THE UNIVERSITY OF HONGKONG
DEPARTMENT OF PHYSICS

Assignment 1

Course: *Quantum Many-Body Computation (PHYS8202)* – Prof. *Zi Yang Meng*
Tutor: *Mr. Min Long, Mr.Shibo Shan*
Due date: *1st October, 2025*

1. **Derivation of the Density of States(DOS) of square lattice model and its limits**

   We learn from the class that the dispersion relation of a square lattice tight-binding model is

   $$\epsilon(\mathbf{k}) = -2t(\cos(k_1) + \cos(k_2))$$

   In the book *Lecture Notes on Electron Correlation and Magnetism* by Patrik Fazekas, page 165, the DOS of the square lattice tight binding band is in a closed form:

   $$\rho(\epsilon) = \frac{1}{2\pi^2 t}\Theta(4t - |\epsilon|)K\left(1 - \frac{\epsilon^2}{16t^2}\right)$$

   where K is the complete elliptic integral of the first kind. We can observe two features: the step-like discontinuities at the band edges, and the logarithmic singularity in the center. The latter can be obtained from the small-ϵ expansion:

   $$\rho(\epsilon) \sim \frac{1}{2\pi^2 t}\ln\frac{t}{|\epsilon|}$$

   (a) Try to derive the DOS of 2D square lattice tight-binding model

   (b) Try to obtain the logarithmic asymptotic behavior of the DOS at $\epsilon \to 0$

2. **The gap equation**

   We learn from the class that the gap equation for square lattice Hubbard model is

   $$1 = U\int_0^{4t}\frac{\rho(\epsilon)d\epsilon}{\sqrt{\epsilon^2 + \Delta^2}}$$

   (a) Try to derive the relation in Jorge E.Hirsch's paper([Hirsch, Phys. Rev. B 31, 4403 (1985)]):

   $$\Delta \sim t e^{-2\pi\sqrt{\frac{t}{U}}}.$$

   (b) Try to perform the momentum-space mean-field simulation and verify such result with your numerical data.