THE UNIVERSITY OF HONG KONG
DEPARTMENT OF PHYSICS

Assignment 2

Course: *Computational Physics (PHYS4150/8150)* – Professor: *Prof. Zi Yang Meng*
Tutor: *Mr. Tim Lok Chau, Mr. Menghan Song*
Due date: *Oct. 19th, 2025 (Sunday)*

1. Three body problem
   The three-body problem is a classic challenge in physics that explores the motion of three masses interacting under mutual gravitational attraction. Unlike the two-body problem, which has an exact analytical solution, the three-body problem exhibits complex behavior. In this question, we will explore a stable solution and a chaotic solution to the problem.

   The three-body motion is governed by Newton's law of gravitation $\overrightarrow{F_{ij}} = -\frac{Gm_i m_j}{|r_{ij}|^2}\frac{\overrightarrow{r_{1ij}}}{|r_{ij}|}$. Denote the mass, position, and velocity vectors of the three bodies as m_i, $r_i = (x, y) = x\hat{i} + y\hat{i}$, $v_i = (v_x, v_y) = v_x\hat{i} + v_y\hat{i}$ respectively, where $i = 0, 1, 2$. We consider $G = 1$ and the unit of mass, length, and time to be dimensionless for simplicity.

   (a) Euler's solution is one of the stable solutions to the three-body problem and the three bodies are collinear under this solution. Set $m_0 = 3$, $m_1 = 1$, $m_2 = 1$, $r_0 = (0, 1)$, $r_1 = (0, -0.83299)$, $r_2 = (0, -2.16700)$, $v_0 = (-0.63034, 0)$, $v_1 = (0.52507, 0)$, $v_2 = (1.36596, 0)$ as the mass, initial positions and velocities of the 3 bodies, difference in time steps $\Delta t = 0.01$, and total time $T = 7$. Solve the systems of equations with the **4th order Runge Kutta method**, and **plot the orbits of the three bodies**. You can refer to the left panel of Fig.1.

   (b) Take the positions of the three bodies in a) at $T = 1, 2, 4, 6$ and prove numerically that **they are collinear** under these time slices.

   (c) Set $m_0 = m_1 = m_2 = 1$ and $r_0 = (3, 0)$, $r_1 = (-1.5, 2.59)$, $r_2 = (-0.6, -2.65)$, $v_0 = (0.1, 0.2)$, $v_1 = (-0.2, -0.1)$, $v_2 = (0.1, -0.1)$ as the initial positions and velocities of the 3 bodies, difference in time steps to be $\Delta t = 0.01$, and total time $T = 20$. Solve the systems of equations with the **4th order Runge Kutta method** again, and **plot the orbits of the three bodies**. You would see that they are chaotic, and you can refer to the right panel of Fig.1.

2. Poisson Equation
   In the lectures, we have learned to use three different relaxation methods to solve Poisson Equation $\Delta\phi(\vec{x}) = -\frac{1}{\epsilon_0}\rho(\vec{x})$ with Dirichlet boundary conditions.
   In this question, the error bound is set to be 10^{-4}, h and ϵ_0 are set to be 1.
   We consider the Dirichlet boundary condition, $\phi(\vec{x})|_{boundary} = 0$ on the 2D plane of two configurations. Using **Jacobi Relaxation, Gauss-Seidel Relaxation and Successive Overrelaxation scheme**, compare the numerical result on the numbers of iterations for each method to the theoretical prediction on computational

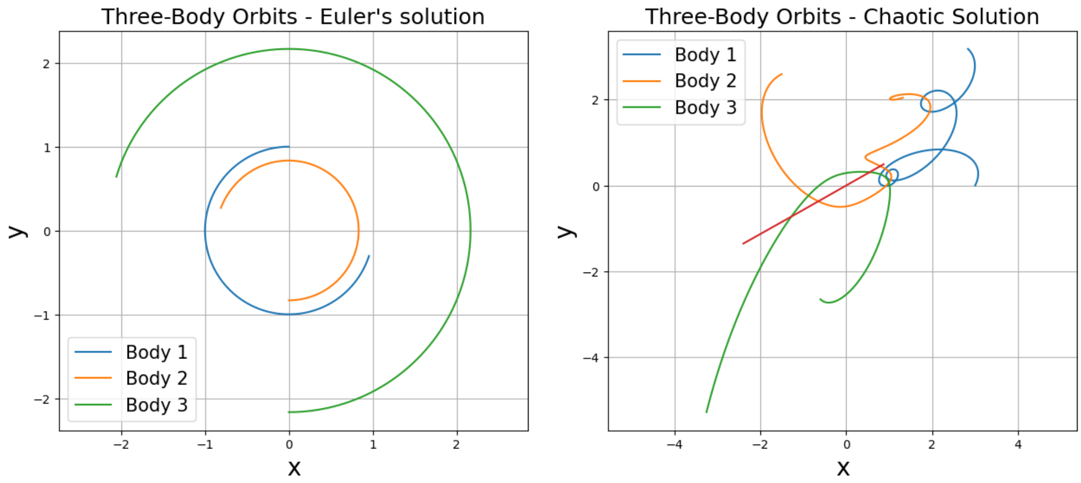

Figure 1: Euler's solution and chaotic solution for the three-body problem.

complexity introduced in the lecture notes (Chap1_3). **Comment on the leading order, as well as its coefficient.**

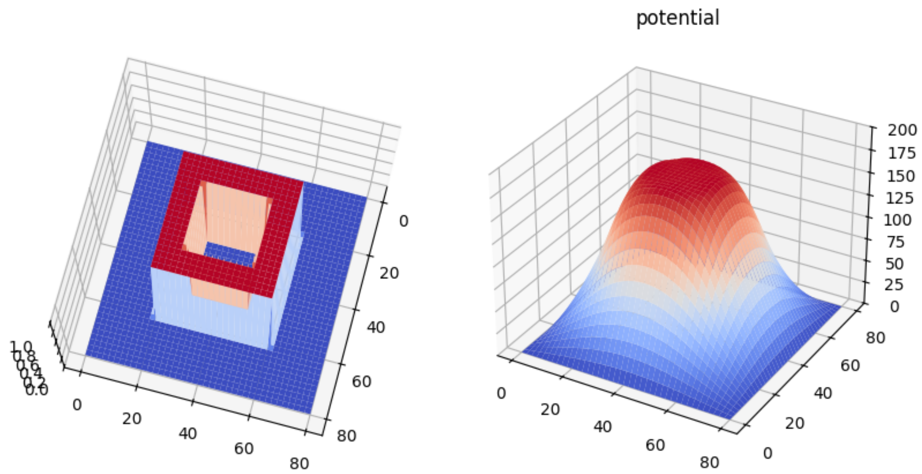

Figure 2: Ring charge distribution and the potential.

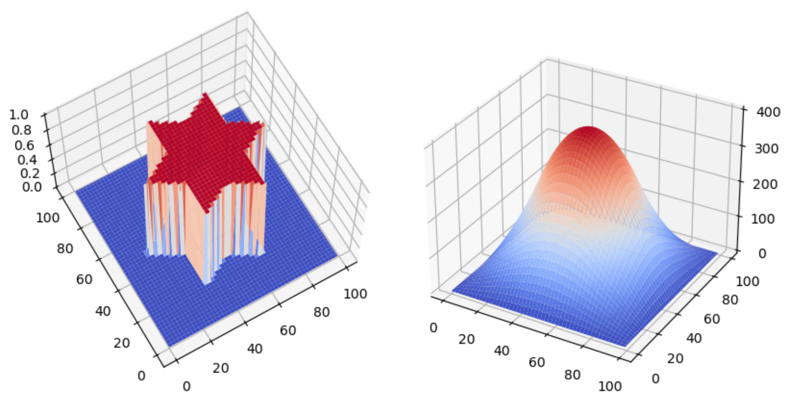

Figure 3: Star of David charge distribution and potential.

(a) We consider a *hollow square* charge distribution with side length N located at the center with an inner square with length $N/2$ being digged out, as shown in Figure 2. The points **in the hollow square (i.e. we do not account for the points on the boundary) have charge 1**, while those outside have no charge. The plane is a square lattice with size (2N+1)×(2N+1). **Make a plot of its potential, as shown in Figure 2. Please consider N = 10, 20, 40 for this configuration.**

(b) We consider a **Star of David** centers on a N×N square lattice, as shown in Figure 3. The **Star of David** can be view as a overlap of two equilateral triangles whose centers of gravity both lie on (N/2, N/2) and put on opposite direction, an example is shown in Figure 2. **The side length of the equilateral triangles is 0.6N**. **Make a plot of its potential, as shown in Figure 3. Please consider N = 10, 20, 40 for this configuration.** We have no specific requirement for the relative orientation of the star of David.