Generic (fractional) quantum anomalous Hall crystals from interaction-driven band folding

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Model and band folding

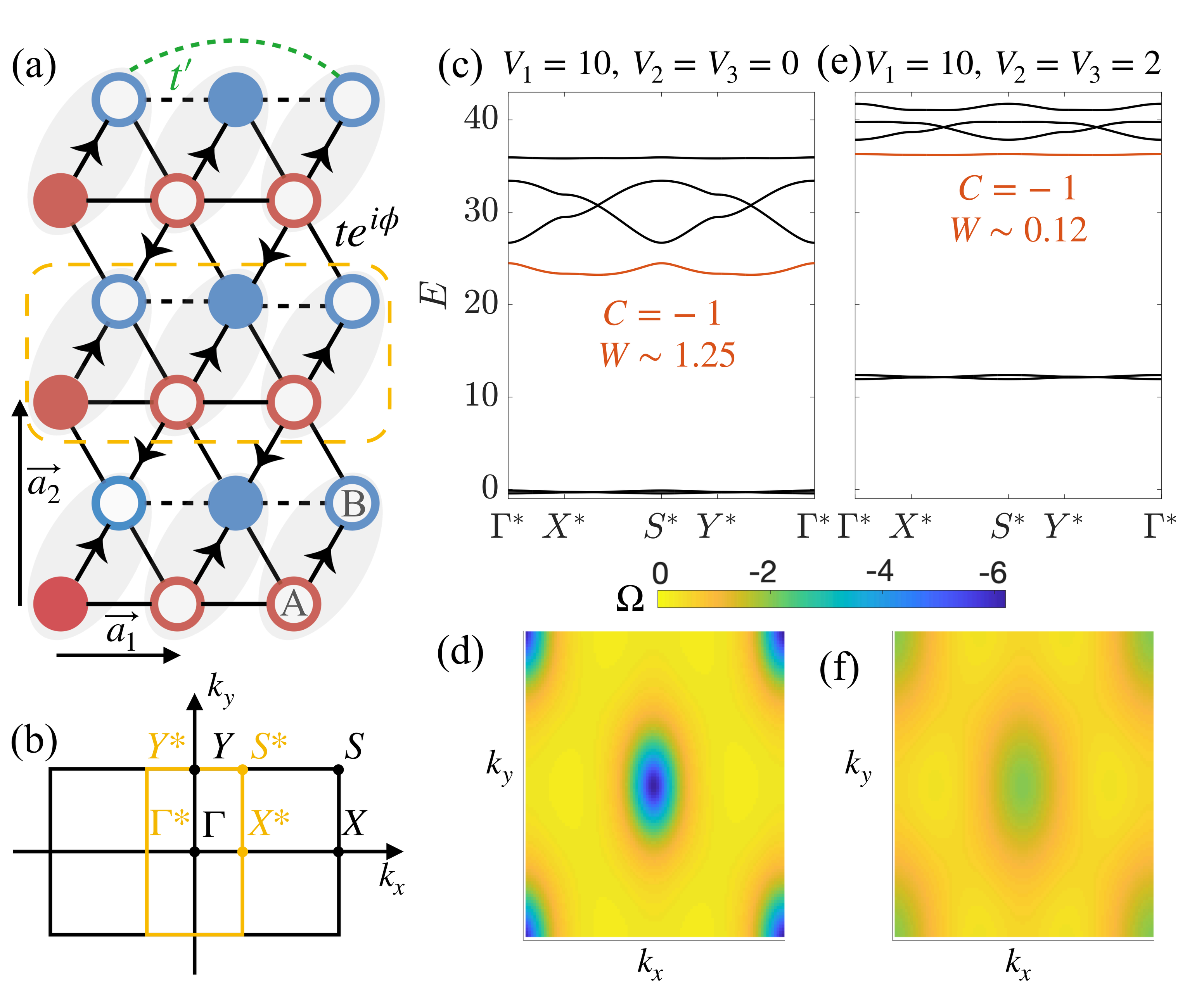

Fig 1: (a) The lattice model with two sites per unit cell and the A/B sublattices are labeled in red/blue. The NN hoppings ($te^{i\phi}$) are labeled by black lines and the phase ϕ is 0 on the solid lines, π on the dashed lines, $\pi/2$ along the arrows. The green dashed line is an example of the uniform NNNN hopping t'. The primitive vector of the original two-orbital unit cell is $\vec{a}_1 = (1, 0)$ and $\vec{a}_2 = (0, \sqrt{3})$. The filled/empty sites refers to a pattern of the interaction-driven CDW order (will be discussed later), and the yellow dashed box refers to the corresponding enlarged (tripled) unit cell. (b) The BZ for the original unit cell and the folded BZ* due to the CDW illustrated in panel (a). The renormalized band structures in the folded BZ* are shown in (c) with only $V_1 = 10$ and in (e) with $V_1 = 10,\ V_2 = V_3 = 2$. The Chern number of the miniband above the CDW gap (labeled in orange) is $C = -1$ in both cases while the bandwidth differs a lot. (d,f) The Berry curvature Ω plotted in the folded BZ* with the corresponding interactions in panels (c,e) respectively.

Topological two-band model

- **Hamiltonian**

$$H = \sum_{\langle i,j \rangle} te^{i\phi_{ij}}(c_i^\dagger c_j + h.c.) + \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} t'(c_i^\dagger c_j + h.c.) + V_1 \sum_{\langle i,j \rangle} n_i n_j + V_2 \sum_{\langle\langle i,j \rangle\rangle} n_i n_j + V_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} n_i n_j, \tag{1}$$

where the nearest-neighbor (NN) hopping is complex with the fixed magnitude $t = 1$ and its phase ϕ_{ij} is illustrated in Fig.1 (a). The next to next nearest-neighbor (NNNN) hopping is uniform and we take $t' = 0.2$, which flattens the lower band. With these parameters, the Chern number of the upper/lower band is $C = \pm 1$, respectively. We will consider the NN, next nearest-neighbor (NNN), and NNNN repulsive interactions $V_1,\ V_2$, and V_3 in this work.

- **Some definitions**

ν is the filling of the original lower Chern band. $\nu^* = 3\nu - 2$ is the filling of the mini Chern band (orange in Fig.1 (c,3)) after folding. We have taken $\frac{e^2}{h} = 1$ for the Hall conductivities.

- **Method**

We mainly use the iDMRG and ED methods for the ground-state results and the exponential tensor renormalization group (XTRG) method for finite-temperature calculations.

Conclusions of results

- The interaction-driven CDW (by V_1) at $\nu = 2/3$ filling of the Chern band could triple the unit cell and fold the original BZ. Above the CDW gap, we find an isolated miniband with Chern number $C = -1$.
- Adding finite competing interactions (V_2 and V_3) would not immediately destroy the CDW order. On the contrary, it could largely flatten the energy dispersion and smoothen the quantum geometry of the miniband.
- Further doping the miniband would lead to a series of (F)QAHC states at (fractional)integer fillings of the miniband, and both the topological and CDW degeneracies contribute to the total ground-state degeneracies. The Hall conductivity of any FQAHC state is determined by the filling of the miniband (ν^*), not the filling of the original band (ν), in that, $|\sigma_\mathrm{H}| = \nu^* \neq \nu$ (for example: $|\sigma_\mathrm{H}| = \nu^* = \frac{1}{3}, \frac{1}{5}$ at $\nu = \frac{7}{9}, \frac{11}{15}$), while it is $|\sigma_\mathrm{H}| = \nu = \nu^* = 1$ for the QAHC state.
- Through the thermodynamic study of the $\sigma_\mathrm{H} = -\frac{1}{3}$ FQAHC state, we find the separation of the finite-temperature translation symmetry breaking (T_CDW) from the lower onset temperature of the incompressibility (T^*) of the FQAHC phase, and thus establish a compressible CDW phase at intermediate temperature range of $T^* < T < T_\mathrm{CDW}$.

(F)QAHC ground states

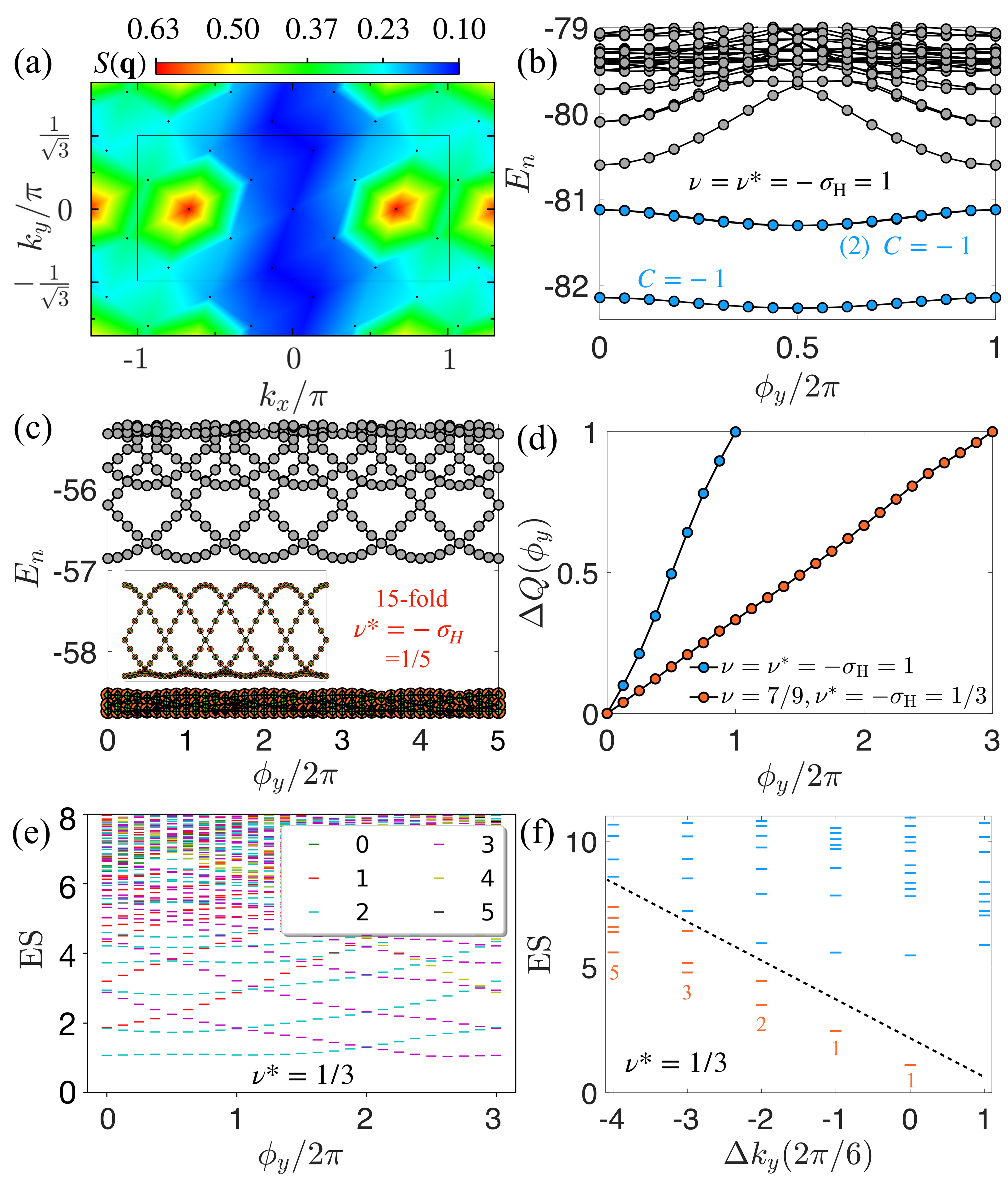

Fig 2: Fixed interactions $V_1 = 10,\ V_2 = V_3 = 2$ for these results. Panels (a-c) are from ED results of a 30-site torus. (a) Structure factor and (b) the spectral flow of the QAHC state at $\nu^* = \nu = 1$. The structure factor shows clear peaks at $(\pm\frac{2\pi}{3}, 0)$ and there are 3-fold (quasi)degenerate ground states. The Chern number of each ground state $C = -1$. (c) The spectral flow of the FQAHC state at $\nu = \frac{11}{15}$. There are totally 15 (quasi)degenerate ground states (zoomed in in the inset) and the Hall conductivity from each ground state is $\sigma_\mathrm{H} = \nu^* = -1/5$. Panels (d-f) are from iDMRG results of $N_y = 6$ cylinders. (d) Charge pumping of the $\sigma_\mathrm{H} = -1$ QAHC and the $\sigma_\mathrm{H} = -1/3$ FQAHC state. One electron is pumped after inserting 2π (6π) fluxes, respectively. (e) The entanglement spectrum (ES) of the $\sigma_\mathrm{H} = -1/3$ FQAHC state in different charge sectors. After inserting 6π fluxes, the ES is shifted by one charge sector. (f) The momentum-resolved ES of the $\sigma_\mathrm{H} = -1/3$ FQAHC state in the charge sector with the largest eigenvalue. The counting of states is consistent with the chiral edge modes of the Laughlin states.

Thermodynamics of the FQAHC state

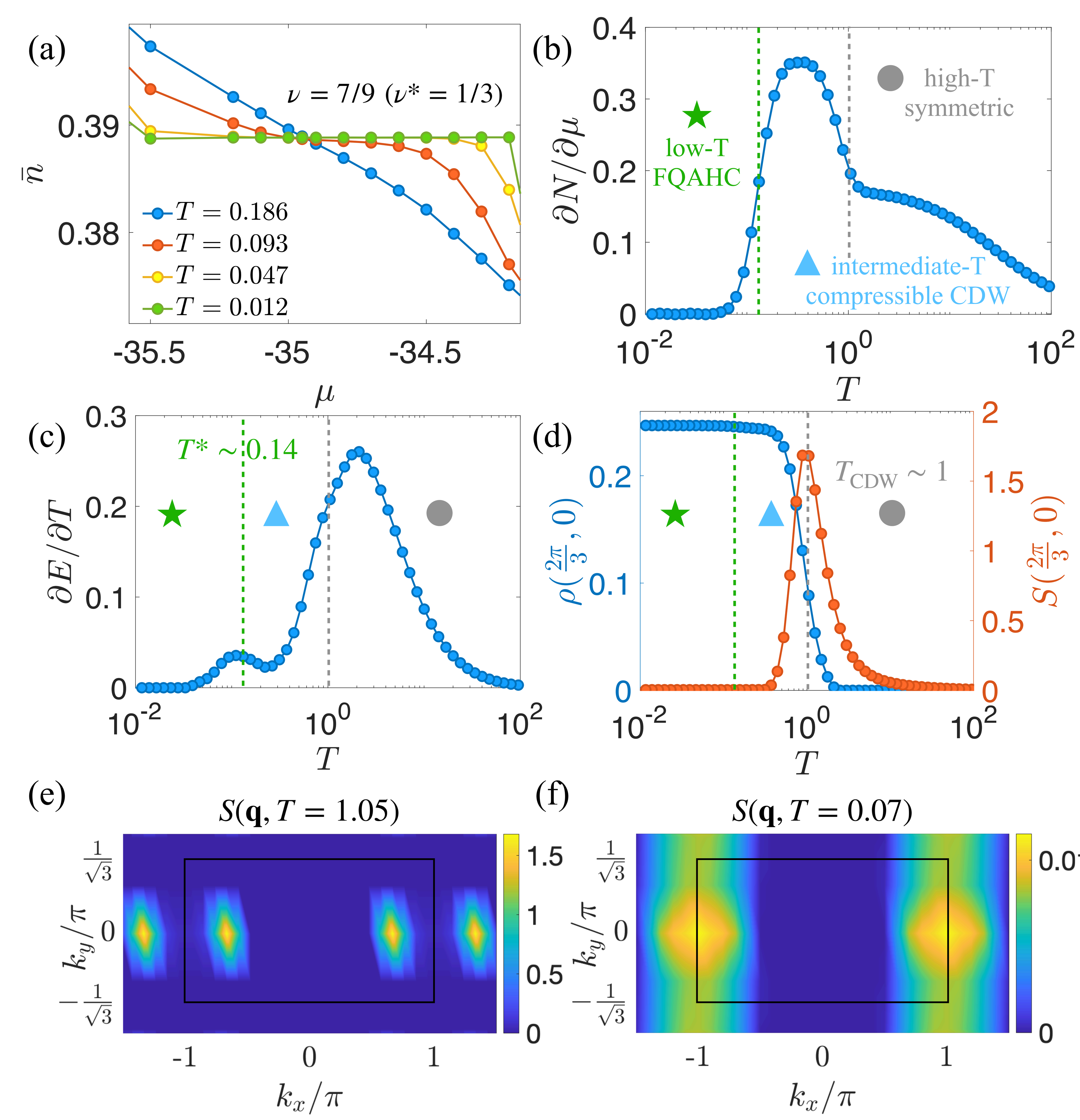

Fig 3: Results obtained from XTRG of a $3 \times 18 \times 2$ cylinder with $V_1 = 10,\ V_2 = V_3 = 2$: (a) The average density as a function of chemical potential at different temperatures, focusing on the plateaus at $\nu = 7/9$. The (b) compressibility, (c) specific heat and (d) CDW order $\rho(\frac{2\pi}{3}, 0)$ and CDW fluctuations $S(\frac{2\pi}{3}, 0)$ as functions of temperature. The green star, blue triangle, and gray circle refer to the low-T FQAHC, intermediate-T compressible CDW, and the high-T symmetric phases, respectively. The green/gray dashed line refers to the onset temperature of the incompressibility and the CDW transition temperature, respectively. The structure factors in the original BZ at different temperatures are shown in panels (e,f).

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