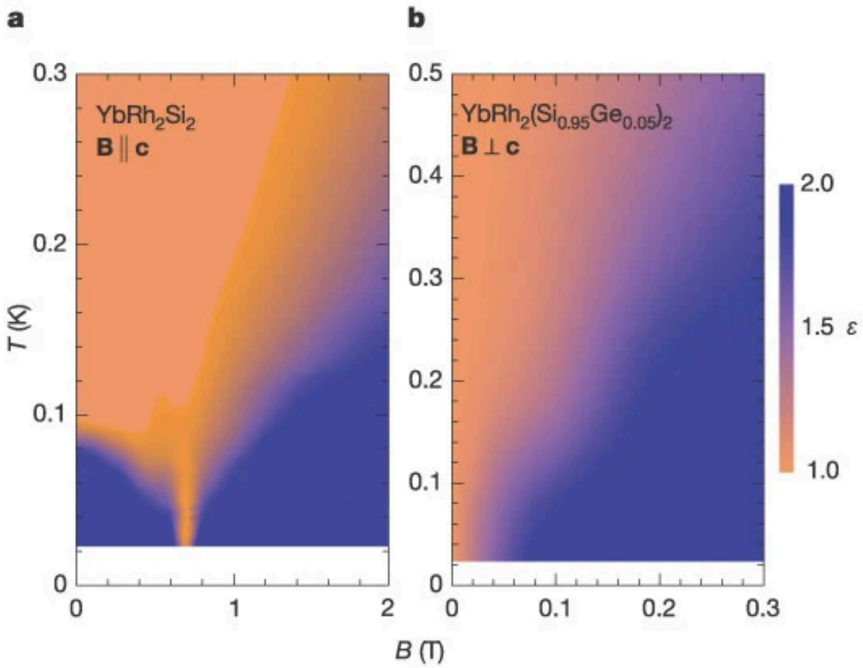
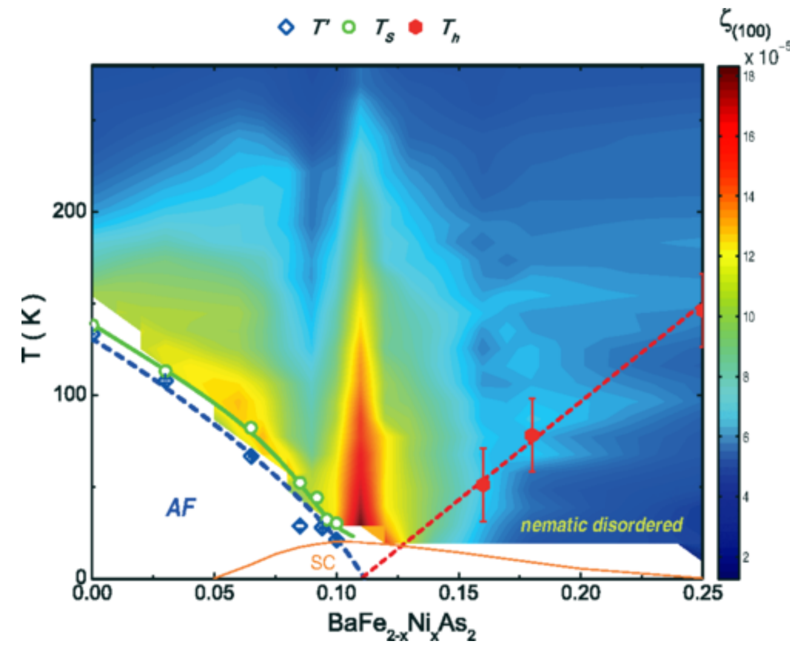
# Yukawa-SYK model & Quantum Entanglement: From Many-Body Computation Perspective

ZI YANG MENG
孟子楊
https://quantummc.xyz/

# Question



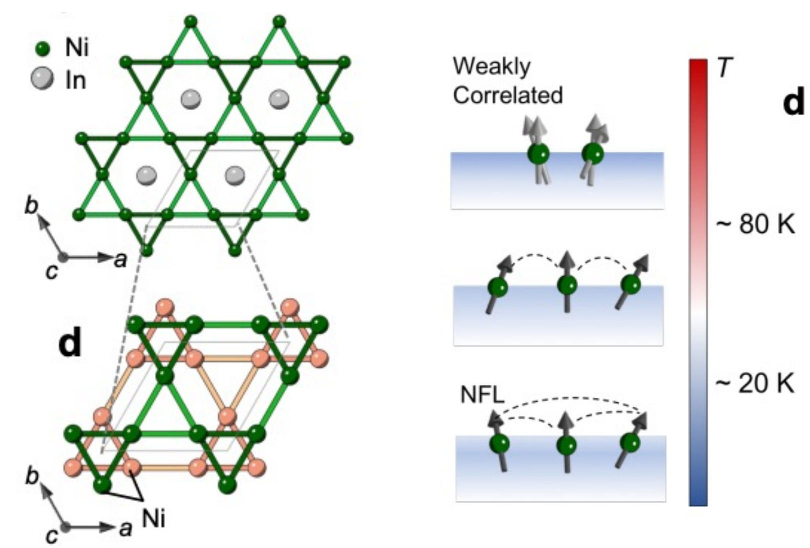

Nature 424, 524 (2003)

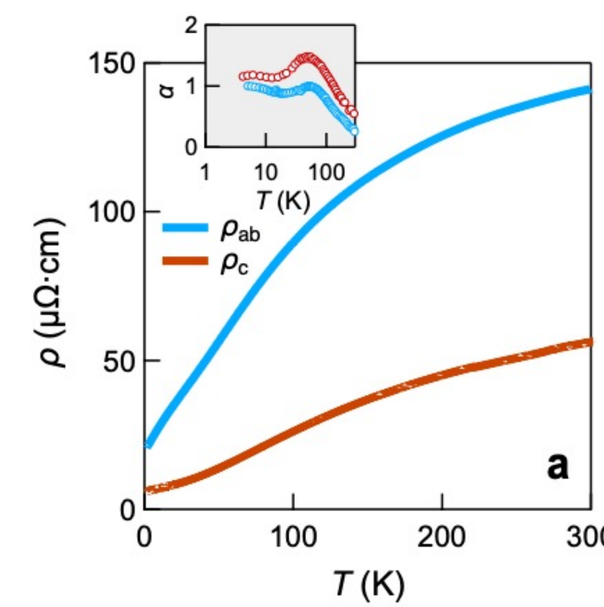


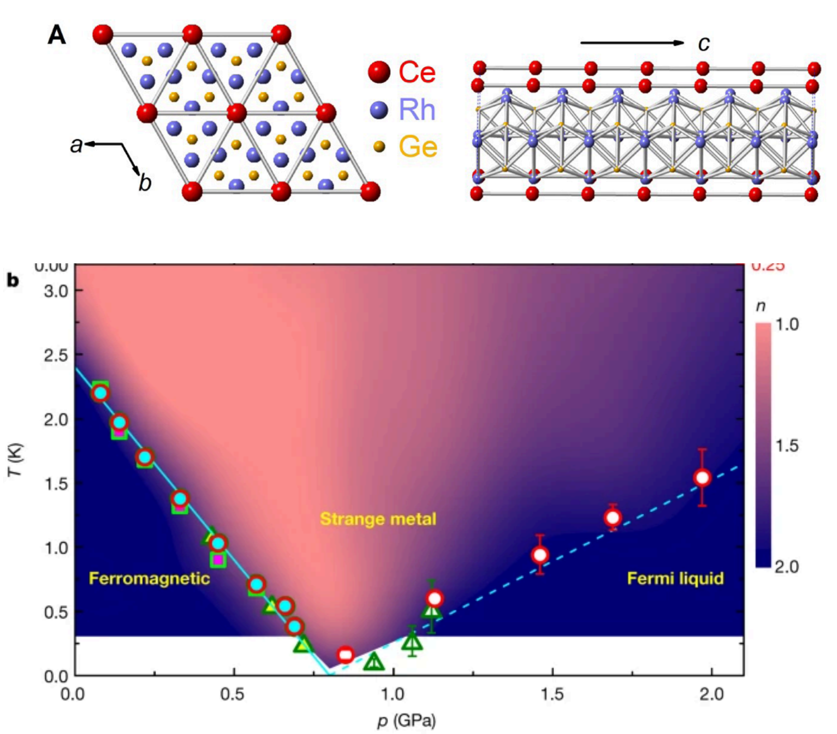

PRL 117, 157002 (2016)

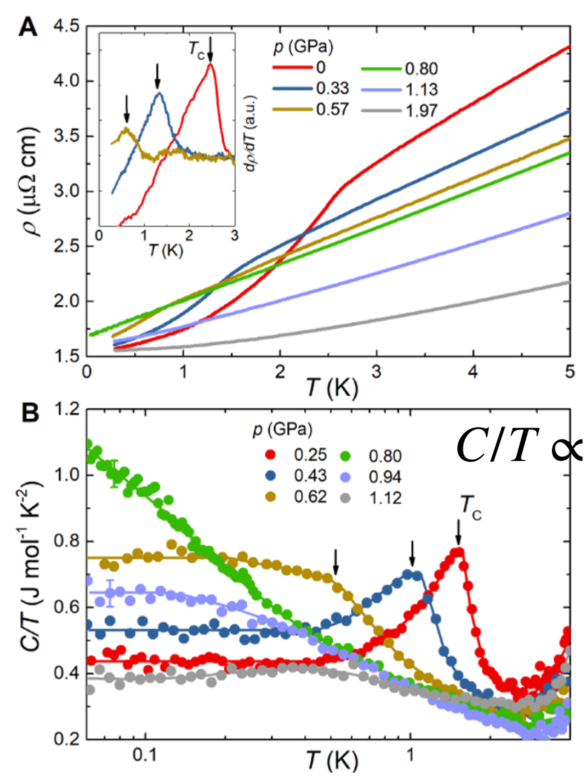
Kagome metal Ni3In

Nature Physics 20, 610 (2024)




$\rho \propto T$

$C/T \propto \ln(\frac{T^*}{T})$

Ce-based heavy fermion metal, Nature 579, 51-55 (2020)

- FM / AFM / Nematic fluctuations
- Non-Fermi liquid, Superconductivity
- Fermionic QCP

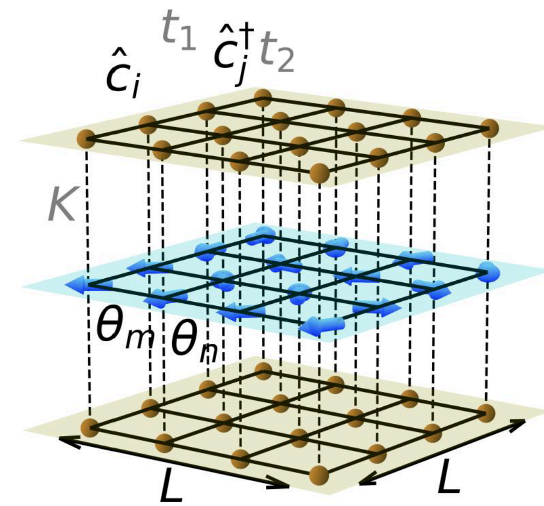
# Fermions couple to critical boson / gauge modes

Non-Fermi-Liquid

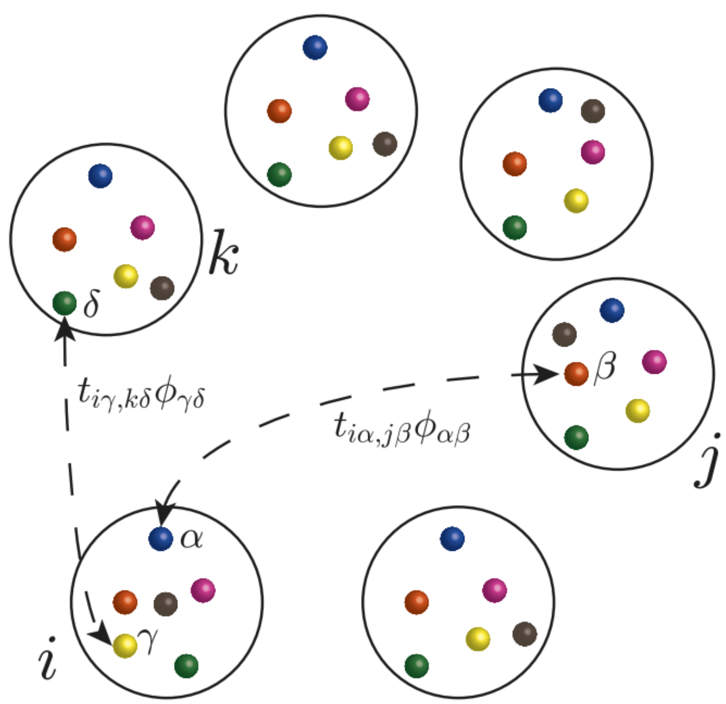
Quantum critical metals

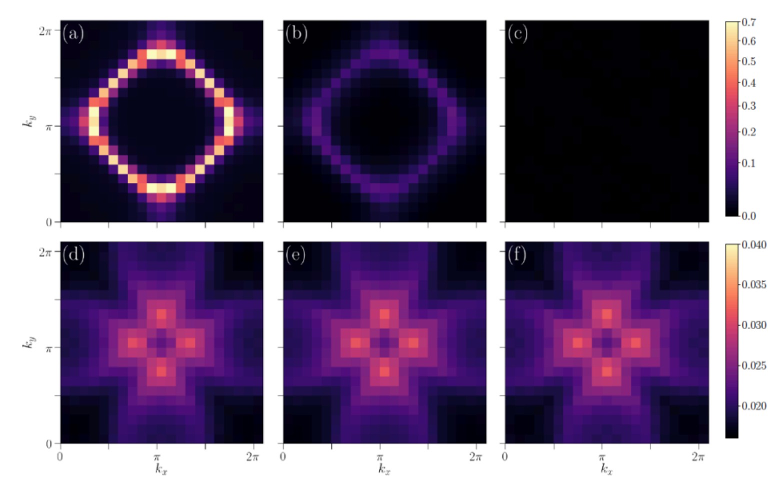
Matter fields coupled to gauge fields

Yukawa-SYK model



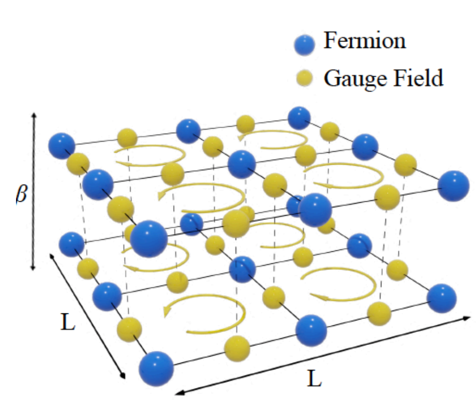

- PRX 7, 031058 (2017)
- PNAS 116 (34), 16760 (2019)
- npj Quantum Materials 5, 65 (2020)
- PRB 105, L041111 (2022)
- Nat. Comm. 13, 2655 (2022)
- ......

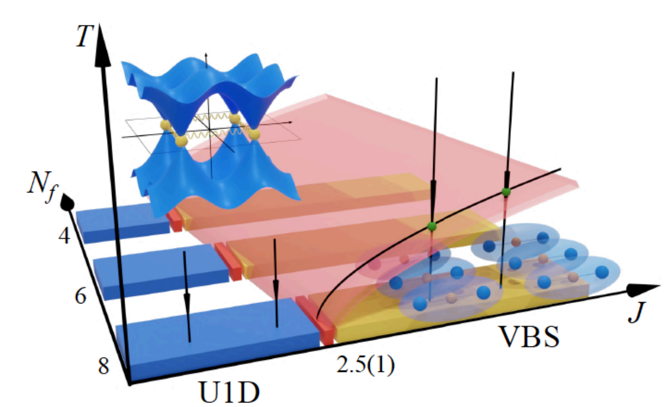


- PRX 9, 021022 (2019)
- PRB 101, 235118 (2020)
- CPL 37, 047103 (2020)
- PRB 103, 165131 (2021)
- ......

- PRR 3, 013250 (2021)
- PRB 103, 195108 (2021)



- Rich analytic literature,
  sum particular series of diagrams
- Obtain exact forms of fermionic
  and bosonic propagators in NFL
- Alternative numerical approaches QMC
- Lattice models, large sizes and low T
- Numerics and Analytics converge

Journal Club for Condensed Matter Physics JCCM_November_2018_02
https://www.condmatjclub.org

# Solving metallic quantum criticality in a casino

1. **Monte Carlo Studies of Quantum Critical Metals**
   Authors: E. Berg, S. Lederer, Y. Schattner, and S. Trebst
   Annual Reviews of Condensed Matter Physics, arXiv:1804.01988 (2018)
2. **Superconductivity mediated by quantum critical antiferromagnetic fluctuations: The rise and fall of hot spots**
   Authors: X. Wang, Y. Schattner, E. Berg, and R. M. Fernandes
   Physical Review B 95, 174520 (2017)
3. **Itinerant Quantum Critical Point with Fermion Pockets and Hot Spots**
   Authors: Z-H Liu, G. Pan, X-Y Xu, K. Sun, and Z-Y Meng
   arXiv:1808.08878 (2018) PNAS 116 (34), 16760 (2019)

*Recommended with a Commentary by Andrey V Chubukov, University of Minnesota*

One of the most extensively studied items in modern physics of correlated metals is whether a Fermi-liquid (FL) behavior can be destroyed in dimensions $D > 1$. Two main roots to non-FL physics have been proposed. One is to increase interactions and bring the system close to a transition to a Mott insulator. Another is to keep interactions relatively weak, but

# Model

$$H = \sum_{\mathbf{k},\alpha} \mathbf{v}_k \cdot (\mathbf{k}-\mathbf{k}_F) c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha} + \sum_{\mathbf{q}} \chi_0^{-1}(\mathbf{q}) \mathbf{S}_{\mathbf{q}} \cdot \mathbf{S}_{-\mathbf{q}} + g \sum_{\mathbf{q},\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k}+\mathbf{q},\alpha} \sigma_{\alpha,\beta} c_{\mathbf{k},\beta} \cdot \mathbf{S}_{-\mathbf{q}}$$

$$S = -\int_0^\beta d\tau \int_0^\beta d\tau' \sum_{k,\alpha} c^\dagger_{\mathbf{k},\sigma} G_0^{-1}(\mathbf{k},\tau-\tau') c_{\mathbf{k},\sigma}(\tau')$$

$$+\frac{1}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \sum_q \chi_0^{-1}(\mathbf{q}) \mathbf{S}_q(\tau) \cdot \mathbf{S}_{-q}(\tau')$$

$$+g \int_0^\beta d\tau \sum_q \mathbf{s}_q(\tau) \cdot \mathbf{S}_{-q}(\tau)$$

$$\chi_0(\mathbf{q},\omega) = \frac{\chi_0}{\xi^{-2} + (\mathbf{q}-\mathbf{Q})^2 - (\omega/v_s)^2}$$

$$G_0^{-1}(\mathbf{k},\tau) = \partial_\tau - \mathbf{v}_k \cdot (\mathbf{k}-\mathbf{k}_F)$$


- Abanov, Chubukov, Schmalian, Adv. in Phys. 52, 119 (2003)
- Metlitski, Sachdev, PRB 82, 075127 (2010)
- Metlitski, Sachdev, PRB 82, 075128 (2010)
- Sung-Sik Lee, Annu. Rev. Condens. Matter Phys 9, 227 (2018)

- Revealing fermionic quantum criticality from new Monte Carlo techniques
  Xiao Yan Xu, Zi Hong Liu, Gaopei Pan, Yang Qi, Kai Sun, ZYM
  J. Phys.: Condens. Matter 31 463001 (2019)
- A sport and a pastime: model design and computation in quantum many-body systems
  Gaopei Pan, Weilun Jiang, ZYM
  Chinese Phys. B 31, 127101 (2022) Topical Review

# Determinant quantum Monte Carlo

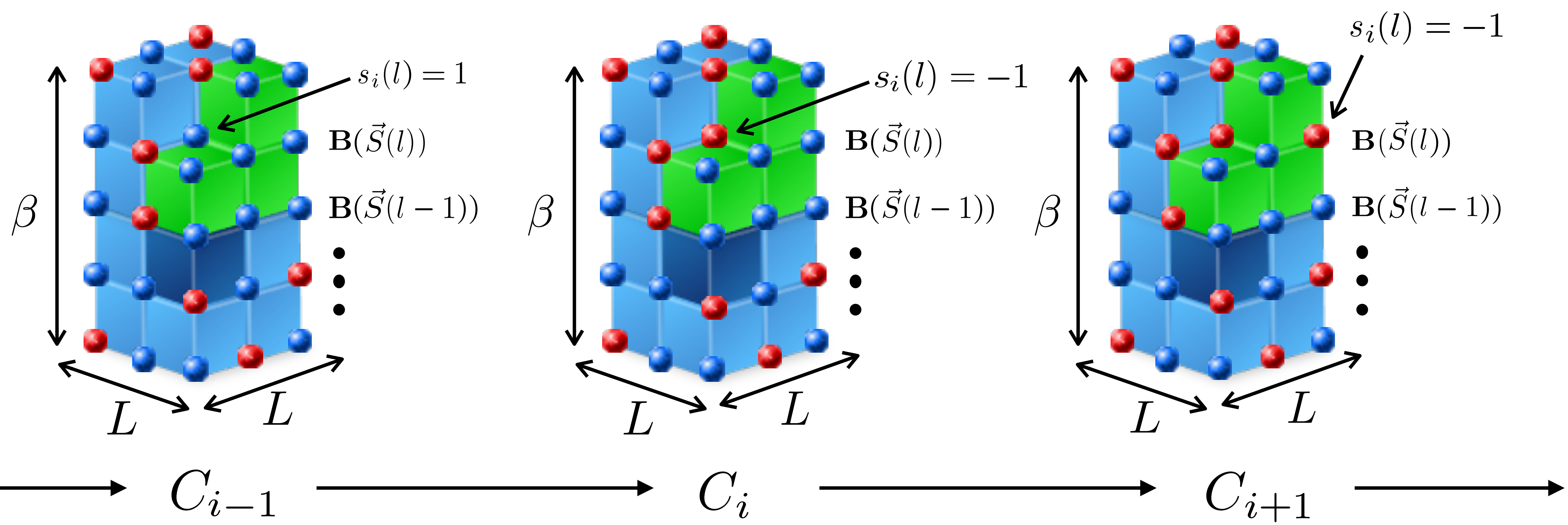
- Write Path-integral into determinant

$$Z = \mathrm{Tr}[\prod_{l=1}^{m} e^{-\Delta\tau \hat{H}_t} e^{-\Delta\tau \hat{H}_U}] = C^m \sum_{\vec{s}_1, \vec{s}_2, \cdots, \vec{s}_m} \det[\mathbf{1} + \mathbf{B}_m \mathbf{B}_{m-1} \cdots \mathbf{B}_1]$$

$$\mathbf{B}_l = e^{-\Delta\tau \mathbf{H_t}} e^{-\Delta\tau \mathbf{H_U}(\vec{s}(l))}$$

- Monte Carlo sampling in configuration space

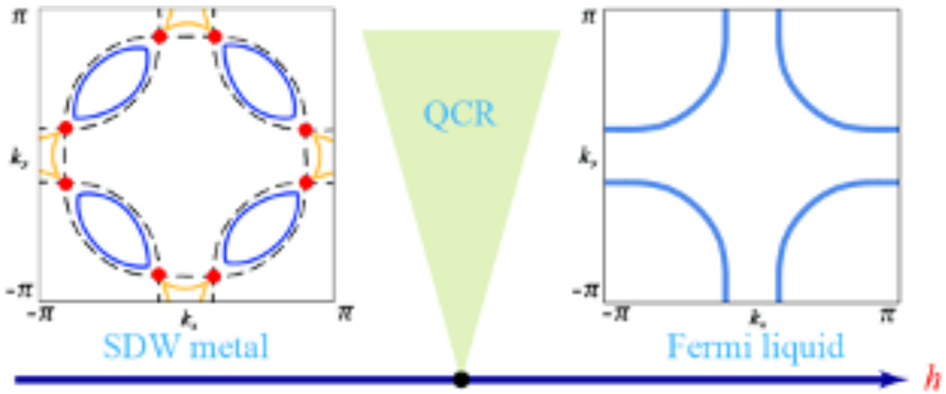
$$\mathbf{H_U}(\vec{s}(l)) \propto \alpha \vec{s}(l)$$



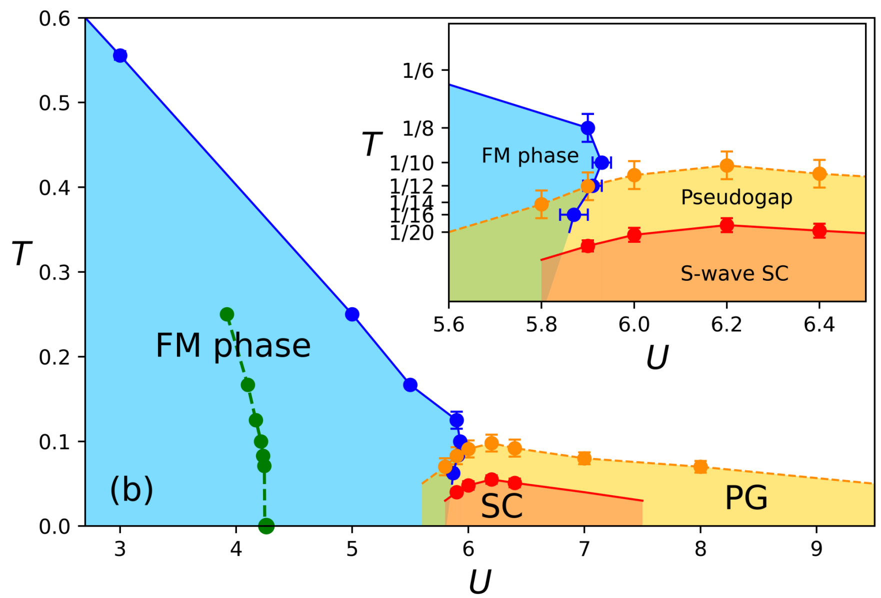

Sign Problem in quantum Monte Carlo simulation, Gaopei Pan & ZYM

[Encyclopedia of Condensed Matter Physics (Second Edition) (2024)]

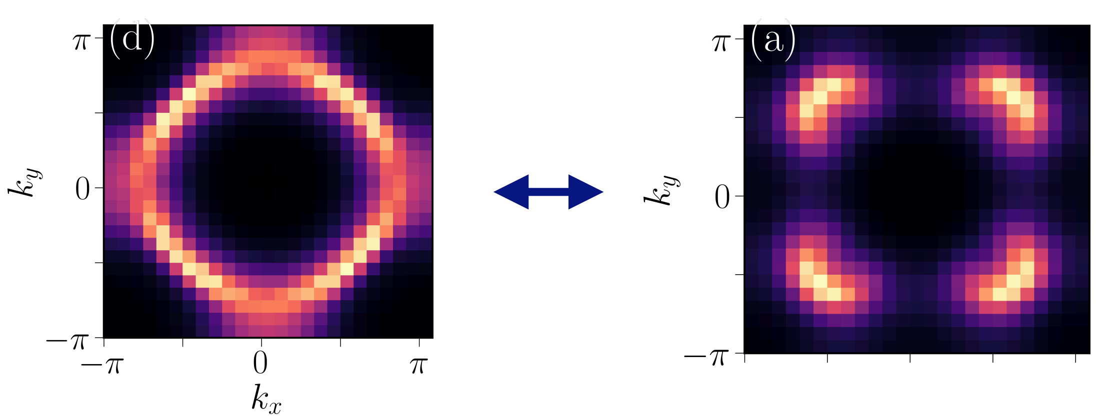
# Fermions couple to critical boson / gauge modes




PNAS 116 (34), 16760 (2019)

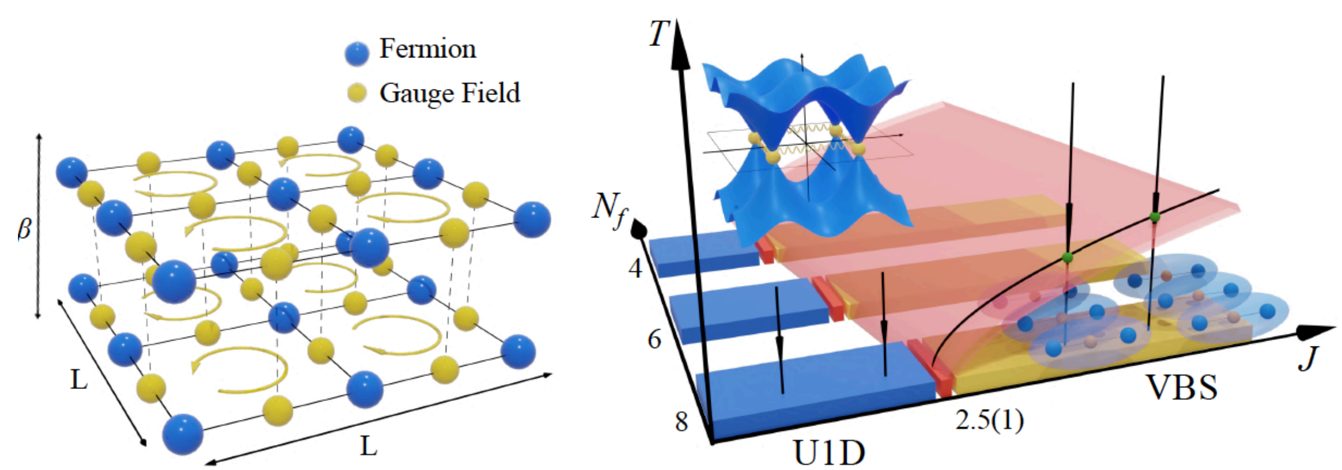



Nat. Comm. in press (2022)




PRB 103, 165131 (2021)

- Itinerant quantum critical point
- Non-Fermi-liquid
- Pseudogap and superconductivity
- Z2 gauge field, orthogonal metal, Fermi arc
- U1 gauge field, Dirac / algebraic spin liquid
- Yukawa-SYK model

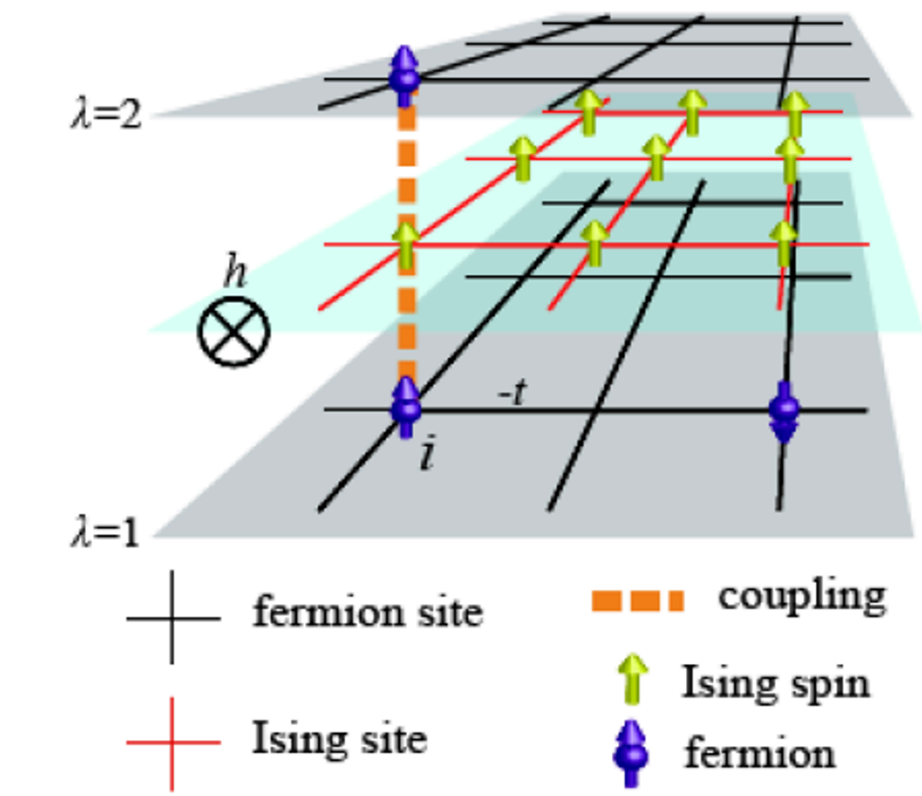


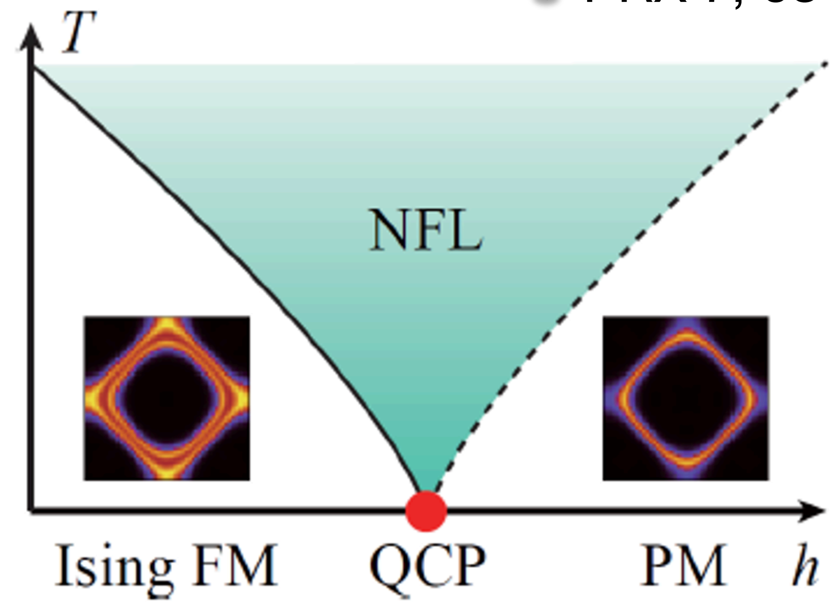

PRX 9, 021022 (2019)

# Non-Fermi Liquid at (2+1)D Ferromagnetic Quantum Critical Point

Xiao Yan Xu,[1] Kai Sun,[2] Yoni Schattner,[3] Erez Berg,[3] and Zi Yang Meng[1]

PRX 7, 031058 (2017)



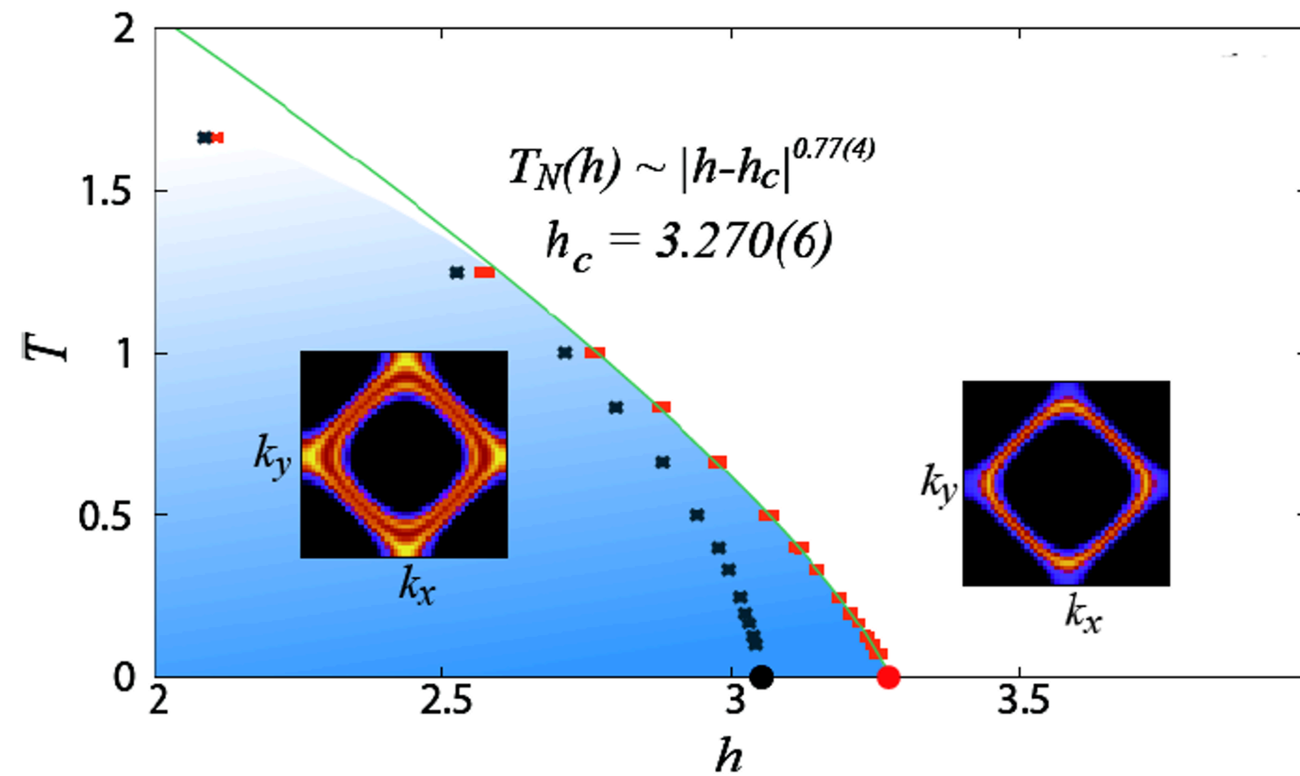

$\underbrace{SU(2)\times SU(2)}_{\text{orbital rotation for }\uparrow\text{ and }\downarrow}$ $\underbrace{U(1)\times U(1)}_{\text{charge conservation for }\uparrow\text{ and }\downarrow}$ $\underbrace{Z_2}_{\text{interchange }\uparrow\text{ and }\downarrow\text{ while flipping }s^z}$

$$\hat{H} = \hat{H}_f + \hat{H}_s + \hat{H}_{sf}$$

$$\hat{H}_f = -t\sum_{\langle ij\rangle\lambda\sigma}\hat{c}^\dagger_{i\lambda\sigma}\hat{c}_{j\lambda\sigma} + h.c. - \mu\sum_{i\lambda\sigma}\hat{n}_{i\lambda\sigma}$$

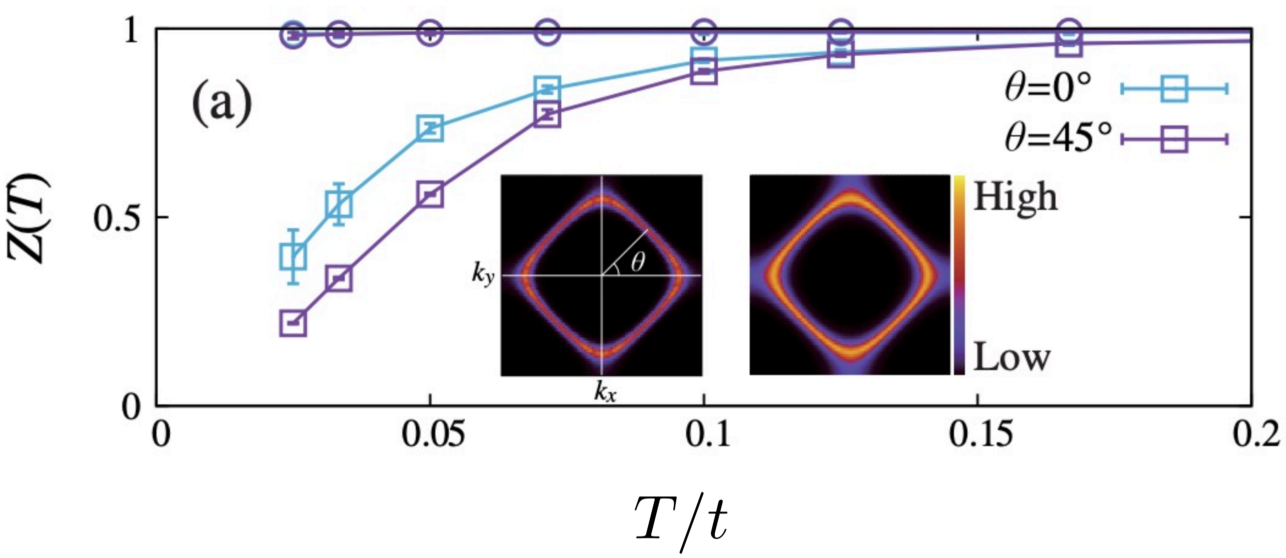
$$\hat{H}_s = -J\sum_{\langle ij\rangle}\hat{s}^z_i\hat{s}^z_j - h\sum_i \hat{s}^x_i$$

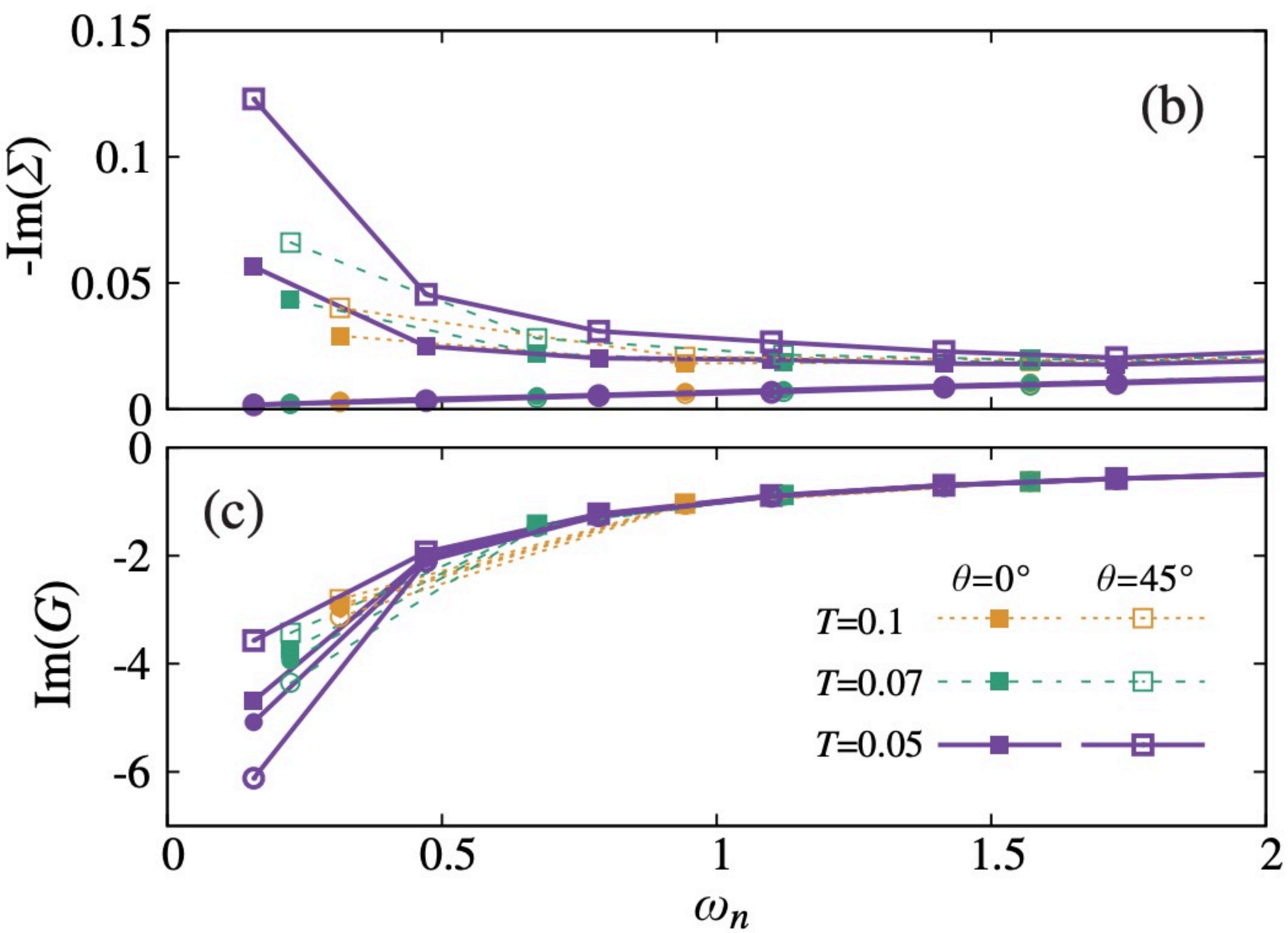
$$\hat{H}_{sf} = -\xi\sum_{i\lambda}s^z_i(n_{i\lambda\uparrow} - n_{i\lambda\downarrow})$$

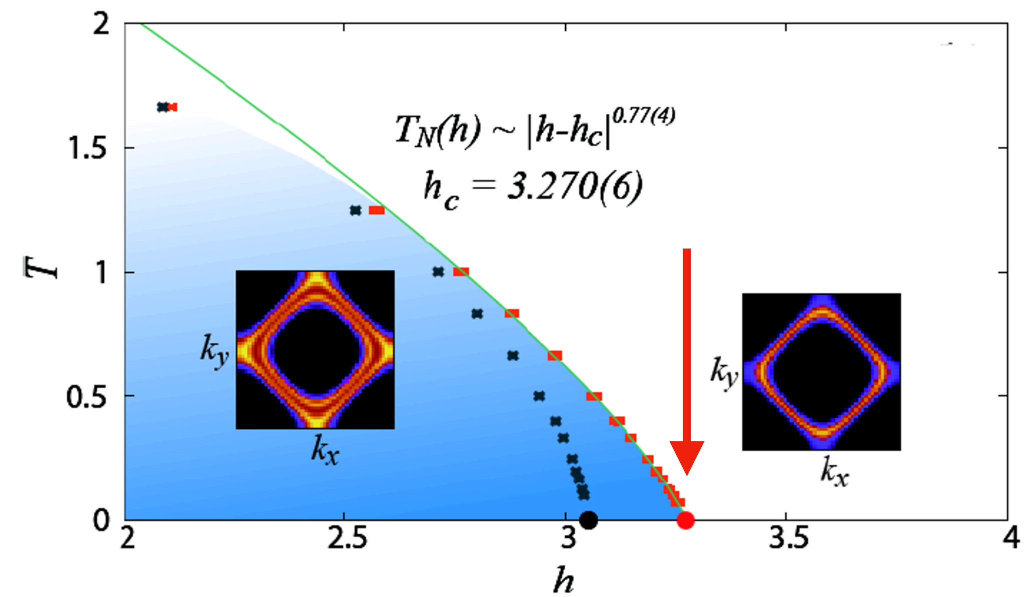


# Non-Fermi-liquid

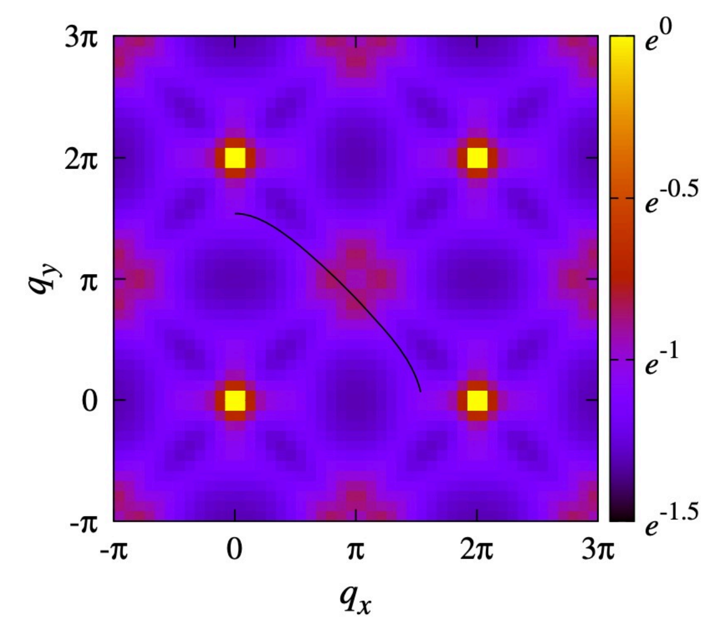
PRX 7, 031058 (2017)



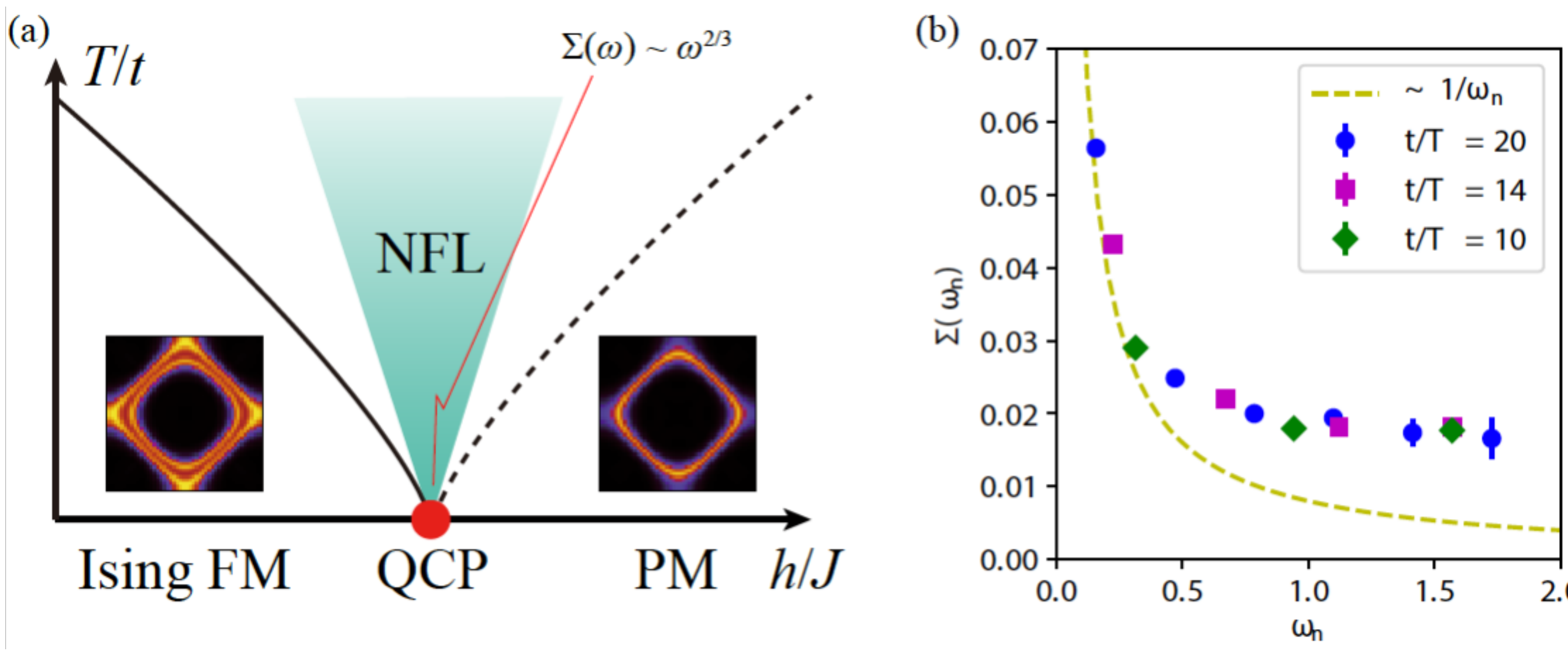

$$Z_{\mathbf{k}_F} \approx \frac{1}{1 - \frac{\mathrm{Im}\Sigma(\mathbf{k}_F, i\omega_0)}{\omega_0}}$$



# Identification of non-Fermi liquid fermionic self-energy from quantum Monte Carlo data

Xiao Yan Xu[1] ✉, Avraham Klein[2], Kai Sun[3], Andrey V. Chubukov[2] and Zi Yang Meng[4,5,6] ✉

npj Quantum Materials 5, 65 (2020)




$$\Sigma(\omega_n) = \Sigma_T(\omega_n) + \Sigma_Q(\omega_n)$$

$$\Sigma_T(\omega_n) \approx \alpha(T)/\omega_n$$

$$\Sigma_Q(\omega_n) \rightarrow \omega_n^{2/3}$$

$\omega_F \quad << \quad \Sigma \quad << \quad \pi T\,,\, \bar{g}\,,\, \omega_b \quad << \quad E_F$

QMC data

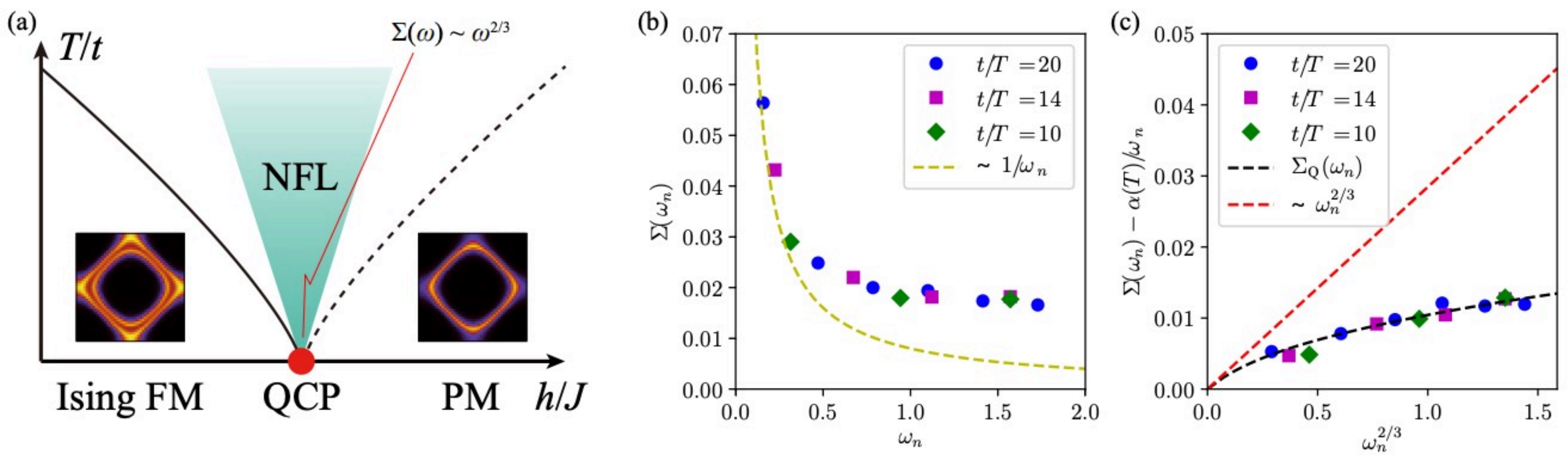
$\bar{g} \ll$ **bandwidth**$/E_F$

$$\Sigma(\omega_n) \ll \omega_n$$

# Identification of non-Fermi liquid fermionic self-energy from quantum Monte Carlo data

Xiao Yan Xu[1]✉, Avraham Klein[2], Kai Sun[3], Andrey V. Chubukov[2] and Zi Yang Meng[4,5,6]✉

npj Quantum Materials 5, 65 (2020)



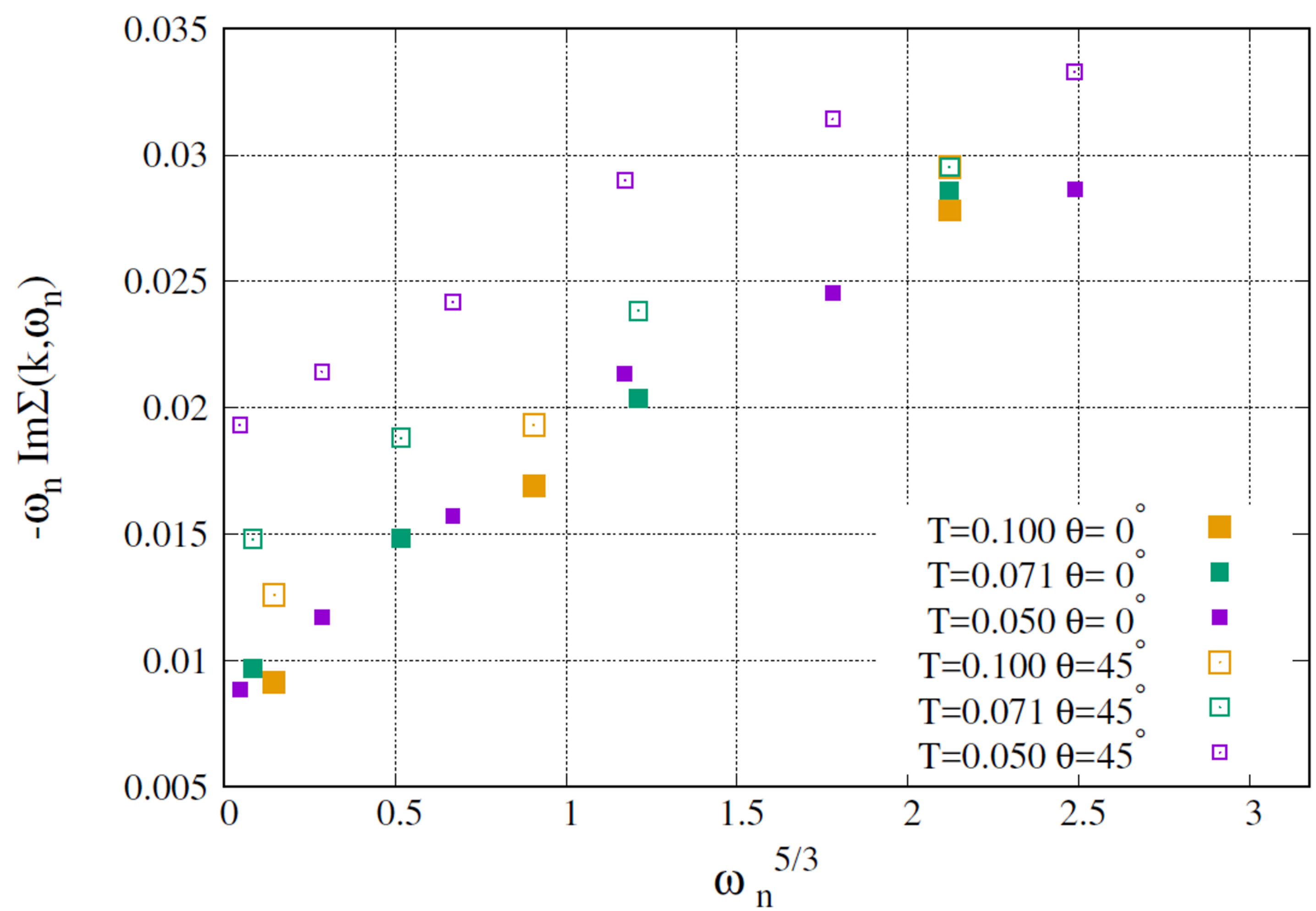

$$\Sigma(\omega_n) = \Sigma_T(\omega_n) + \Sigma_Q(\omega_n)$$

$$\Sigma_T(\omega_n) = \alpha(T)/\omega_n$$

$$\Sigma_Q(\omega_n) = \omega_F^{1/3}\omega_n^{2/3} + \cdots$$

$$\omega_F = \frac{\bar{g}^2}{8\pi^2 3^{3/2}\mathcal{V}_F v_F^2 N_f}$$

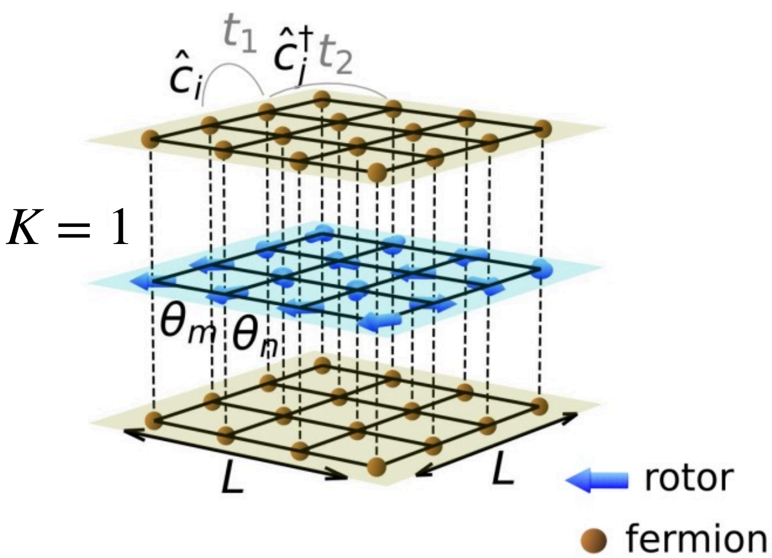
$$\omega_n\Sigma(\omega_n) = \alpha(T) + \omega_F^{1/3}\omega_n^{5/3} + \cdots$$

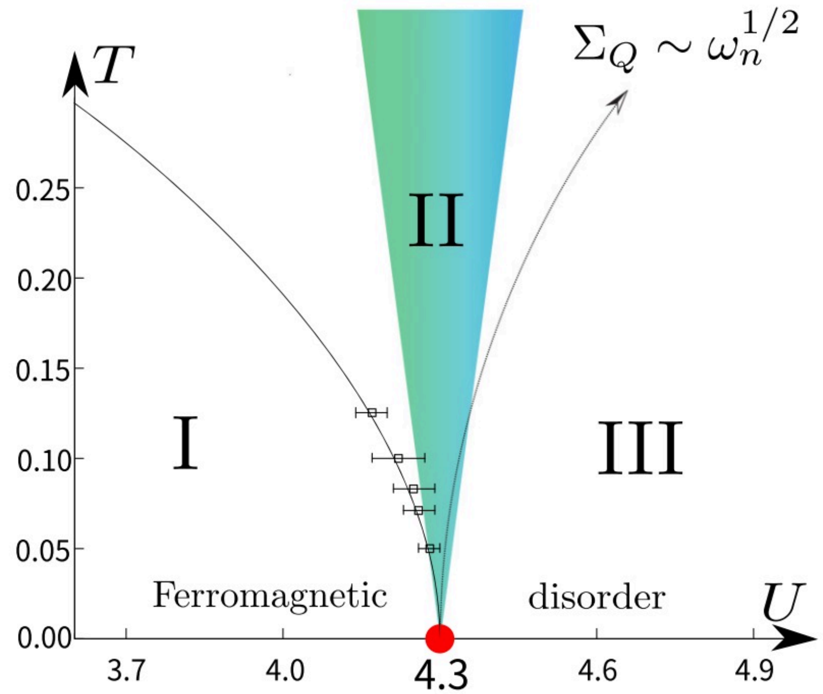


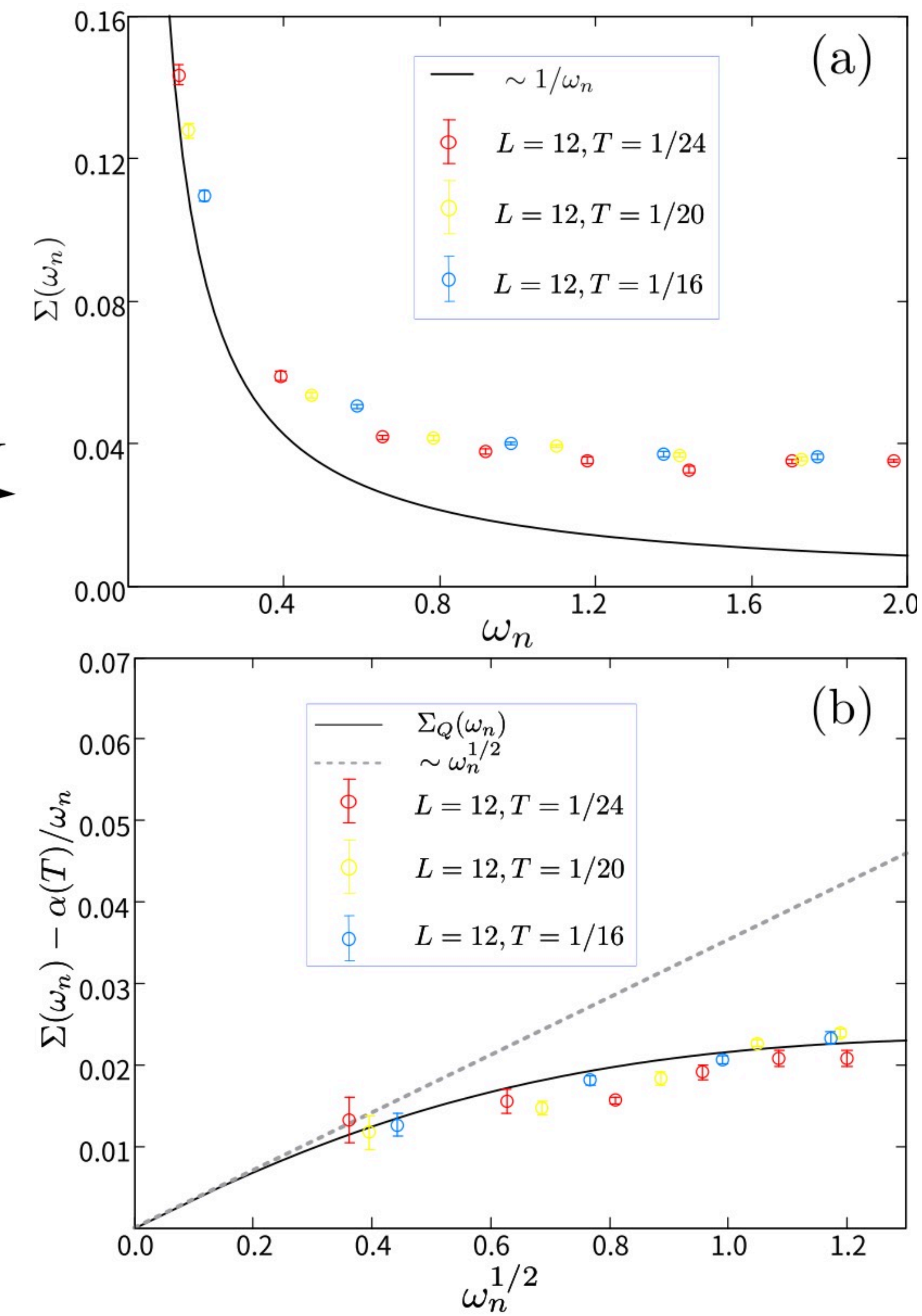
# Dynamical exponent of a quantum critical itinerant ferromagnet: A Monte Carlo study

Yuzhi Liu[1,2] Weilun Jiang,[1,2] Avraham Klein,[3] Yuxuan Wang,[4] Kai Sun,[5] Andrey V. Chubukov,[6] and Zi Yang Meng[7,1,*]

PRB 105, L041111 (2022)




$$H = H_f + H_{qr} + H_{qr-f}$$

$$\hat{H}_{\rm f} = -t_1 \sum_{\langle i,j\rangle\sigma\lambda} \hat{c}^+_{i\sigma\lambda}\hat{c}_{j\sigma\lambda} - t_2 \sum_{\langle\langle i,j\rangle\rangle\sigma\lambda} \hat{c}^+_{i\sigma\lambda}\hat{c}_{j\sigma\lambda} - \mu \sum_{i\sigma\lambda} \hat{n}_{i\sigma\lambda}$$

$$\hat{H}_{\rm qr} = \frac{U}{2}\sum_i \hat{L}_i^2 - t_b \sum_{\langle i,j\rangle} \cos\left(\hat{\theta}_i - \hat{\theta}_j\right)$$

$$\hat{H}_{\text{qr-f}} = -\frac{K}{2}\sum_{i\lambda} \hat{c}^+_{i\lambda}\boldsymbol{\sigma}\hat{c}_{i\lambda}\cdot\hat{\boldsymbol{\theta}}_i = -\frac{K}{2}\sum_{i\lambda}\left(\hat{c}^+_{i\lambda}\sigma^x\hat{c}_{i\lambda}\cdot\cos\hat{\theta}_i + \hat{c}^+_{i\lambda}\sigma^y\hat{c}_{i\lambda}\cdot\sin\hat{\theta}_i\right)$$

$$\Sigma_T(\omega_n) \sim \alpha(T)/\omega_n$$

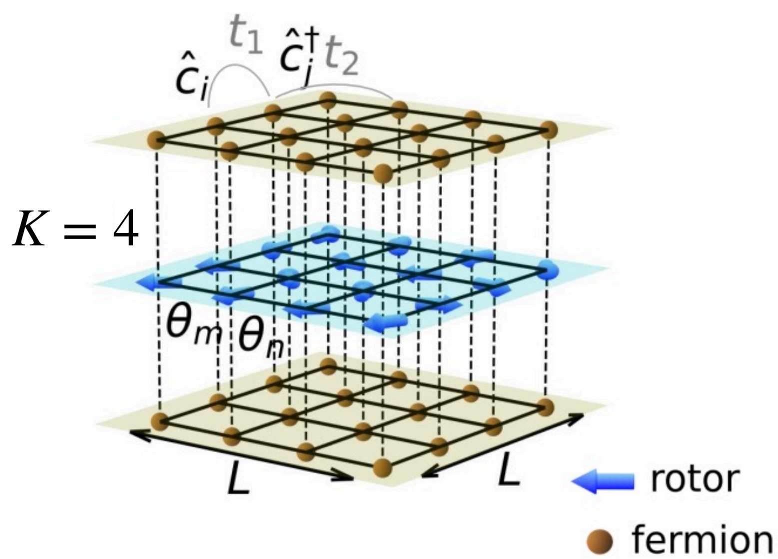
$$\Sigma(\omega_n) = \Sigma_T(\omega_n) + \Sigma_Q(\omega_n)$$

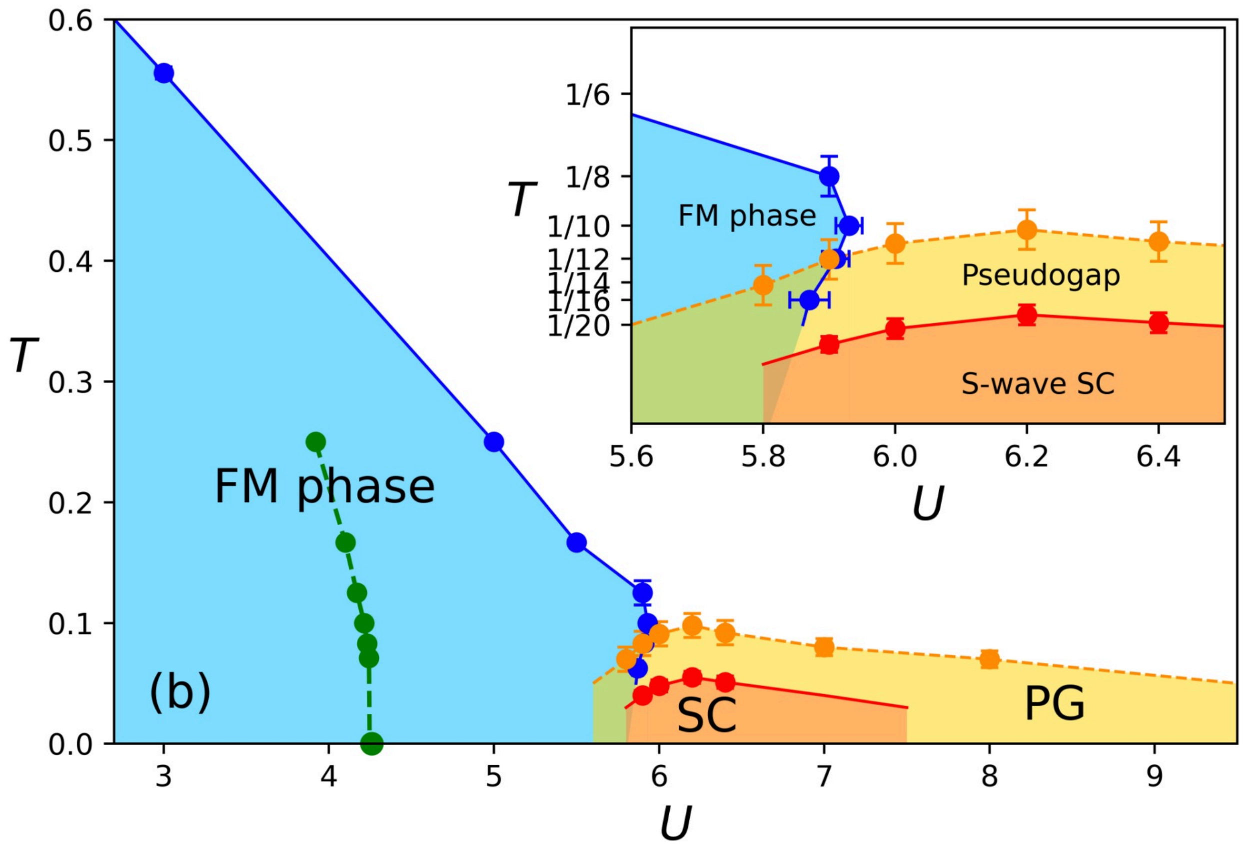
$$\Sigma_Q(\omega_n) \sim (\omega_n)^{1/2} f(\omega_n/\omega_c) + \cdots$$

# Pseudogap and superconductivity emerging from quantum magnetic fluctuations: a Monte Carlo study

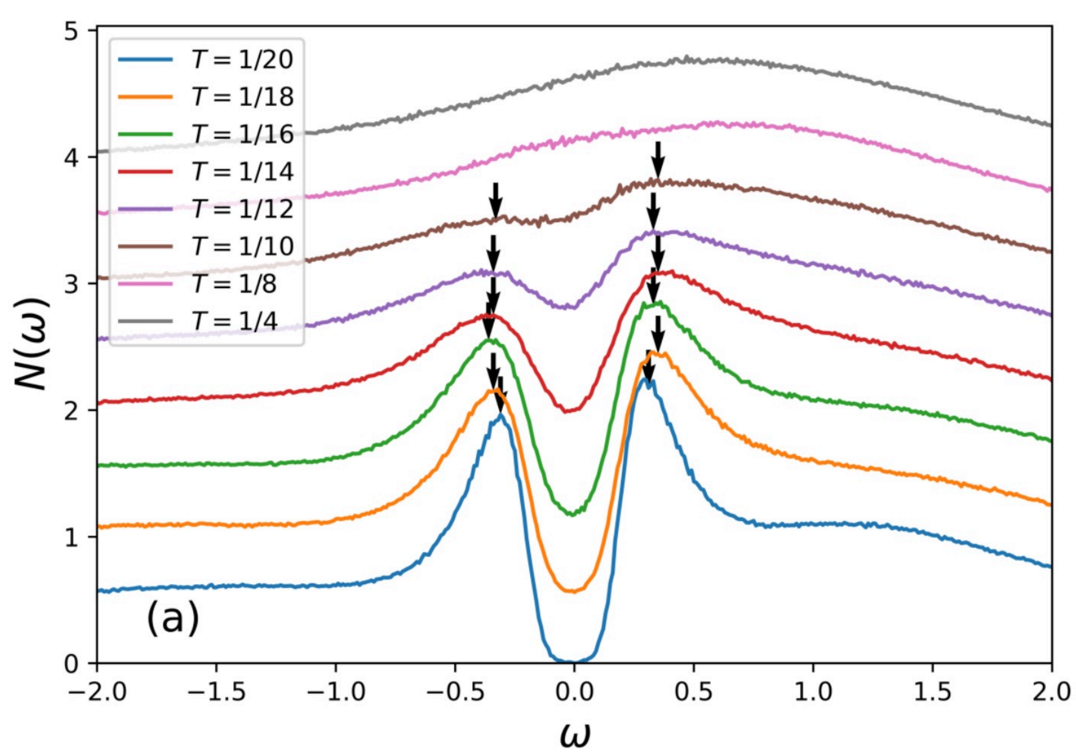
Weilun Jiang,[1,2] Yuzhi Liu,[1,2] Avraham Klein,[3] Yuxuan Wang,[4] Kai Sun,[5] Andrey V. Chubukov,[6] and Zi Yang Meng[7,1,*]

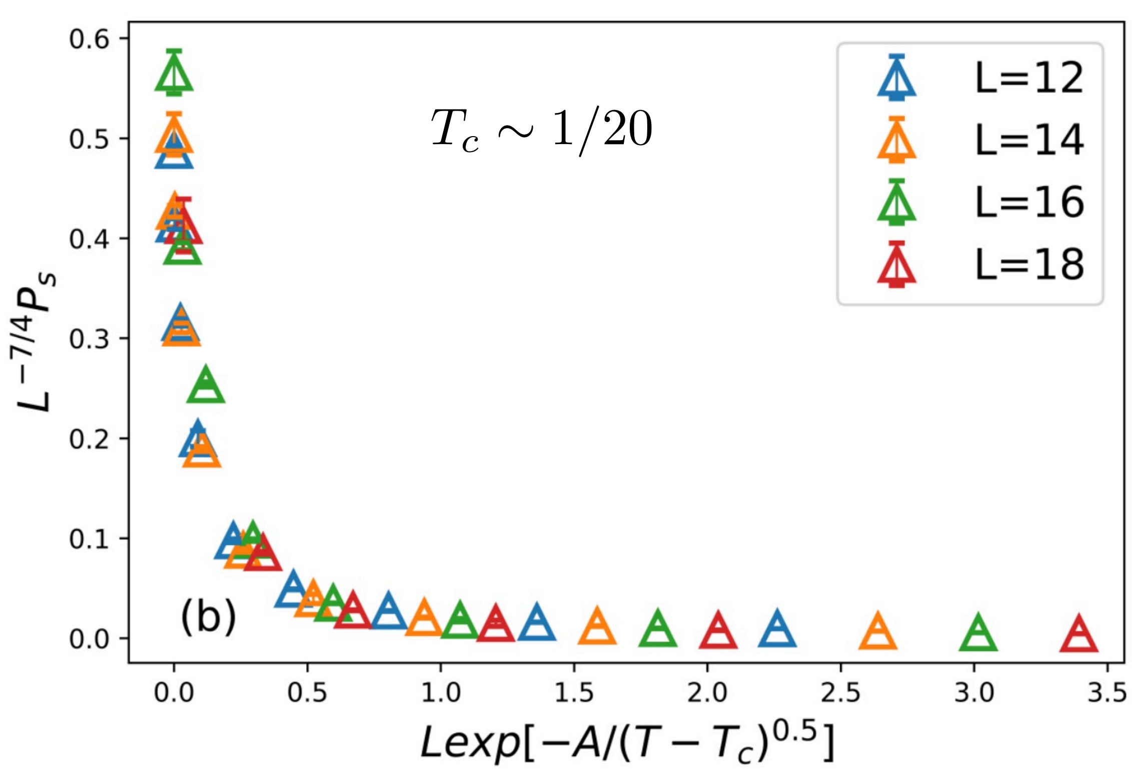
Nat. Comm. 13, 2655 (2022)




- **Pseudogap filling, not BCS**
- **DOS progressively depleted upon lowering T**



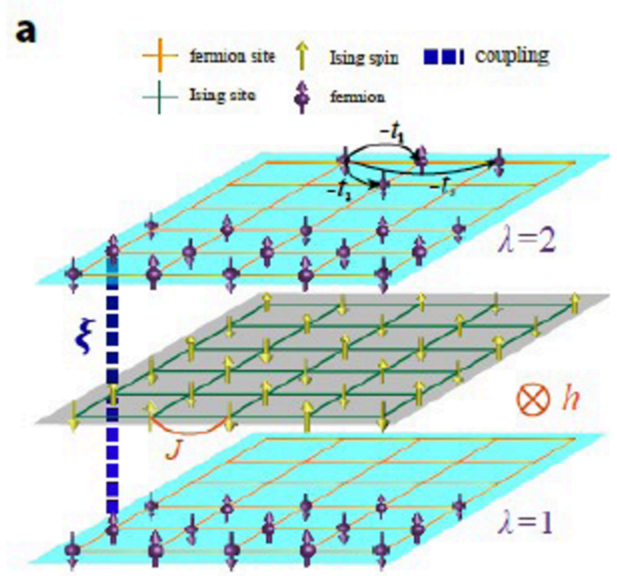



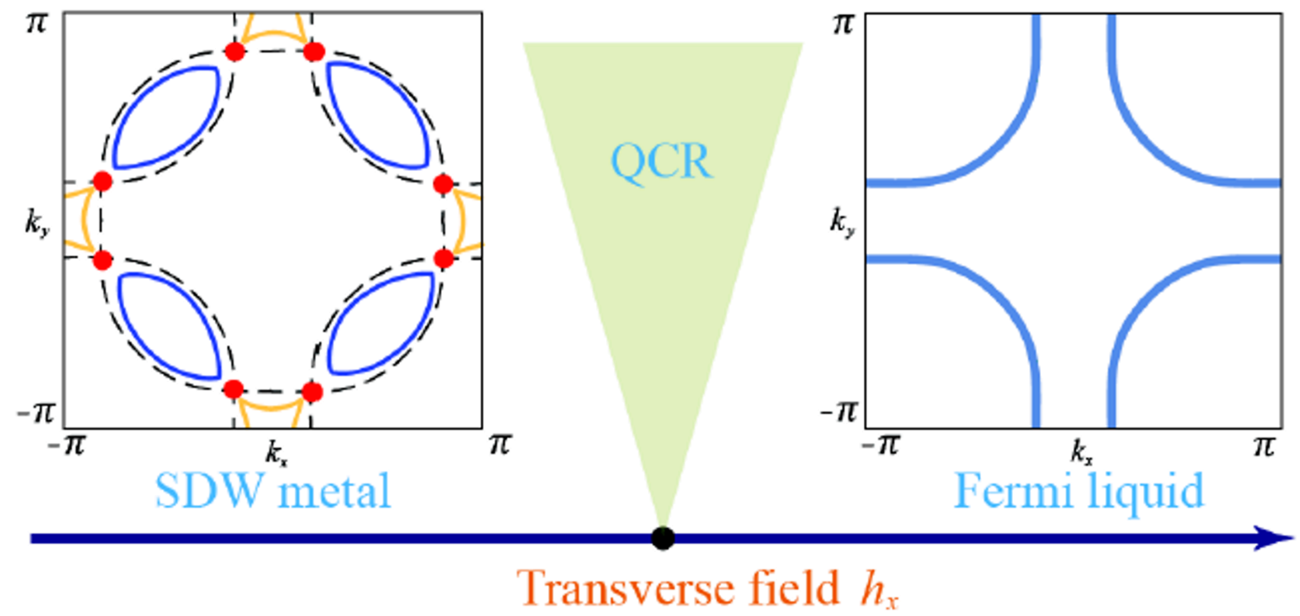
# Itinerant quantum critical point with fermion pockets and hotspots

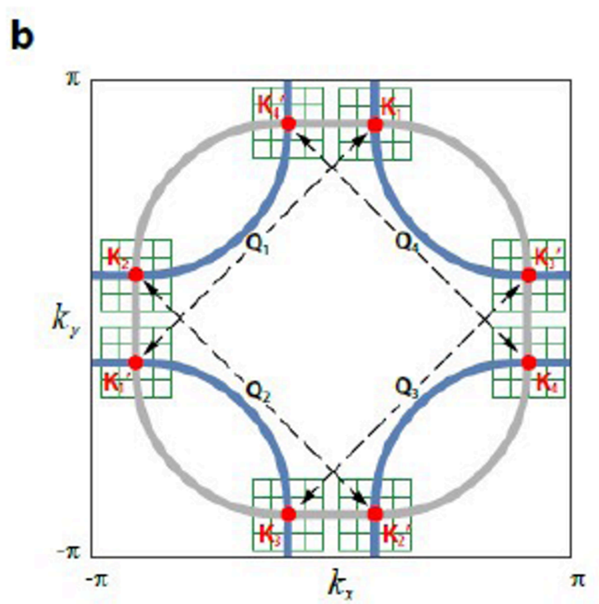
Zi Hong Liu[a,b], Gaopei Pan[a,b], Xiao Yan Xu[c], Kai Sun[d], and Zi Yang Meng[e,a,f,g,h,1]

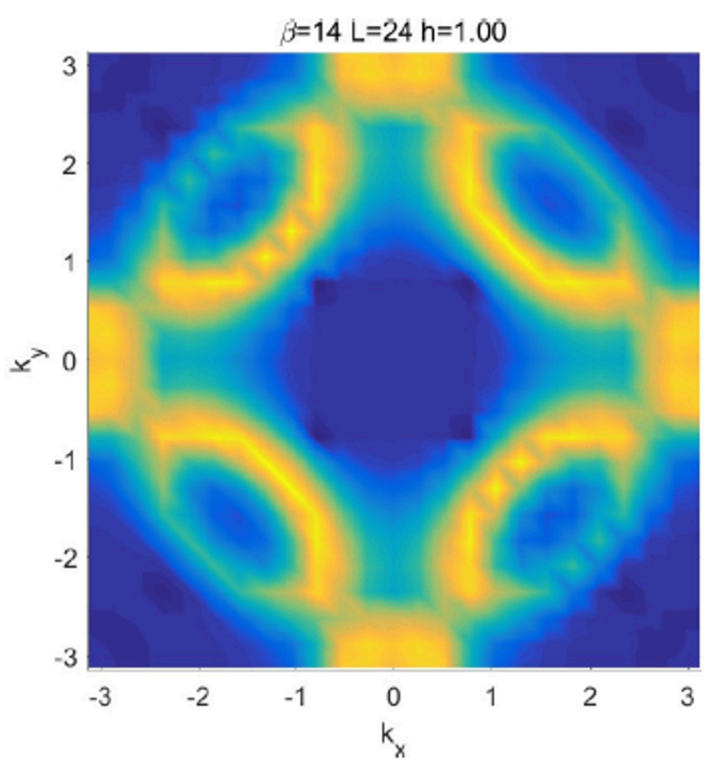
PNAS 116, 16760 (2019)

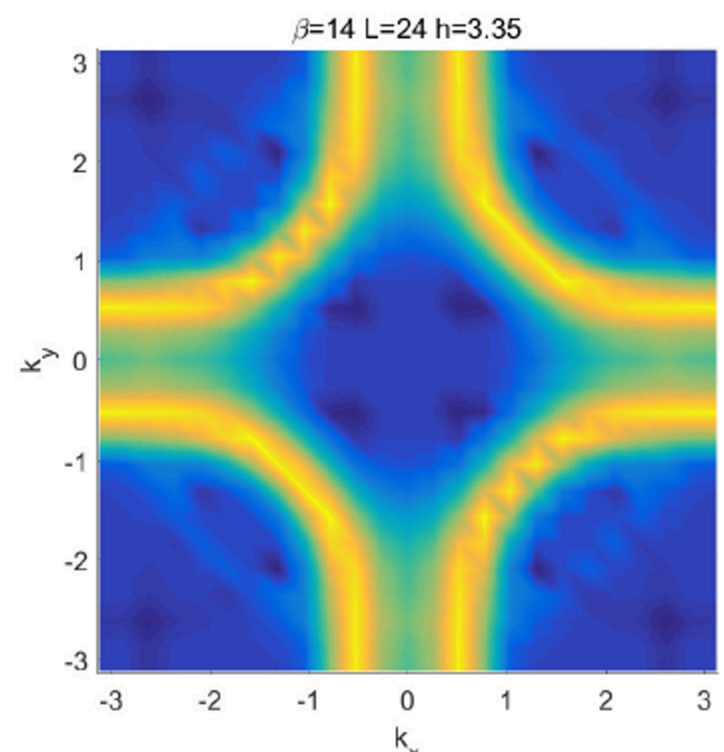
nFL and critical scaling $\chi(T,h,\mathbf{q},\omega_n) = \dfrac{1}{c_t T^{a_t}(c_q|\mathbf{q}|^2 + c_\omega \omega)^{1-\eta} + c'_\omega \omega^2}$ $\eta \sim \dfrac{1}{8}$



# FM / AFM quantum critical scaling with QMC

PRX 7, 031058 (2017)
npj Quantum Materials 5, 65 (2020)

- Ferromagnetic/nematic QCP $\text{zero } \mathbf{Q}$

$$\chi(T,h,\mathbf{q},\omega_n) \propto \frac{1}{c_t T^{a_t} + c_h|h-h_c|^\gamma + (c_q|\mathbf{q}|^2 + c_\omega \omega^2)^{a_q/2} + \Delta(\mathbf{q},\omega)}$$

$$a_q = 2-\eta \text{ with } \eta = 0.15(3)$$

- Antiferromagnetic QCP $\text{finite } \mathbf{Q}$
  - Triangle lattice $3\mathbf{Q}_{\mathrm{AF}} = \Gamma$

PRB 98, 045116 (2018)

$$\chi(T,h,\mathbf{q},\omega_n) \propto \frac{1}{(c_t T + c_t' T^2) + c_h|h-h_c|^\gamma + c_q|\mathbf{q}|^2 + (c_\omega \omega + c_\omega' \omega^2)}$$

  - Square lattice $2\mathbf{Q}_{\mathrm{AF}} = \Gamma$

PNAS 116, 16760 (2019)

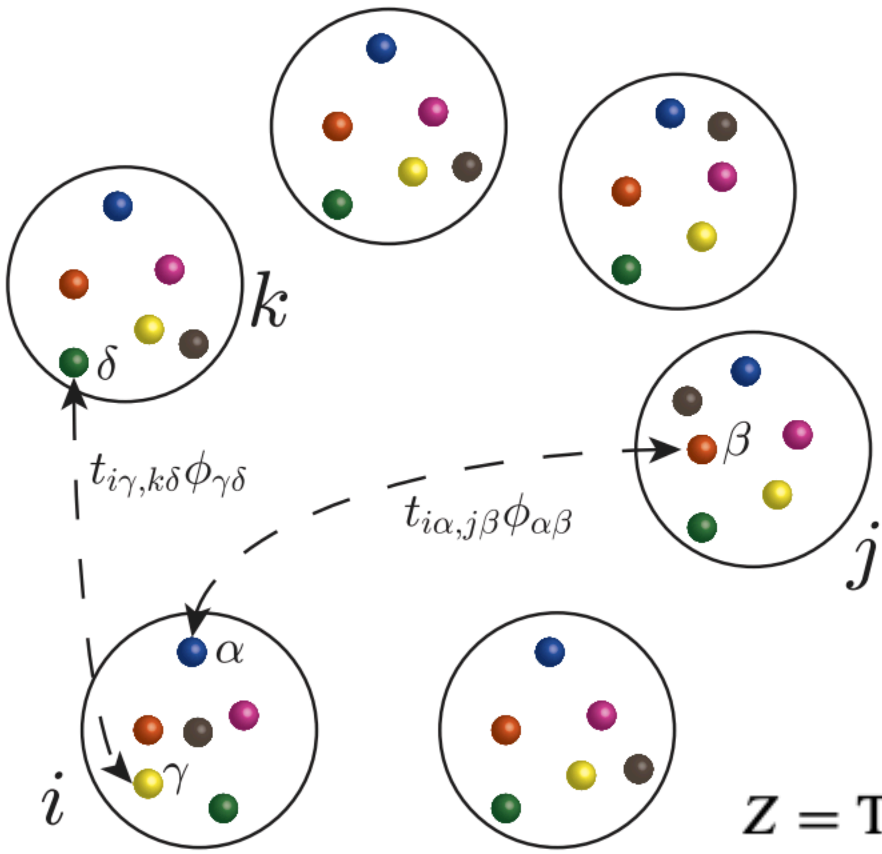
$$\chi(T,h,\mathbf{q},\omega_n) \propto \frac{1}{c_t T^{a_t} + c_h|h-h_c|^\gamma + (c_q|\mathbf{q}|^2 + c_\omega \omega)^{1-\eta} + c_\omega' \omega^2}$$

$$\eta = \frac{2}{N_{hs}} = 0.125 \text{ (with } N_{hs} = 16)$$

PHYSICAL REVIEW RESEARCH 3, 013250 (2021)

**Yukawa-SYK model and self-tuned quantum criticality**

Gaopei Pan[1,2] Wei Wang[1,2] Andrew Davis[3] Yuxuan Wang[3,*] and Zi Yang Meng[1,4,†]




$$H = \sum_{i,j=1}^{M}\sum_{\alpha,\beta=1}^{N}\sum_{m,n}^{\uparrow,\downarrow}\left(\frac{i}{\sqrt{MN}}t_{i\alpha,j\beta}\phi_{\alpha\beta}c^{\dagger}_{i\alpha;m}\sigma^{z}_{m,n}c_{j\beta;n}\right) + \sum_{\alpha,\beta=1}^{N}\left(\frac{1}{2}\pi^2_{\alpha\beta} + \frac{m_0^2}{2}\phi^2_{\alpha\beta}\right)$$

$M$ quantum dots

$N$ fermion flavors

$\phi_{\alpha,\beta}$ $N^2$ matrix bosons

$\phi_{\alpha,\beta} = -\phi_{\beta,\alpha}$ hermiticity

random coupling

$\langle t_{i,\alpha;j,\beta}\rangle = 0$

$\langle t_{i,\alpha;j,\beta}\, t_{k,\gamma;l,\delta}\rangle = (\delta_{i,k}\delta_{\alpha,\gamma}\delta_{j,l}\delta_{\beta,\delta} + \delta_{i,l}\delta_{\alpha,\delta}\delta_{j,k}\delta_{\beta,\gamma})\ \omega_0^3$

$\omega_0 = 1$ as the unit

$$Z = \mathrm{Tr}\{e^{-\beta\hat{H}}\} = \mathrm{Tr}\{(e^{-\Delta\tau\hat{H}})^{L_\tau}\} = \int\left(\prod_{\alpha\beta}d\phi_{\alpha\beta}\right)\mathrm{Tr}_{\mathbf{F}}\,\langle\phi_{11}\cdots\phi_{NN}|(e^{-\Delta\tau\hat{H}})^{L_\tau}|\phi_{11}\cdots\phi_{NN}\rangle$$

$$= \int\prod_{l=1}^{L_\tau}d\vec{\Phi}_l\ \underbrace{C^{L_\tau}\left(\prod_{l=1}^{L_\tau}\prod_{\alpha,\beta=1}^{N}e^{-\Delta\tau\frac{m_0^2}{2}\phi^2_{\alpha\beta,l}}\right)\left(\prod_{\langle l,l'\rangle}\prod_{\alpha,\beta=1}^{N}e^{-\frac{(\phi_{\alpha\beta,l}-\phi_{\alpha\beta,l'})^2}{2\Delta\tau}}\right)}_{\mathcal{W}_b}\ \underbrace{\mathrm{Tr}_{\mathbf{F}}\{e^{-\Delta\tau\hat{H}_{fb}(\vec{\Phi}_{L_\tau})}\cdots e^{-\Delta\tau\hat{H}_{fb}(\vec{\Phi}_1)}\}}_{\mathcal{W}_{fb}}$$

$\frac{\omega_0}{m_0} < 1$ weakly coupled regime

$\frac{\omega_0}{m_0} > 1$ strongly coupled regime

$$\det[I + \mathbf{B}^{L_\tau}\mathbf{B}^{L_\tau-1}\cdots\mathbf{B}^{l}\cdots\mathbf{B}^2\mathbf{B}^1]$$

$\{\vec{\Phi}\}:\ N\times N\times L_\tau$

$B^l = e^{-\Delta\tau V(\Phi_l)}$

$V(\vec{\Phi}_l) = \frac{i}{\sqrt{MN}}\sigma^z_{2\times 2}\otimes(t_{i\alpha,j\beta}\phi_{\alpha\beta,l})_{MN\times MN}$

$\sim 20$ realisations

Time reversal symm. $\det[\mathbf{1}+B^{\uparrow}(\beta,0)]\det[\mathbf{1}+B^{\downarrow}(\beta,0)] = |\det[\mathbf{1}+B^{\uparrow}(\beta,0)]|^2$

PHYSICAL REVIEW RESEARCH **3**, 013250 (2021)

**Yukawa-SYK model and self-tuned quantum criticality**

Gaopei Pan[1,2] Wei Wang[1,2] Andrew Davis[3] Yuxuan Wang[3,*] and Zi Yang Meng[1,4,†]

$$N \to \infty,\ M \to \infty, T = 0$$

$$m_0 \sim \omega_0;\ \ \omega, \Omega \ll \omega_0$$

$$\Sigma(\omega) = -G_f(\omega)^{-1} = ic\,\mathbf{sgn}(\omega)|\omega|^x\,\omega_0^{1-x}$$

$$0 < x < 1/2$$

$$\Pi(\Omega) = G_b(\Omega)^{-1} = -m_0 + c^{-2}\alpha(x)|\Omega|^{1-2x}\omega_0^{1+2x}$$

$$m_0^2 - \Pi(\Omega = 0) = 0$$

boson is always critical
renormalise mass to zero

$$\frac{4M}{N} = \frac{1/x - 2}{1 + \sec(\pi x)} \qquad \alpha(x) = -\frac{\Gamma^2(-x)}{4\pi\Gamma(-2x)}$$

$$M = N, x \approx 0.098 \qquad 4M = N, x \approx 0.231$$

$$\Sigma(\tau,\tau') \propto |\tau - \tau'|^{-(1+x)}\mathbf{sgn}(\tau - \tau')$$

$$\Pi(\tau,\tau') \propto |\tau - \tau'|^{-(2-2x)}$$

$$G_b(\tau,\tau') \propto |\tau - \tau'|^{-2x}$$

$$G_f(\tau,\tau') \propto |\tau - \tau'|^{x-1}\mathbf{sgn}(\tau - \tau')$$

$$\omega_F = \frac{\omega_0^3}{m_0^2} \qquad T = \frac{1}{\beta} \ll \omega_F$$

low temperature and long time limit

reparametrization symm. transformation $\tau \to f(\tau) = \tan(\frac{\pi\tau}{\beta})$

$$G_f(\tau,0) \propto \left(\frac{\pi}{\beta\sin(\frac{\pi\tau}{\beta})}\right)^{1-x}$$

$$G_b(\tau,0) \propto \left(\frac{\pi}{\beta\sin(\frac{\pi\tau}{\beta})}\right)^{2x}$$

PHYSICAL REVIEW RESEARCH **3**, 013250 (2021)

**Yukawa-SYK model and self-tuned quantum criticality**

Gaopei Pan,[1,2] Wei Wang,[1,2] Andrew Davis,[3] Yuxuan Wang,[3,*] and Zi Yang Meng[1,4,†]

$N = 4M \to \infty \quad x \approx 0.23$

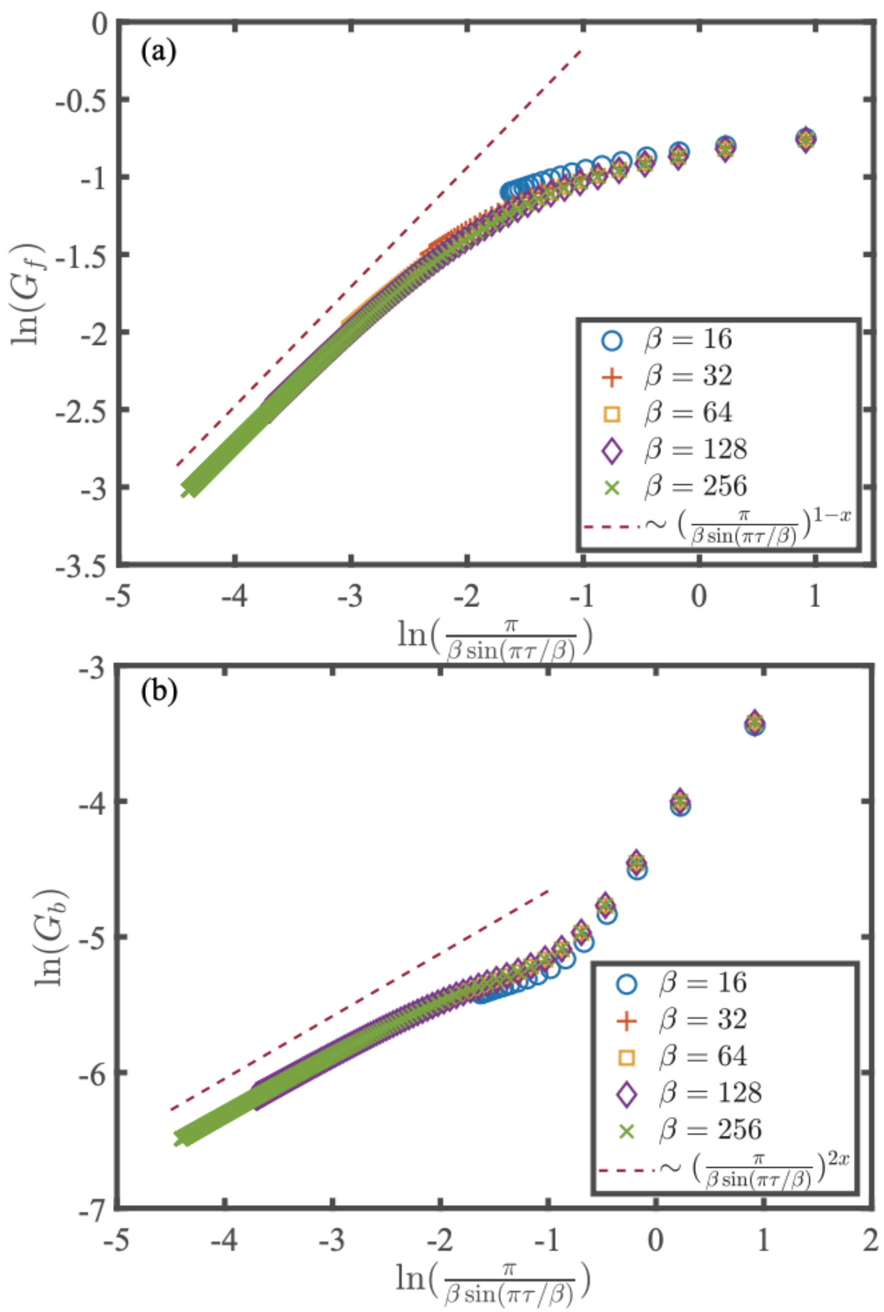
$\omega_0 = 1,\ m_0 = 2$

compact

$\omega_m, \Omega_n \in (-\frac{\pi}{\Delta\tau}, \frac{\pi}{\Delta\tau})$

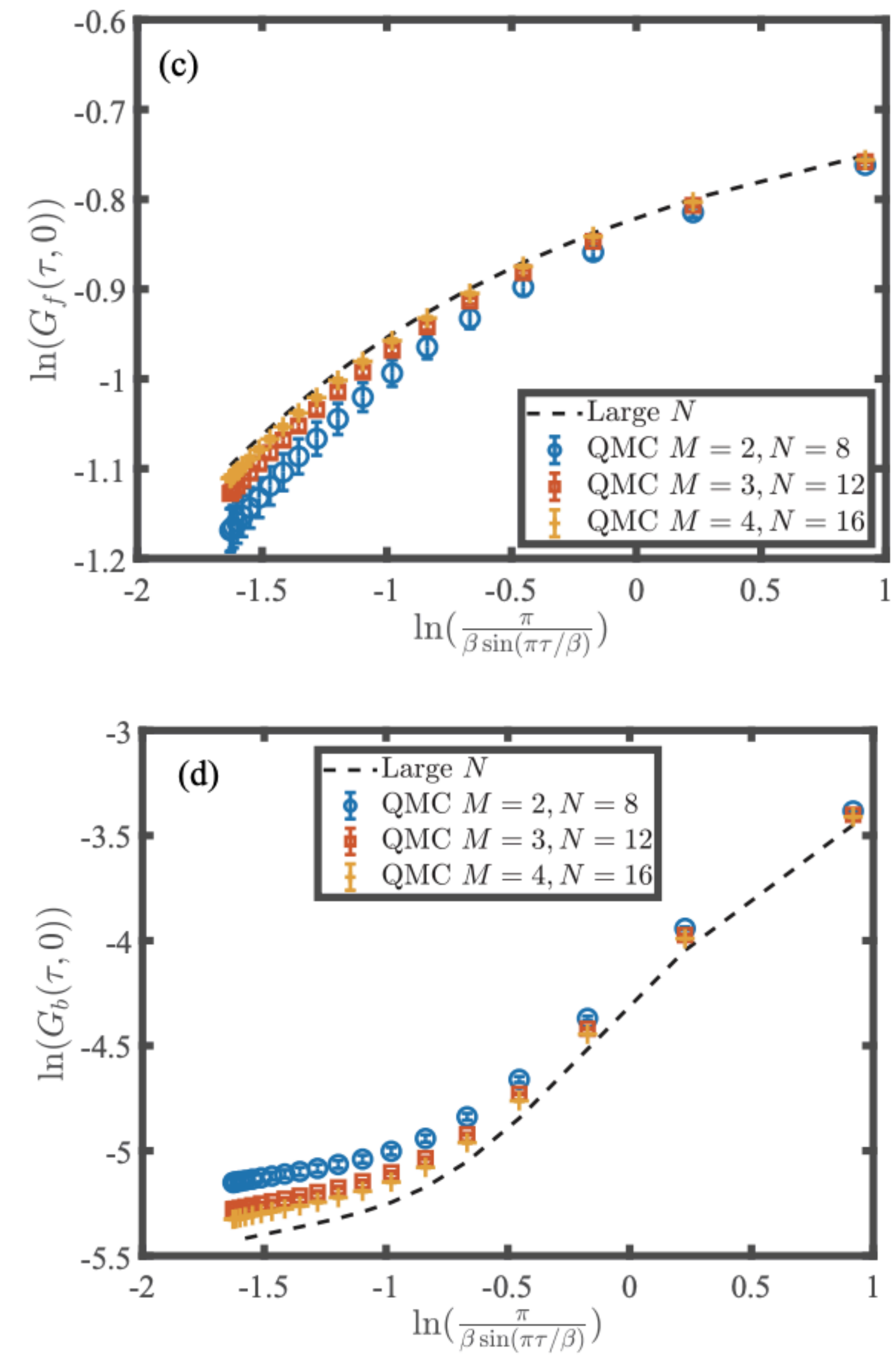
$N = 4M \quad M = 2,3,4$

$\omega_0 = 1,\ m_0 = 2,\ \beta = 16$




$$G_f(\tau,0) \propto \left(\frac{\pi}{\beta\sin(\frac{\pi\tau}{\beta})}\right)^{1-x}$$

$$G_b(\tau,0) \propto \left(\frac{\pi}{\beta\sin(\frac{\pi\tau}{\beta})}\right)^{2x}$$



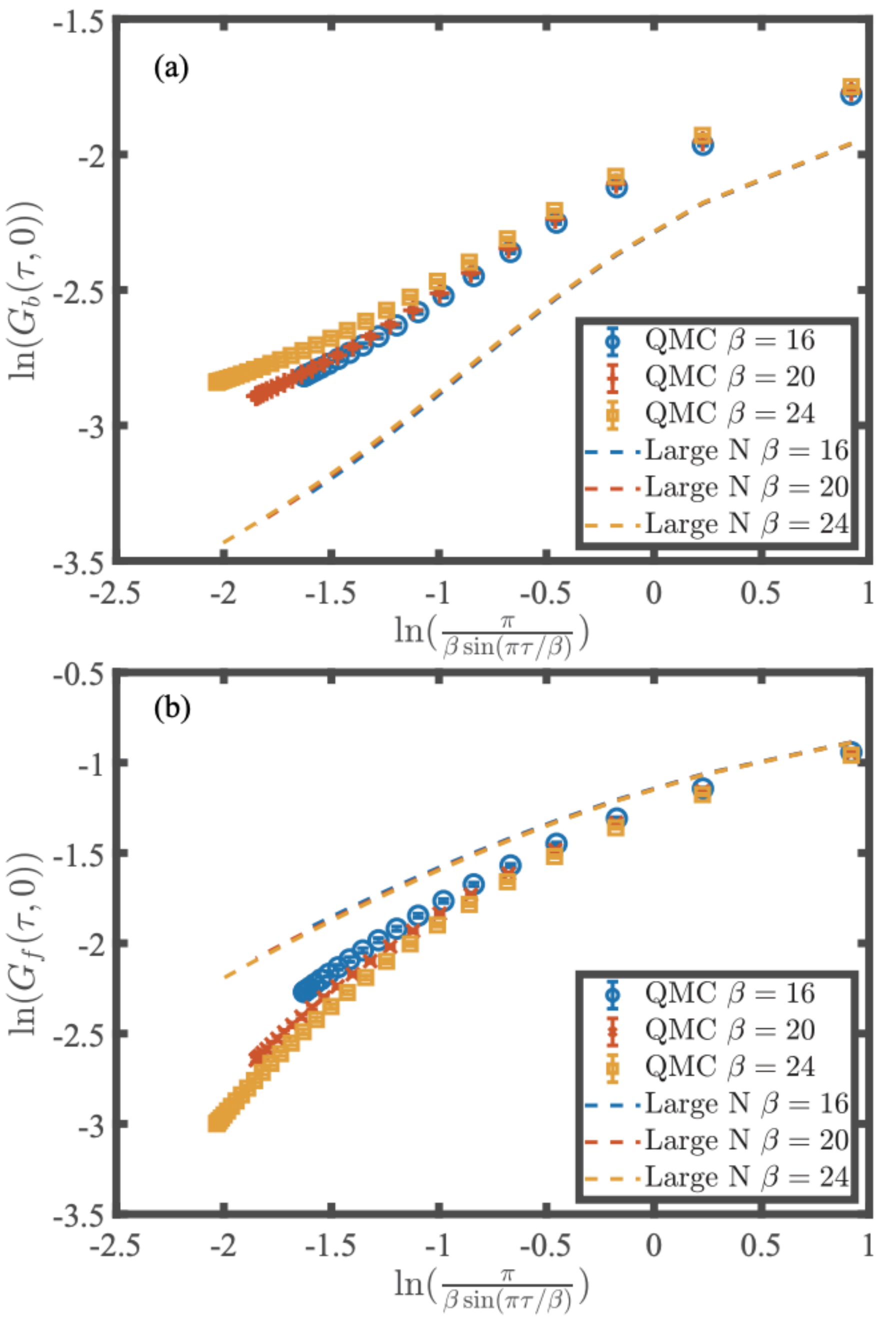
PHYSICAL REVIEW RESEARCH 3, 013250 (2021)

**Yukawa-SYK model and self-tuned quantum criticality**

Gaopei Pan[1,2] Wei Wang[1,2] Andrew Davis[3] Yuxuan Wang[3,*] and Zi Yang Meng[1,4,†]

$M = 4, N = 16$

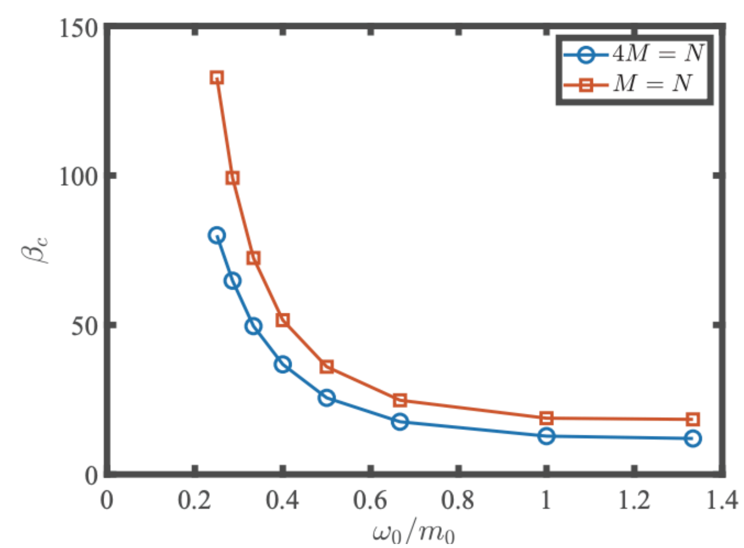
$\omega_0 = m_0 = 1$




$$\Delta \sim \sum_{i,\alpha} \langle c^\dagger_{i,\alpha,\uparrow} c^\dagger_{i,\alpha,\downarrow} \rangle$$

$$\frac{\omega_F}{\omega_0} \sim \left(\frac{\omega_0}{m_0}\right)^2 \qquad T_c \leq \omega_F$$

Instability fo the NFL

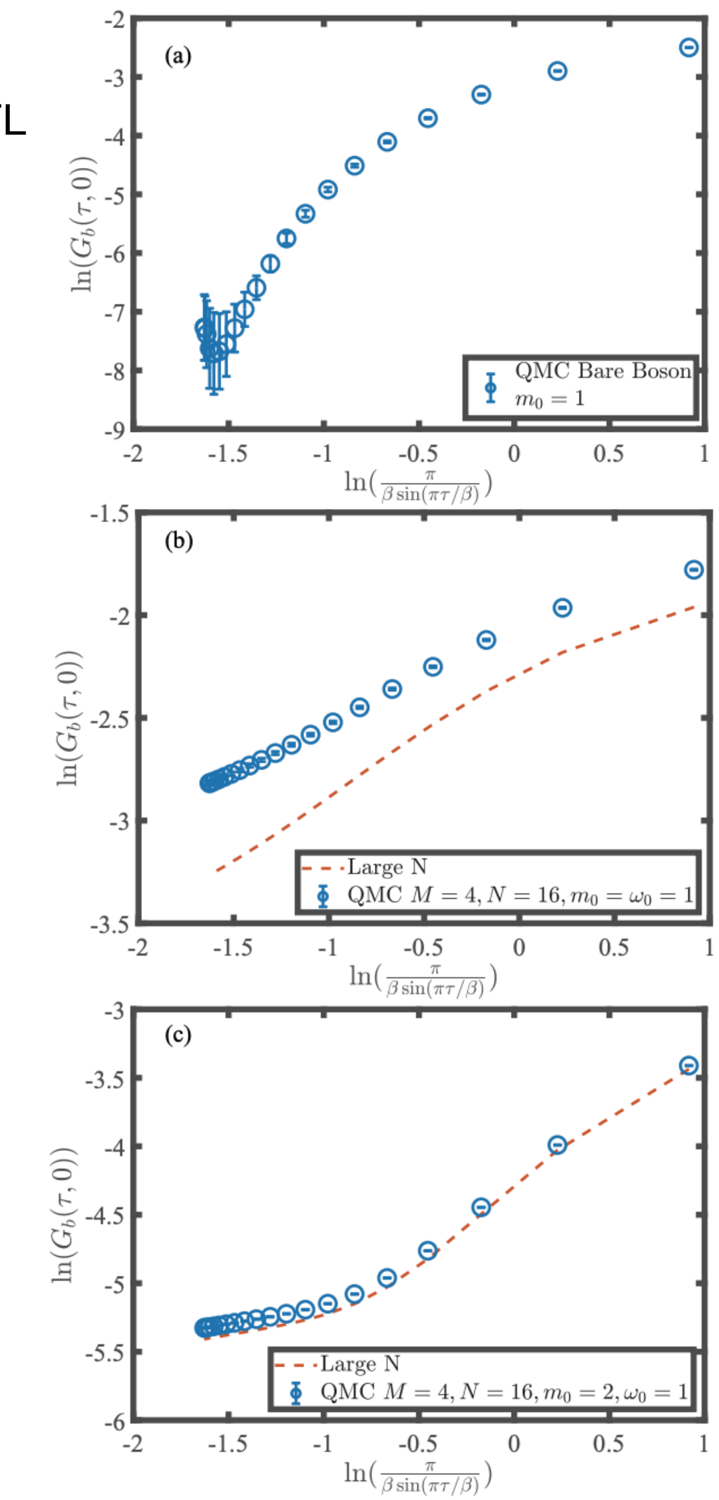


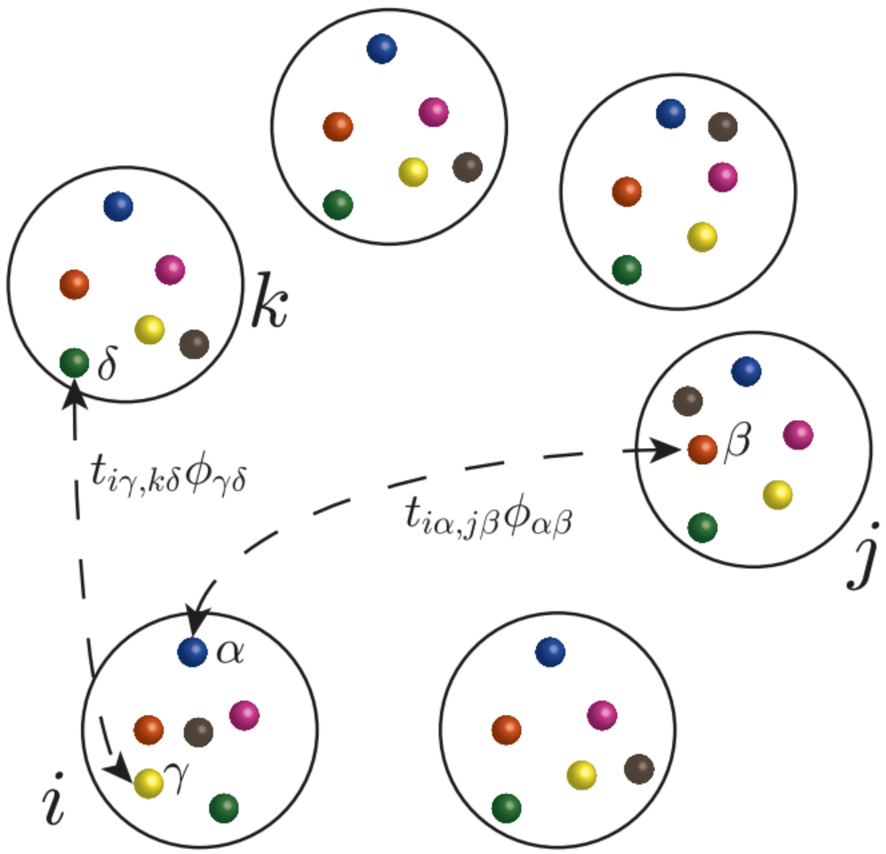

replica-off-diagonal fluctuations
G_b static

enhanced pairing fluctuations

self-tuned quantum criticality

$\beta = 16$






$$H = \sum_{i,j=1}^{M}\sum_{\alpha,\beta=1}^{N}\sum_{m,n}^{\uparrow,\downarrow}\left(\frac{i}{\sqrt{MN}}t_{i,j}\phi_{\alpha\beta}c^{\dagger}_{i\alpha;m}\sigma^{z}_{m,n}c_{j\beta;n}\right) + \sum_{\alpha,\beta=1}^{N}\left(\frac{1}{2}\pi^2_{\alpha\beta} + \frac{m_0^2}{2}\phi^2_{\alpha\beta}\right)$$

$M$ quantum dots

fermion flavors

$\phi_{\alpha,\beta}$ $N^2$ matrix bosons

$\phi_{\alpha,\beta} = -\phi_{\beta,\alpha}$ hermiticity

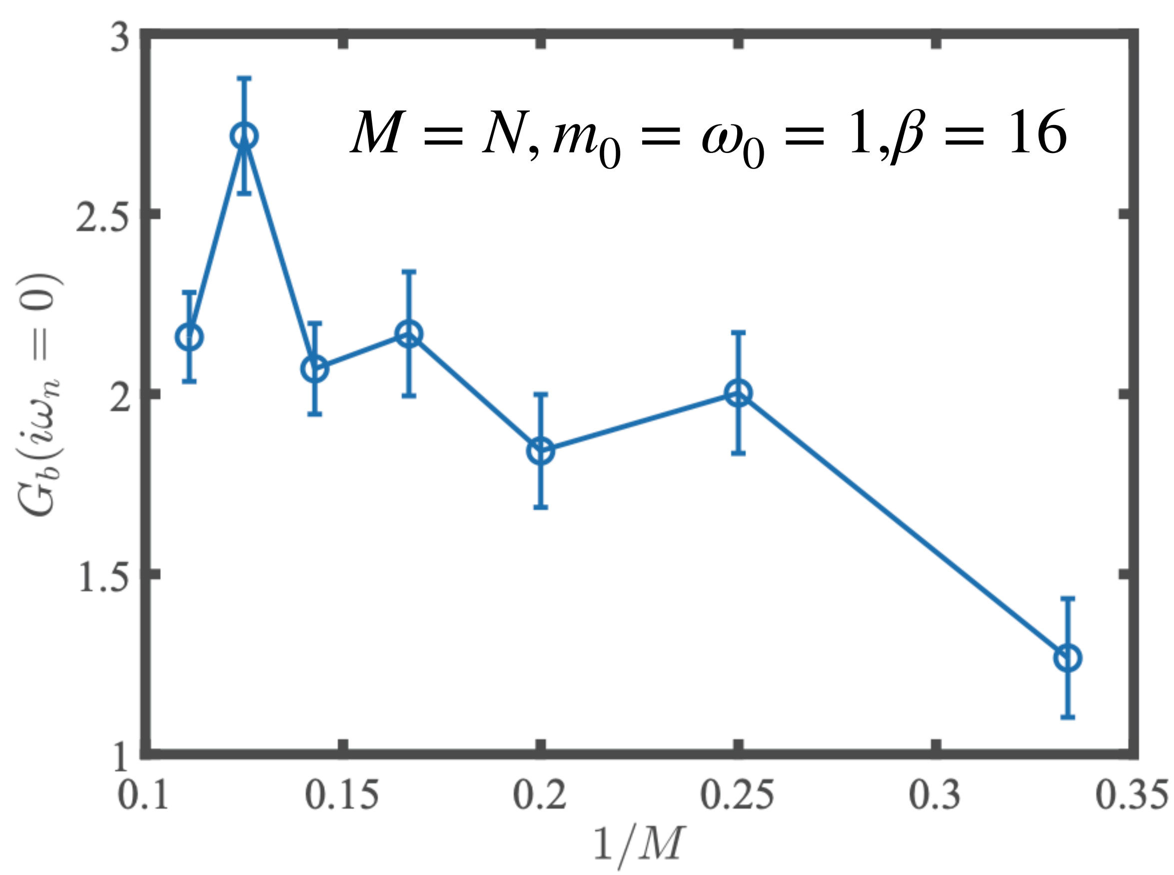
random coupling of lower rank

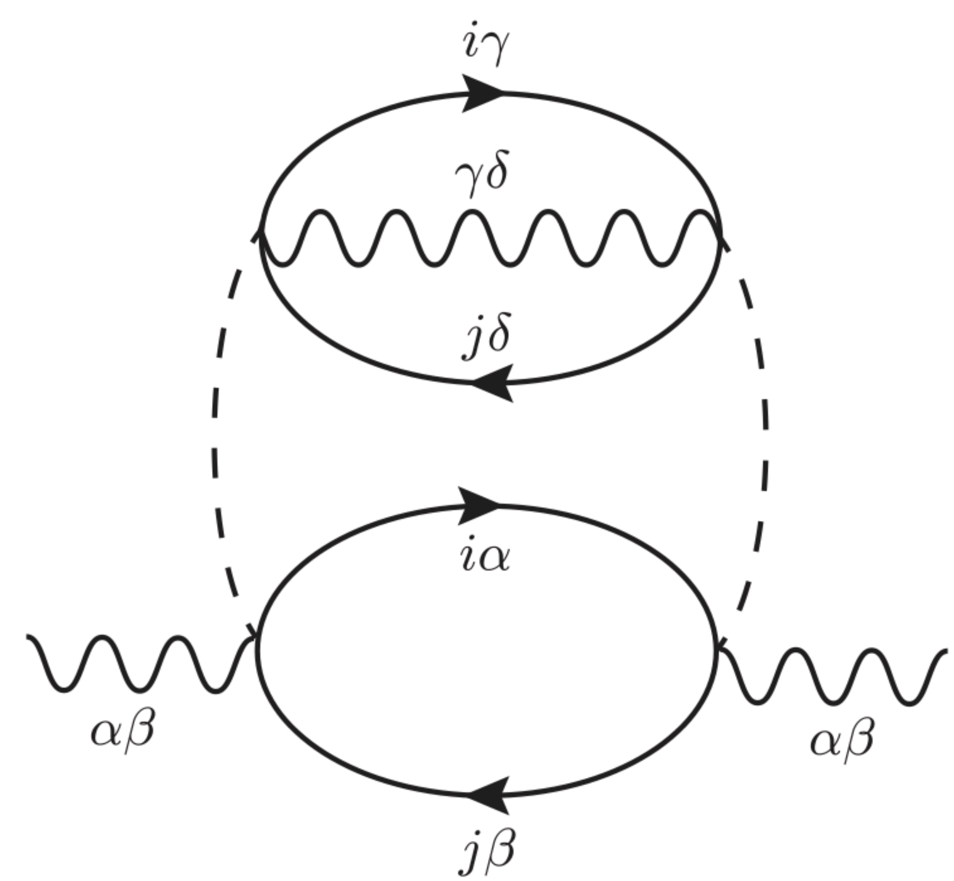
$$\langle t_{i,j}\rangle = 0$$

$$\langle t_{i,j}\ t_{k,l}\rangle = (\delta_{i,k}\delta_{j,l} + \delta_{i,l}\delta_{j,k})\ \omega_0^3$$

$\omega_0 = 1$ as the unit

Edwards-Anderson order parameter of the spin-glass phase




Replica-off-diagonal process that not suppressed by 1/MN;
Two fermion loops carry different replicon indices

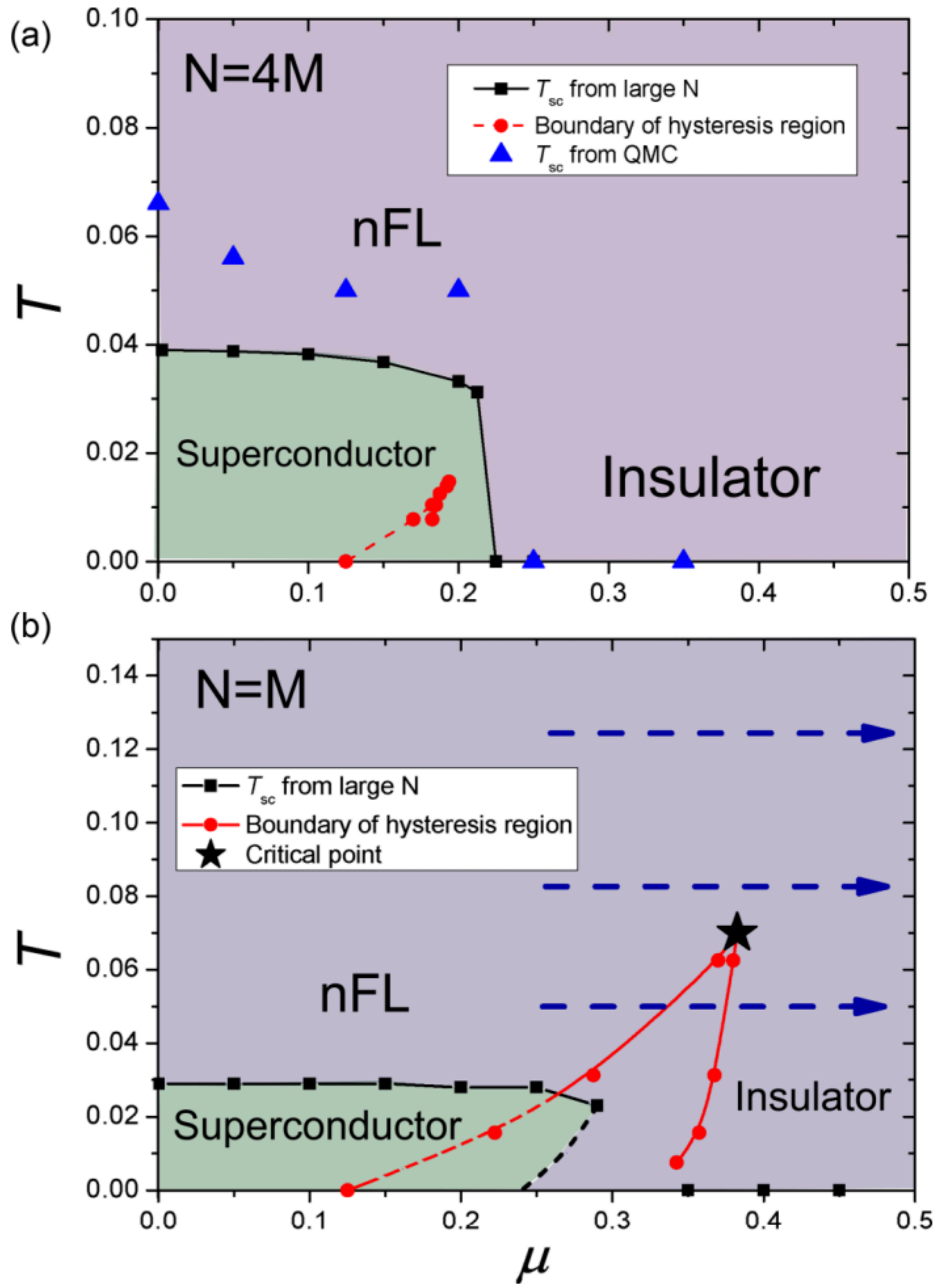
PHYSICAL REVIEW B **103**, 195108 (2021)

# Phase diagram of the spin-$\frac{1}{2}$ Yukawa–Sachdev-Ye-Kitaev model: Non-Fermi liquid, insulator, and superconductor

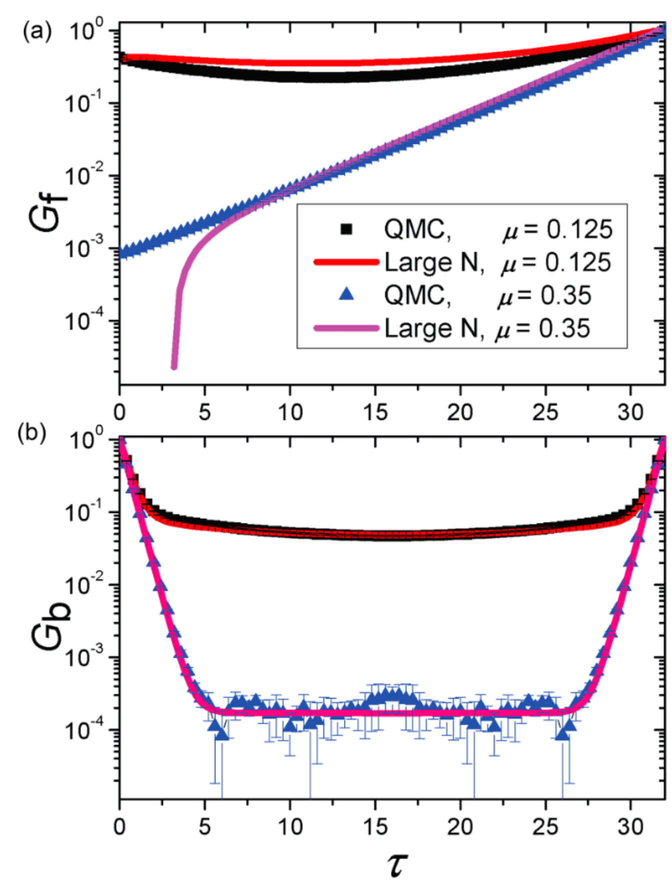
Wei Wang [1,2,*] Andrew Davis [3,*] Gaopei Pan [1,2] Yuxuan Wang [3,†] and Zi Yang Meng [4,1,‡]

$$H=\sum_{i,j=1}^{M}\sum_{\alpha,\beta=1}^{N}\sum_{m,n}^{\uparrow,\downarrow}\left(\frac{i}{\sqrt{MN}}t_{i\alpha,j\beta}\phi_{\alpha\beta}c^{\dagger}_{i\alpha;m}\sigma^{z}_{m,n}c_{j\beta;n}\right)+\sum_{\alpha,\beta=1}^{N}\left(\frac{1}{2}\pi^{2}_{\alpha\beta}+\frac{m_0^2}{2}\phi^{2}_{\alpha\beta}\right)-\mu\sum_{i=1}^{M}\sum_{\alpha=1}^{N}\sum_{m=\uparrow,\downarrow}c^{\dagger}_{i\alpha m}c_{i\alpha m}$$

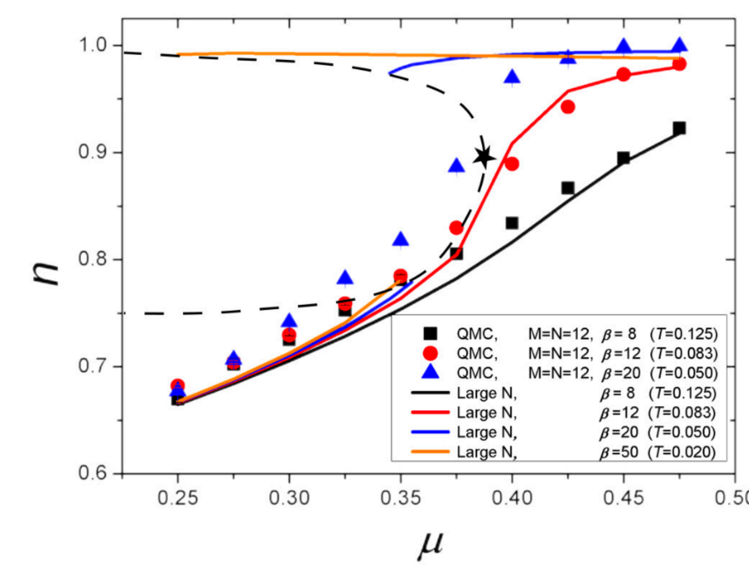
$\omega_0=1,\ m_0=2$




$M=4, N=16, \beta=32$




$M=N=12$



PHYSICAL REVIEW B **103**, 195108 (2021)

# Phase diagram of the spin-$\frac{1}{2}$ Yukawa–Sachdev-Ye-Kitaev model: Non-Fermi liquid, insulator, and superconductor

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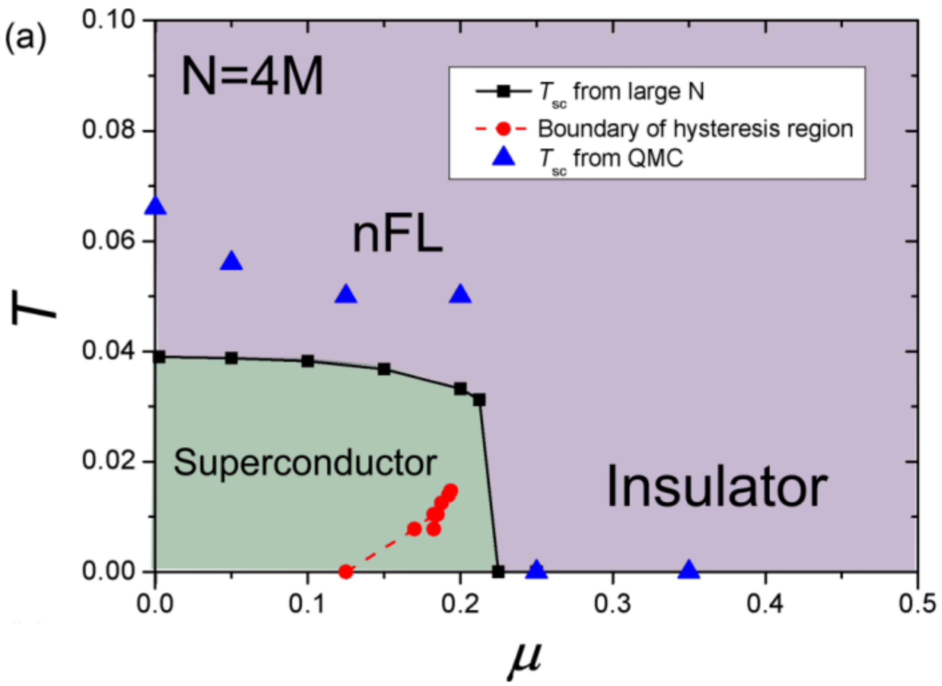
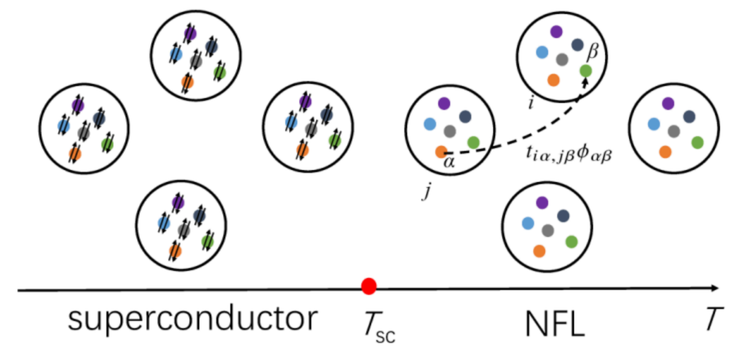



$$P_s = \int_0^\beta d\tau \langle \Delta(\tau)\Delta^\dagger(0)\rangle \qquad \Delta^\dagger = \frac{1}{\sqrt{MN}} \sum_{i=1}^{M}\sum_{\alpha=1}^{N} c^\dagger_{i,\alpha,\uparrow} c^\dagger_{i,\alpha,\downarrow}$$

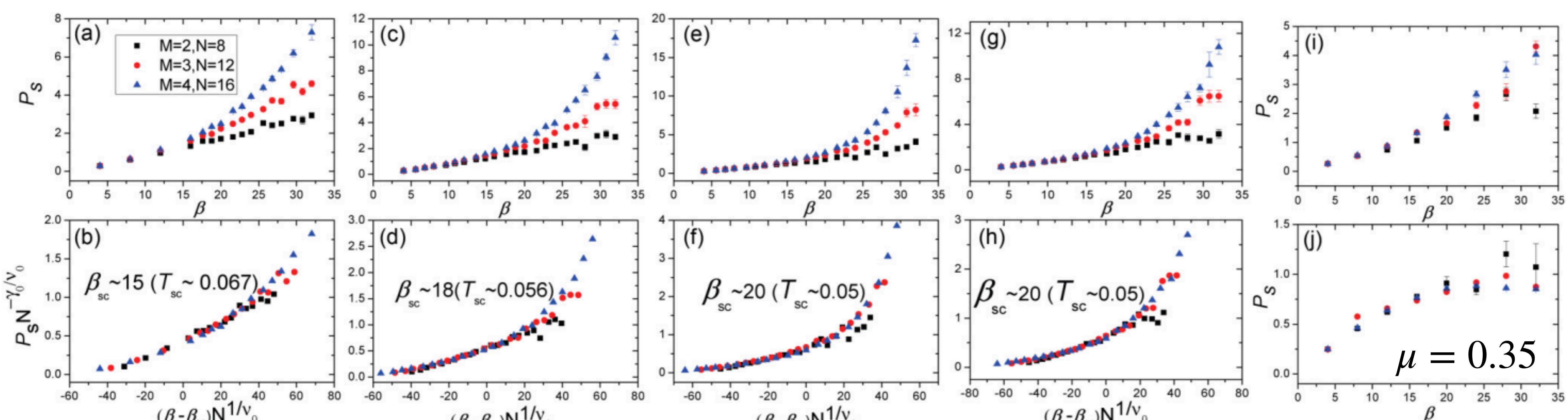
mean-field exponents

$$P_s \sim N^{\gamma/\nu} f(N^{1/\nu}(T - T_{sc}))$$

$$\gamma = 1,\ \nu = 2$$

$$N = 4M, \omega_0 = 1, m_0 = 2$$

$\mu = 0$ $\mu = 0.05$ $\mu = 0.125$ $\mu = 0.2$ $\mu = 0.25$



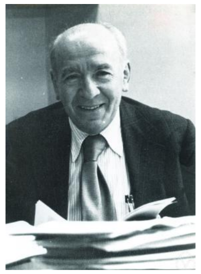
# Yukawa-SYK model & Quantum Entanglement: From Many-Body Computation Perspective

ZI YANG MENG
孟子楊
https://quantummc.xyz/

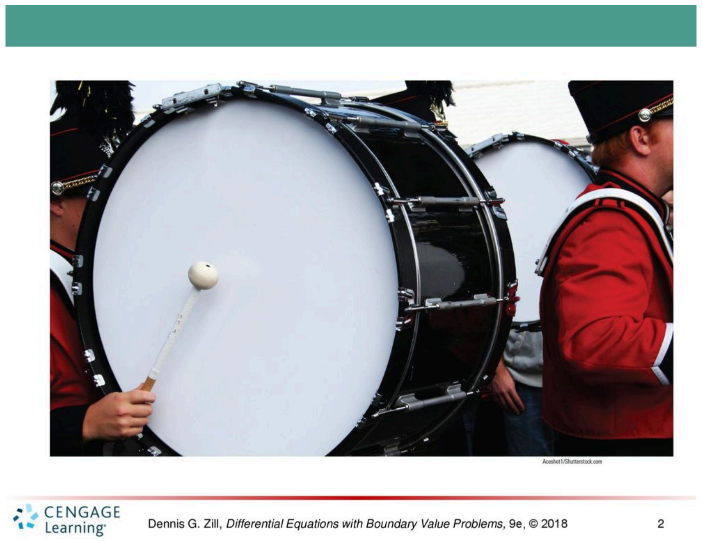
# CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

To George Eugene Uhlenbeck on the occasion of his sixty-fifth birthday

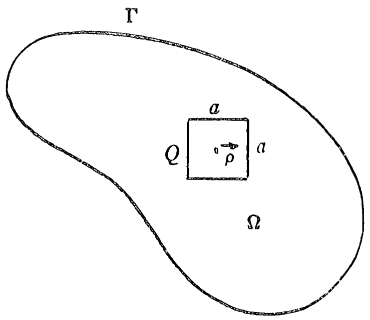


Mark Kac, Polish American mathematician 1914 - 1984




Eigenvalues of Dirichlet problem for Laplacian

Am. Math. Mon. 73, 1 (1966)




$$\tfrac{1}{2}\nabla^2 U + \lambda U = 0 \ \text{ in } \ \Omega,$$
$$U = 0 \ \text{ on } \ \Gamma.$$

Volume — Length of circumference — Number of holes

$$\sum_{n=1}^{\infty} e^{-\lambda_n t} \sim \frac{|\Omega|}{2\pi t} - \frac{L}{4}\frac{1}{\sqrt{2\pi t}} + (1-r)\,\frac{1}{6}$$

## FINITE-SIZE DEPENDENCE OF THE FREE ENERGY IN TWO-DIMENSIONAL CRITICAL SYSTEMS

Nucl. Phys. B 300, 377 (1988)

John L. CARDY and Ingo PESCHEL*

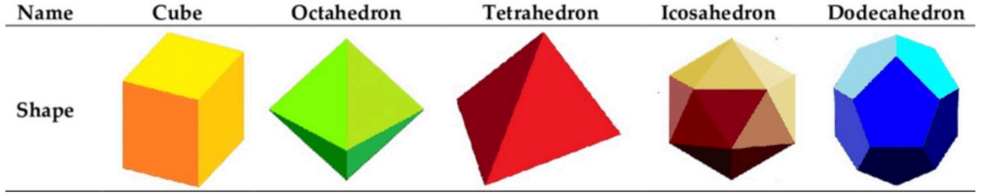
Conformal anomaly number (central charge)

$$F = f_b|A| + f_s L - \frac{1}{6}c\chi \ln L + O(1)$$

Euler characteristic

Platonic solids: homeomorphic to sphere

$$\chi = V - E + F = 2$$




sphere / polyhedron

$\chi = 2$

torus / cylinder / annuls

$\chi = 2 - 2g = 0$

Projective plane / disc

$\chi = 1$

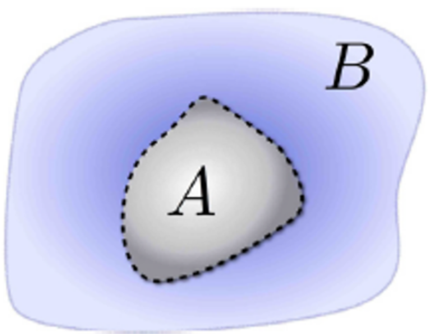
Klein bottle / moebius

$\chi = 0$

**Entanglement Entropy of 2D Conformal Quantum Critical Points: Hearing the Shape of a Quantum Drum**

Eduardo Fradkin[1] and Joel E. Moore[2,3]

Phys. Rev. Lett. 97, 050404 (2006)

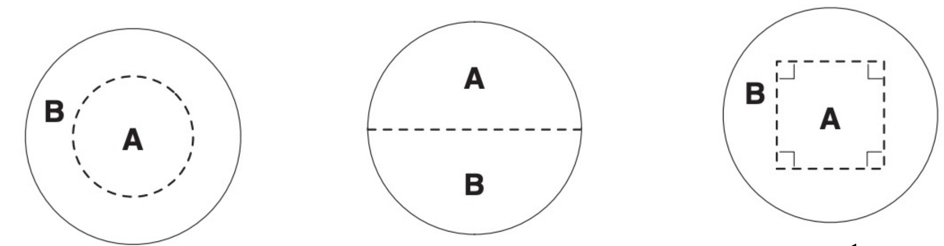



$$S = F_A + F_B - F_{A\cup B}$$

central charge

$$S = 2f_s\, L - \frac{1}{6} c\, (\chi_A + \chi_B - \chi_{A\cup B}) \ln(L) + O(1)$$

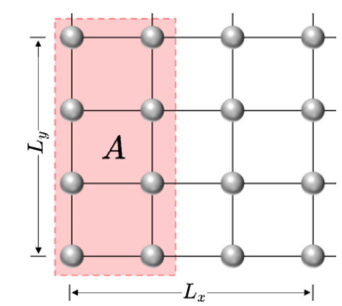
Geometric properties of the partition



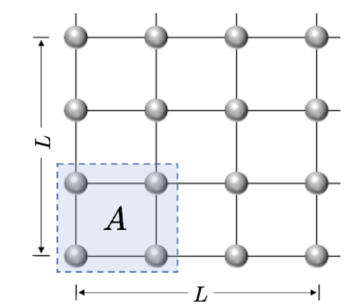

$S_{\ln} = 0$ $\quad S_{\ln} = -\frac{1}{4} c \ln(L)$ $\quad S_{\ln} = -\frac{1}{9} c \ln(L)$

$$S_A(l) = al - s\ln(\frac{l}{\epsilon}) - \gamma + O(1/l)$$

Smooth boundary, no log




Corner, log

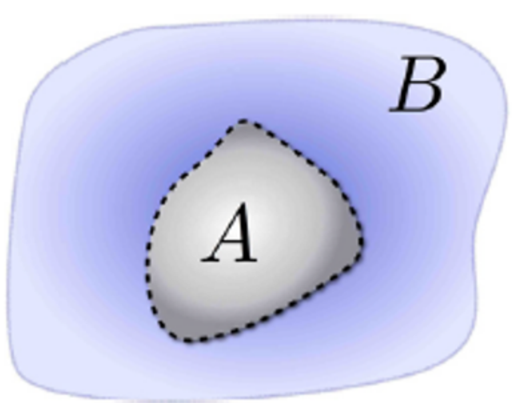



| | | | |
|---|---|---|---|
| d=1 CFT | $S \sim c\ln(l)$ | Heisenberg chain, Luttinger liquid | DMRG |
| d=2 QCP | $S \sim al - s_C \ln(l) - \gamma$ | Wilson-Fisher O(N), GNY | QMC |
| SSB | $S \sim al - (s_G + s_C)\ln(l) - \gamma$ | Antiferromagnet, Superfluid | QMC |
| Topological order | $S \sim al - \gamma_{top}$ | Z2 top ord, Kitaev QSL | Toy model, QMC |
| Fermi surface | $S \sim l\ln(l) + al - \cdots$ | free fermion, interaction ? | |

# Entanglement entropy and quantum field theory

J. Stat. Mech. (2004) P06002

Pasquale Calabrese[1,3] and John Cardy[1,2]




$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B\, \rho$$

$$S_A = -\mathrm{Tr}_A\, \rho_A \ln(\rho_A)$$

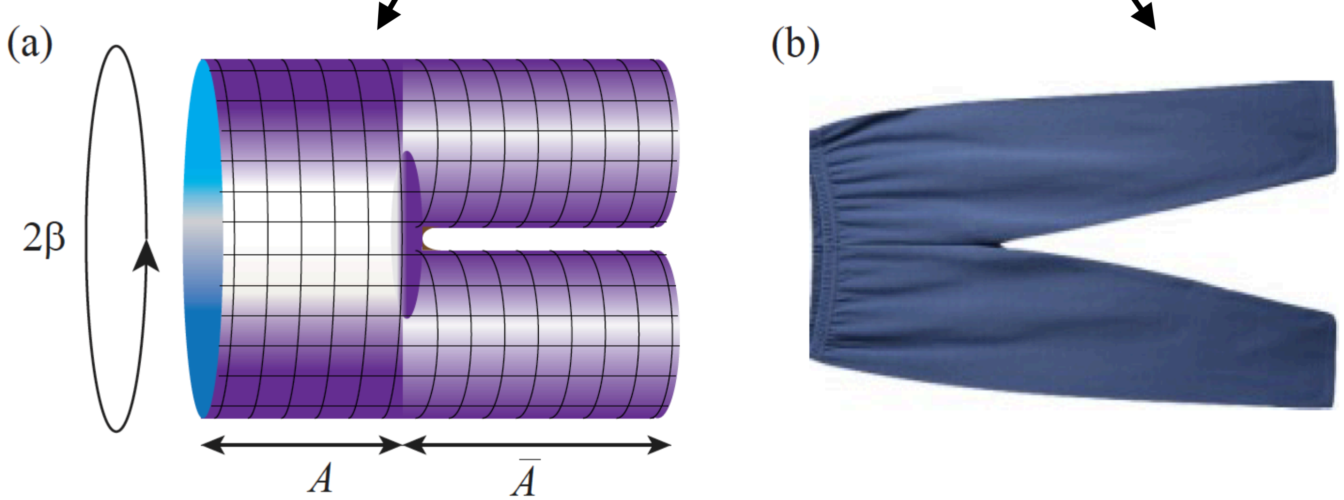
$$S_A^{(n)} = \frac{1}{1-n} \ln(\mathrm{Tr}_A(\rho_A^{(n)}))$$

" discuss entropy in terms of the **Euclidean path integral** on an n-sheeted Riemann surface. "

$$S_A^{(2)} = -\ln(\mathrm{Tr}_A(\rho_A^{(2)})) = -\ln(\frac{Z_A^{(2)}}{Z_\varnothing^2}) = \beta(F(Z_A^{(2)}) - F(Z_\varnothing^{(2)}))$$

" Renyi **EE** is **the difference in free energy** between partition functions with different trace topologies " (in equilibrium)

**" Qiu Ku is the $Z_A^{(2)}$ "**



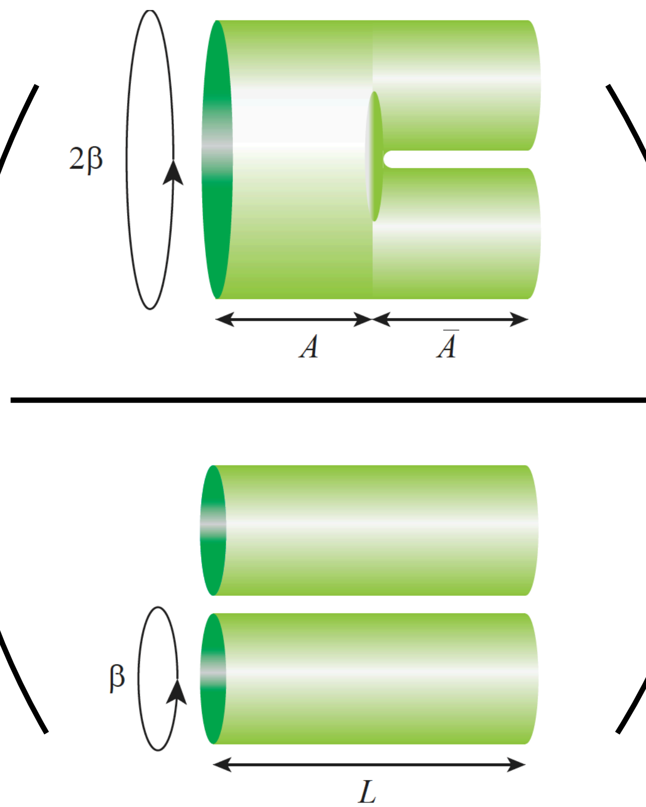

- Hastings, González, Kallin, Melko, PRL 104, 157201 (2010).
- Isakov, Hastings, Melko, Nat. Phys. 7, 772 (2011).
- Humeniuk, Roscilde, PRB 86, 235116 (2012).
- Kallin, Stoudenmire, Fendley, Singh, Melko, J. Stat. Mech. (2014) P06009
- Helmes and Wessel, PRB 89, 245120 (2014).
- Kulchytskyy, Herdman, Inglis, Melko, PRB 92, 115146 (2015).
- ……

- Grover, PRL 111, 130402 (2013).
- Assaad, Lang, Toldin, PRB 89, 125121 (2014).
- Broecker, Trebst, PRB 94, 075144 (2016).
- ……

# Entanglement entropy with incremental (Qiu Ku) method

$$S_A^{(2)} = -\ln(\mathrm{Tr}_A(\rho_A^2)) = -\ln(\frac{Z_A^{(2)}}{Z_\varnothing^{(2)}}) = \beta(F(Z_A^{(2)}) - F(Z_\varnothing^{(2)}))$$

$= -\ln \Big($




$\Big)$

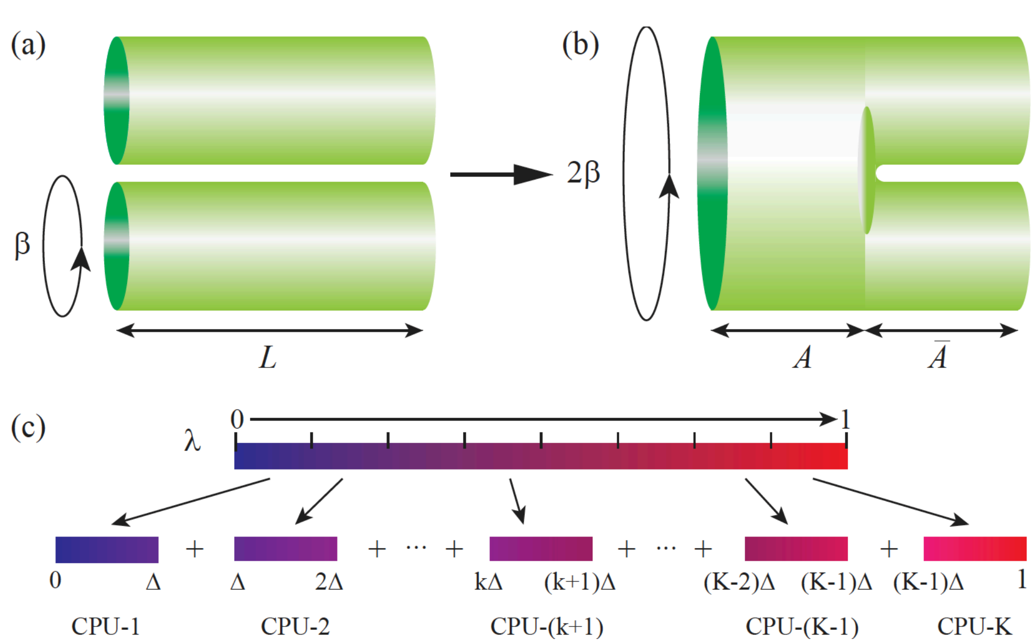
$$= -\int_0^1 d\lambda \frac{\partial \ln Z(\lambda)}{\partial \lambda} = -\sum_{k=1,2,\cdots,N_\lambda} \int_{(k-1)\Delta}^{k\Delta} d\lambda \frac{\partial \ln Z(\lambda_k)}{\partial \lambda}$$

↕

$$-\ln(\langle e^{-\beta W_A^{(2)}} \rangle) \qquad - \sum_{k=1,2,\cdots,N_\lambda} \ln(\langle e^{-\beta W_{k,A}^{(2)}} \rangle)$$

- V. Alba, PRE 95, 062132 (2017)
- J. D'Emidio, PRL 124, 110602 (2020)
- J. Zhao, …, M. Cheng, ZYM, PRL 128, 010601 (2022)
- J. Zhao, …, M. Cheng, ZYM, npj Quantum Materials 7, 69 (2022)

$$Z(\lambda = 0) = Z_\varnothing^{(2)} \qquad Z(\lambda = 1) = Z_A^{(2)}$$




$$e^{-S_A^{(2)}} = \frac{Z(1)}{Z(0)} := \frac{Z(\lambda_1)}{Z(0)} \frac{Z(\lambda_2)}{Z(\lambda_1)} \cdots \frac{Z(\lambda_k)}{Z(\lambda_{k-1})} \cdots \frac{Z(1)}{Z(\lambda_{N_\lambda - 1})}$$

- G. Pan, Y. D. Liao, J. D'Emidio, ZYM, PRB 108, L081123 (2023)
- J. D'Emidio, et al., PRL 132, 076502 (2024)
- Y.D. Liao, G.Pan, W. Jiang, Y. Qi, ZYM, arXiv:2302.11742

nature communications

Article

https://doi.org/10.1038/s41467-023-37756-7

# Unlocking the general relationship between energy and entanglement spectra via the wormhole effect

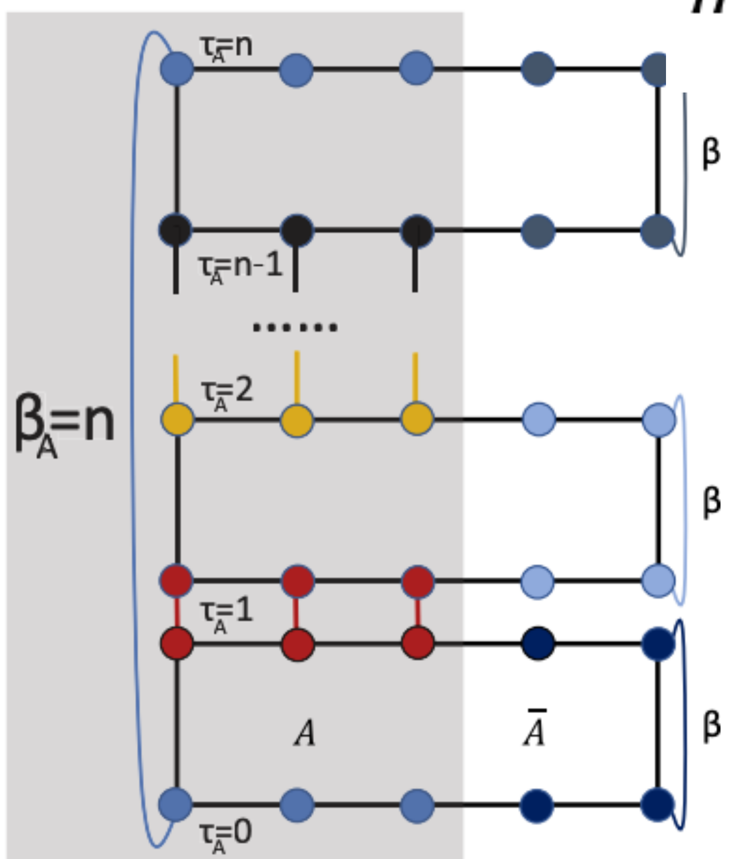
Received: 21 September 2022

Zheng Yan [1,2,3] & Zi Yang Meng [1]

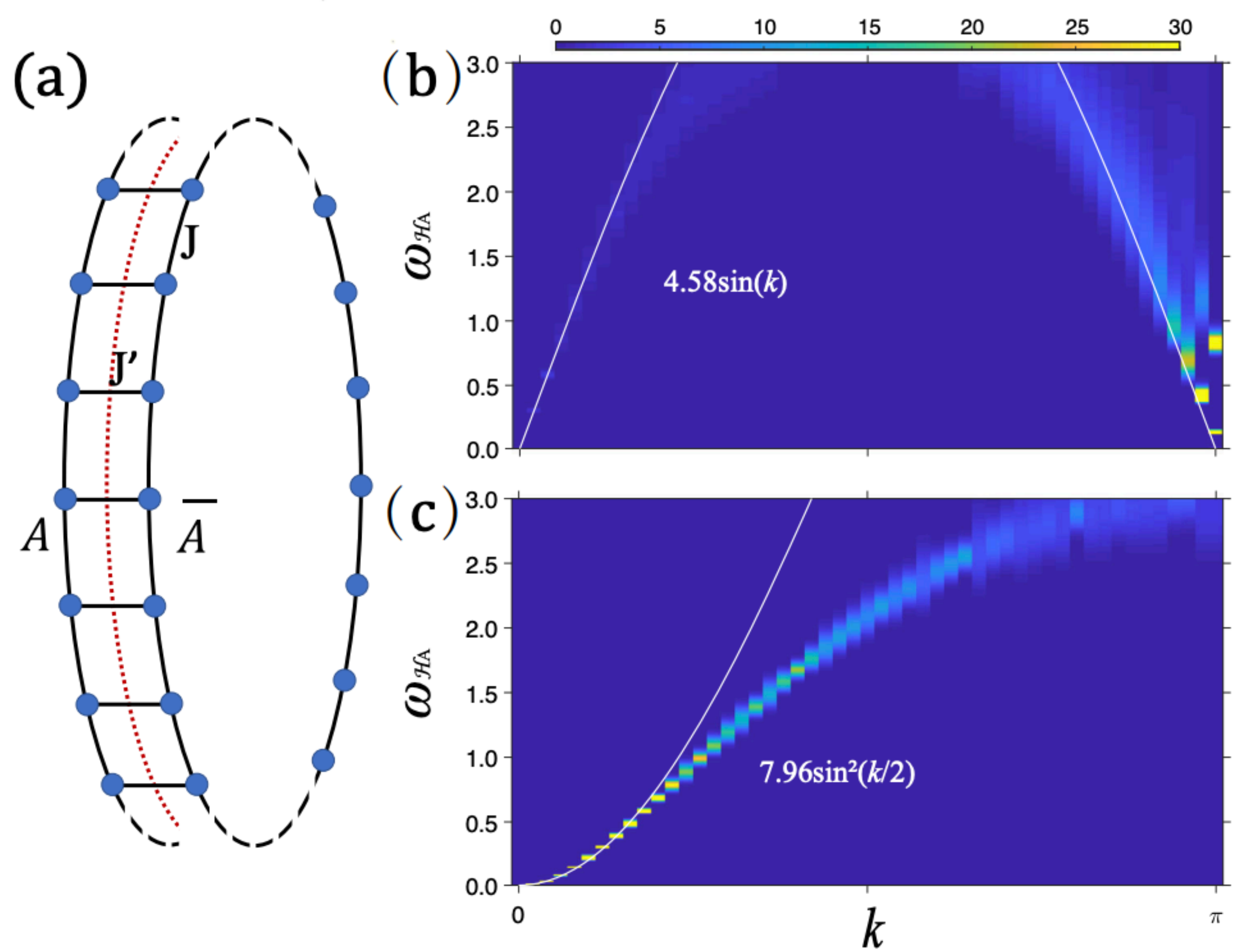
$$Z_A^{(n)} = \mathrm{Tr}[\rho_A^n] = \mathrm{Tr}[e^{-nH_A}]$$

$$n = 1,2,3,4,\cdots = \beta_A$$

$$H = J\sum_{\langle ij\rangle}(S_{A,i}S_{A,j} + S_{\bar{A},i}S_{\bar{A},j}) + J'\sum_i S_{A,i}S_{\bar{A},i}$$




$L = 100, J' = 1.732, J = 1, \beta = 100, \beta_A = 200$




$L = 100, J' = 1.732, J = -1, \beta = 100, \beta_A = 800$

$$G_k(\tau_A) = \langle S^z_{-k}(\tau_A) S^z_k(0)\rangle$$

$$S(\omega_{H_A}(k))$$

nature communications

Article

https://doi.org/10.1038/s41467-023-37756-7

# Unlocking the general relationship between energy and entanglement spectra via the wormhole effect

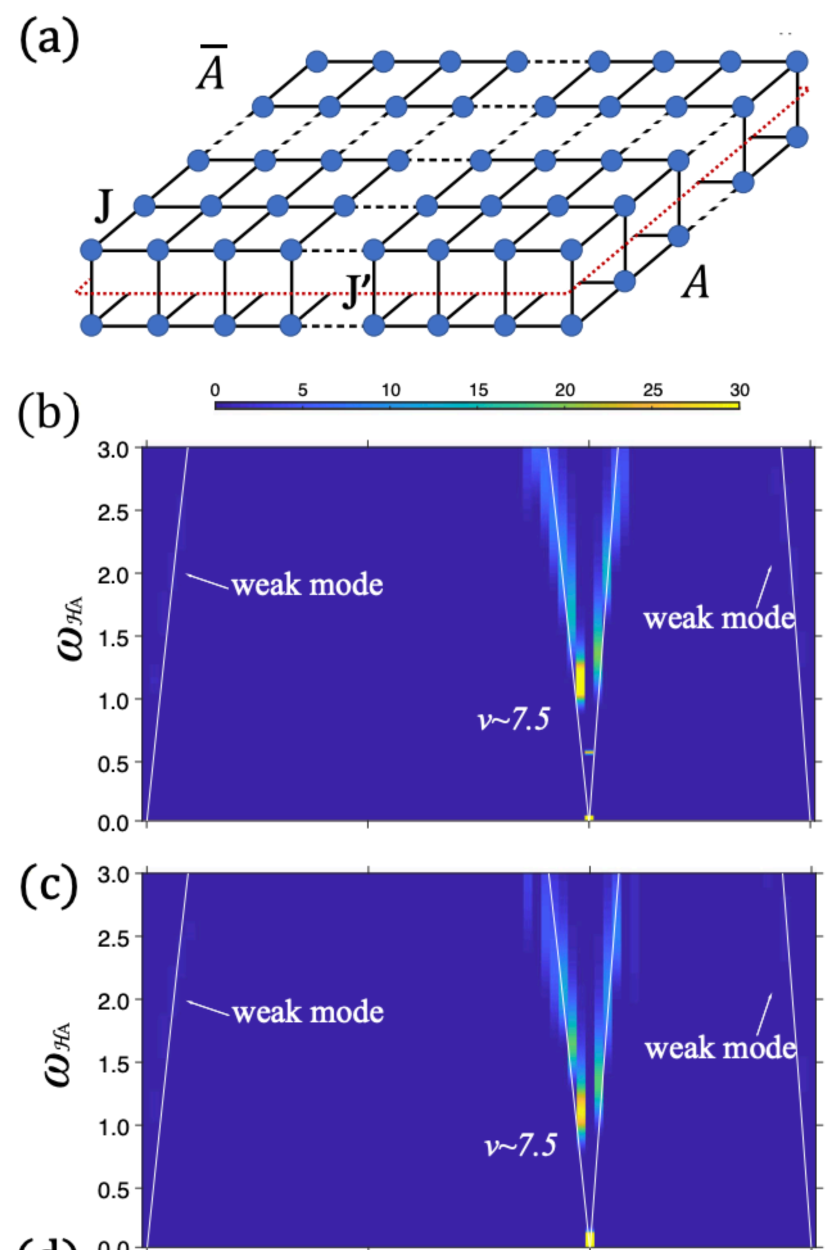
Received: 21 September 2022

Zheng Yan [1,2,3] & Zi Yang Meng [1]

$L = 50, \beta = 100, \beta_A = 32$

$J' = 1.732, J = 1$

$J' = 2.522, J = 1$



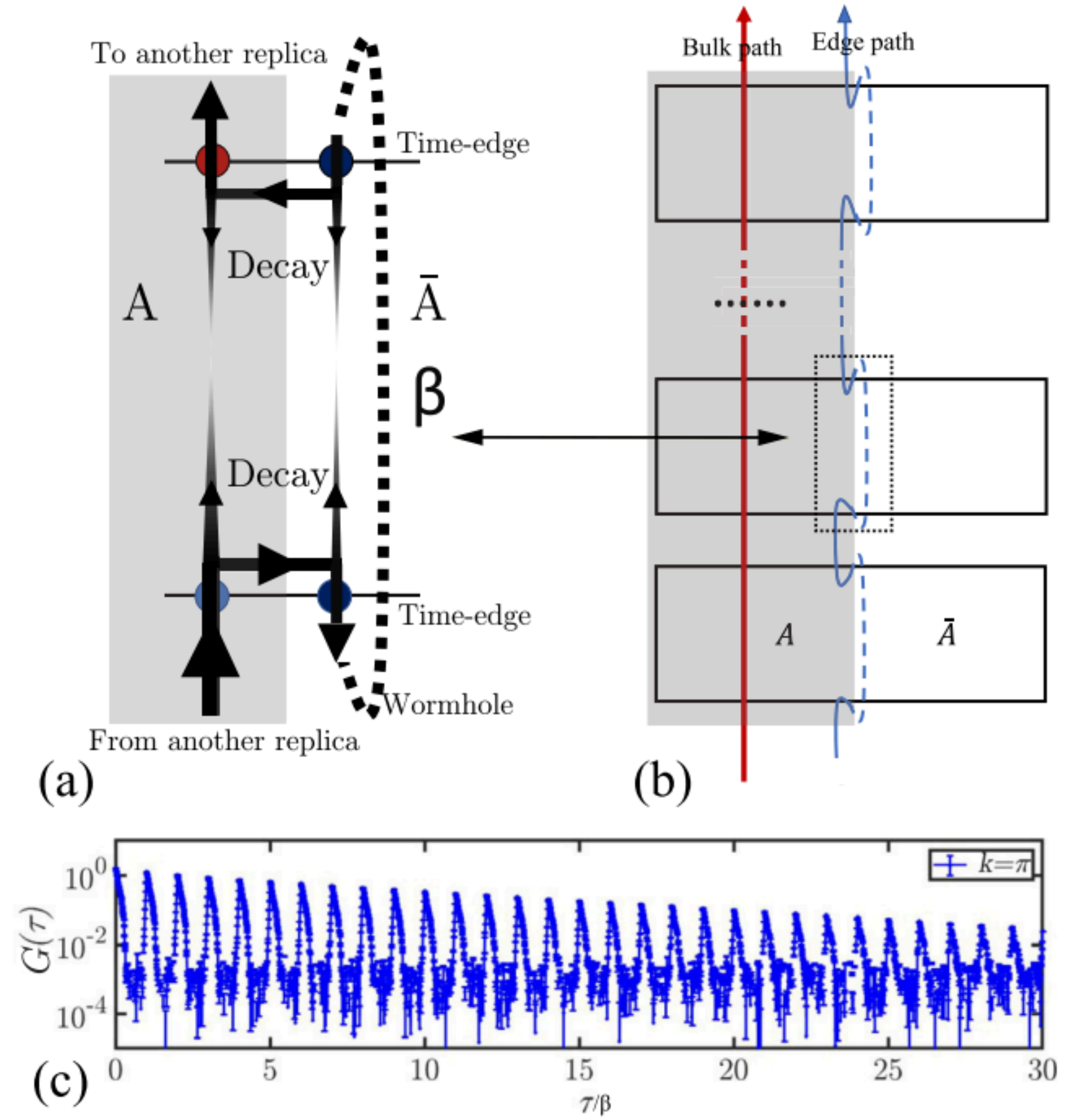




**Fig. 5 | The wormhole effect. a** Wormhole effect of worldlines going through a replica. It is a zoom-in of the region inside dotted box of (**b**). The gray part is the subsystem $A$ and the white part is the environment $\bar{A}$. The arrows go into bulk will decay to zero as $\beta \to \infty$. At the same time, the arrows go through the imaginary-time-edge of environment will reach the other side through "wormhole" without much attenuation. **b** The path integral of replica system which has both bulk and edge.

$G_{k=\pi}(\tau)$

$L = 100, J' = 1.732, J = 1, \beta = 100, \beta_A = 100$

**Controllable Incremental Algorithm for Entanglement Entropy in Quantum Monte Carlo Simulations**

npj Quantum Inf 11, 64 (2025)

Yuan Da Liao[1,2]

**An integral algorithm of exponential observables for interacting fermions in quantum Monte Carlo simulation**

Xu Zhang,[1] Gaopei Pan,[1] Bin-Bin Chen,[1] Kai Sun,[2,*] and Zi Yang Meng[1,†]

PRB 109, 205147 (2024)

$$S_A^{(2)} = -\ln(\mathrm{Tr}_A(\rho_A^{(2)})) = -\ln(\frac{Z(\lambda=1)}{Z(\lambda=0)}) = -\ln(\frac{\sum_{s_1,s_2} P_{s_1}P_{s_2}\ \mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2})}{\sum_{s_1,s_2} P_{s_1}P_{s_2}})$$

Entanglement partition function

$$Z(\lambda) = \sum_{s_1,s_2} P_{s_1}P_{s_2}(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))^{\lambda}$$

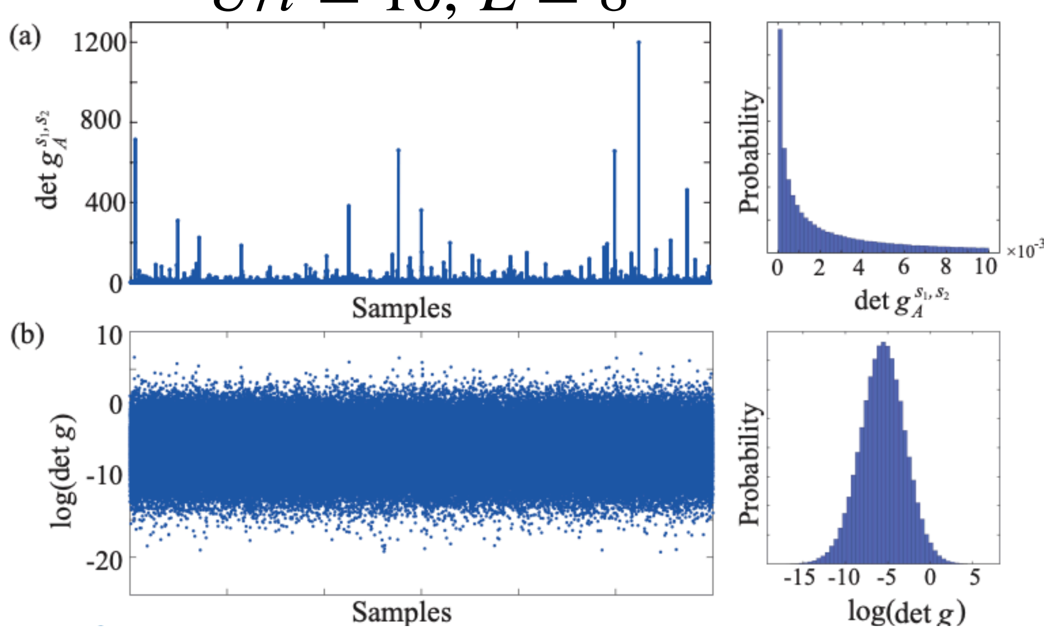
$$= -\int_0^1 d\lambda \frac{\partial \ln(Z(\lambda))}{\partial\lambda} = -\int_0^1 d\lambda \frac{\sum_{s_1,s_2} P_{s_1}P_{s_2}(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))^{\lambda}\ \ln(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))}{\sum_{s_1,s_2} P_{s_1}P_{s_2}(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))^{\lambda}} = -\langle \ln(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))\rangle_{s_1,s_2,\lambda}$$

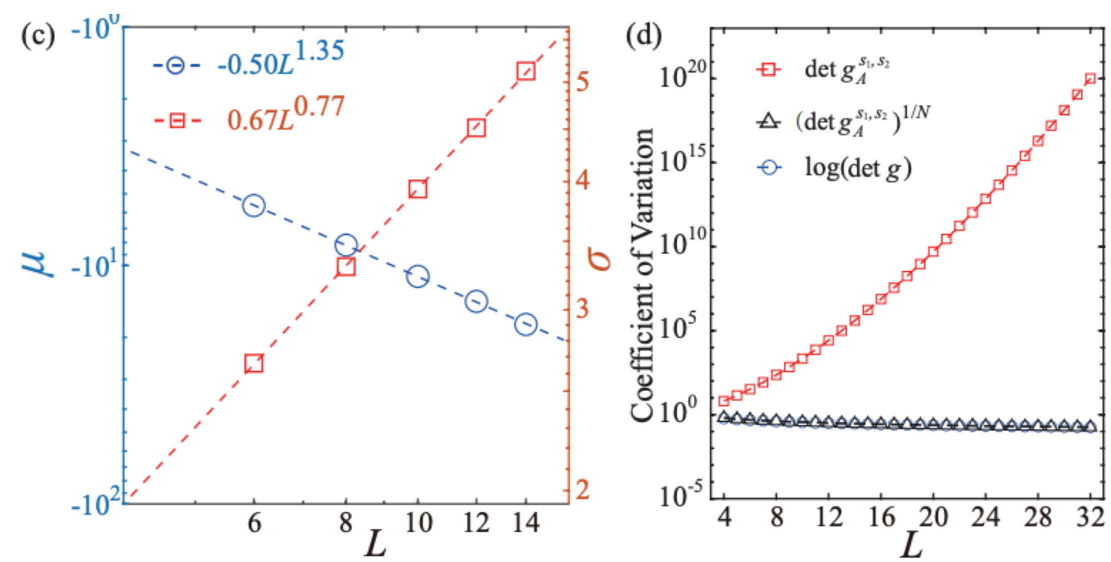
$$\frac{da^{\lambda}}{d\lambda} = \frac{de^{\lambda\ln(a)}}{d\lambda} = a^{\lambda}\ln(a)$$

Convert exponential complexity to polynomial complexity

$$CV[x] = \frac{SD[x]}{E[x]} = \sqrt{e^{\sigma^2}-1}$$

$U/t = 10,\ L = 8$



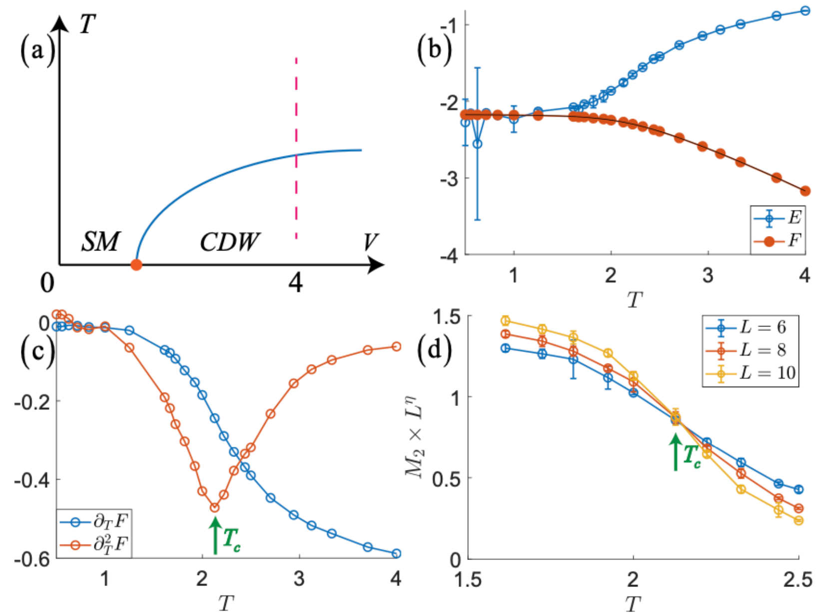
## An integral algorithm of exponential observables for interacting fermions in quantum Monte Carlo simulation

PRB 109, 205147 (2024)

Xu Zhang,[1] Gaopei Pan,[1] Bin-Bin Chen,[1] Kai Sun,[2,*] and Zi Yang Meng[1,†]

$$S_A^{(n)} = -\frac{1}{n-1}\ln(\mathrm{Tr}(\rho_A^n)) = -\frac{1}{n-1}\int_o^1 d\lambda \frac{\sum P_s^n \mathrm{Tr}(\rho_{A,s}^n)^\lambda \ \ln(\mathrm{Tr}(\rho_{A,s}^n))}{\sum P_s^n \mathrm{Tr}(\rho_{A,s}^n)^\lambda} = -\frac{1}{n-1}\langle \ln(\mathrm{Tr}(\rho_{A,s}^n))\rangle_{s_1,s_2,\cdots,s_n,\lambda}$$

$$F = -\frac{1}{\beta}\ln(Z) = -\frac{1}{\beta}\int_0^1 d\lambda \frac{\sum P_s^\lambda \ \ln(P_s)}{\sum P_s^\lambda} = -\frac{1}{\beta}\langle \ln(P_s)\rangle_{s,\lambda}$$

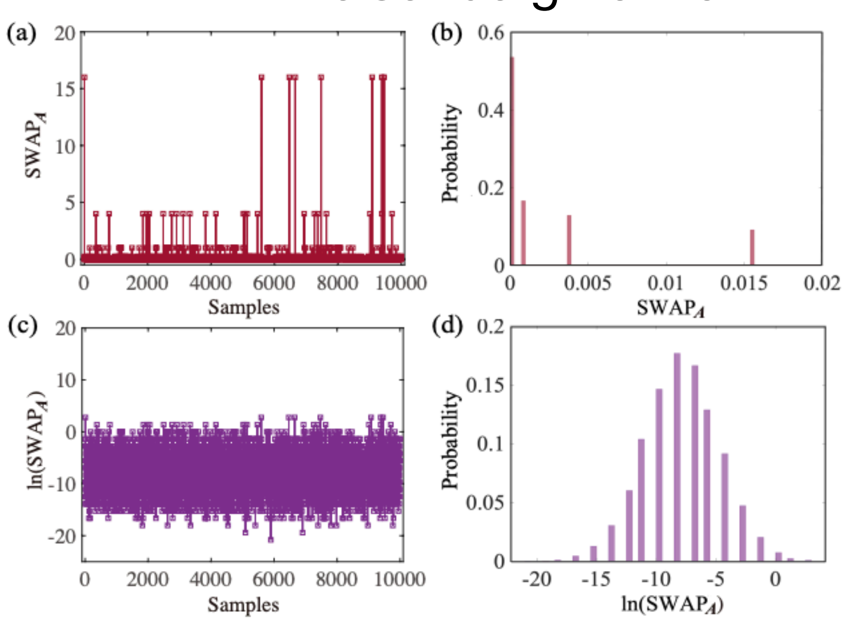


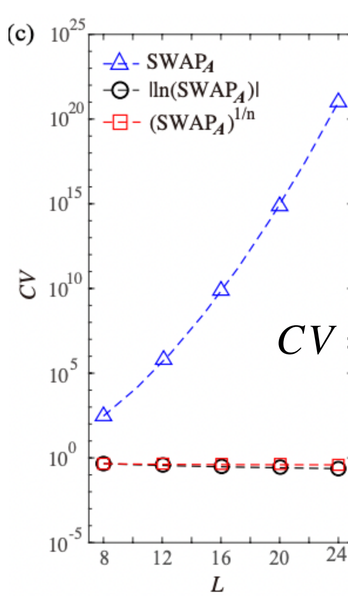

## Incremental SWAP Operator for Entanglement Entropy: Application for Exponential Observables in Quantum Monte Carlo Simulation

Xuan Zhou,[1,2] Zi Yang Meng,[3] Yang Qi,[1,2,4,*] and Yuan Da Liao[1,2,3,†]

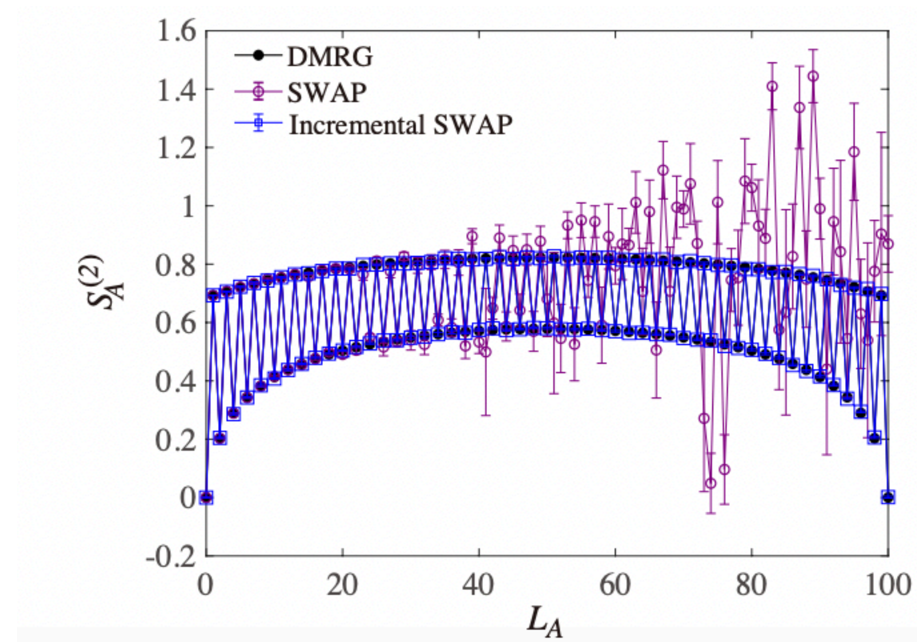
PRB 109, 165106 (2024)

AF Heisenberg 10x10



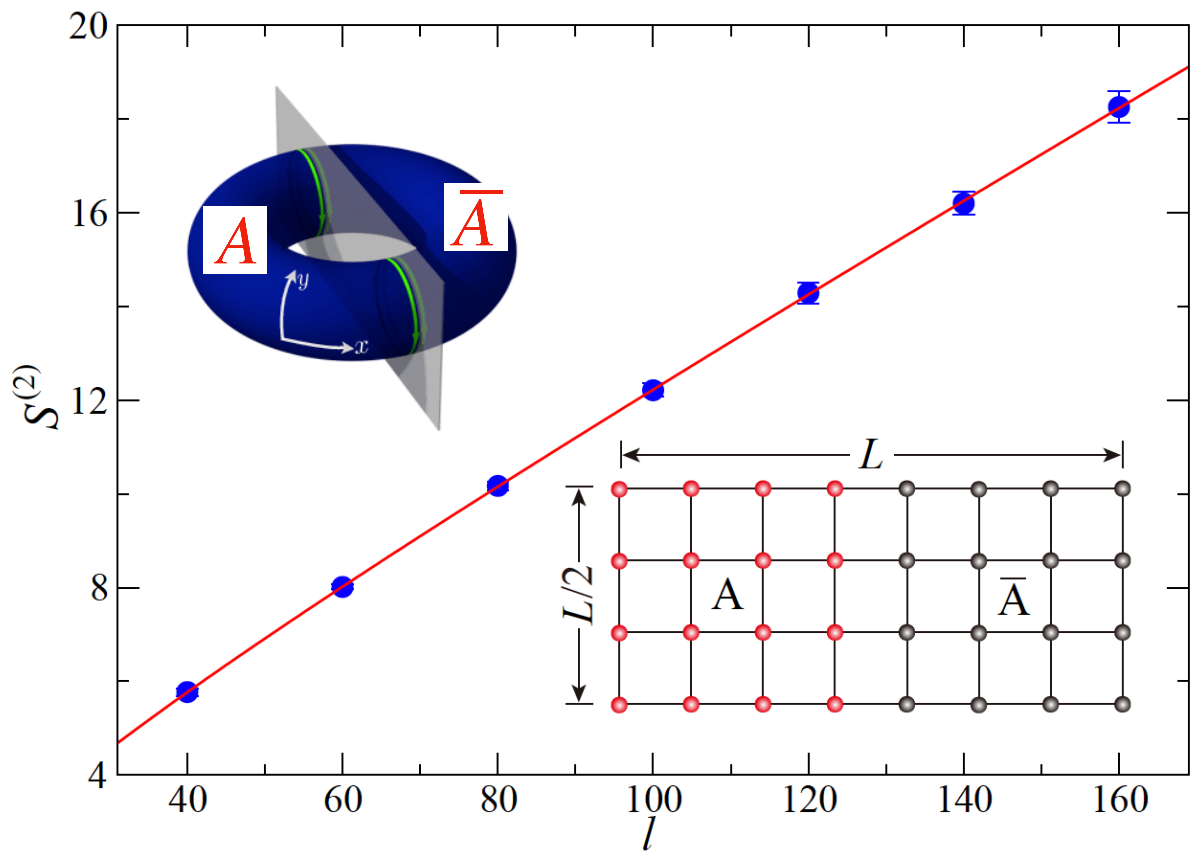




$$CV = \frac{\sigma_{swap}}{\mu_{swap}} = \sqrt{e^{\sigma^2} - 1}$$



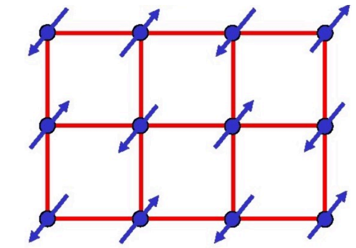
# Spontaneous symmetry breaking phases: smooth boundary

- J. Zhao, B.-B Chen, Y.-C. Wang, Z. Yan, M. Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)




Square lattice Heisenberg model

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Smooth boundary $\quad S_A^{(2)}(l) = al - s\ln(l) + c$

$$S_A^{(2)}(l) = 0.092(1)l + 1.0(1)\ln(l) - 1.63(3)$$

$$l \in [40,\ 160]$$

$$s = -\frac{N_G}{2}$$

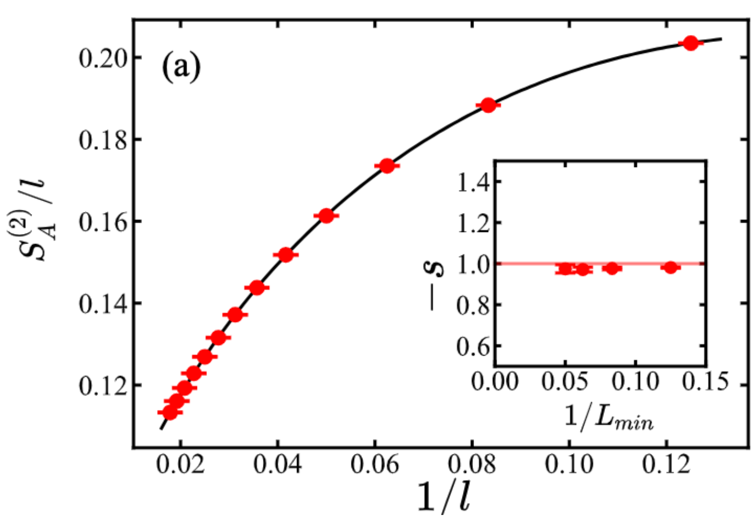
- Metlitski & Grover, arXiv:1112.5166

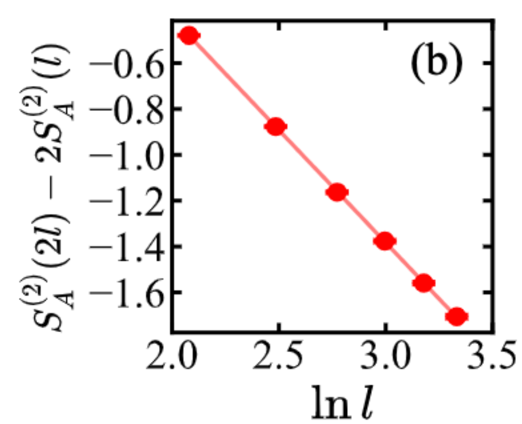
$$\frac{S_A^{(2)}(l)}{l} = s\frac{\ln(1/l)}{l} + \frac{c}{l} + a$$

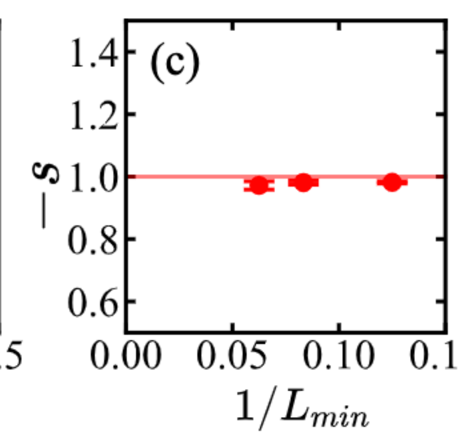
$$\frac{d^2y}{dx^2} = \frac{s}{x}$$

$$s < 0$$

concave




Subtracted EE

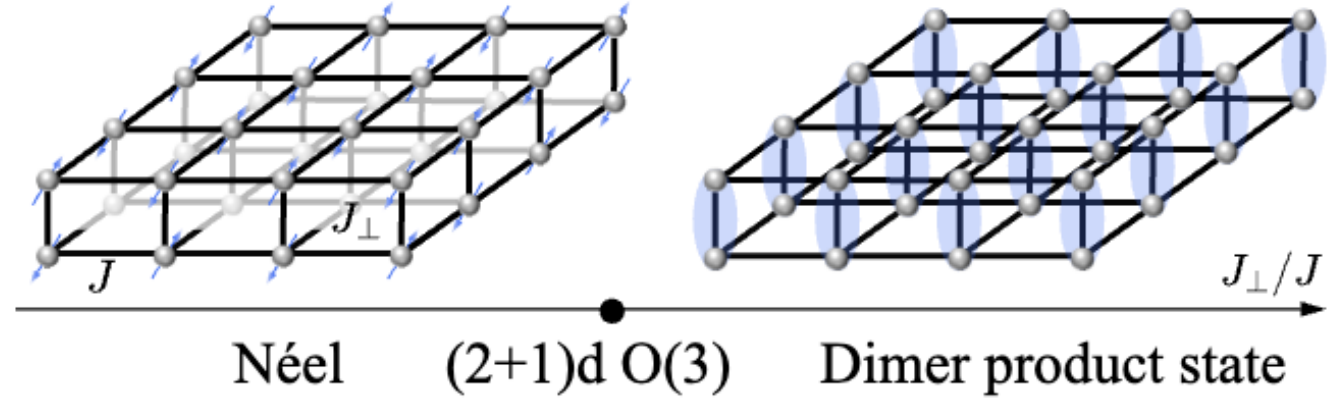
$$S^s(l) = S_A(2l) - 2S_A(l)$$

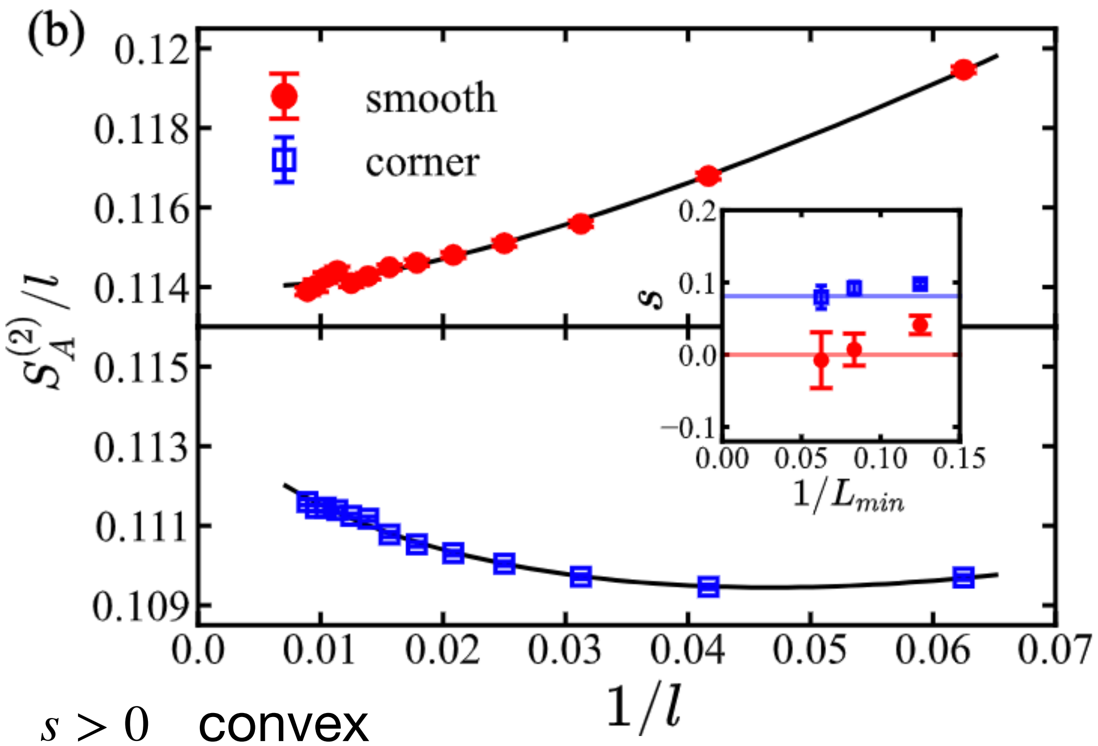
$$S^s(l) = s\ln(l) - c$$

# (2+1)d O(3) quantum critical points: smooth & corner

- J. Zhao, B.-B Chen, Y.-C. Wang, Z. Yan, M. Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)

$$H = J\sum_{\langle i,j\rangle}(\mathrm{S}_{i,1}\cdot\mathrm{S}_{j,1}+\mathrm{S}_{i,2}\cdot\mathrm{S}_{i,2}) + J_\perp\sum_i \mathrm{S}_{i,1}\cdot\mathrm{S}_{i,2}$$




$s > 0$ convex

$$\frac{S_A^{(2)}(l)}{l} = a + \frac{c}{l}$$

$$\frac{S_A^{(2)}(l)}{l} = a - s\frac{\ln l}{l} + \frac{c}{l}$$

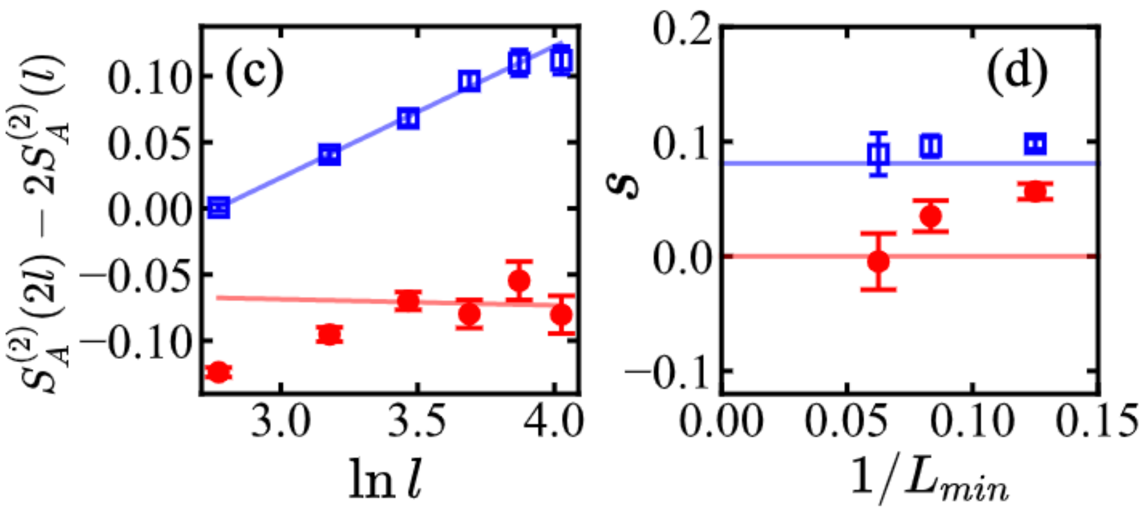
## Subtracted EE

$$S^s(l) = S_A(2l) - 2S_A(l)$$

$$S^s(l) = -\frac{3b}{2l} - c$$

$$S^s(l) = s\ln(l) - c$$

| | $s$ with $L_{\min}=8$ | $s$ with $L_{\min}=16$ | $\chi^2/k$ | |
|---|---|---|---|---|
| | | | $\ln l$ | $1/l$ |
| O(3), smooth | 0.056(7) | −0.004(24) | 2.38 | 1.30 |
| O(3), corner | 0.098(4) | 0.088(18) | 0.23 | 2.66 |




- A. Kallin, et. al, J. Stat. Mech. P06009 (2014)
- J. Helmes, S. Wessel, Phys. Rev. B 89, 245120 (2014)

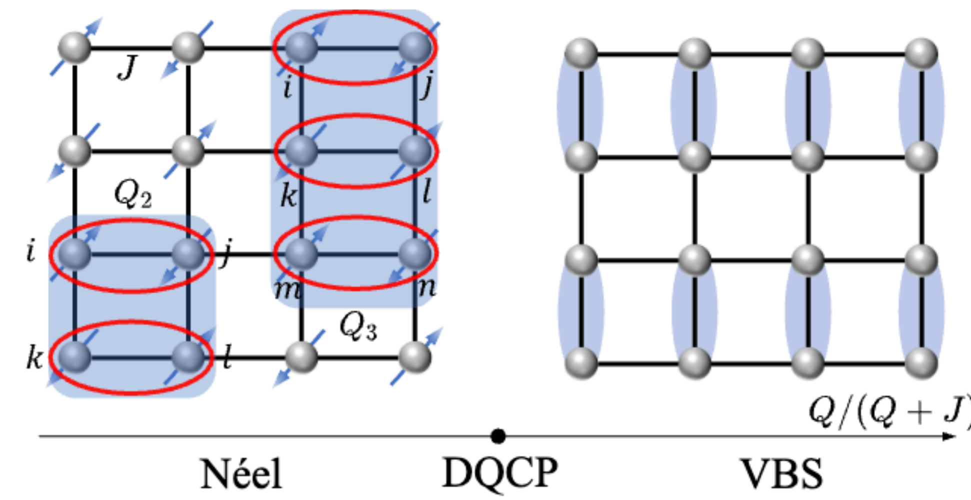
$$s_C = 0.07(2)$$

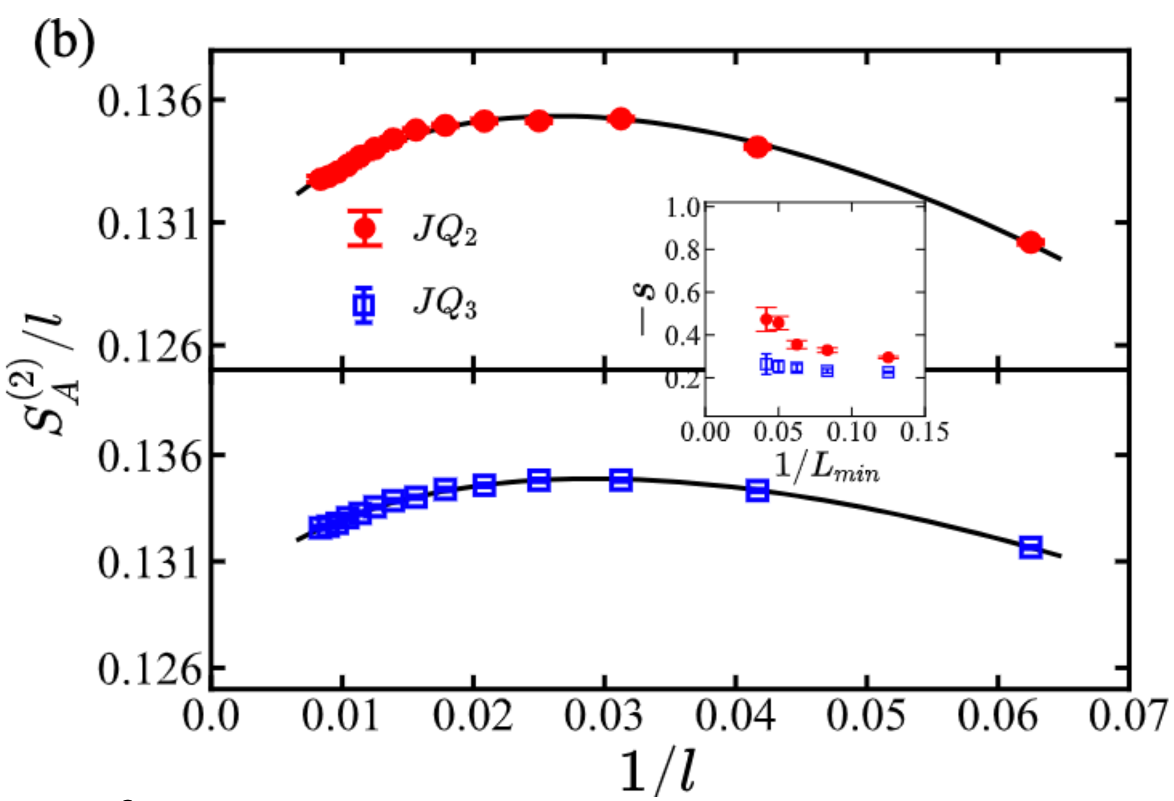
# Deconfined quantum critical points: Smooth boundary

M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)

JQ2 model: $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q\sum_{\langle ijkl\rangle} P_{ij}P_{kl}$

JQ3 model: $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q\sum_{\langle ijklmn\rangle} P_{ij}P_{kl}P_{mn}$




$$\frac{S_A^{(2)}(l)}{l} = a + \frac{c}{l}$$

$$\frac{S_A^{(2)}(l)}{l} = a - s\frac{\ln l}{l} + \frac{c}{l}$$

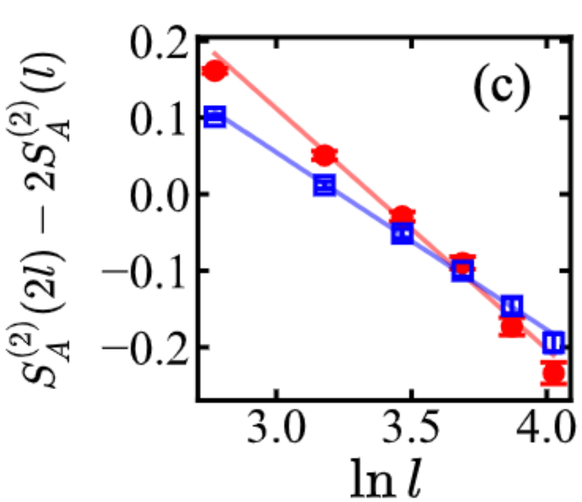
Subtracted EE

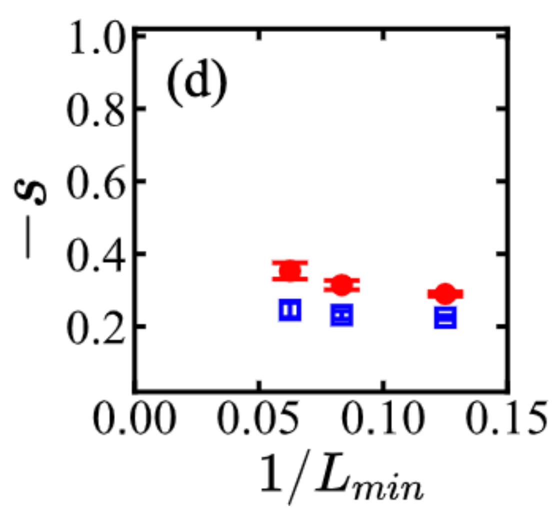
$$S^s(l) = S_A(2l) - 2S_A(l)$$

$$S^s(l) = -\frac{3b}{2l} - c$$

$$S^s(l) = s\ln(l) - c$$

$s < 0$ concave



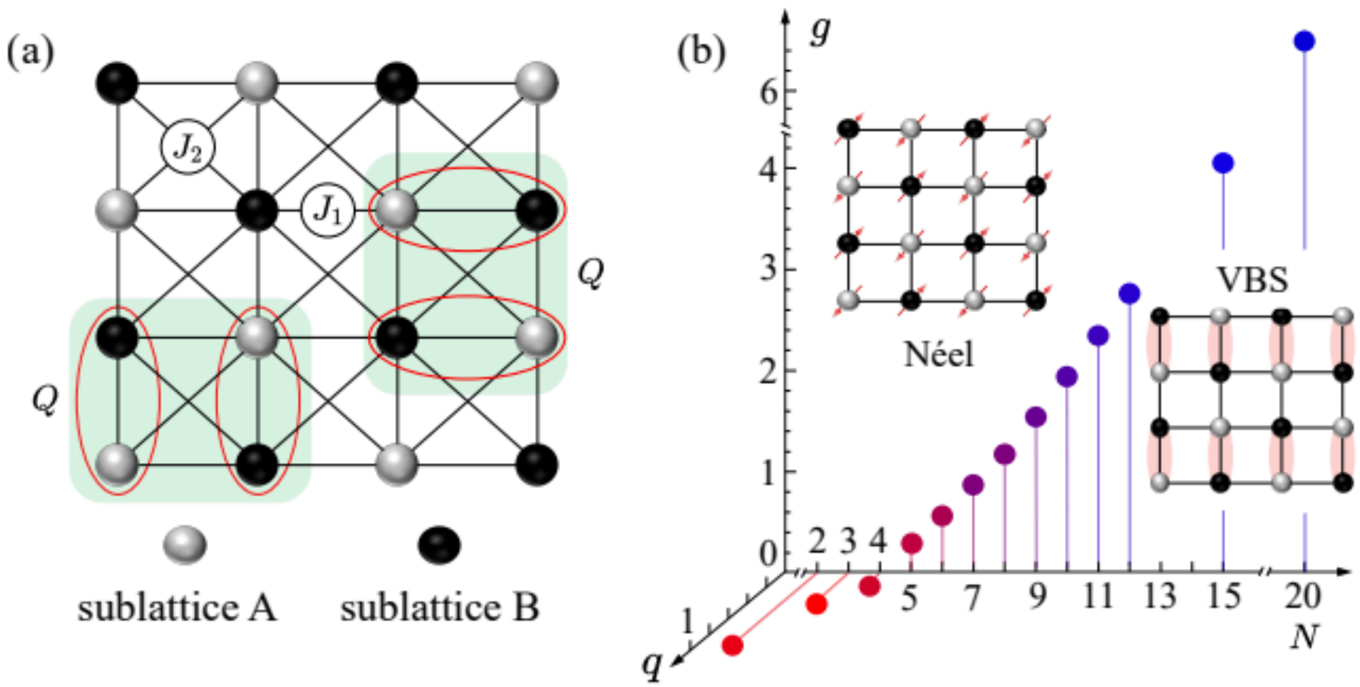

| | $s$ with $L_{\min}=8$ | $s$ with $L_{\min}=16$ | $\chi^2/k$ | |
|---|---|---|---|---|
| | | | $\ln l$ | $1/l$ |
| J-Q$_2$, smooth | −0.289(6) | −0.35(2) | 3.38 | 25.2 |
| J-Q$_3$, smooth | −0.224(5) | −0.24(2) | 0.49 | 13.1 |

$-s > 0$ **Not a CFT, behave like Goldstone mode.**

# Evolution of entanglement entropy at SU($N$) deconfined quantum critical points

Menghan Song,[1] Jiarui Zhao,[1] Meng Cheng,[2] Cenke Xu,[3] Michael M. Scherer,[4] Lukas Janssen,[5] and Zi Yang Meng[1,*]

Sci. Adv. 11, adr0634 (2025)




$$H = -\frac{J_1}{N}\sum_{\langle ij\rangle} P_{ij} - \frac{J_2}{N}\sum_{\langle\langle ij\rangle\rangle} \Pi_{ij} - \frac{Q}{N}\sum_{\langle ij\rangle,\langle kl\rangle} P_{ij}P_{kl}$$

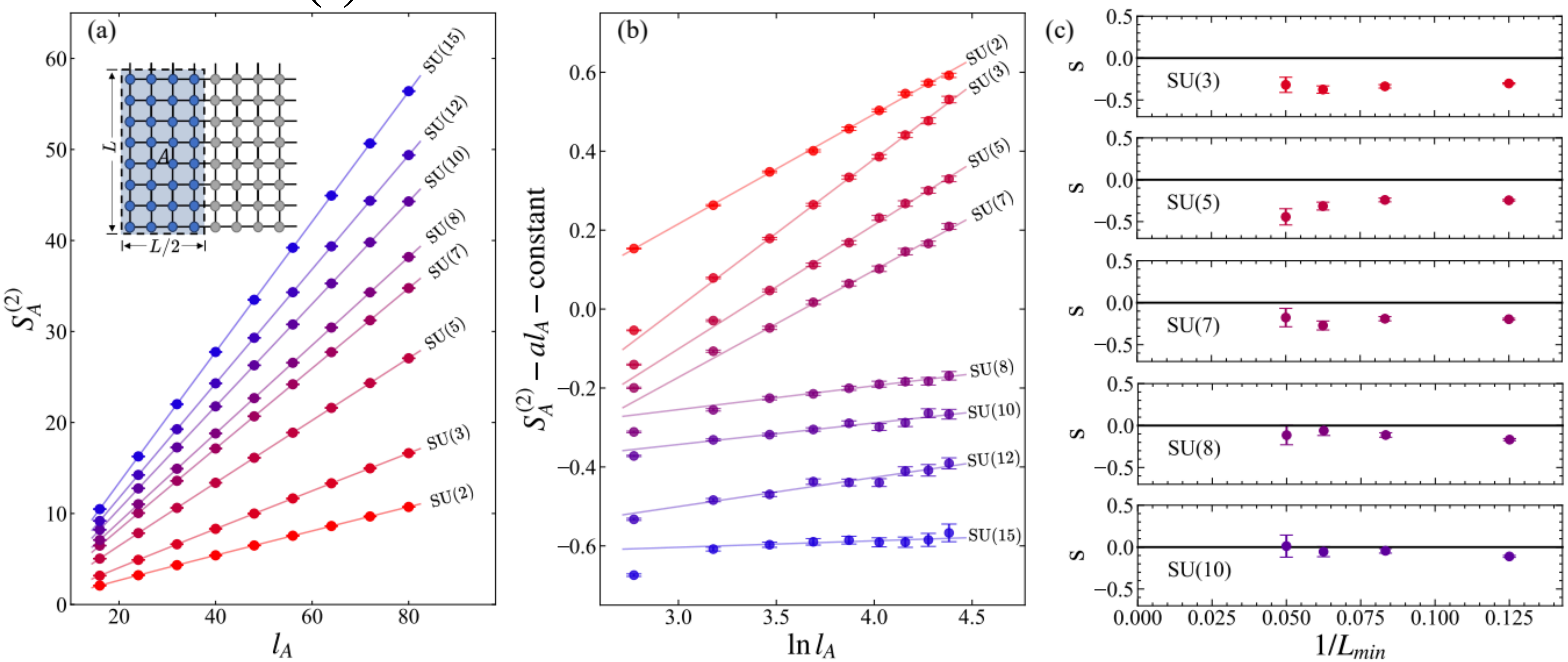
$P_{ij}$ SU(N) singlet projection

$\Pi_{ij}|\alpha\beta\rangle = |\beta\alpha\rangle$ SU(N) permutation with the same rep

Kaul, Sandvik, PRL 108, 137201 (2012)

Block, Melko, Kaul, PRL 111, 137202 (2013)

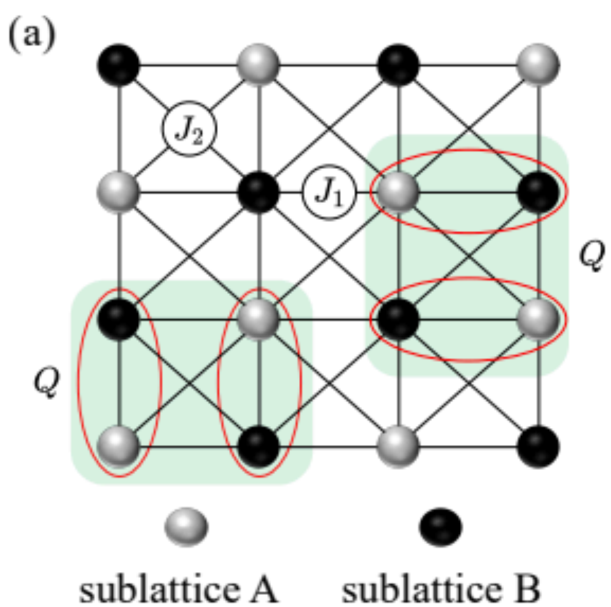
$$S_A^{(2)} = al - s\ln(l) + c$$

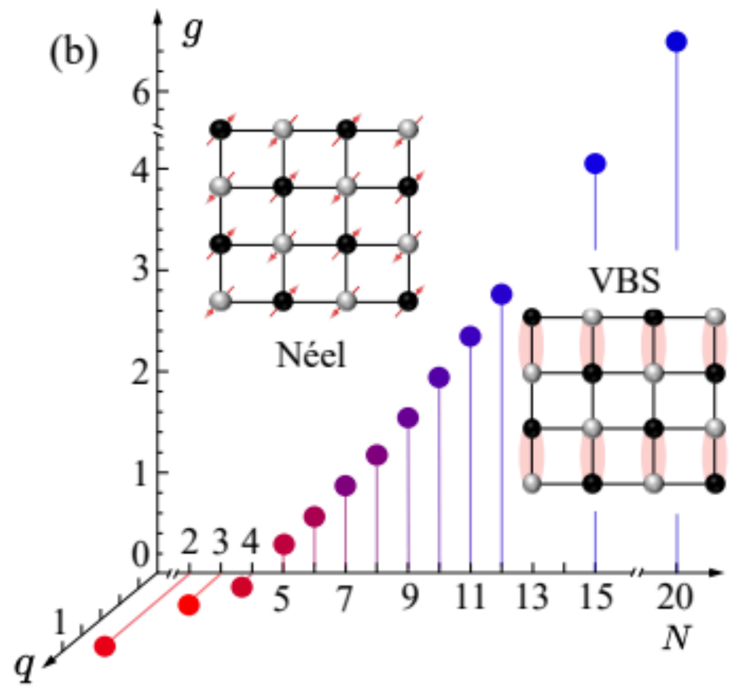


## Deconfined quantum criticality lost

Menghan Song,[1] Jiarui Zhao,[1] Meng Cheng,[2] Cenke Xu,[3] Michael M. Scherer,[4] Lukas Janssen,[5] and Zi Yang Meng[1,*]

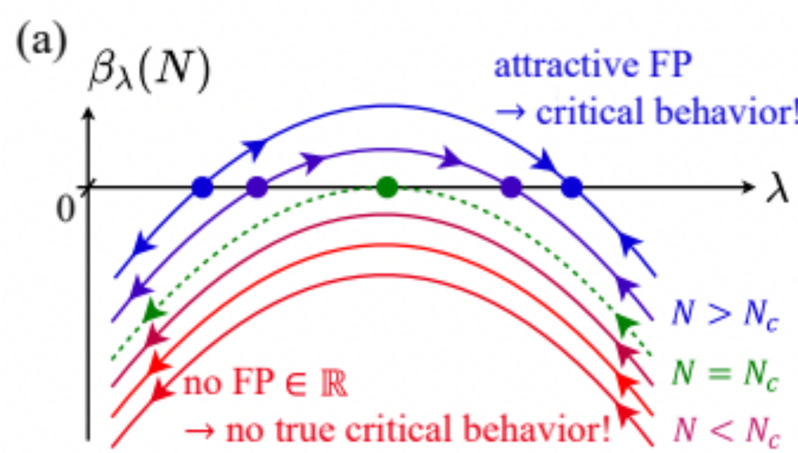
Sci. Adv. 11, adr0634 (2025)

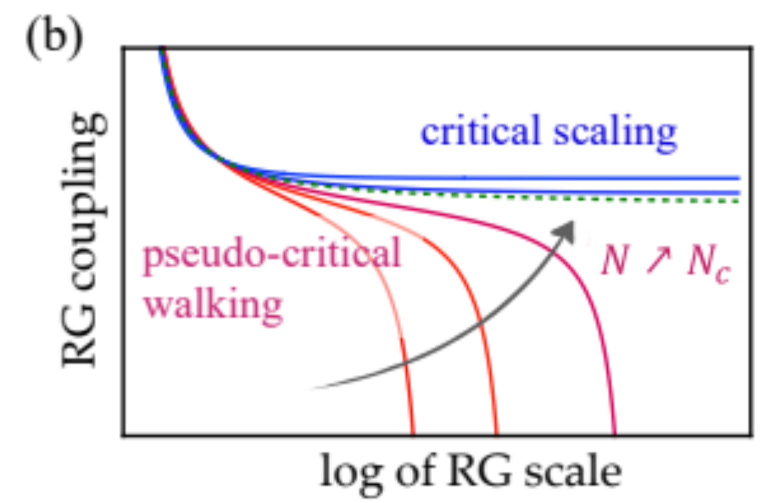



$$S_A^{(2)} = al - s\ln(l) + c$$

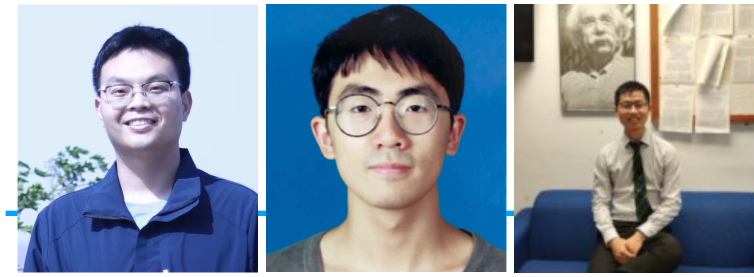
**Walking**




**Goldstone mode**

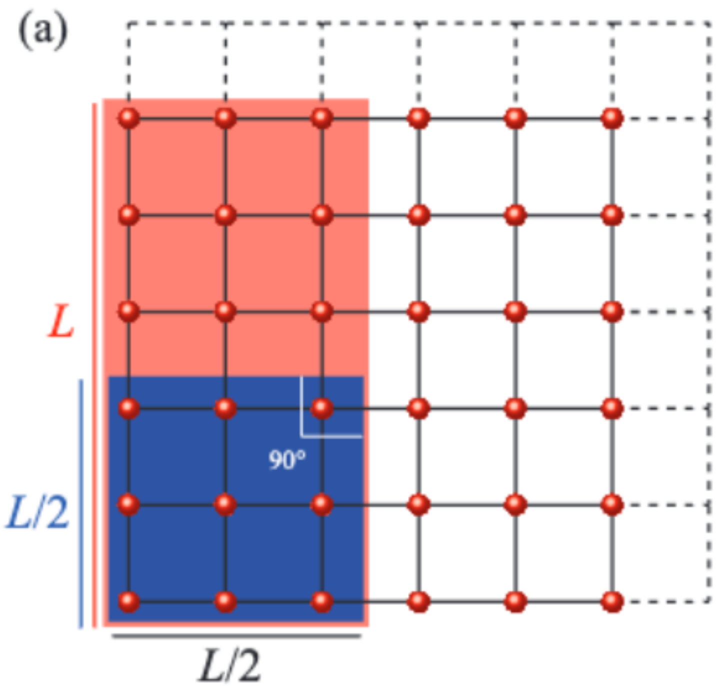
**Near-marginal renormalisation group flow on the entanglement cut**
(unlikely due to defect renormalization group flows)



# Extracting Universal Corner Entanglement Entropy during the Quantum Monte Carlo Simulation

Yuan Da Liao,[1] Menghan Song,[1] Jiarui Zhao,[1] and Zi Yang Meng[1]

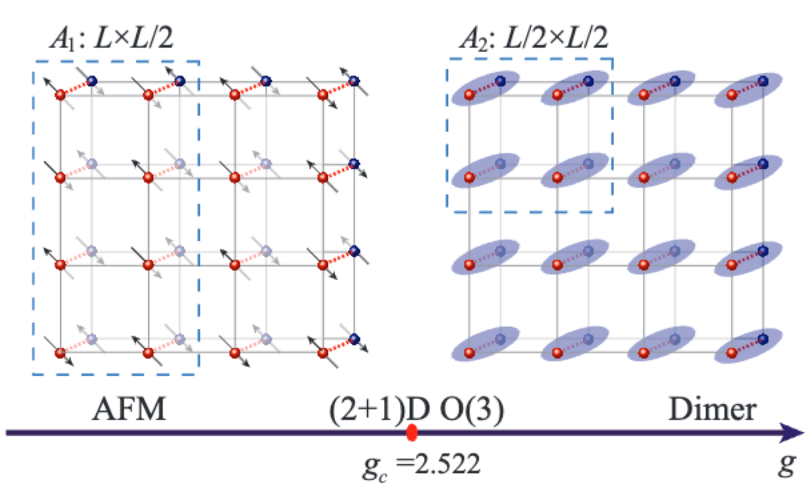
PRB 110, 235111 (2024)

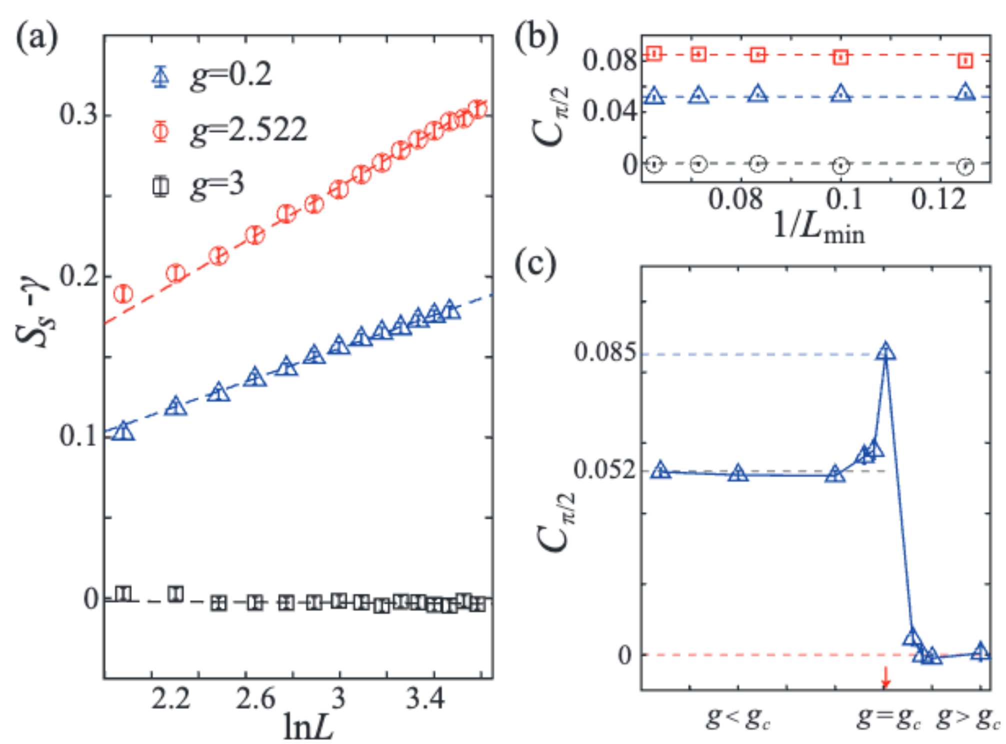


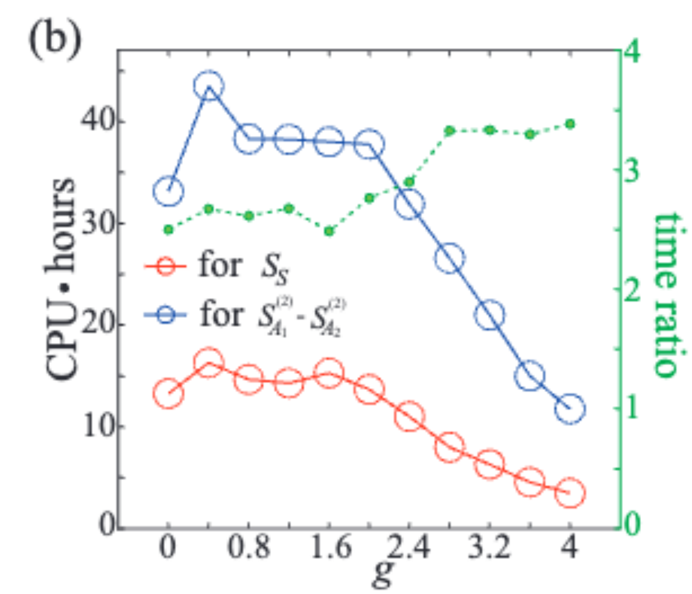

$$S_s = -\ln\left(\frac{Z_{A_1}}{Z_{A_2}}\right) = -\int_0^1 d\lambda \frac{\partial \ln(Z(\lambda))}{\partial \lambda}$$

$$Z(\lambda) = \sum_{s,s'} P_s P_{s'} \, (\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s}))^{\lambda} (\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s}))^{1-\lambda}$$

$$= -\int_0^1 d\lambda \frac{\sum_{s,s'} P_s P_{s'} \, (\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s'}))^{\lambda} (\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s'}))^{1-\lambda} \, \{\ln(\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s'})) - \ln(\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s'}))\}}{\sum_{s,s'} P_s P_{s'} \, (\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s}))^{\lambda} (\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s}))^{1-\lambda}}$$

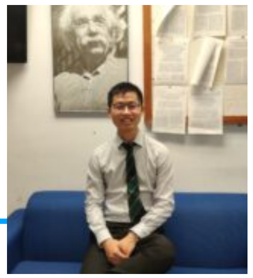


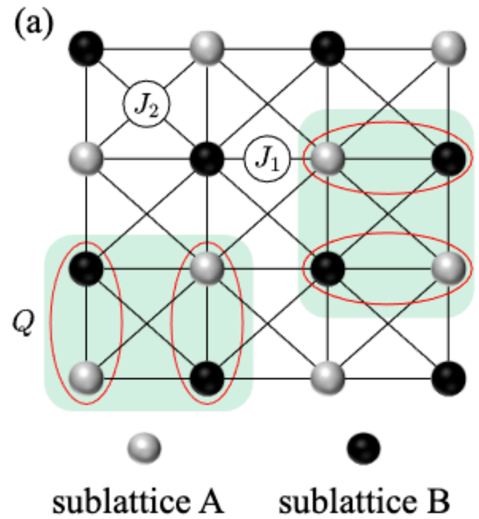



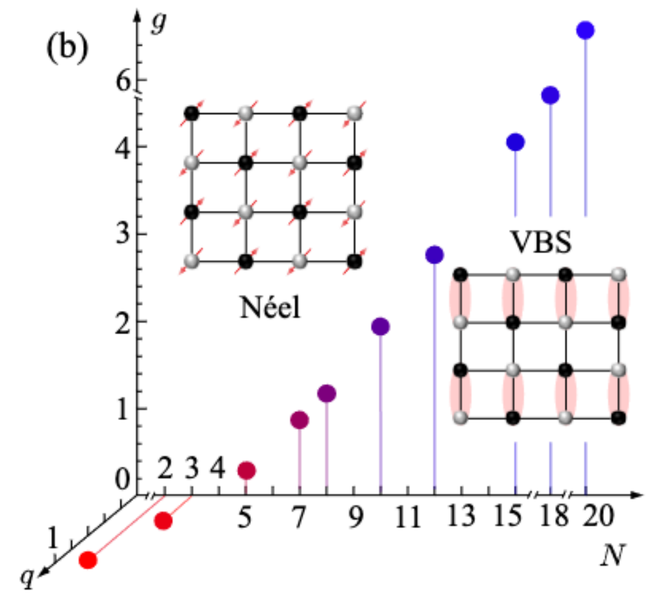
# Evolution of entanglement entropy at SU($N$) deconfined quantum critical points

Menghan Song,[1] Jiarui Zhao,[1] Meng Cheng,[2] Cenke Xu,[3] Michael M. Scherer,[4] Lukas Janssen,[5] and Zi Yang Meng[1,*]

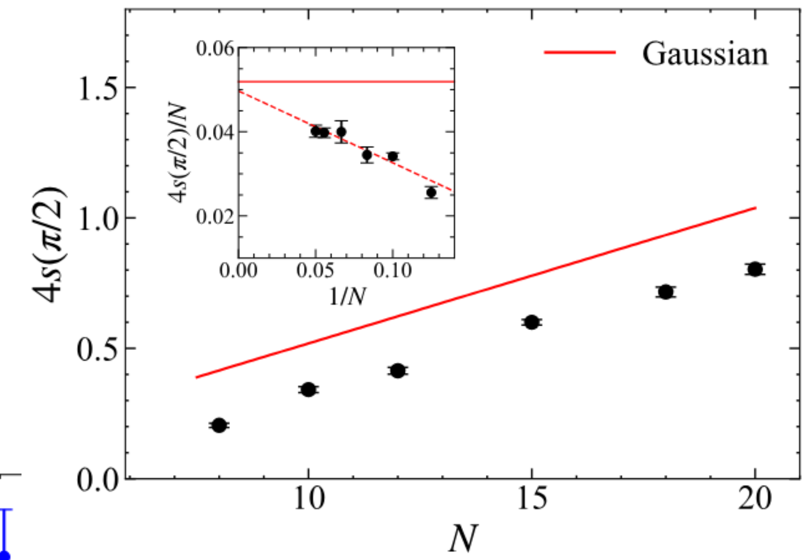


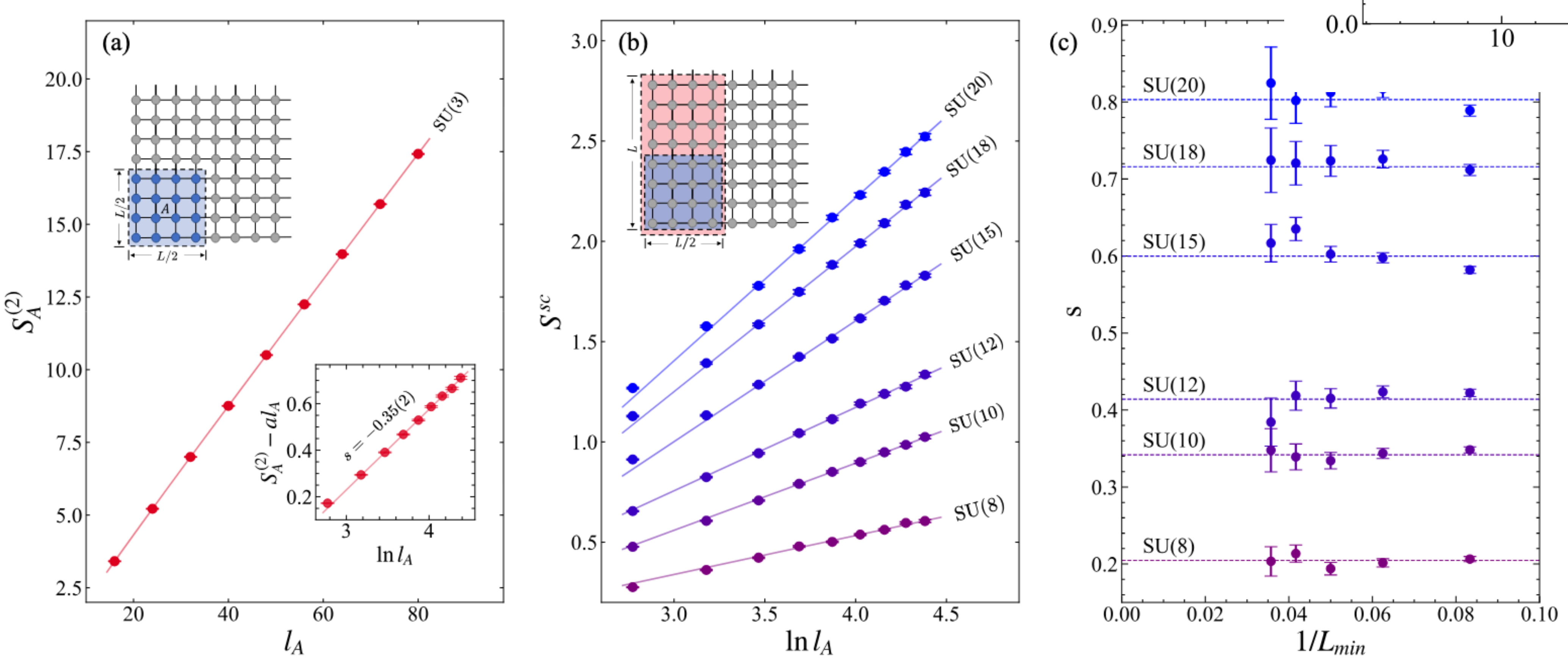
Sci. Adv. 11, adr0634 (2025)




$$H = -\frac{J_1}{N}\sum_{\langle ij\rangle} P_{ij} - \frac{J_2}{N}\sum_{\langle\langle ij\rangle\rangle} \Pi_{ij} - \frac{Q}{N}\sum_{\langle ij\rangle,\langle kl\rangle} P_{ij}P_{kl}$$



## Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

PRL 132, 246503 (2024)

### Wess-Zumino-Witten Terms in Graphene Landau Levels

Junhyun Lee and Subir Sachdev
Phys. Rev. Lett. **114**, 226801 – Published 1 June 2015

$$S = \frac{1}{g}\int d^3x(\nabla\hat{\varphi})^2 + S_{\rm WZW} + \cdots$$

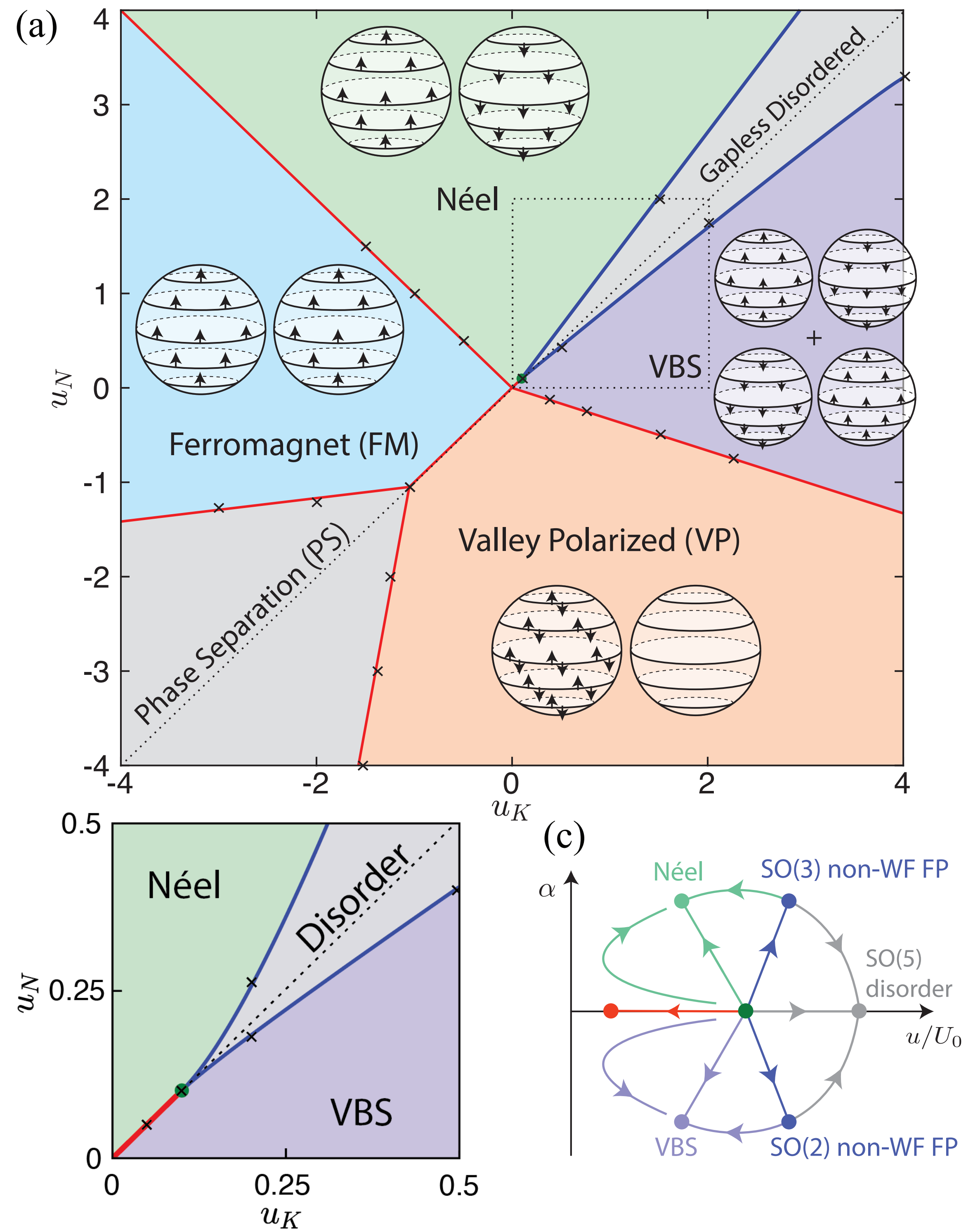
$$H = \frac{1}{2}\int d\Omega\{U_0[\psi^\dagger(\Omega)\psi(\Omega)-2]^2 - \sum_{i=1}^{5} u_i[\psi^\dagger(\Omega)\Gamma^i\psi(\Omega)]^2\}$$

$\psi_{\tau\sigma}(\Omega)$ $\quad \Gamma^i = \{\tau_x\otimes\mathbb{I}, \tau_y\otimes\mathbb{I}, \tau_z\otimes\sigma_x, \tau_z\otimes\sigma_y, \tau_z\otimes\sigma_z\}$

magnet monople inside a sphere $\quad 4\pi s$

Projected to the LLL with degeneracy $\quad N = 2s+1$

$$\psi(\Omega) = \sum_{m=-s}^{s}\Phi_m(\Omega)c_m \qquad \Phi_m(\Omega) \propto e^{im\phi}\cos^{s+m}(\frac{\theta}{2})\sin^{s-m}(\frac{\theta}{2})$$

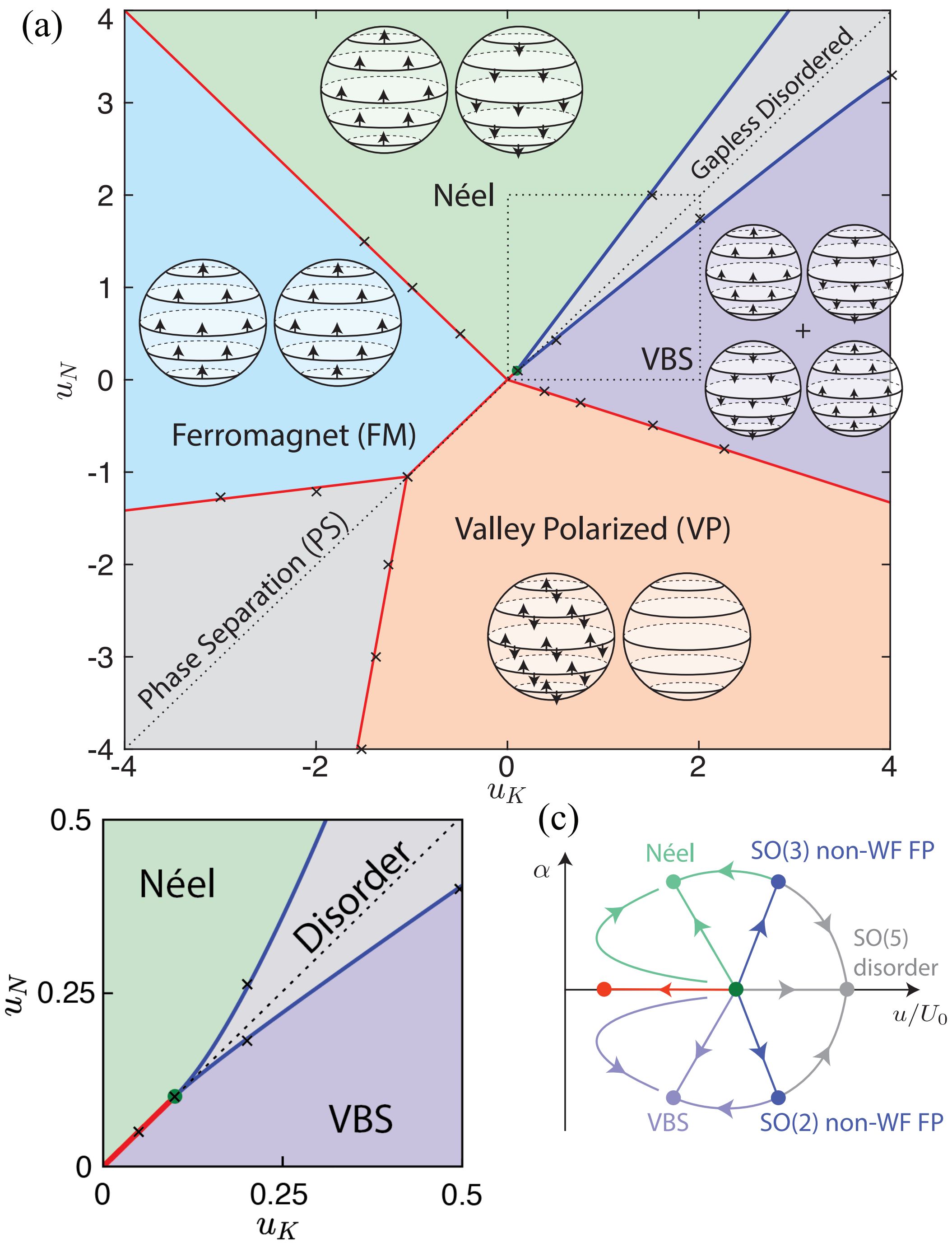



- M. Ippoliti, R. Mong, F. Assaad, M. Zaletel, PRB 98, 235108 (2018)
- Z. Wang, M. Zaletel, R. Mong, F. Assaad, PRL 126, 045701 (2021)
- Z. Zhou, L. Hu, W. Zhu, and Y.-C. He, PRX 14, 021044 (2024)

# Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

PRL 132, 246503 (2024)

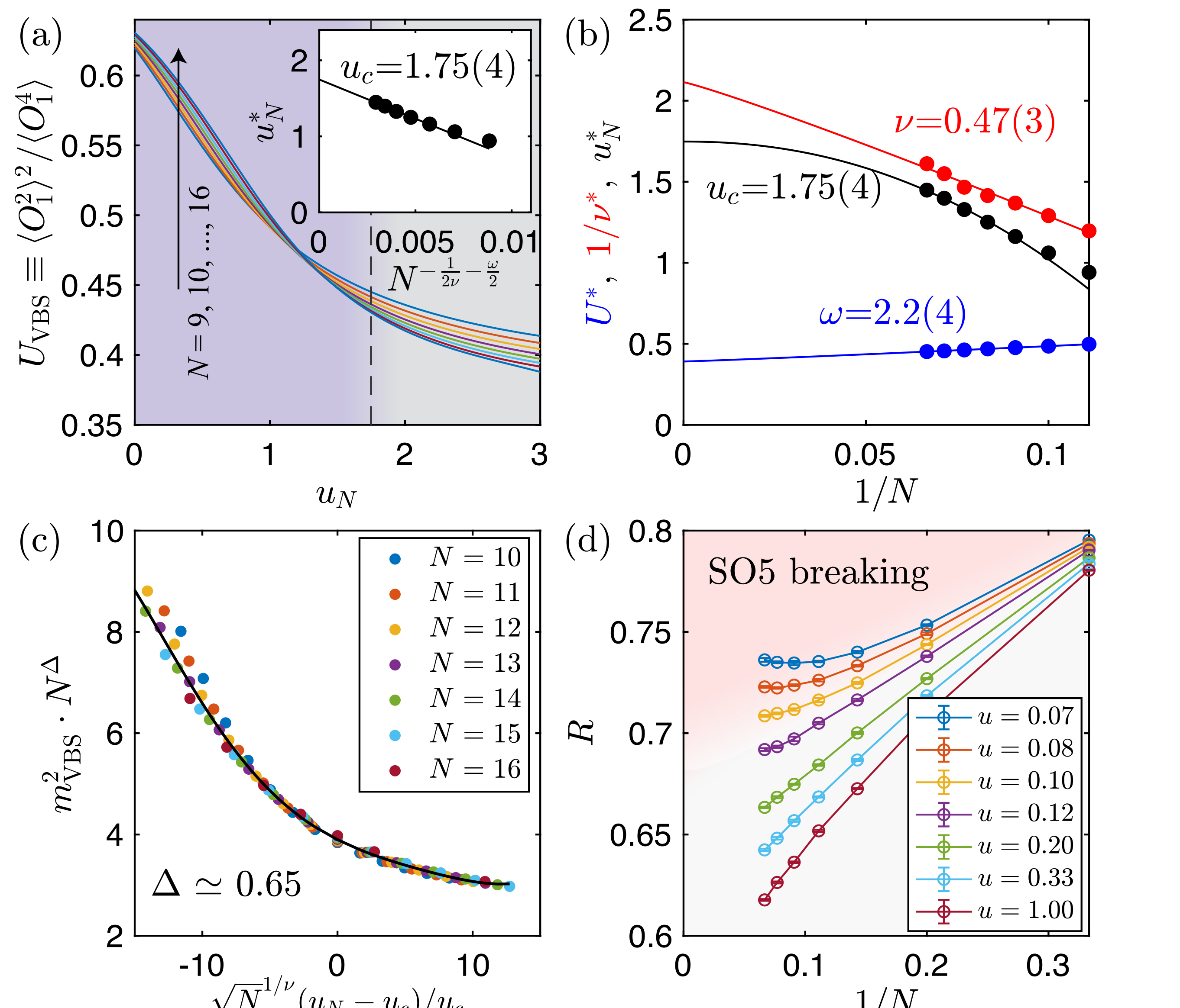



$$U_0 = 1,\ u_1 = u_2 = u_K, u_3 = u_4 = u_5 = u_N$$

$$\langle O_i \rangle = \int d\Omega \psi^\dagger(\Omega)\Gamma^i\psi(\Omega) = \sum_{m=-s}^{s} c_m^\dagger \Gamma^i c_m$$

$$m^2_{VBS} = \frac{1}{2N^2}\langle (O_1^2 + O_2^2)\rangle \qquad m^2_{Neel} = \frac{1}{3N^2}\langle (O_3^2 + O_4^2 + O_5^2)\rangle$$

$$u_K = 2$$



# Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

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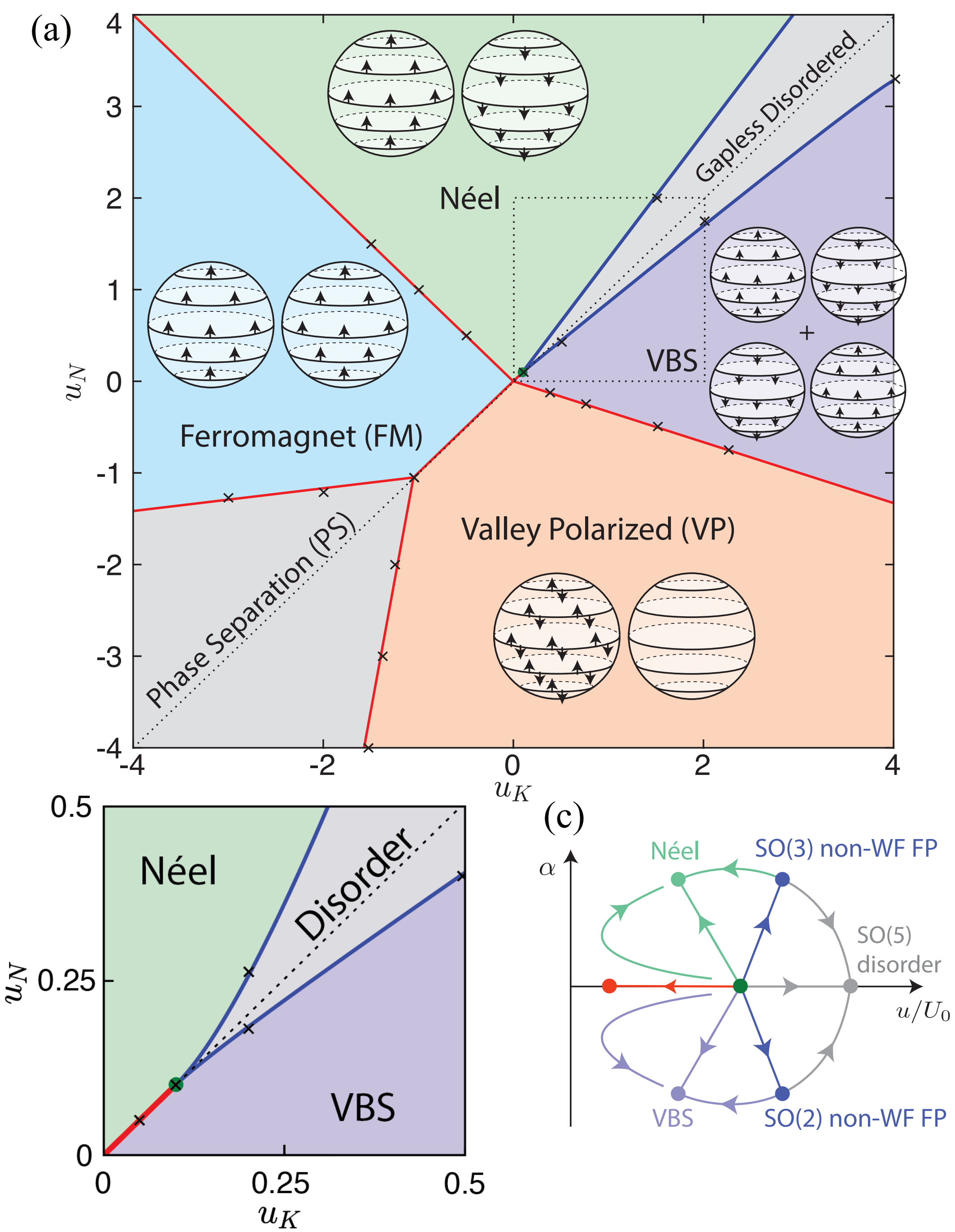
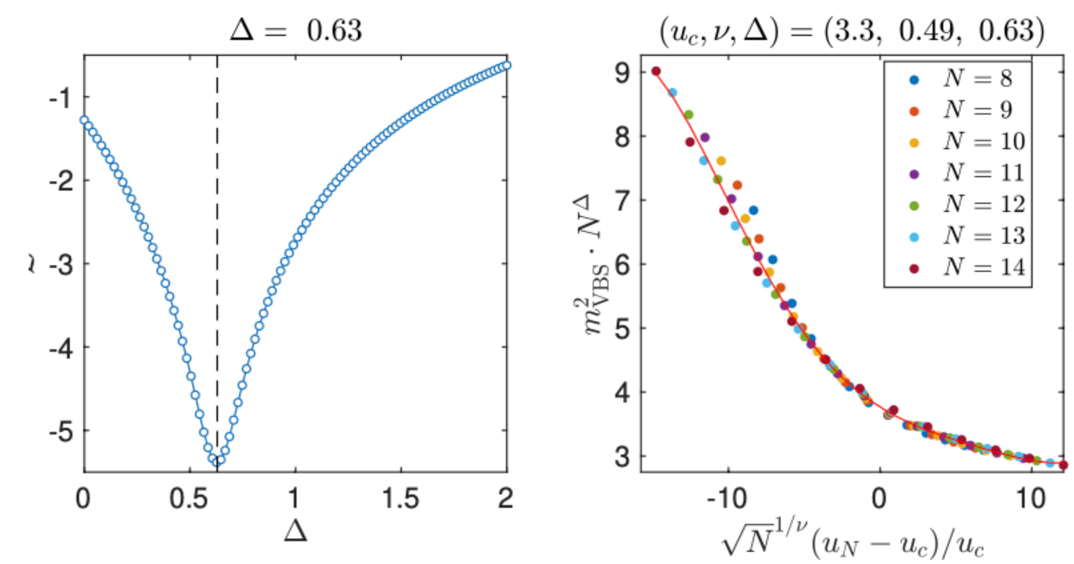



$$U_0 = 1,\ u_1 = u_2 = u_K, u_3 = u_4 = u_5 = u_N$$

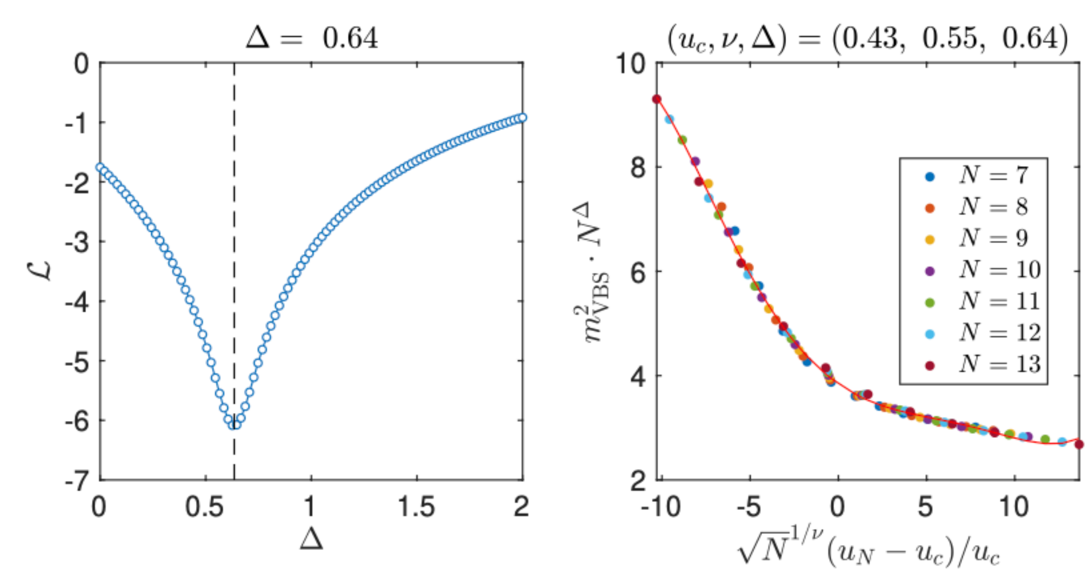
$$\langle O_i \rangle = \int d\Omega \psi^\dagger(\Omega)\Gamma^i \psi(\Omega) = \sum_{m=-s}^{s} c_m^\dagger \Gamma^i c_m$$

$$m_{VBS}^2 = \frac{1}{2N^2}\langle (O_1^2 + O_2^2)\rangle \qquad m_{Neel}^2 = \frac{1}{3N^2}\langle (O_3^2 + O_4^2 + O_5^2)\rangle$$

$u_K = 4$

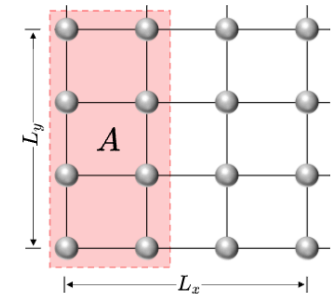



$u_K = 0.5$

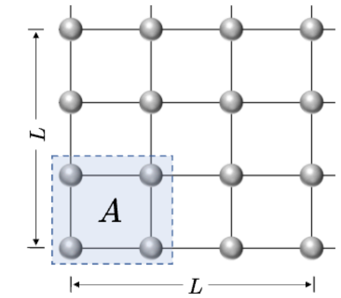


$$S_A(l) = al - s\ln(\frac{l}{\epsilon}) - \gamma + O(1/l)$$

Smooth boundary, no log




Corner, log




| | | | |
|---|---|---|---|
| d=1 CFT | $S \sim c\ln(l)$ | Heisenberg chain, Luttinger liquid | DMRG |
| d=2 QCP | $S \sim al - s_C\ln(l) - \gamma$ | Wilson-Fisher O(N), GNY | QMC |
| SSB | $S \sim al - (s_G + s_C)\ln(l) - \gamma$ | Antiferromagnet, Superfluid | QMC |
| Topological order | $S \sim al - \gamma_{top}$ | Z2 top ord, Kitaev QSL | QMC |
| Fermi surface | $S \sim l\ln(l) + al - \cdots$ | free fermion, interaction ? | |