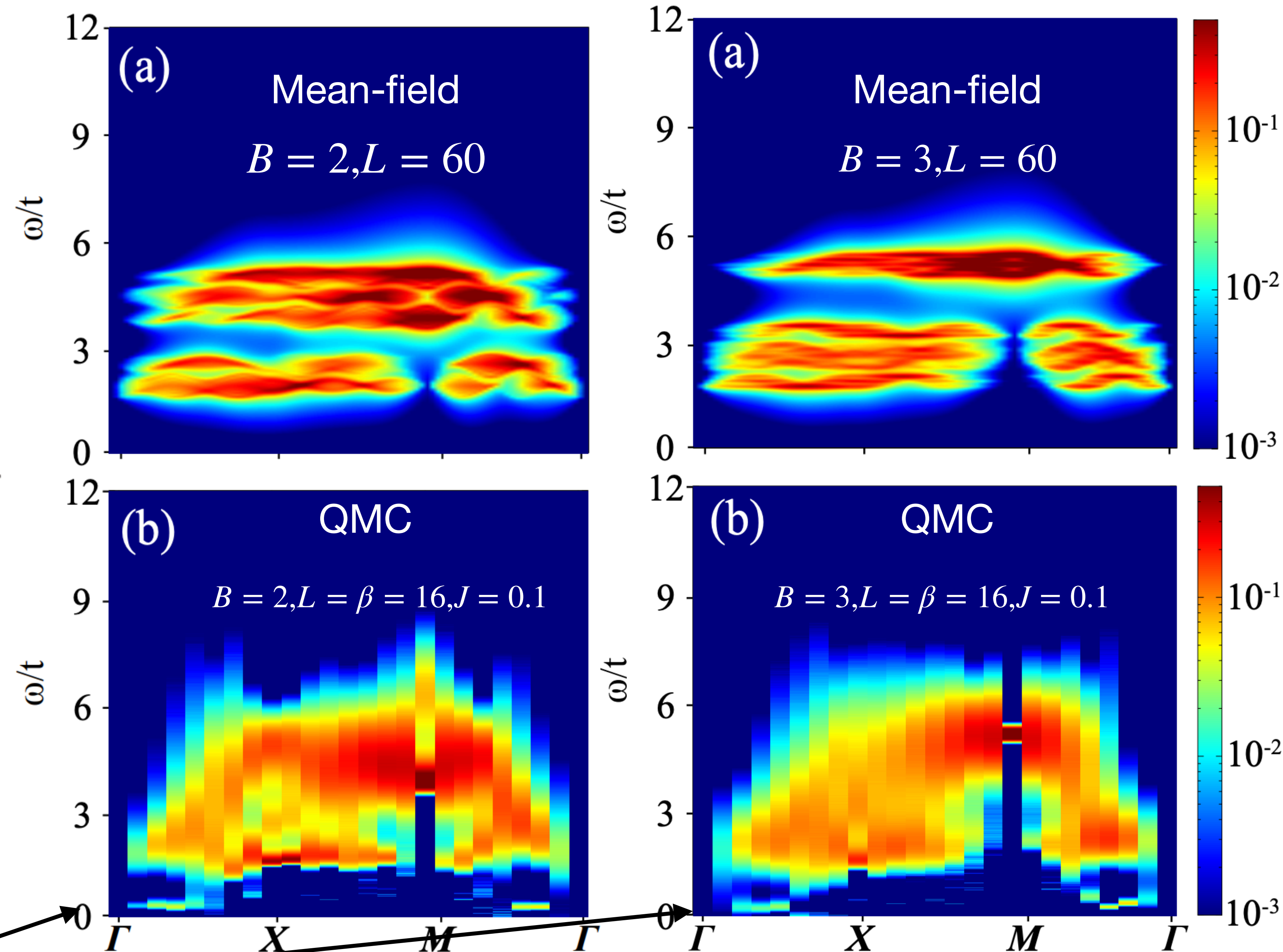
$$S^{zz}(\mathbf{q}, \omega)$$

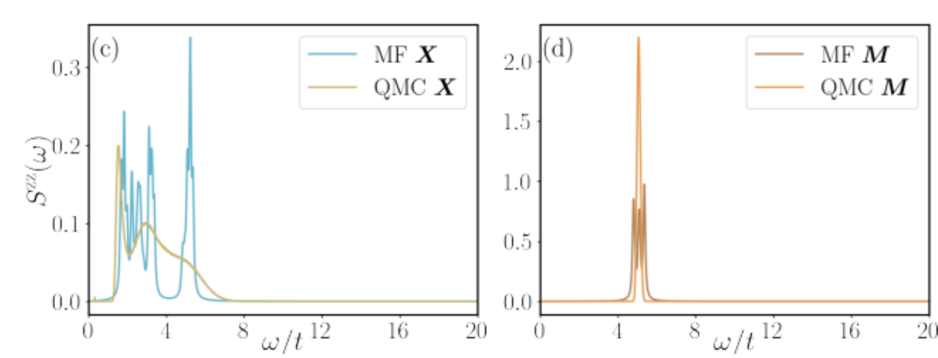
$$S^z \sim \nabla \times a^c$$

$$\langle S^z S^z \rangle_{\omega,\mathbf{q}} \sim \frac{1}{2} \chi c q \delta(\omega - cq)$$

spin corr. probes emergent magnetic flux

gapless gauge photon





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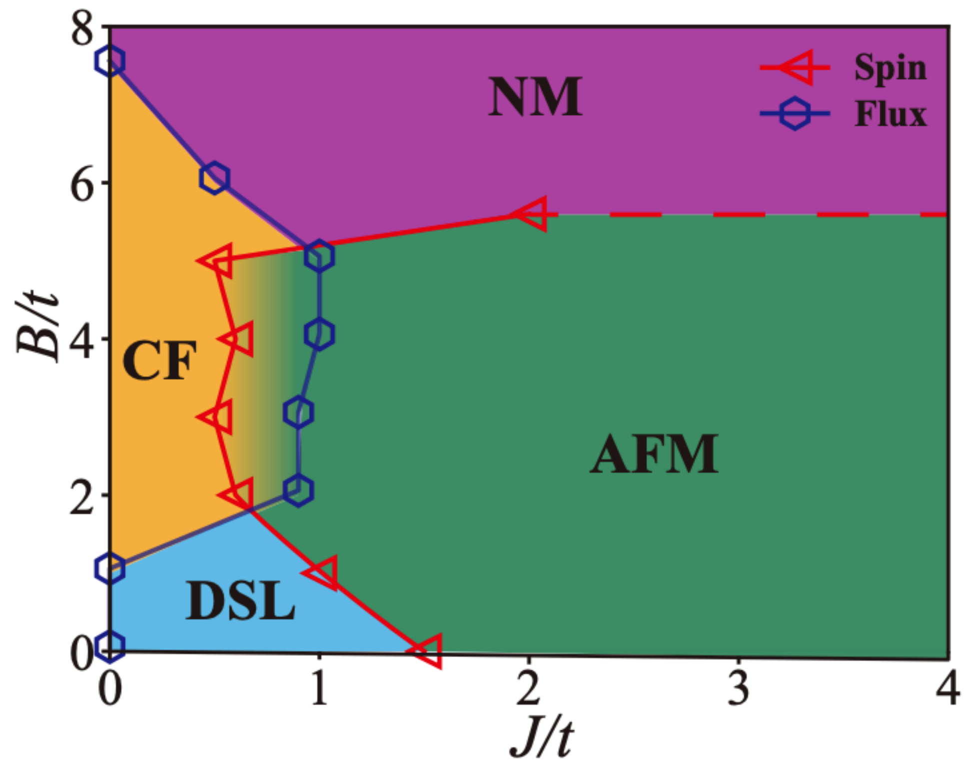
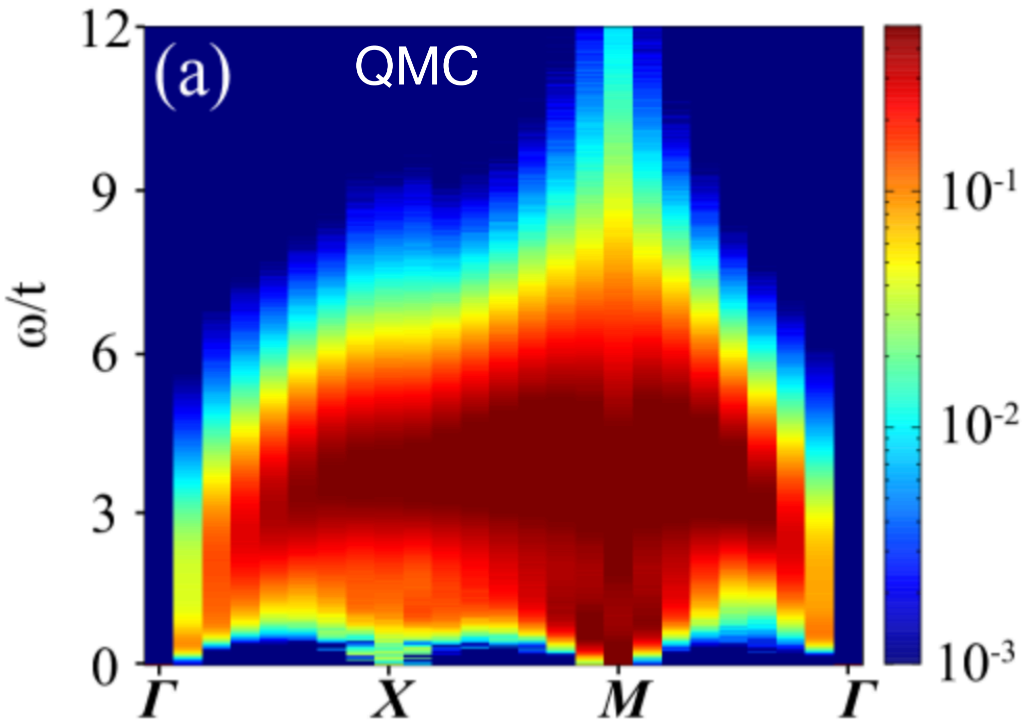
Chuang Chen,[1,2] Urban F. P. Seifert,[3] Kexin Feng,[1] Oleg A. Starykh,[4] Leon Balents,[5,6,7] and Zi Yang Meng[1]

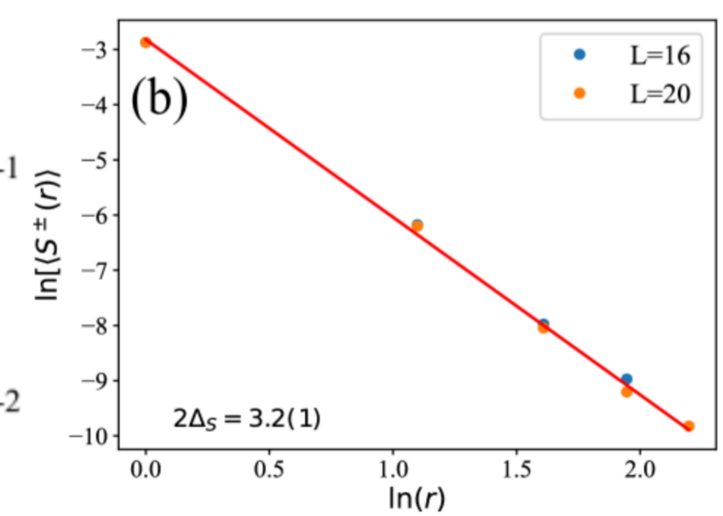
arXiv: 2508.08528



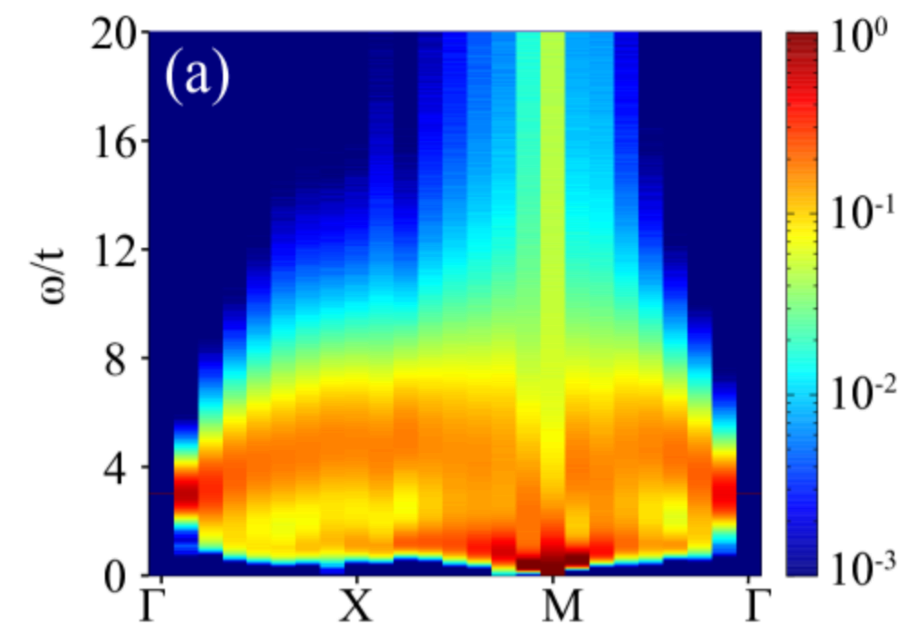
$S^{\pm}(\mathbf{q},\omega)$ $B=0, L=\beta=16, J=1$



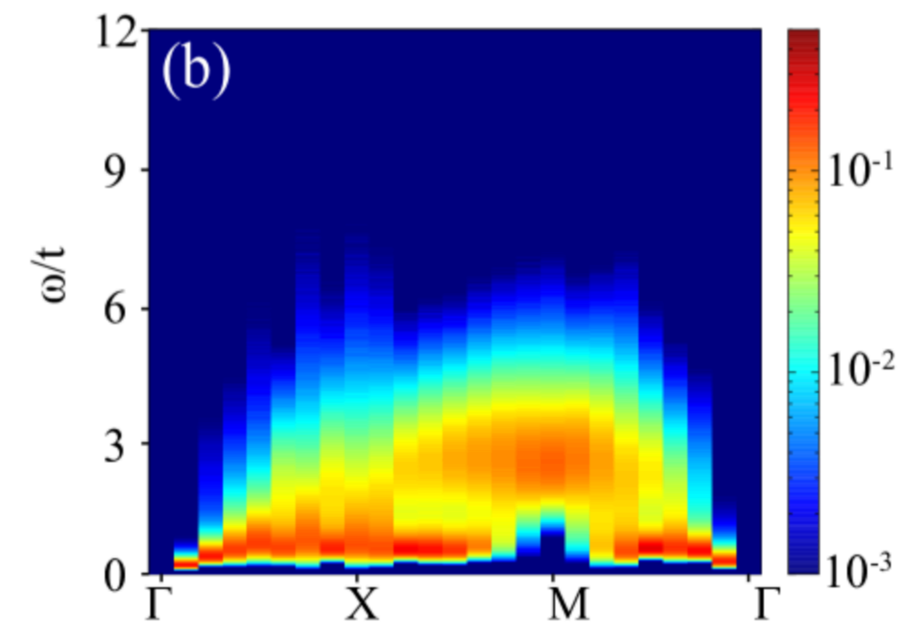
- fermion bilinears or mass terms: a singlet $M_s = \bar{\Psi}\Psi$ and a set of adjoints $M_a = \bar{\Psi}\mathrm{T}_a\Psi$, where $a = 1\ldots15$ range over the SU(4) generators. *A priori*, these two sets have independent scaling dimensions, Δ_s and $\Delta_{\rm adj}$, respectively. An estimate from Ref. [5] is $\Delta_s \approx 2.3$, $\Delta_{\rm adj} \in (1.4, 1.7)$.



$S^{\pm}(\mathbf{q},\omega)$ $B=3, L=\beta=16, J=3$



$S^{zz}(\mathbf{q},\omega)$ $B=3, L=\beta=16, J=3$

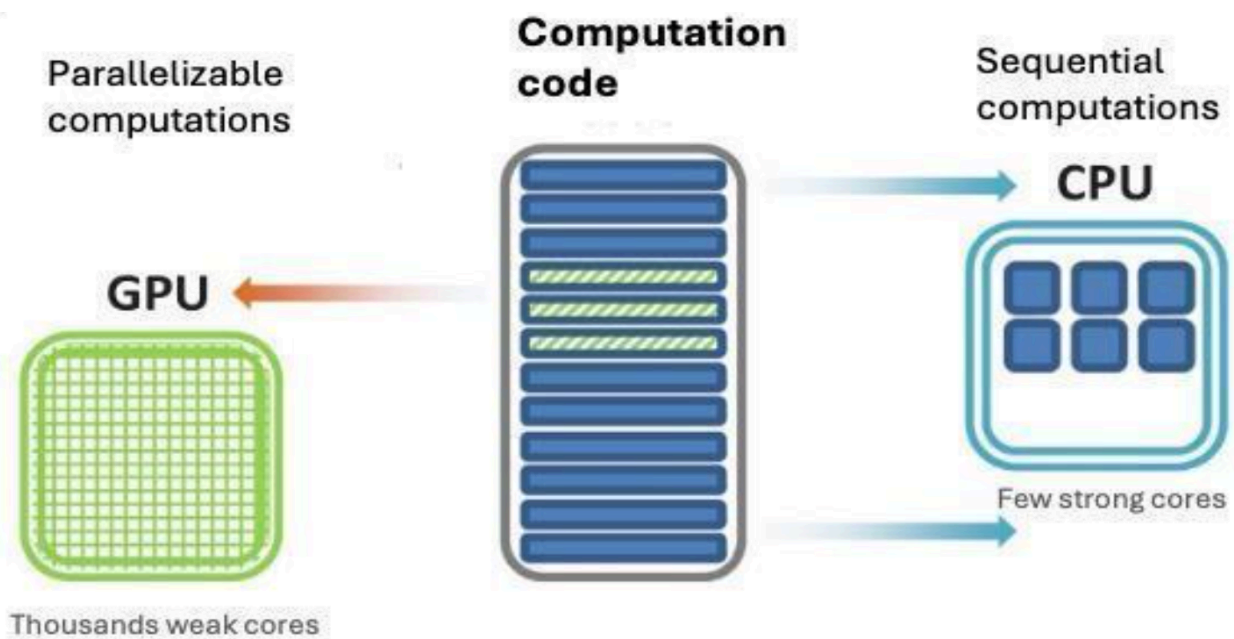


Scalable hybrid quantum Monte Carlo simulation of U(1) gauge field coupled to fermions on GPU

Kexin Feng,[1] Chuang Chen,[1] and Zi Yang Meng[1,*]

arXiv: 2508.16298

How to reduce the complexity $O(\beta \times (L \times L)^3) \sim O(L^7)$



$$Z = \int [\delta\phi] e^{-S_B(\phi)} \prod_\alpha \det M_\alpha(\phi)$$

$$= \int [\delta\phi\delta\eta] e^{-S_B(\phi) - \eta^\dagger [M^\dagger(\phi) M(\phi)]^{-1}\eta}$$

$$= \int [\delta\phi\delta p\delta\eta] e^{\underbrace{-(S_B(\phi) + \sum_{ij,\tau} p_{ij,\tau}^2 + \eta^\dagger [M^\dagger(\phi)M(\phi)]^{-1}\eta)}_{H(\phi,p,\eta)}}$$

$$\mathbf{Tr}_\psi[e^{-S_F}] = \prod_\alpha \det(\mathbf{1} + \prod_\tau^\beta B_{\tau,\alpha}) = \prod_\alpha \det(M_\alpha)$$

$N_\tau(\beta) \times V_s(L \times L) \sim L^3$

$$M_\alpha = \begin{pmatrix} \mathbf{1} & 0 & 0 & \cdots & 0 & B_{N_\tau,\alpha} \\ -B_{1,\alpha} & \mathbf{1} & 0 & \cdots & 0 & 0 \\ 0 & -B_{2,\alpha} & \mathbf{1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{1} & 0 \\ 0 & 0 & 0 & \cdots & -B_{N_\tau - 1,\alpha} & \mathbf{1} \end{pmatrix}$$

$N_\tau(\beta) \times V_s(L \times L) \sim L^3$

(i) Generate $\phi_0 \in \mathbb{R}^{V_s N_\tau}$, $p \in \mathbb{R}^{V_s N_\tau}$ and $R := [M^\dagger(\phi_0)]^{-1}\eta \in \mathbb{C}^{V_s N_\tau}$

solve for $\eta = M^\dagger(\phi_0) R$

(ii) Hamiltonian dynamics

$$\dot{p}_{ij,\tau} = -\frac{\partial H}{\partial \phi_{ij,\tau}}$$

Leap-frog

$$\dot{\phi}_{ij,\tau} = \frac{\partial H}{\partial p_{ij,\tau}} = 2 p_{ij,\tau}$$

(iii) Acceptance

$$r = \min(1, e^{H(\phi_0, p_0, \eta) - H(\phi_t, p_t, \eta)})$$

(iv) Back to (i) with ϕ_t

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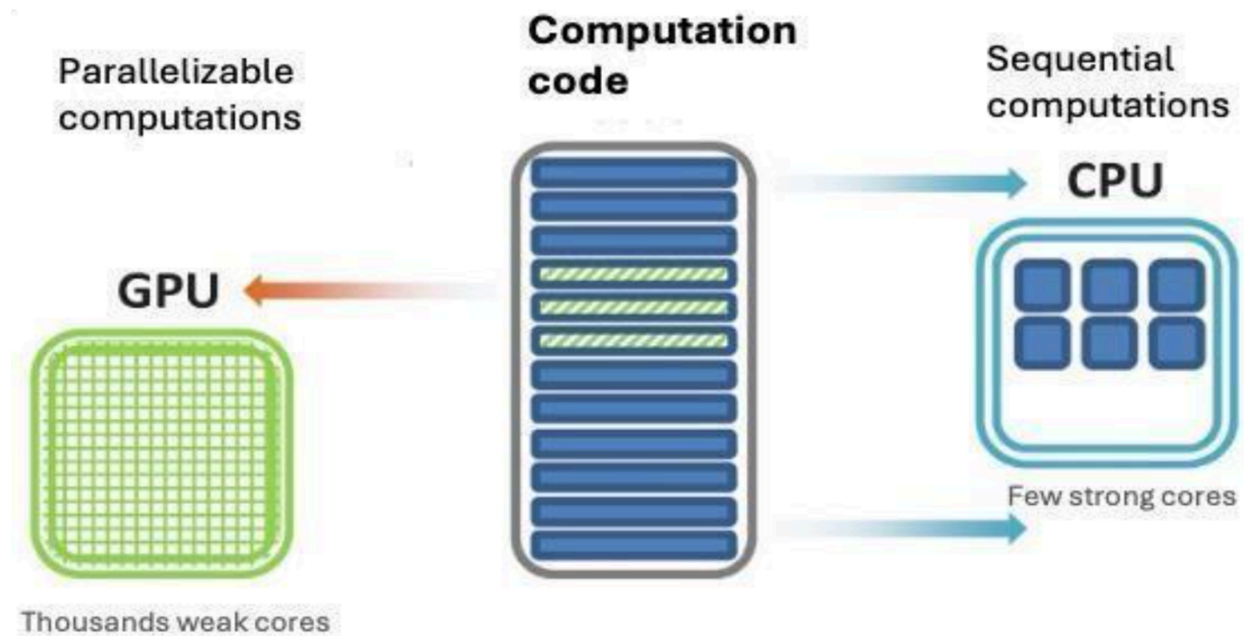
In step (ii), Leap-frog

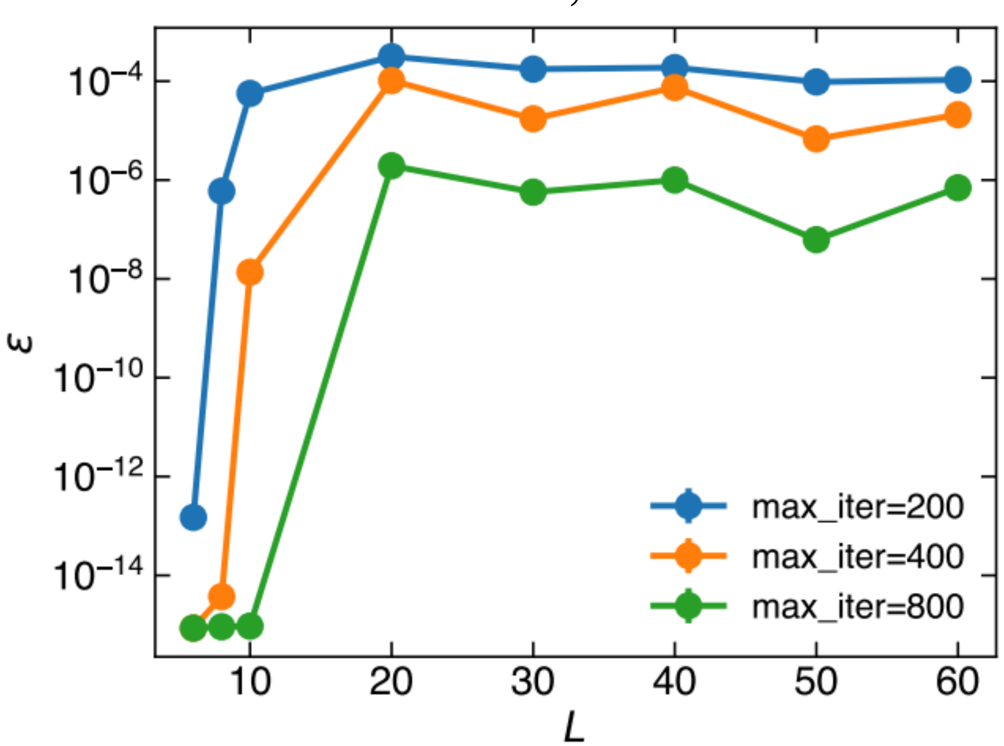
$$
\begin{aligned}
p_{n-\frac{1}{2}} &= p_{n-1} + F\left(\phi_{n-1}\right)\frac{\Delta t}{2} \\
\phi_n &= \phi_{n-1} + 2p_{n-\frac{1}{2}}\Delta t \\
p_n &= p_{n-\frac{1}{2}} + F\left(\phi_n\right)\frac{\Delta t}{2}.
\end{aligned}
$$



$$F(\phi) := -\frac{\partial H}{\partial \phi} = -\frac{\partial S_B}{\partial \phi} - \frac{\partial}{\partial \phi}\{\eta^\dagger (M^\dagger(\phi)M(\phi))^{-1}\eta\}$$

$$
\begin{aligned}
F_{i,\tau} &= \frac{\partial}{\partial \phi_{i,\tau}}\eta^\dagger \left(M^\dagger(\phi)M(\phi)\right)^{-1}\eta \\
&= -\eta^\dagger\left(M^\dagger(\phi)M(\phi)\right)^{-1}\left[M^\dagger(\phi)\frac{\partial}{\partial\phi_{i,\tau}}M(\phi) + \left(\frac{\partial}{\partial\phi_{i,\tau}}M^\dagger(\phi)\right)M(\phi)\right]\left(M^\dagger(\phi)M(\phi)\right)^{-1}\eta
\end{aligned}
$$

$J = 1.25, \Delta\tau = 0.1$



$$[M^\dagger M]^{-1}\eta = X$$

$$[M^\dagger M]X = \eta$$

$$
\begin{aligned}
X_{n+1} &= X_n + \zeta_n d_n \\
r_{n+1} &= r_n - \zeta_n O d_n \\
\rho_n &= \left\langle r_{n+1}, \tilde{O}^{-1} r_{n+1}\right\rangle / \left\langle r_n, \tilde{O}^{-1} r_n\right\rangle \\
d_{n+1} &= \tilde{O}^{-1} r_{n+1} + \rho_n d_n \\
\zeta_{n+1} &= \left\langle r_{n+1}, \tilde{O}^{-1} r_{n+1}\right\rangle / \left\langle d_{n+1}, O d_{n+1}\right\rangle,
\end{aligned}
$$

solving a linear system with preconditioned conjugate gradient algorithm

$$O(N_{pCG}) \sim O(1)$$

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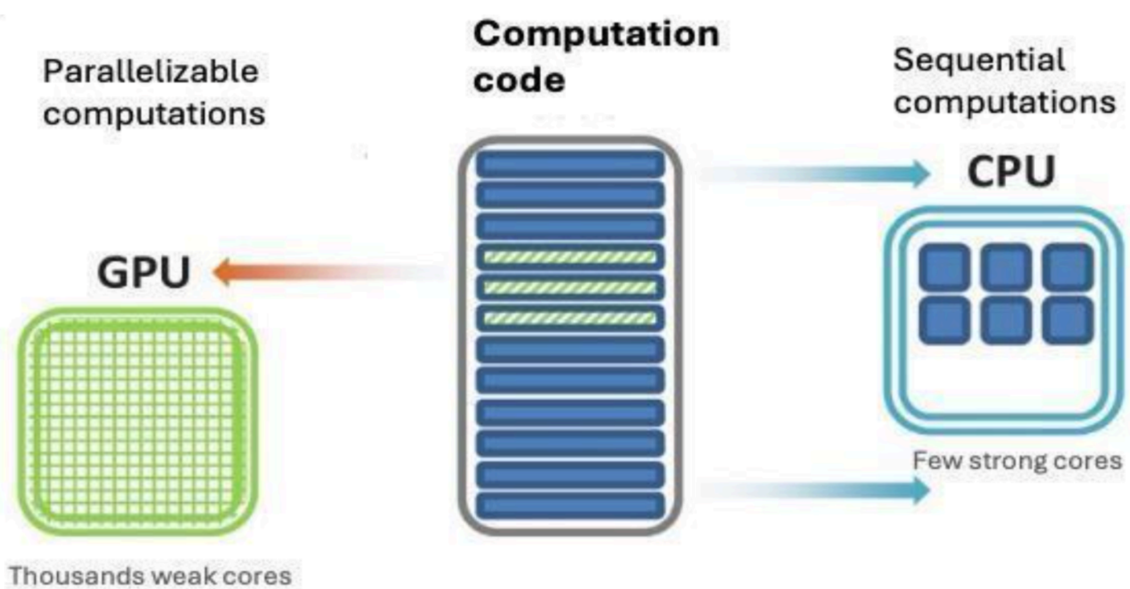
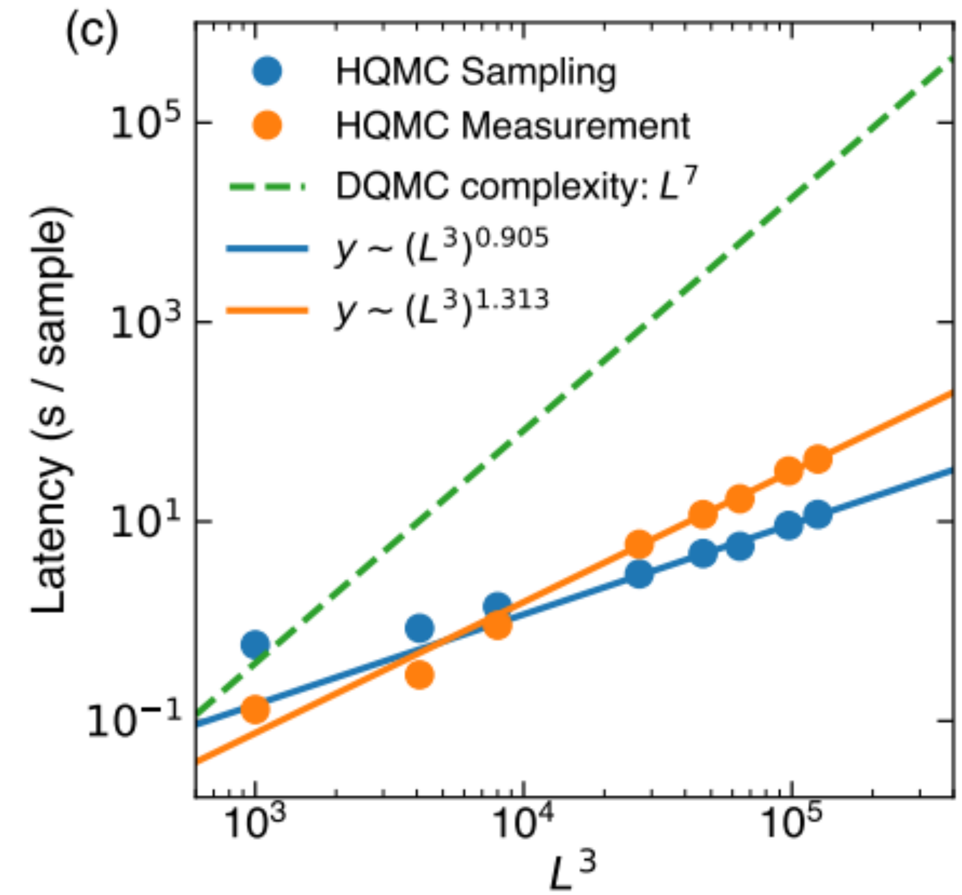


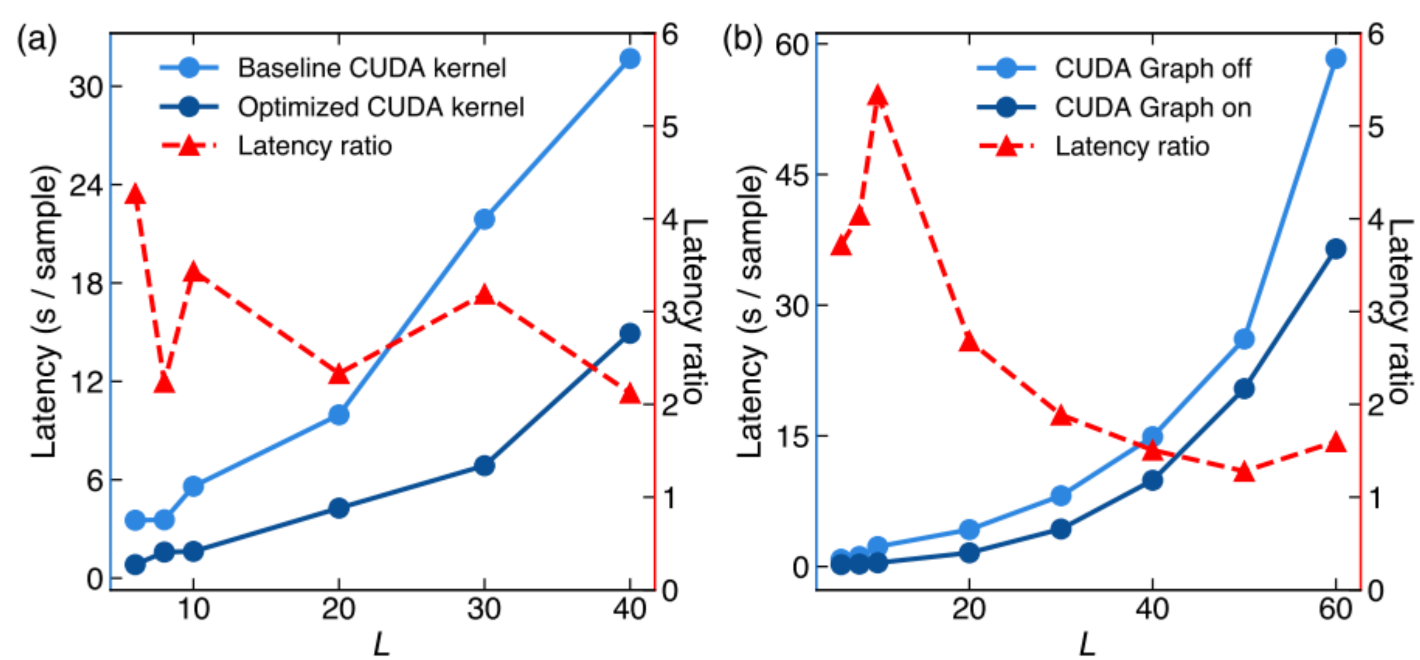
$$T_{\text{sample}} = N_{\text{leapfrog}} \cdot N_{\text{pCG}} \cdot O(N_\tau V_s) \sim O(L^3)$$



Customised CUDA kernel for matrix-vector multiplication

$$\begin{aligned}
X_{n+1} &= X_n + \zeta_n d_n \\
r_{n+1} &= r_n - \zeta_n O d_n \\
\rho_n &= \left\langle r_{n+1}, \tilde{O}^{-1} r_{n+1} \right\rangle / \left\langle r_n, \tilde{O}^{-1} r_n \right\rangle \\
d_{n+1} &= \tilde{O}^{-1} r_{n+1} + \rho_n d_n \\
\zeta_{n+1} &= \left\langle r_{n+1}, \tilde{O}^{-1} r_{n+1} \right\rangle / \langle d_{n+1}, O d_{n+1} \rangle ,
\end{aligned}$$

- Matrix-free representation of $M^\dagger M \nu$ multiplication
- Preconditioner-vector multiplication $\tilde{O}^{-1} \nu$



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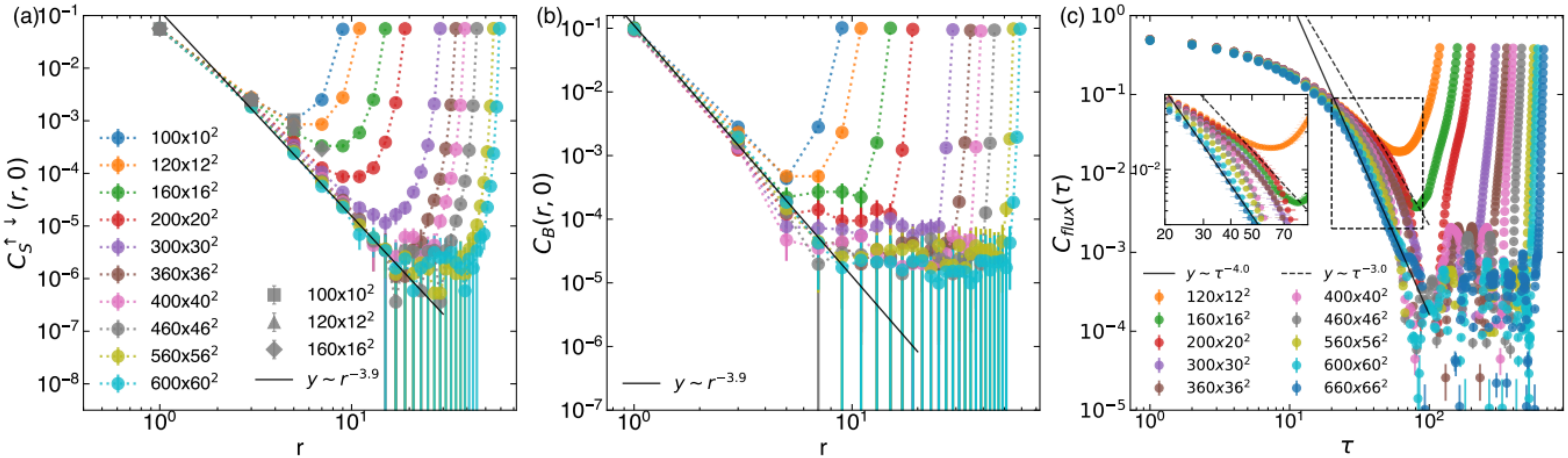
Non-compact DSL

$K = 0, B = 0, J = 1.25$



Compact DSL

$K = 1, B = 0, J = 1.25$