THE UNIVERSITY OF HONG KONG
DEPARTMENT OF PHYSICS

Assignment 2

Course: *Quantum Many-Body Computation (PHYS8202)* – Prof. *Zi Yang Meng*
Tutor: *Mr. Min Long, Mr.Shibo Shan*
Due date: *17th October, 2025*

1. **Derivation of U_{cr} and the critical behavior of the order parameter m in the vicinity of U_{cr} in Honeycomb Lattice Hubbard model.**

   (a) Derive the linear dispersion relation (Dirac cone) in the vicinity of the K (K′) point of the honeycomb lattice.

   (b) Approximate $\epsilon(\mathbf{k})$ using the linear dispersion to calculate the corresponding density of states.

   (c) Employ the approximate density of states to determine U_{cr} and derive the relation between U and m from the gap equation. Find the critical behavior of m in the vicinity of U_{cr} .

2. **Mean-field simulation at van Hove singularity point in Honeycomb lattice Hubbard model.**

   In the density of states (DOS) of a honeycomb lattice, there are two van Hove singularity points: $\epsilon = -t$ and $\epsilon = t$. The corresponding filling numbers (per spin) at these points are $1/4$ and $3/4$, respectively.

   (a) Verify that the filling number is $1/4$ when the Fermi energy is $\epsilon = -t$ (either numerically or analytically).

   (b) **Bonus:** Investigate the literature to understand the phase behavior at finite U at the van Hove singularity point ($\epsilon = -t$, filling number $= 1/4$) in Honeycomb lattice. Can you perform a mean-field simulation to verify this understanding?