Quantum Entanglement & Fractional Chern Insulator

Recent Topics in

Quantum Many-Body Computation

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https://quantummc.xyz/

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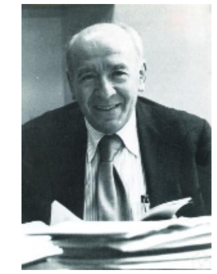
CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

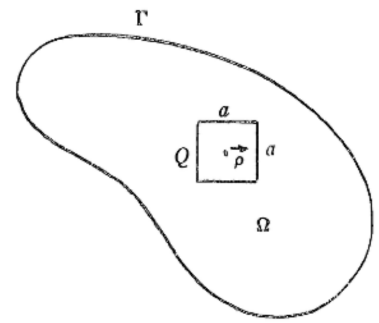
To George Eugene Uhlenbeck on the occasion of his sixty-fifth birthday



Mark Kac, Polish American mathematician 1914 - 1984

Eigenvalues of Dirichlet problem for Laplacian

Am. Math. Mon. 73, 1 (1966)



$$\tfrac{1}{2}\nabla^2 U + \lambda U = 0 \quad \text{in} \quad \Omega,$$
$$U = 0 \quad \text{on} \quad \Gamma.$$

Volume — Length of circumference — Number of holes

$$\sum_{n=1}^{\infty} e^{-\lambda_n t} \sim \frac{|\Omega|}{2\pi t} - \frac{L}{4}\frac{1}{\sqrt{2\pi t}} + (1-r)\,\tfrac{1}{6}$$

FINITE-SIZE DEPENDENCE OF THE FREE ENERGY IN TWO-DIMENSIONAL CRITICAL SYSTEMS

John L. CARDY and Ingo PESCHEL*

Nucl. Phys. B 300, 377 (1988)

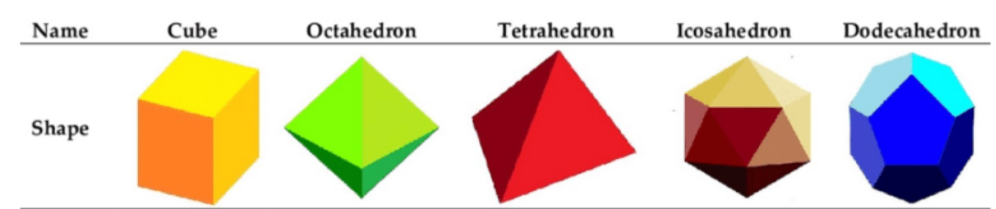
Conformal anomaly number (central charge)

$$F = f_b|A| + f_s L - \frac{1}{6} c\chi \ln L + O(1)$$

Euler characteristic

Platonic solids: homeomorphic to sphere

$$\chi = V - E + F = 2$$



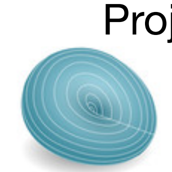
sphere / polyhedron

$\chi = 2$

torus / cylinder / annuls

$\chi = 2 - 2g = 0$

Projective plane / disc



$\chi = 1$

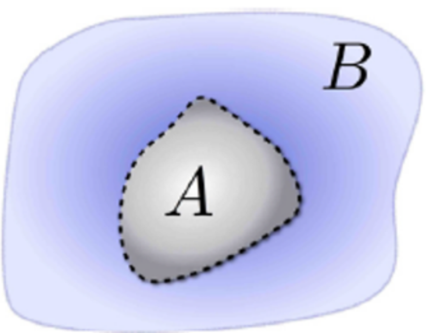
Klein bottle / moebius

$\chi = 0$

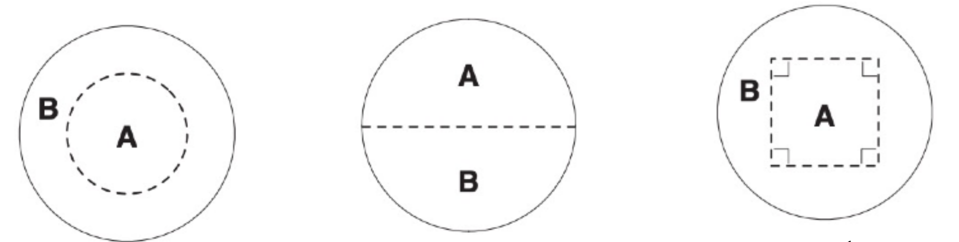
Entanglement Entropy of 2D Conformal Quantum Critical Points:
Hearing the Shape of a Quantum Drum

Eduardo Fradkin[1] and Joel E. Moore[2,3]

Phys. Rev. Lett. 97, 050404 (2006)



$$S = F_A + F_B - F_{A\cup B}$$



$S_{\ln} = 0$ $S_{\ln} = -\frac{1}{4} c \ln(L)$ $S_{\ln} = -\frac{1}{9} c \ln(L)$

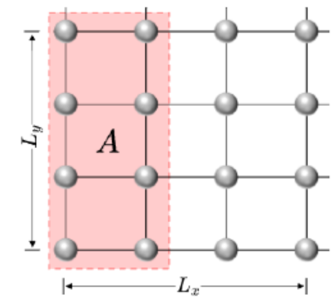
central charge

$$S = 2f_s L - \frac{1}{6} c \underbrace{(\chi_A + \chi_B - \chi_{A\cup B})}\ln(L) + O(1)$$

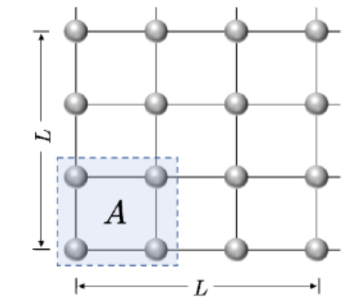
Geometric properties of the partition

$$S_A(l) = al - s\ln(\frac{l}{\epsilon}) - \gamma + O(1/l)$$

Smooth boundary, no log



Corner, log

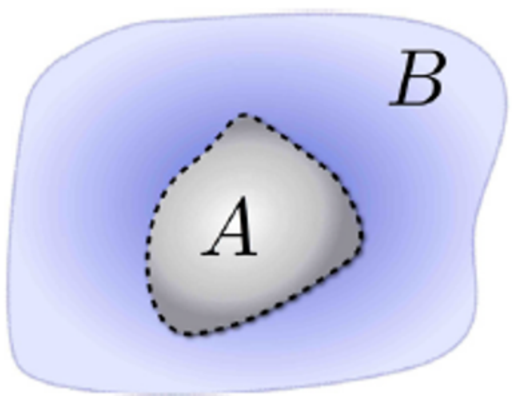


d=1 CFT	$S \sim c\ln(l)$	Heisenberg chain, Luttinger liquid	DMRG
d=2 QCP	$S \sim al - s_C\ln(l) - \gamma$	Wilson-Fisher O(N), GNY	QMC
SSB	$S \sim al - (s_G + s_C)\ln(l) - \gamma$	Antiferromagnet, Superfluid	QMC
Topological order	$S \sim al - \gamma_{top}$	Z2 top ord, Kitaev QSL	Toy model, QMC
Fermi surface	$S \sim l\ln(l) + al - \cdots$	free fermion, interaction ?	

Entanglement entropy and quantum field theory

Pasquale Calabrese[1,3] and John Cardy[1,2]

J. Stat. Mech. (2004) P06002



$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B\, \rho$$

$$S_A = -\,\mathrm{Tr}_A\, \rho_A \ln(\rho_A)$$

$$S_A^{(n)} = \frac{1}{1-n} \ln(\mathrm{Tr}_A(\rho_A^{(n)}))$$

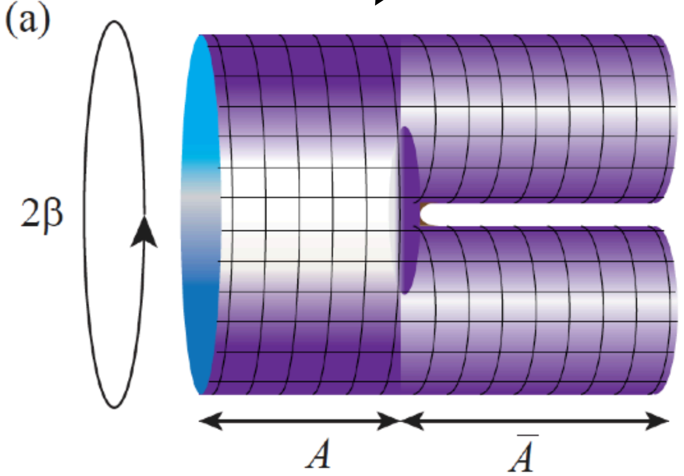
“ discuss entropy in terms of the **Euclidean path integral** on an n-sheeted Riemann surface. ”

$$S_A^{(2)} = -\ln(\mathrm{Tr}_A(\rho_A^{(2)})) = -\ln\left(\frac{Z_A^{(2)}}{Z_\varnothing^2}\right) = \beta(F(Z_A^{(2)}) - F(Z_\varnothing^{(2)}))$$

“ Renyi **EE** is **the difference in free energy** between partition functions with different trace topologies ” (in equilibrium)

“ Qiu Ku is the $Z_A^{(2)}$ ”

(a)



(b)

- Hastings, González, Kallin, Melko, PRL 104, 157201 (2010).
- Isakov, Hastings, Melko, Nat. Phys. 7, 772 (2011).
- Humeniuk, Roscilde, PRB 86, 235116 (2012).
- Kallin, Stoudenmire, Fendley, Singh, Melko, J. Stat. Mech. (2014) P06009
- Helmes and Wessel, PRB 89, 245120 (2014).
- Kulchytskyy, Herdman, Inglis, Melko, PRB 92, 115146 (2015).
- ……

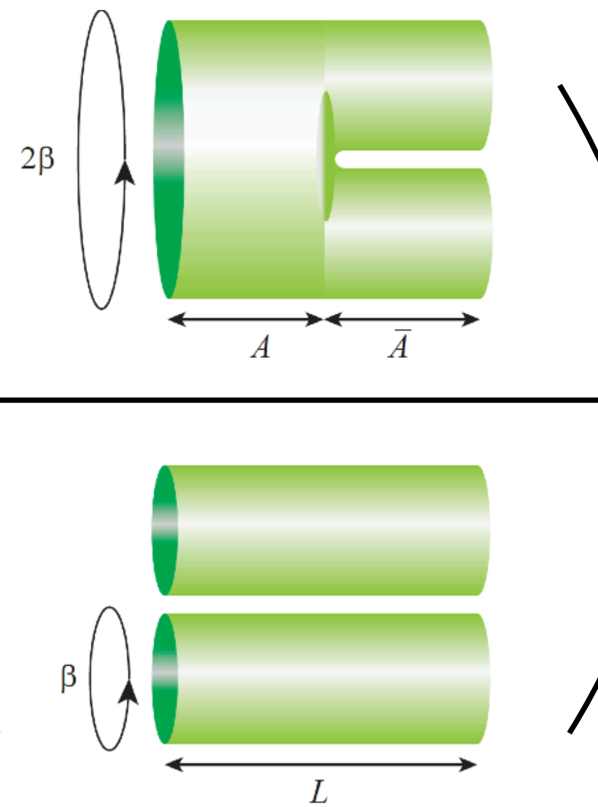
- Grover, PRL 111, 130402 (2013).
- Assaad, Lang, Toldin, PRB 89, 125121 (2014).
- Broecker, Trebst, PRB 94, 075144 (2016).
- ……

Entanglement entropy with incremental (Qiu Ku) method

$$S_A^{(2)} = -\ln(\mathrm{Tr}_A(\rho_A^2)) = -\ln\left(\frac{Z_A^{(2)}}{Z_\varnothing^{(2)}}\right) = \beta(F(Z_A^{(2)}) - F(Z_\varnothing^{(2)}))$$

- V. Alba, PRE 95, 062132 (2017)
- J. D'Emidio, PRL 124, 110602 (2020)
- J. Zhao, …, M. Cheng, ZYM, PRL 128, 010601 (2022)
- J. Zhao, …, M. Cheng, ZYM, npj Quantum Materials 7, 69 (2022)

$$= -\ln\left(\frac{\text{[figure]}}{}\right)$$

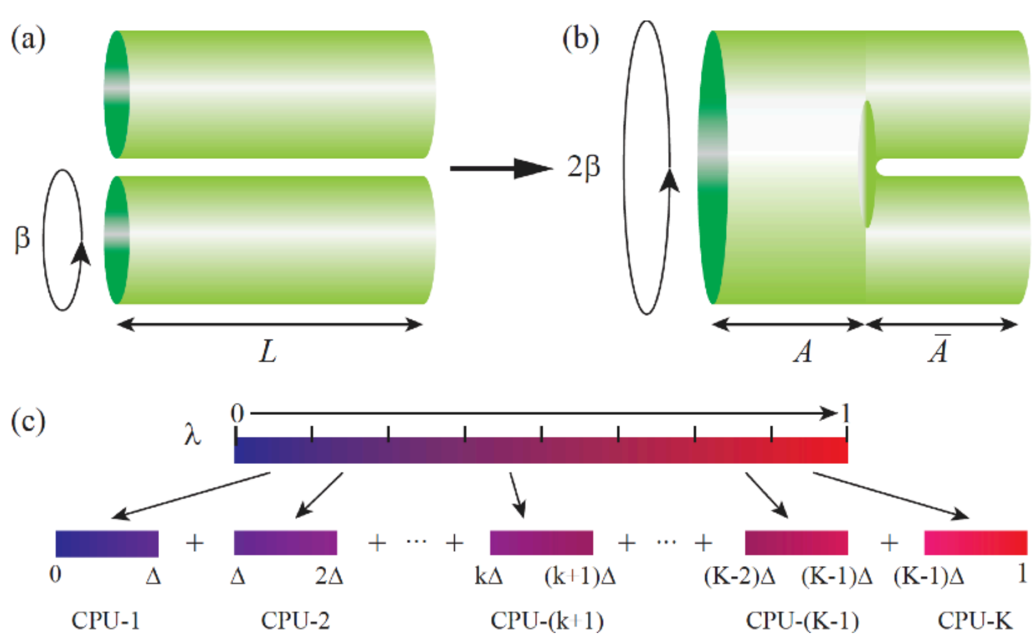


$$= -\int_0^1 d\lambda \frac{\partial \ln Z(\lambda)}{\partial \lambda} = -\sum_{k=1,2,\cdots,N_\lambda} \int_{(k-1)\Delta}^{k\Delta} d\lambda \frac{\partial \ln Z(\lambda_k)}{\partial \lambda}$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$-\ln(\langle e^{-\beta W_A^{(2)}} \rangle) \qquad - \sum_{k=1,2,\cdots,N_\lambda} \ln(\langle e^{-\beta W_{k,A}^{(2)}} \rangle)$$

$$Z(\lambda = 0) = Z_\varnothing^{(2)} \qquad Z(\lambda = 1) = Z_A^{(2)}$$



$$e^{-S_A^{(2)}} = \frac{Z(1)}{Z(0)} := \frac{Z(\lambda_1)}{Z(0)} \frac{Z(\lambda_2)}{Z(\lambda_1)} \cdots \frac{Z(\lambda_k)}{Z(\lambda_{k-1})} \cdots \frac{Z(1)}{Z(\lambda_{N_\lambda - 1})}$$

- G. Pan, Y. D. Liao, J. D'Emidio, ZYM, PRB 108, L081123 (2023)
- J. D'Emidio, et al., PRL 132, 076502 (2024)
- Y.D. Liao, G.Pan, W. Jiang, Y. Qi, ZYM, arXiv:2302.11742

Controllable Incremental Algorithm for Entanglement Entropy in Quantum Monte Carlo Simulations

arXiv:2307.10602

Yuan Da Liao[1,2]

An integral algorithm of exponential observables for interacting fermions in quantum Monte Carlo simulation

PRB 109, 205147 (2024)

Xu Zhang,[1] Gaopei Pan,[1] Bin-Bin Chen,[1] Kai Sun,[2,*] and Zi Yang Meng[1,†]

$$S_A^{(2)} = -\ln(\mathrm{Tr}_A(\rho_A^{(2)})) = -\ln\left(\frac{Z(\lambda=1)}{Z(\lambda=0)}\right) = -\ln\left(\frac{\sum_{s_1,s_2} P_{s_1}P_{s_2}\ \mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2})}{\sum_{s_1,s_2} P_{s_1}P_{s_2}}\right)$$

Entanglement partition function

$$Z(\lambda) = \sum_{s_1,s_2} P_{s_1}P_{s_2}(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))^{\lambda}$$

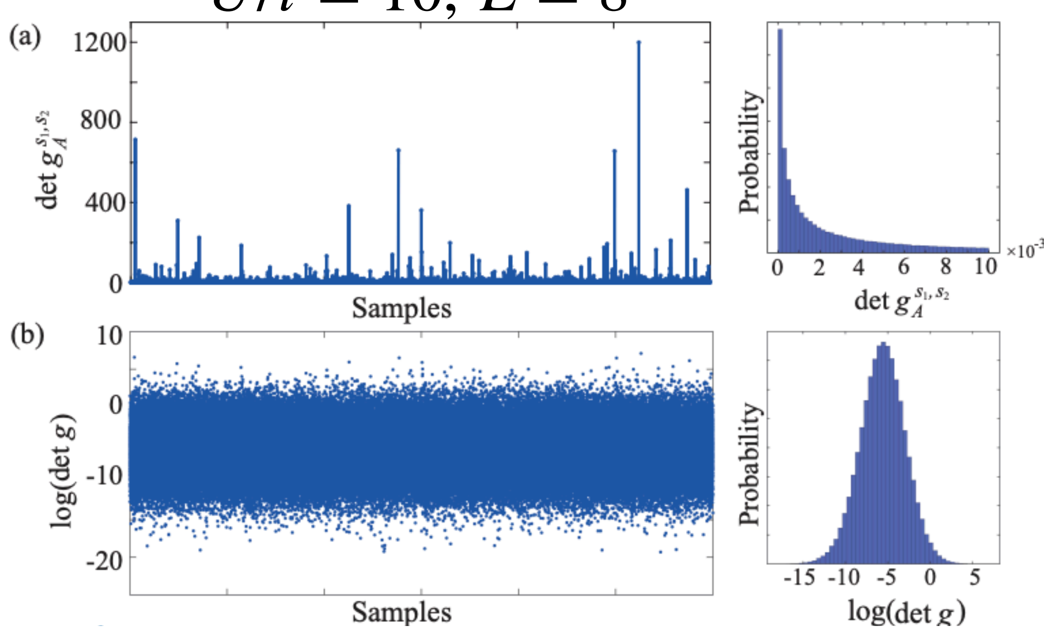
$$= -\int_0^1 d\lambda \frac{\partial \ln(Z(\lambda))}{\partial \lambda} = -\int_0^1 d\lambda \frac{\sum_{s_1,s_2} P_{s_1}P_{s_2}(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))^{\lambda}\ \ln(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))}{\sum_{s_1,s_2} P_{s_1}P_{s_2}(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))^{\lambda}} = -\langle \ln(\mathrm{Tr}(\rho_{A,s_1}\rho_{A,s_2}))\rangle_{s_1,s_2,\lambda}$$

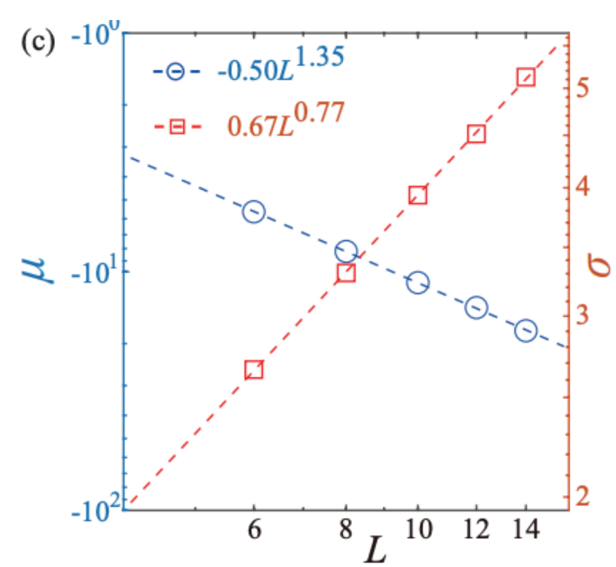
$$\frac{da^{\lambda}}{d\lambda} = \frac{de^{\lambda\ln(a)}}{d\lambda} = a^{\lambda}\ln(a)$$

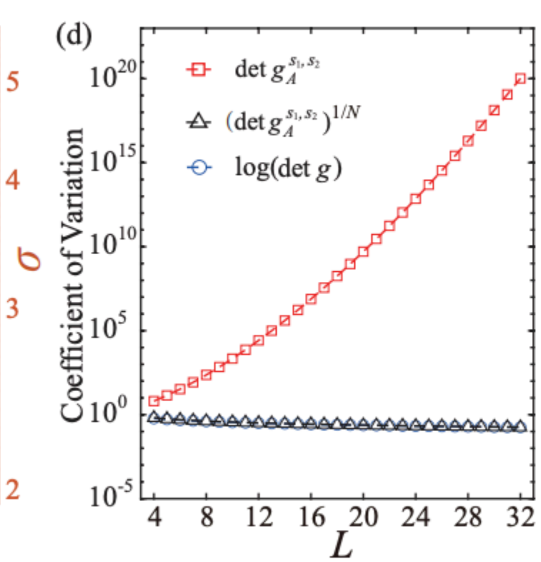
Convert
exponential complexity
to
polynomial complexity

$$CV[x] = \frac{SD[x]}{E[x]} = \sqrt{e^{\sigma^2}-1}$$

$U/t = 10,\ L = 8$







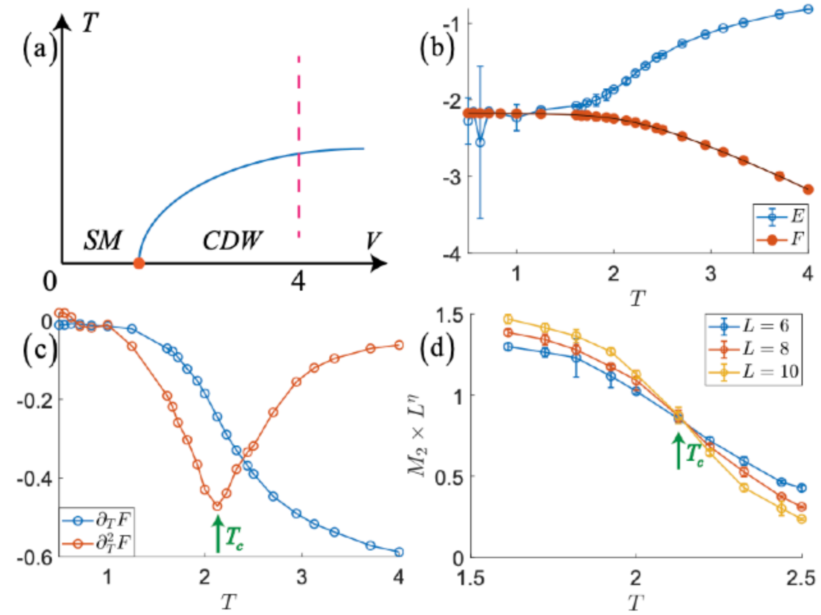
An integral algorithm of exponential observables for interacting fermions in quantum Monte Carlo simulation

PRB 109, 205147 (2024)

Xu Zhang,[1] Gaopei Pan,[1] Bin-Bin Chen,[1] Kai Sun,[2,*] and Zi Yang Meng[1,†]

$$S_A^{(n)} = -\frac{1}{n-1}\ln(\mathrm{Tr}(\rho_A^n)) = -\frac{1}{n-1}\int_o^1 d\lambda \frac{\sum P_s^n \mathrm{Tr}(\rho_{A,s}^n)^\lambda \ \ln(\mathrm{Tr}(\rho_{A,s}^n))}{\sum P_s^n \mathrm{Tr}(\rho_{A,s}^n)^\lambda} = -\frac{1}{n-1}\langle \ln(\mathrm{Tr}(\rho_{A,s}^n))\rangle_{s_1,s_2,\cdots,s_n,\lambda}$$

$$F = -\frac{1}{\beta}\ln(Z) = -\frac{1}{\beta}\int_0^1 d\lambda \frac{\sum P_s^\lambda \ \ln(P_s)}{\sum P_s^\lambda} = -\frac{1}{\beta}\langle \ln(P_s)\rangle_{s,\lambda}$$

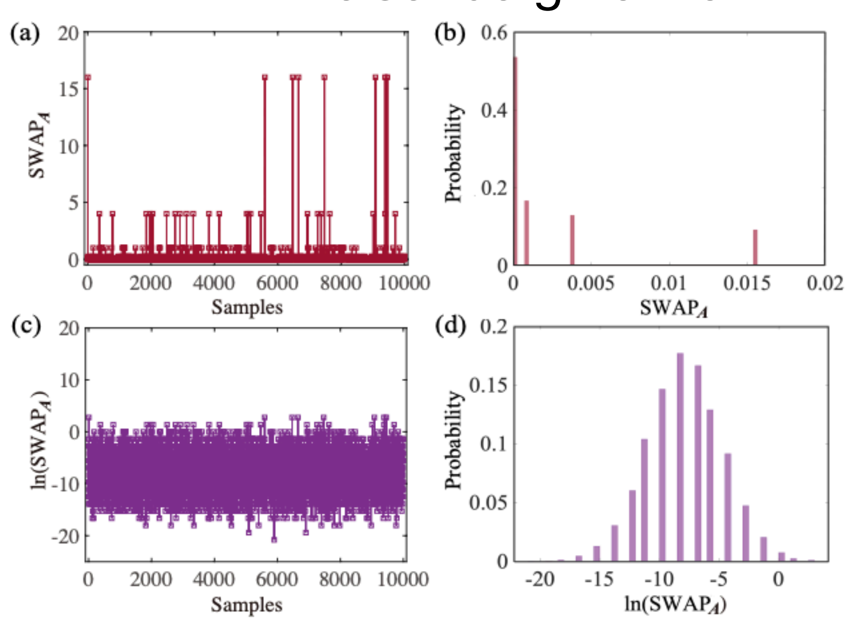


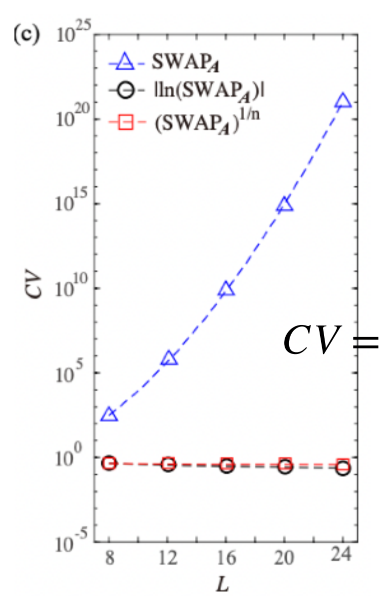
Incremental SWAP Operator for Entanglement Entropy: Application for Exponential Observables in Quantum Monte Carlo Simulation

Xuan Zhou,[1,2] Zi Yang Meng,[3] Yang Qi,[1,2,4,*] and Yuan Da Liao[1,2,3,†]

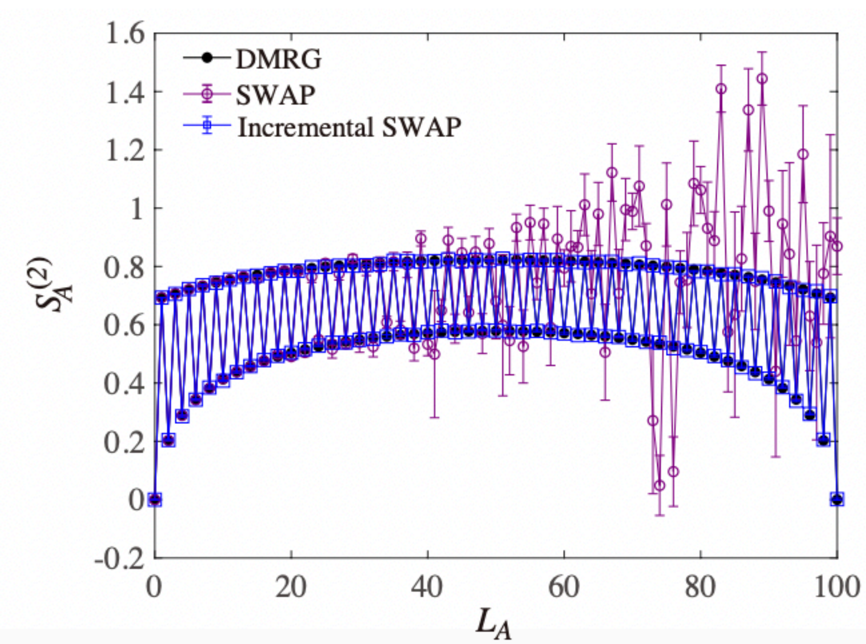
PRB 109, 165106 (2024)

AF Heisenberg 10x10



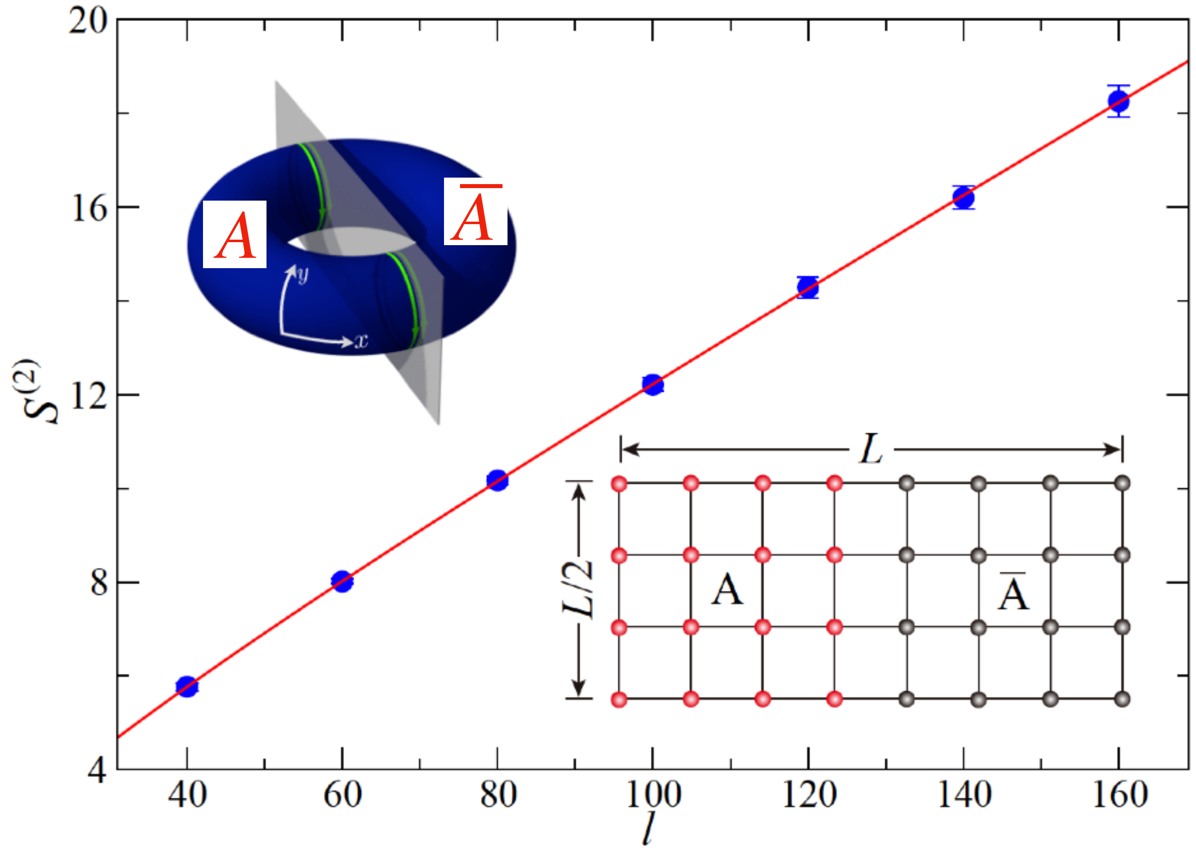


$$CV = \frac{\sigma_{swap}}{\mu_{swap}} = \sqrt{e^{\sigma^2} - 1}$$



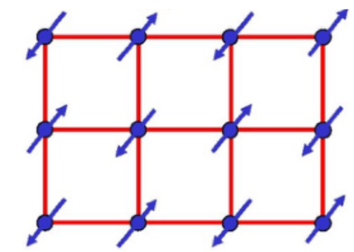
Spontaneous symmetry breaking phases: smooth boundary

- J. Zhao, B.-B Chen, Y.-C. Wang, Z. Yan, M. Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)



Square lattice Heisenberg model

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Smooth boundary $S_A^{(2)}(l) = al - s\ln(l) + c$

$$S_A^{(2)}(l) = 0.092(1)l + 1.0(1)\ln(l) - 1.63(3)$$

$l \in [40,\ 160]$

$$s = -\frac{N_G}{2}$$

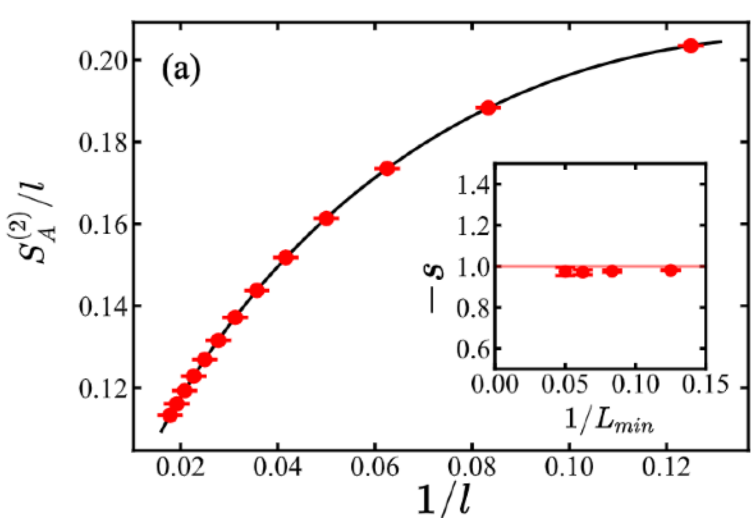
- Metlitski & Grover, arXiv:1112.5166

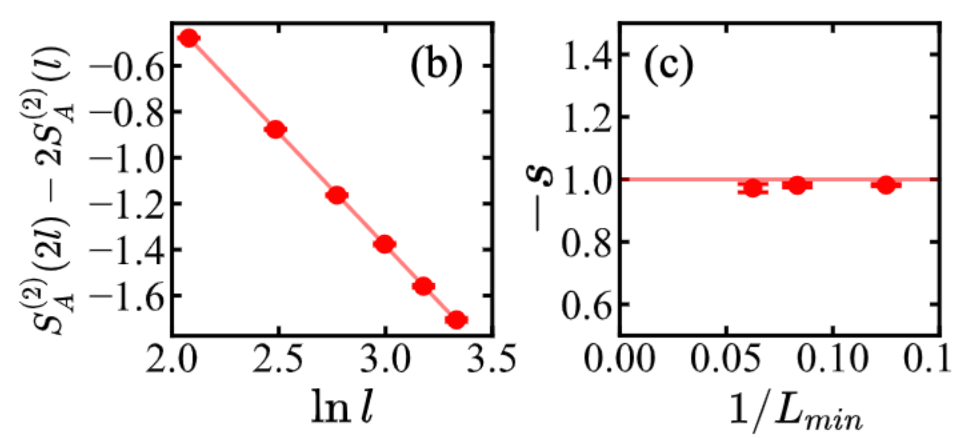
$$\frac{S_A^{(2)}(l)}{l} = s\frac{\ln(1/l)}{l} + \frac{c}{l} + a$$

$$\frac{d^2y}{dx^2} = \frac{s}{x}$$

$s < 0$

concave





Subtracted EE

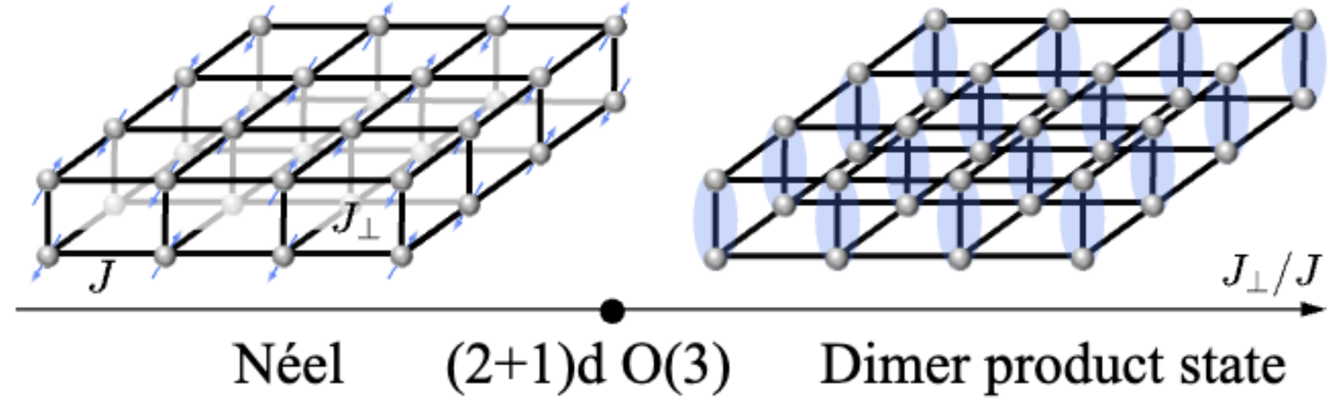
$$S^s(l) = S_A(2l) - 2S_A(l)$$

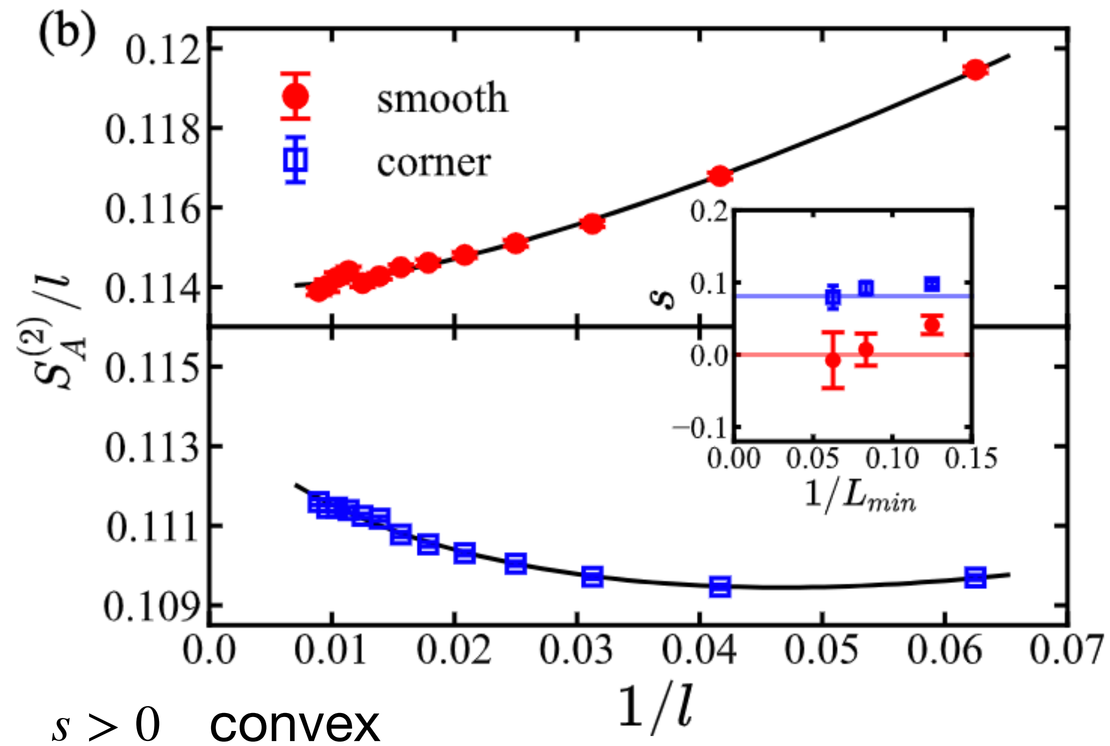
$$S^s(l) = s\ln(l) - c$$

(2+1)d O(3) quantum critical points: smooth & corner


- J. Zhao, B.-B Chen, Y.-C. Wang, Z. Yan, M. Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)

$$H = J\sum_{\langle i,j\rangle}(\mathrm{S}_{i,1}\cdot\mathrm{S}_{j,1}+\mathrm{S}_{i,2}\cdot\mathrm{S}_{i,2}) + J_\perp\sum_i \mathrm{S}_{i,1}\cdot\mathrm{S}_{i,2}$$





$s > 0$ convex

$$\frac{S_A^{(2)}(l)}{l} = a + \frac{c}{l} + \frac{b}{l^2} \qquad \frac{1}{l} \text{ finite size correction}$$

$$\frac{S_A^{(2)}(l)}{l} = a - s\frac{\ln l}{l} + \frac{c}{l}$$

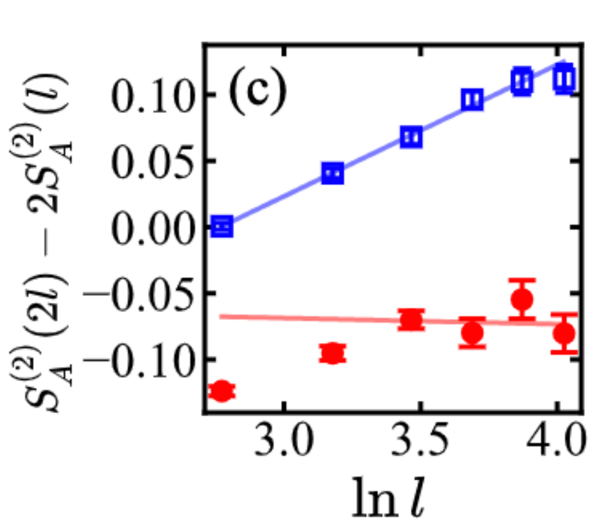
Subtracted EE

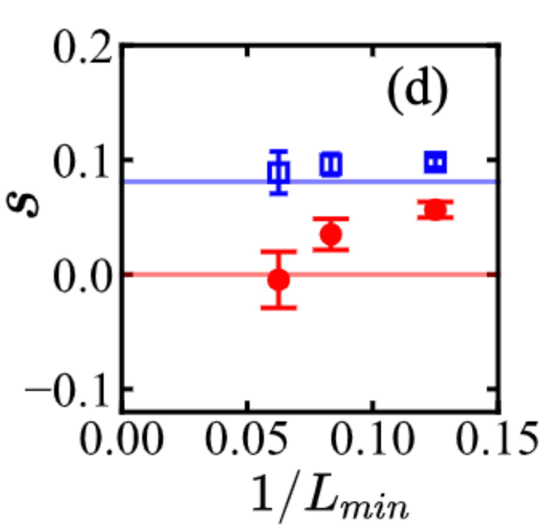
$$S^s(l) = S_A(2l) - 2S_A(l)$$

$$S^s(l) = -\frac{3b}{2l} - c$$

$$S^s(l) = s\ln(l) - c$$

	s with $L_{\min}=8$	s with $L_{\min}=16$	χ^2/k	
			$\ln l$	$1/l$
O(3), smooth	0.056(7)	−0.004(24)	2.38	1.30
O(3), corner	0.098(4)	0.088(18)	0.23	2.66





- A. Kallin, et. al, J. Stat. Mech. P06009 (2014)
- J. Helmes, S. Wessel, Phys. Rev. B 89, 245120 (2014)

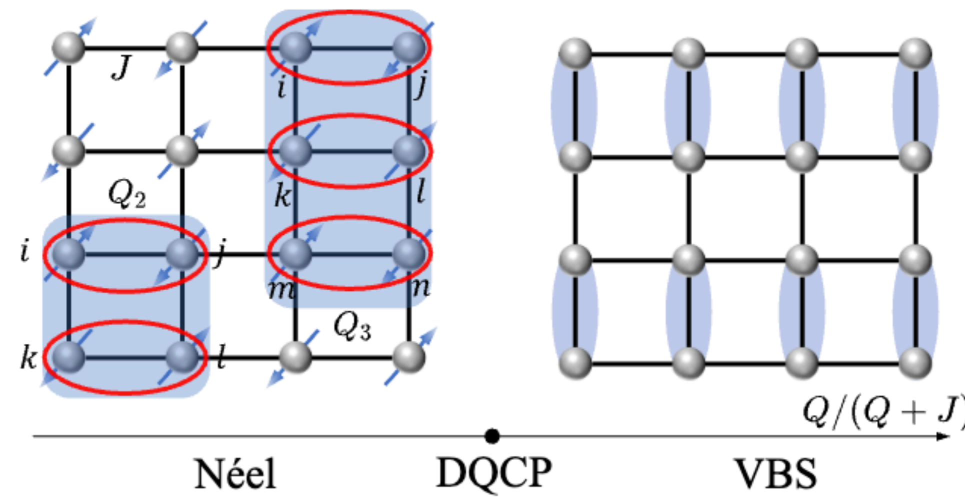
$$s_C = 0.07(2)$$

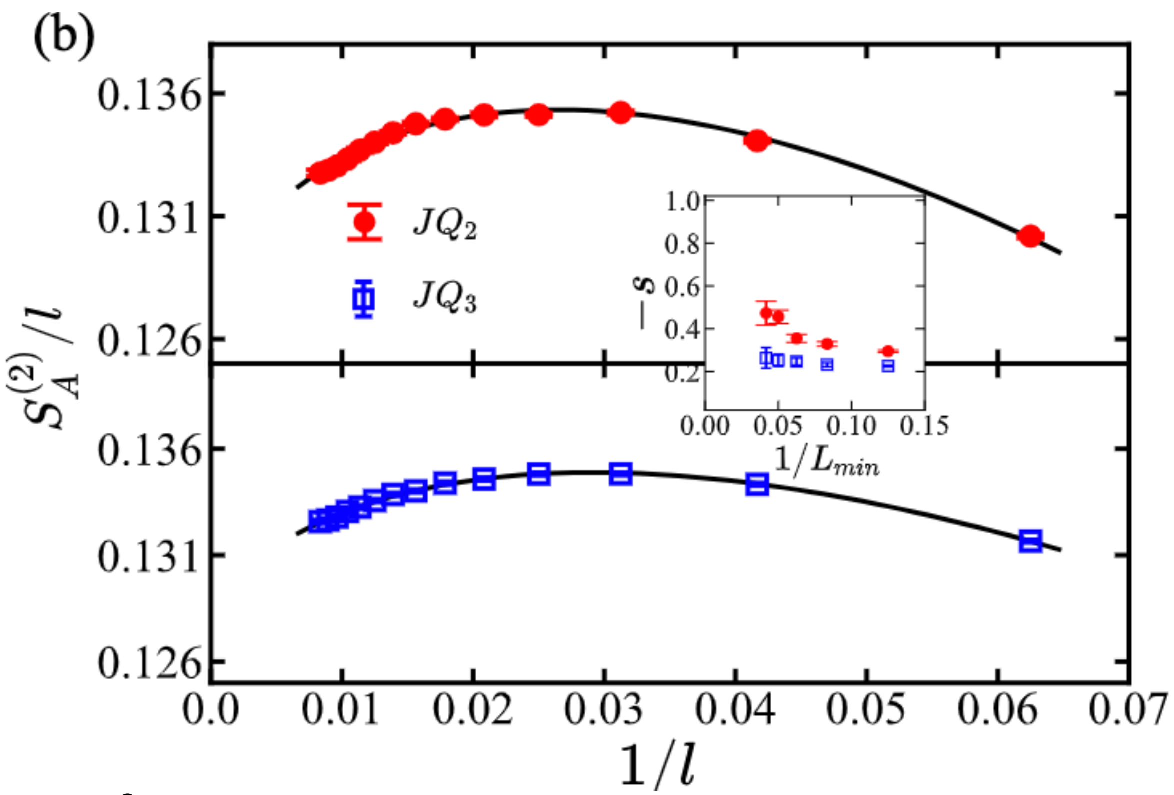
Deconfined quantum critical points: Smooth boundary

M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)

JQ2 model: $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q\sum_{\langle ijkl\rangle} P_{ij}P_{kl}$

JQ3 model: $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q\sum_{\langle ijklmn\rangle} P_{ij}P_{kl}P_{mn}$





$$\frac{S_A^{(2)}(l)}{l} = a + \frac{c}{l} + \frac{b}{l^2}$$

$\frac{1}{l}$ finite size correction

$$\frac{S_A^{(2)}(l)}{l} = a - s\frac{\ln l}{l} + \frac{c}{l}$$

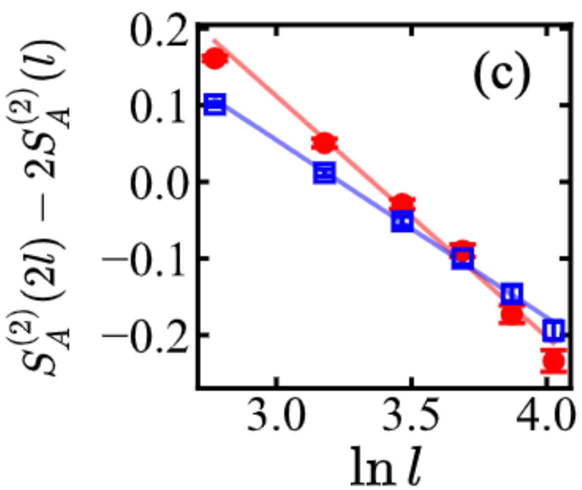
Subtracted EE

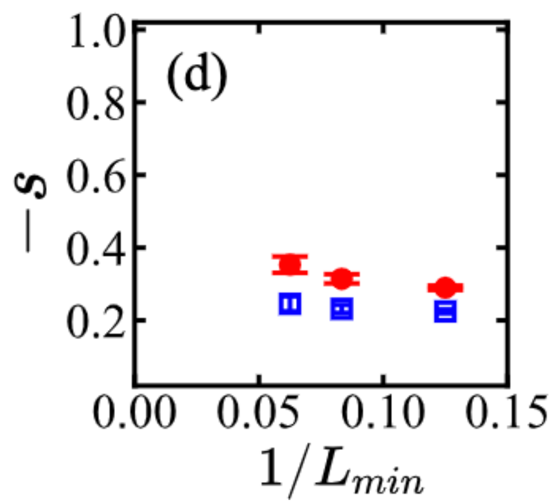
$$S^s(l) = S_A(2l) - 2S_A(l)$$

$$S^s(l) = -\frac{3b}{2l} - c$$

$$S^s(l) = s\ln(l) - c$$

$s < 0$ concave





	s with $L_{\min}=8$	s with $L_{\min}=16$	χ^2/k	
			$\ln l$	$1/l$
J-Q_2, smooth	−0.289(6)	−0.35(2)	3.38	25.2
J-Q_3, smooth	−0.224(5)	−0.24(2)	0.49	13.1

$-s > 0$ **Not a CFT, behave like Goldstone mode.**

Deconfined quantum critical points: Smooth boundary


M. Song, J. Zhao, ZYM, C. Xu, M. Cheng, SciPost Phys. 17, 010 (2024)

JQ2 model: $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q\sum_{\langle ijkl\rangle} P_{ij}P_{kl}$

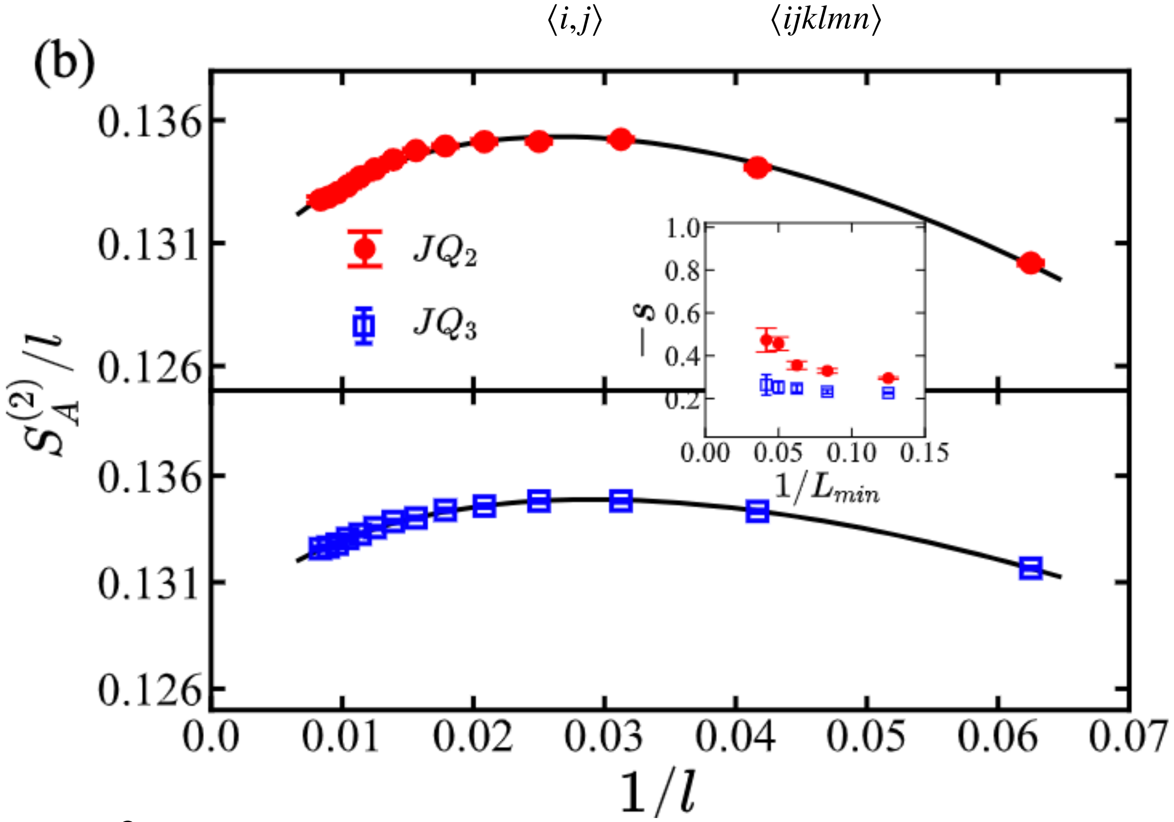
JQ3 model: $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q\sum_{\langle ijklmn\rangle} P_{ij}P_{kl}P_{mn}$

$$S_A(L) = aL + \frac{n_G}{2}\ln\left[(\rho_s(L)L)^{1/2}(I(L)L)^{1/2}\right] + c. \quad (20)$$

$$\rho_s(L) = \rho_s + \frac{u}{L} + \cdots \qquad I(L) = I + \frac{\nu}{L} + \cdots \qquad when \quad L \gg \frac{u}{\rho_s}, \frac{\nu}{I}$$

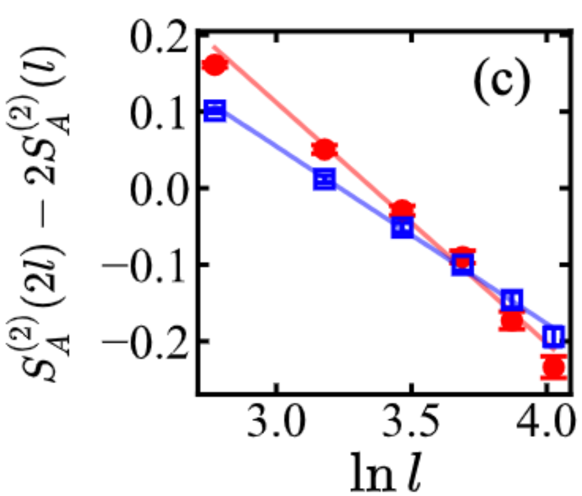
$$S_A(L) = aL + \underline{\frac{n_G}{2}}\ln L + c' + O\left(\frac{1}{L}\right). \qquad (21)$$

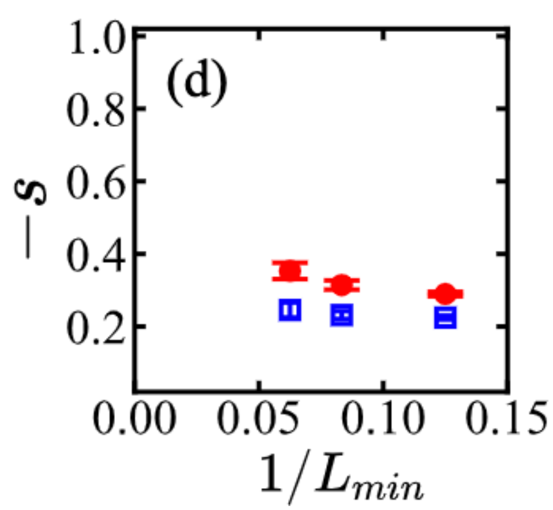
$$\rho_s(L)L \sim I(L)L \sim L^x \quad with \quad x \neq 1$$



will instead becomes $\frac{n_G}{2}x$. This kind of unconventional finite-size scaling has been indeed observed in the J-Q_2 model [92] where both $\rho_s(L)L$ and $I(L)L$ scale as $L^{0.285}$ at the DQCP. It would then give $s = -0.285$ (for $n_G = 2$) when L is big enough, which is close to the value found in this work. It would be worth investigating this picture further in the future, as well as other possible explanations of the logarithmic correction.

$s < 0$ concave





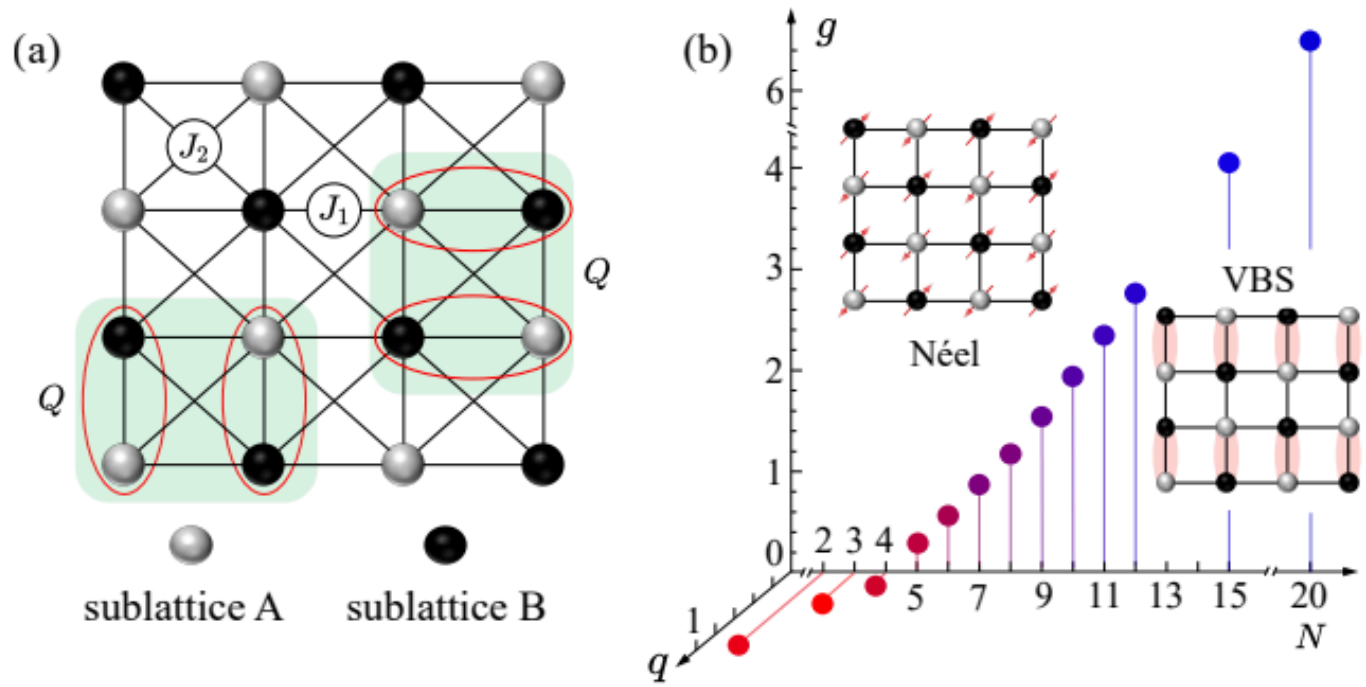
	s with $L_{\min} = 8$	s with $L_{\min} = 16$	χ^2/k	
			$\ln l$	$1/l$
J-Q_2, smooth	−0.289(6)	−0.35(2)	3.38	25.2
J-Q_3, smooth	−0.224(5)	−0.24(2)	0.49	13.1

$-s > 0$ **Not a CFT, behave like Goldstone mode.**

Deconfined quantum criticality lost

Menghan Song,[1] Jiarui Zhao,[1] Meng Cheng,[2] Cenke Xu,[3] Michael M. Scherer,[4] Lukas Janssen,[5] and Zi Yang Meng[1,*]

arXiv: 2307.02547



$$H = -\frac{J_1}{N}\sum_{\langle ij\rangle} P_{ij} - \frac{J_2}{N}\sum_{\langle\langle ij\rangle\rangle} \Pi_{ij} - \frac{Q}{N}\sum_{\langle ij\rangle,\langle kl\rangle} P_{ij}P_{kl}$$

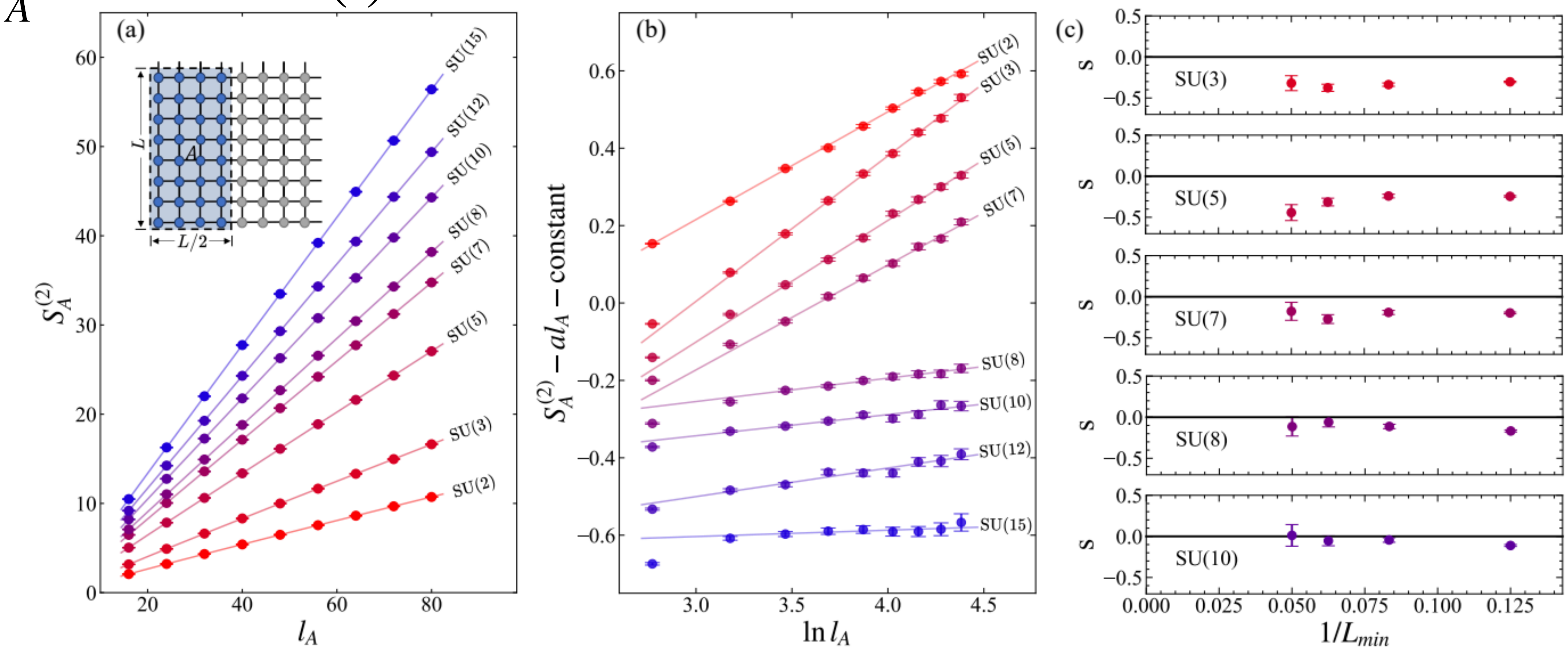
P_{ij} SU(N) singlet projection

$\Pi_{ij}|\alpha\beta\rangle = |\beta\alpha\rangle$ SU(N) permutation with the same rep

Kaul, Sandvik, PRL 108, 137201 (2012)

Block, Melko, Kaul, PRL 111, 137202 (2013)

$$S_A^{(2)} = al - s\ln(l) + c$$



Deconfined quantum criticality lost

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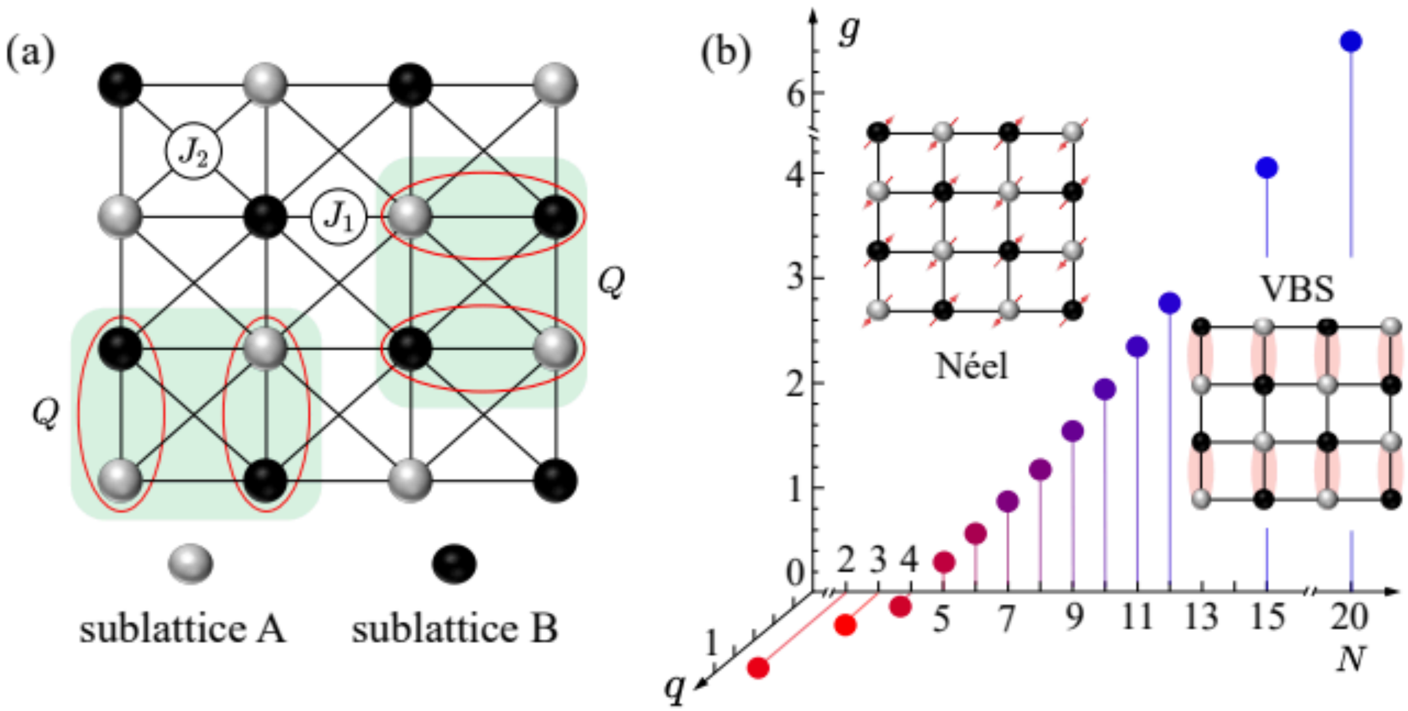
arXiv: 2307.02547

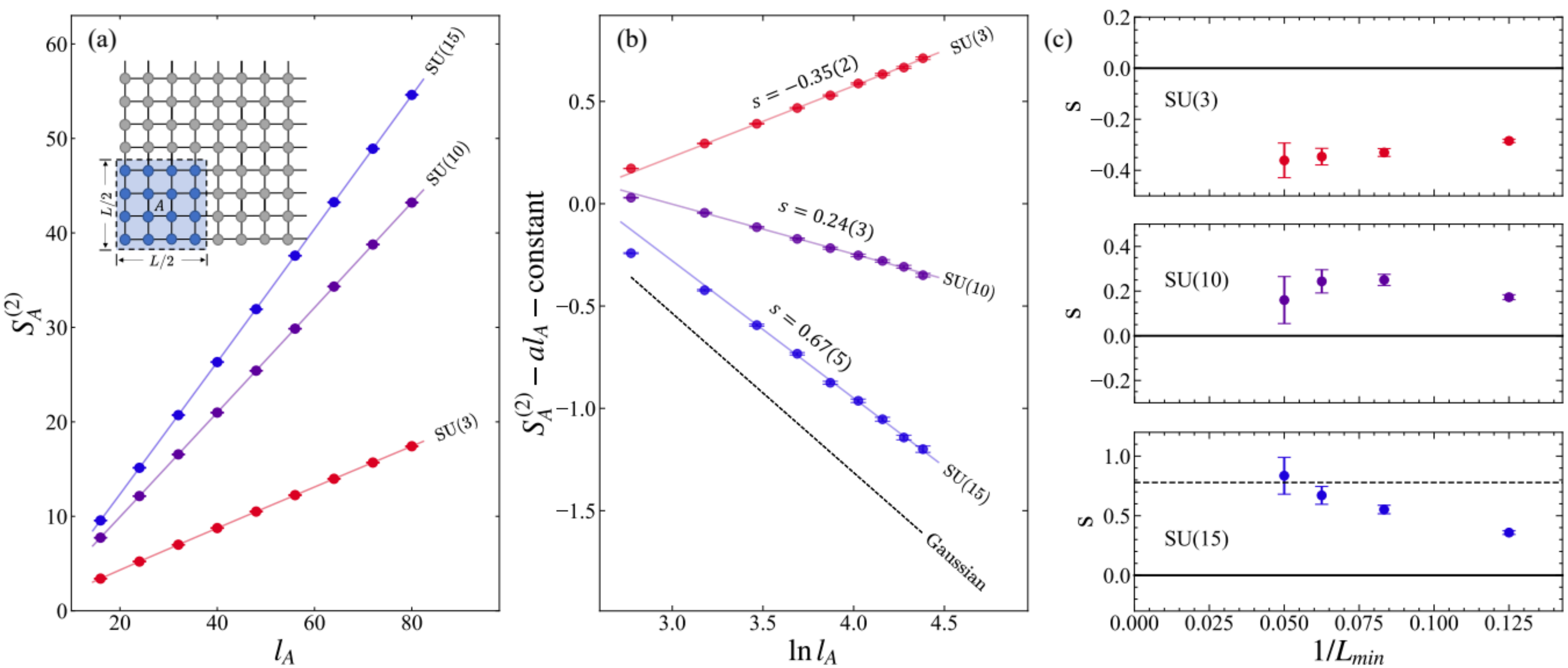


$$H = -\frac{J_1}{N}\sum_{\langle ij\rangle} P_{ij} - \frac{J_2}{N}\sum_{\langle\langle ij\rangle\rangle} \Pi_{ij} - \frac{Q}{N}\sum_{\langle ij\rangle,\langle kl\rangle} P_{ij}P_{kl}$$

P_{ij} SU(N) singlet projection

$\Pi_{ij}|\alpha\beta\rangle = |\beta\alpha\rangle$ SU(N) permutation with the same rep

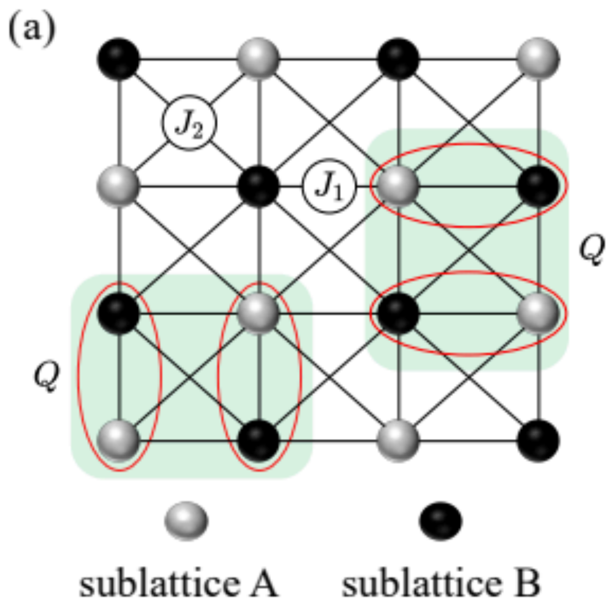
$$S_A^{(2)} = al - s_C \ln(l) + c$$

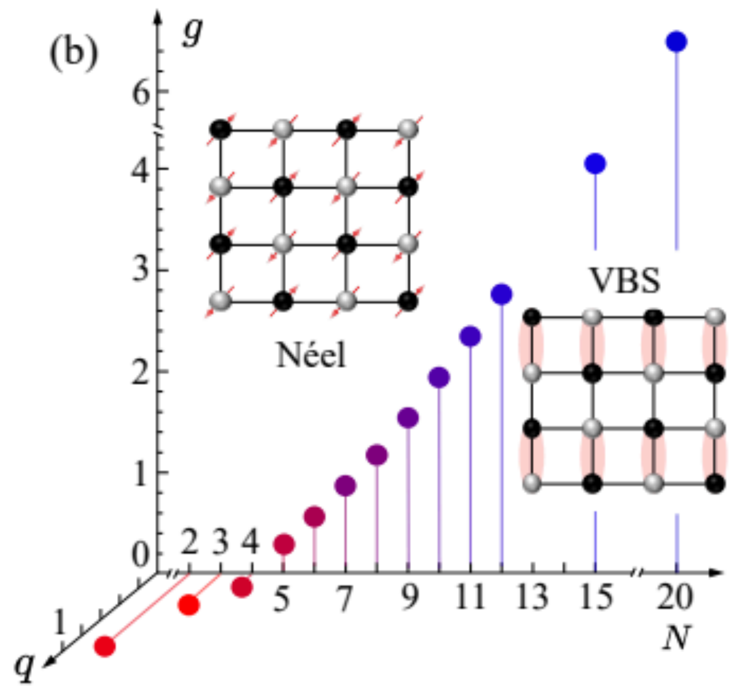


Deconfined quantum criticality lost

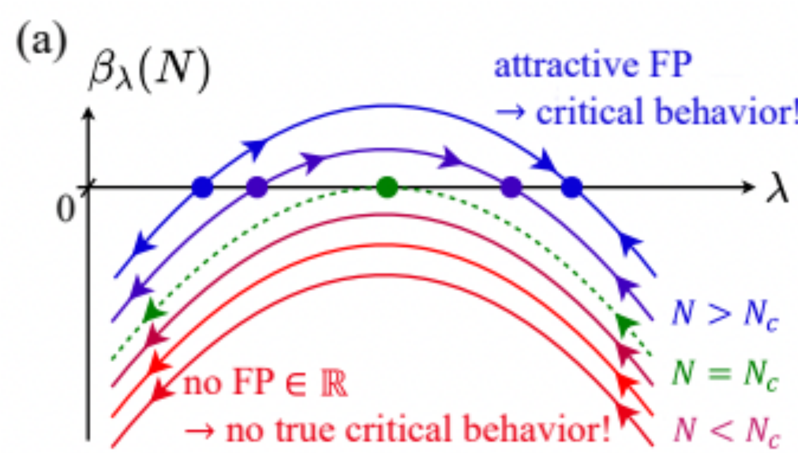
Menghan Song,[1] Jiarui Zhao,[1] Meng Cheng,[2] Cenke Xu,[3] Michael M. Scherer,[4] Lukas Janssen,[5] and Zi Yang Meng[1,*]

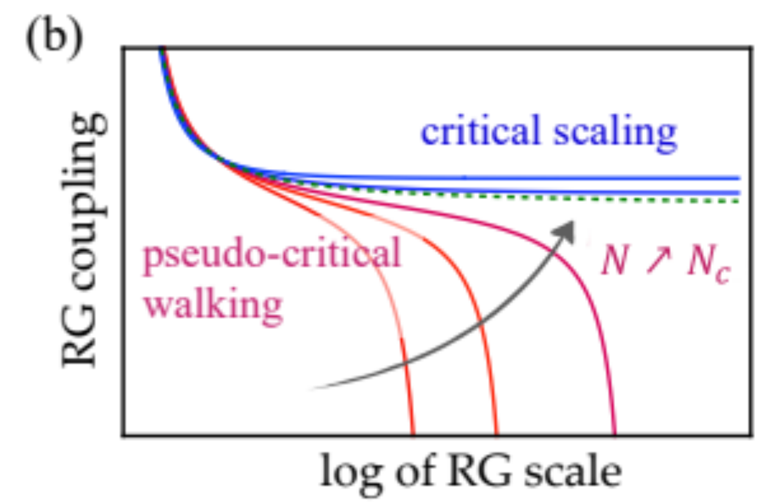

arXiv: 2307.02547





$$S_A^{(2)} = al - s\ln(l) + c$$





Walking

Goldstone mode

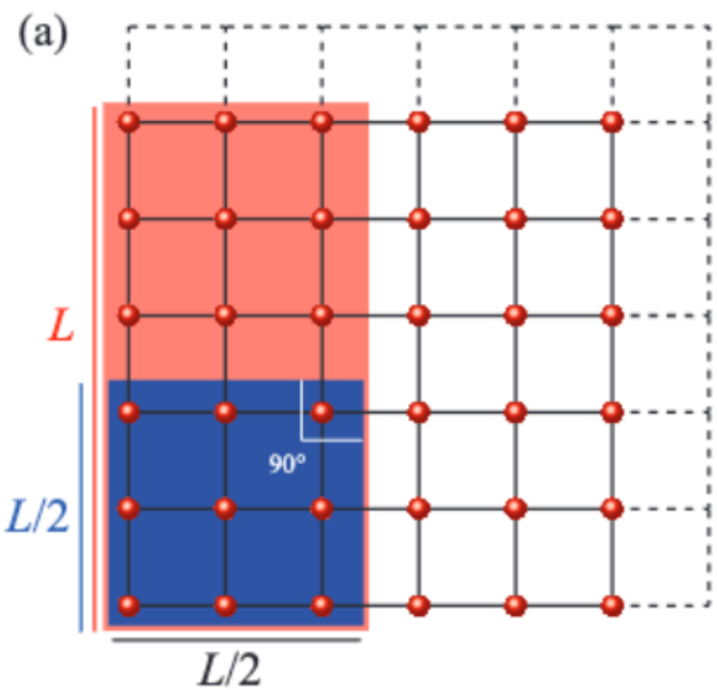
Near-marginal renormalisation group flow on the entanglement cut

(unlikely due to defect renormalization group flows)

Extracting Universal Corner Entanglement Entropy during the Quantum Monte Carlo Simulation

Yuan Da Liao,[1] Menghan Song,[1] Jiarui Zhao,[1] and Zi Yang Meng[1]

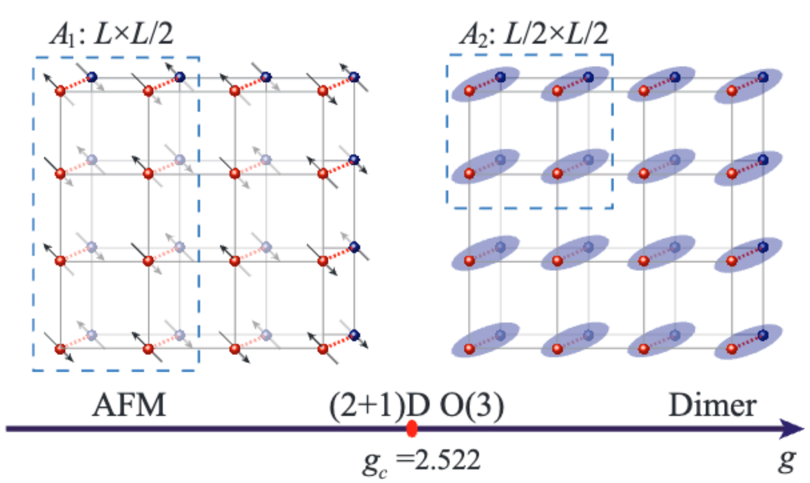
arXiv: 2404.13876

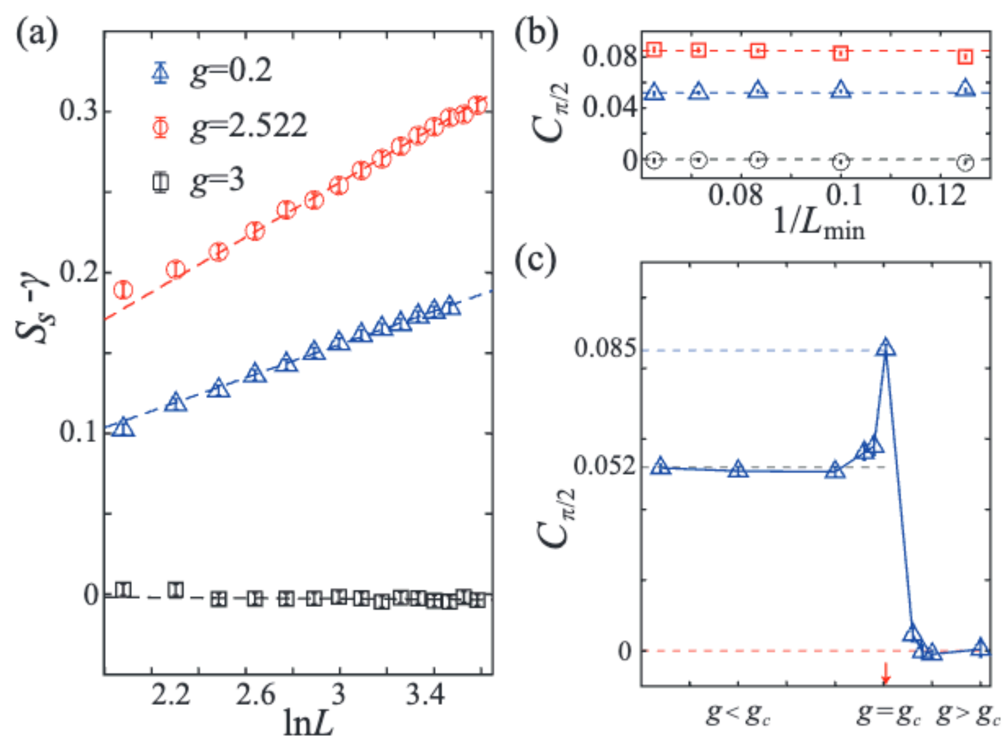


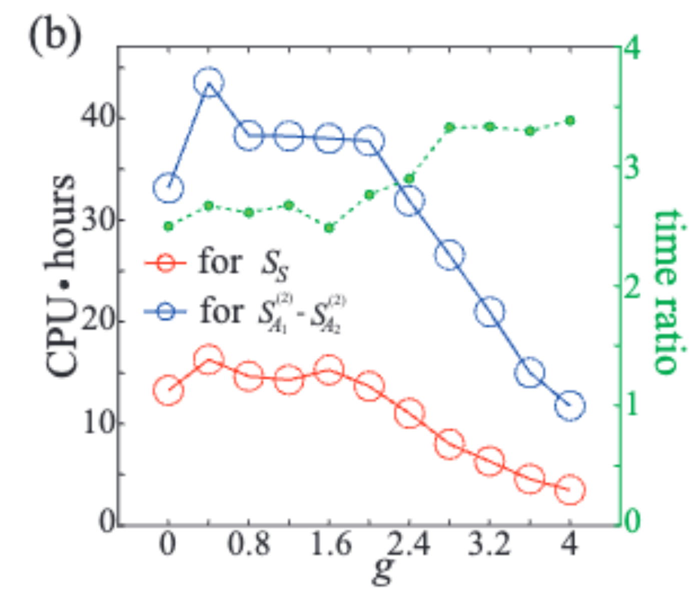
$$S_s = -\ln\left(\frac{Z_{A_1}}{Z_{A_2}}\right) = -\int_0^1 d\lambda \frac{\partial \ln(Z(\lambda))}{\partial \lambda}$$

$$Z(\lambda) = \sum_{s,s'} P_s P_{s'} \left(\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s})\right)^{\lambda} \left(\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s})\right)^{1-\lambda}$$

$$= -\int_0^1 d\lambda \frac{\sum_{s,s'} P_s P_{s'} \left(\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s'})\right)^{\lambda}\left(\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s'})\right)^{1-\lambda} \{\ln(\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s'})) - \ln(\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s'}))\}}{\sum_{s,s'} P_s P_{s'} \left(\mathrm{Tr}(\rho_{A_1,s}\rho_{A_1,s})\right)^{\lambda}\left(\mathrm{Tr}(\rho_{A_2,s}\rho_{A_2,s})\right)^{1-\lambda}}$$







Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

PRL 132, 246503 (2024)

Editors' Suggestion

Wess-Zumino-Witten Terms in Graphene Landau Levels

Junhyun Lee and Subir Sachdev
Phys. Rev. Lett. **114**, 226801 – Published 1 June 2015

$$S = \frac{1}{g}\int d^3x(\nabla\hat{\boldsymbol{\varphi}})^2 + S_{\rm WZW} + \cdots$$

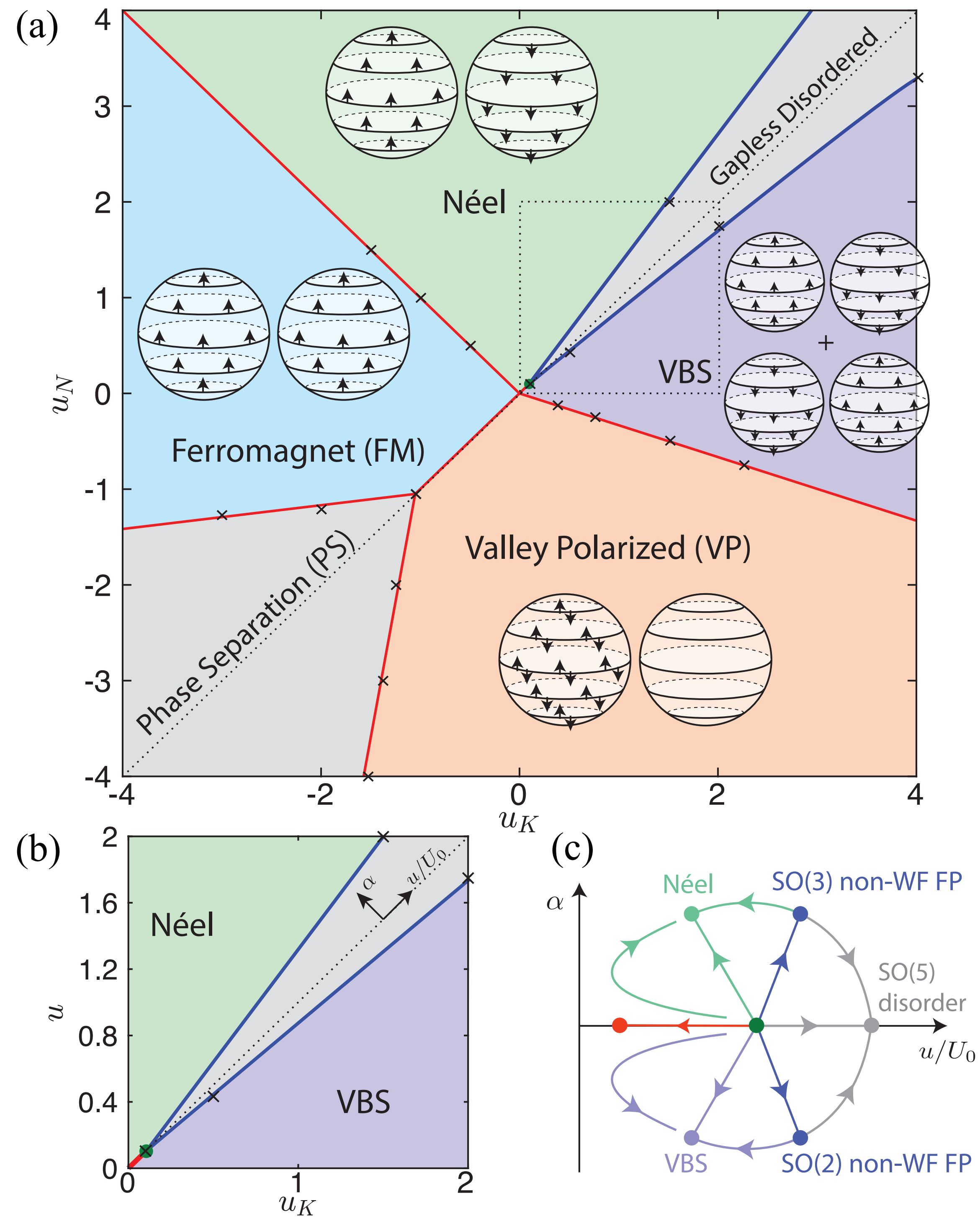
$$H = \frac{1}{2}\int d\Omega\{U_0[\psi^\dagger(\Omega)\psi(\Omega)-2]^2 - \sum_{i=1}^{5} u_i[\psi^\dagger(\Omega)\Gamma^i\psi(\Omega)]^2\}$$

$$\psi_{\tau\sigma}(\Omega) \qquad \Gamma^i = \{\tau_x\otimes\mathbb{I}, \tau_y\otimes\mathbb{I}, \tau_z\otimes\sigma_x, \tau_z\otimes\sigma_y, \tau_z\otimes\sigma_z\}$$

magnet monople inside a sphere $4\pi s$

Projected to the LLL with degeneracy $N = 2s+1$

$$\psi(\Omega) = \sum_{m=-s}^{s}\Phi_m(\Omega)c_m \qquad \Phi_m(\Omega) \propto e^{im\phi}\cos^{s+m}(\frac{\theta}{2})\sin^{s-m}(\frac{\theta}{2})$$

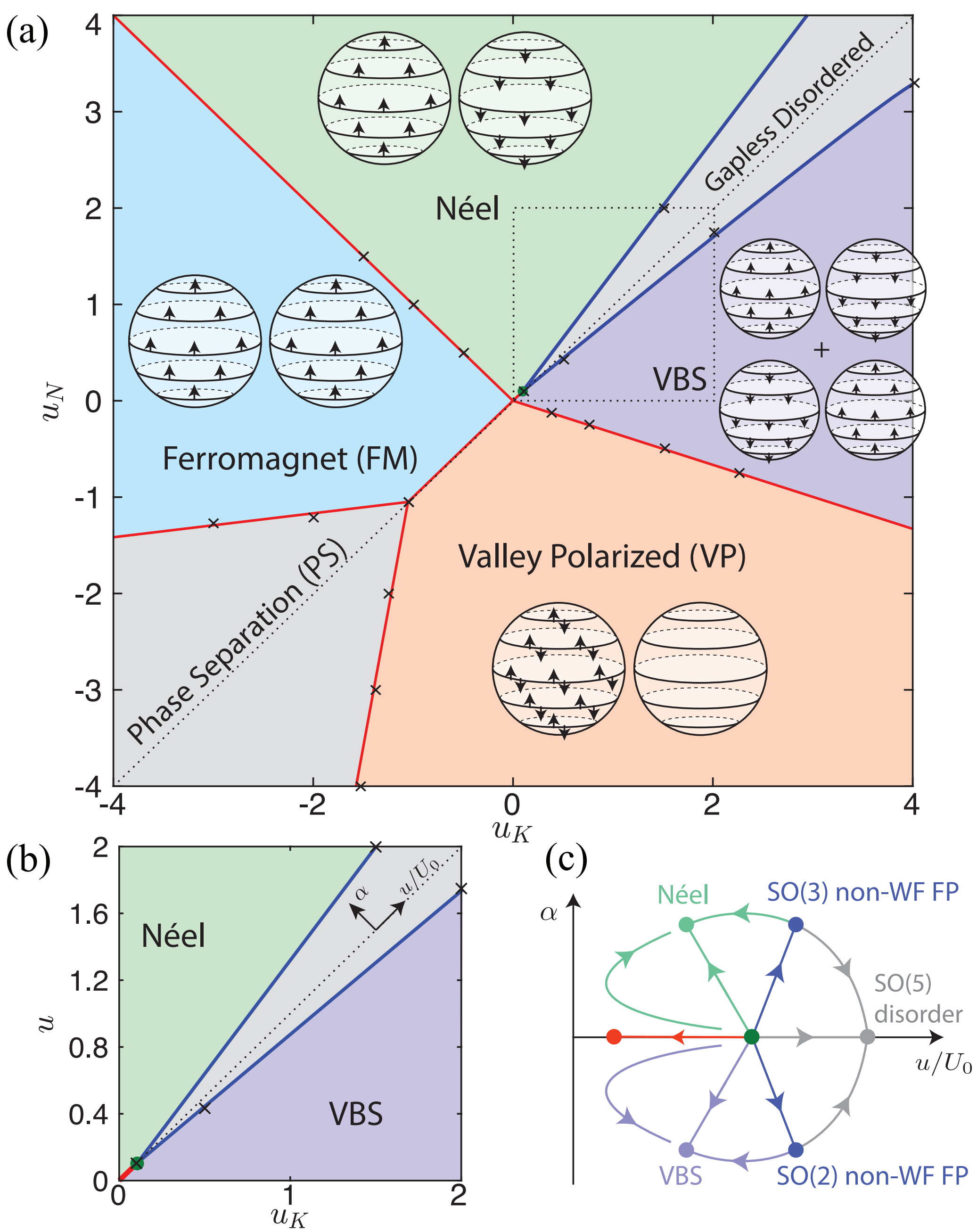


- M. Ippoliti, R. Mong, F. Assaad, M. Zaletel, PRB 98, 235108 (2018)
- Z. Wang, M. Zaletel, R. Mong, F. Assaad, PRL 126, 045701 (2021)
- Z. Zhou, L. Hu, W. Zhu, and Y.-C. He, PRX 14, 021044 (2024)

Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

PRL 132, 246503 (2024)

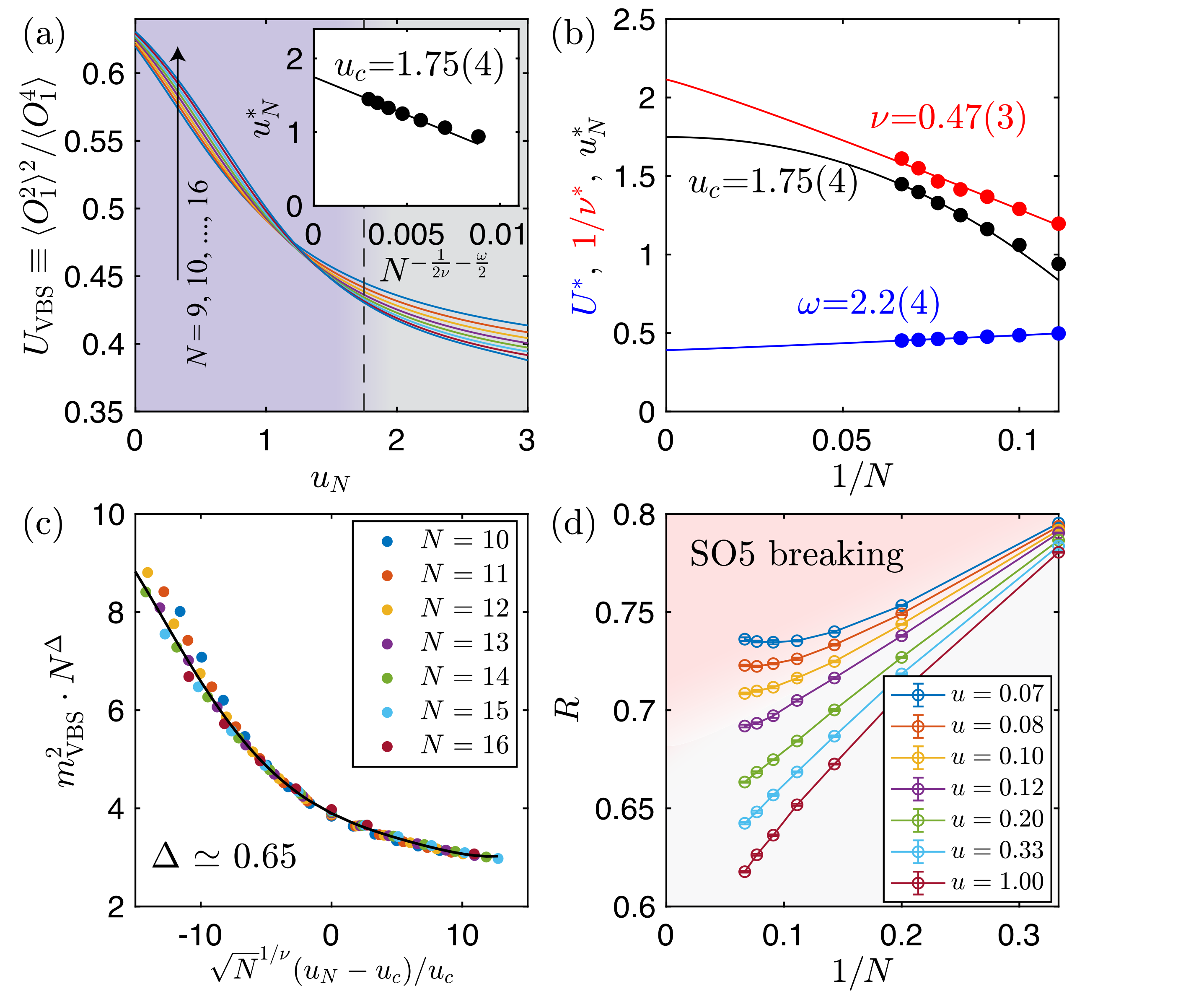


$$U_0 = 1,\ u_1 = u_2 = u_K, u_3 = u_4 = u_5 = u_N$$

$$\langle O_i \rangle = \int d\Omega \psi^\dagger(\Omega)\Gamma^i \psi(\Omega) = \sum_{m=-s}^{s} c_m^\dagger \Gamma^i c_m$$

$$m_{VBS}^2 = \frac{1}{2N^2}\langle (O_1^2 + O_2^2)\rangle \qquad m_{Neel}^2 = \frac{1}{3N^2}\langle (O_3^2 + O_4^2 + O_5^2)\rangle$$

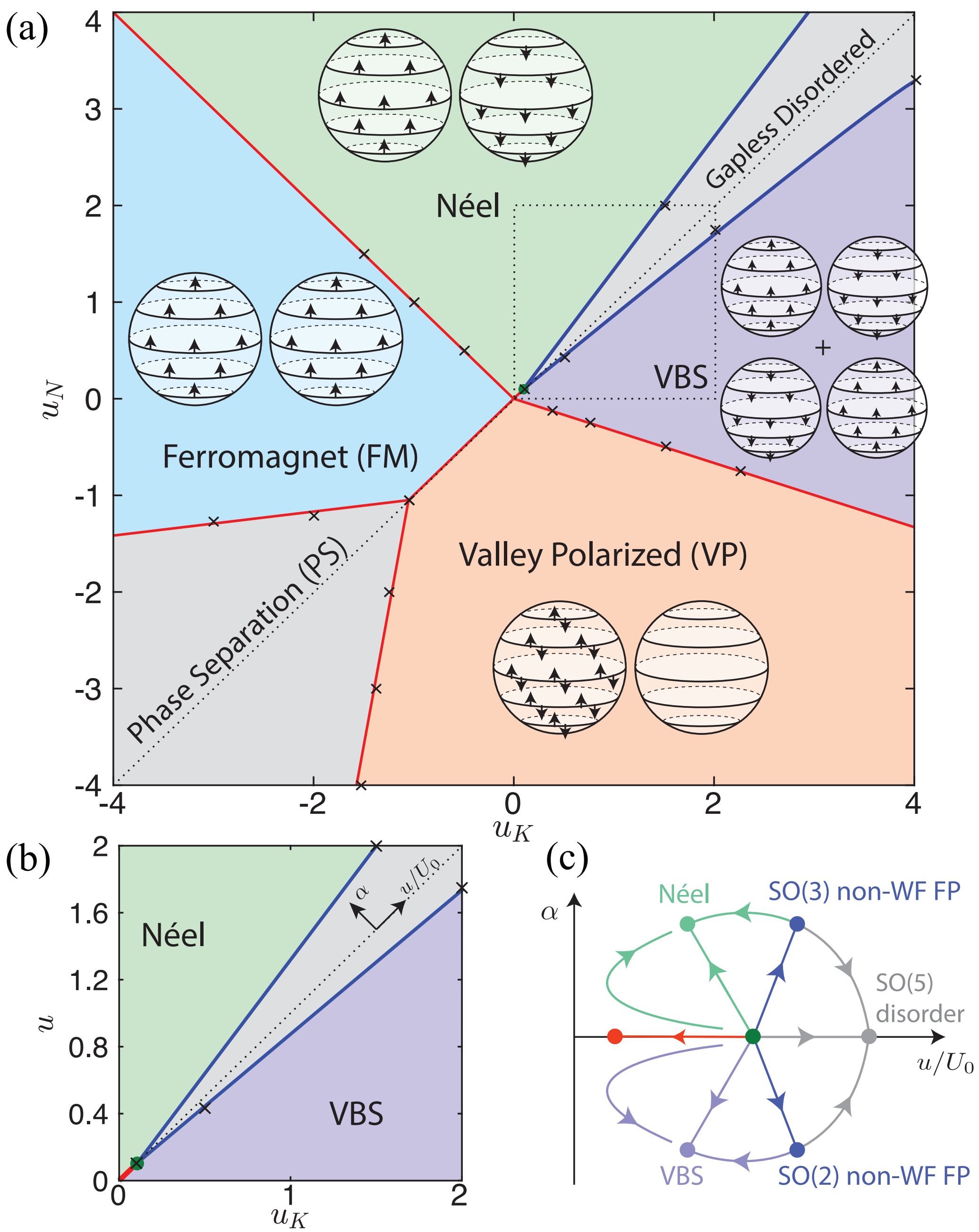
$u_K = 2$



Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

PRL 132, 246503 (2024)



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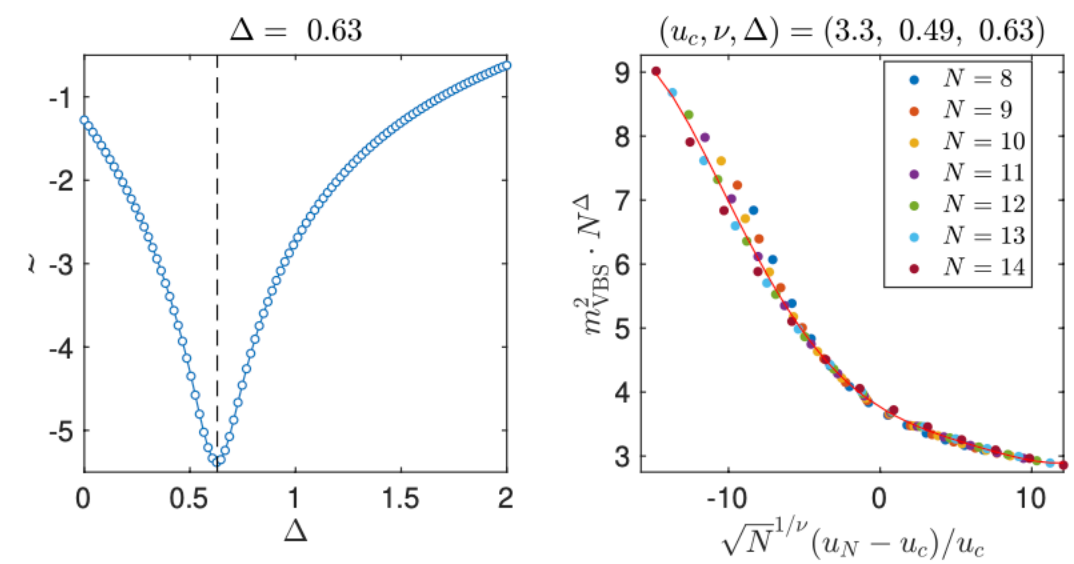
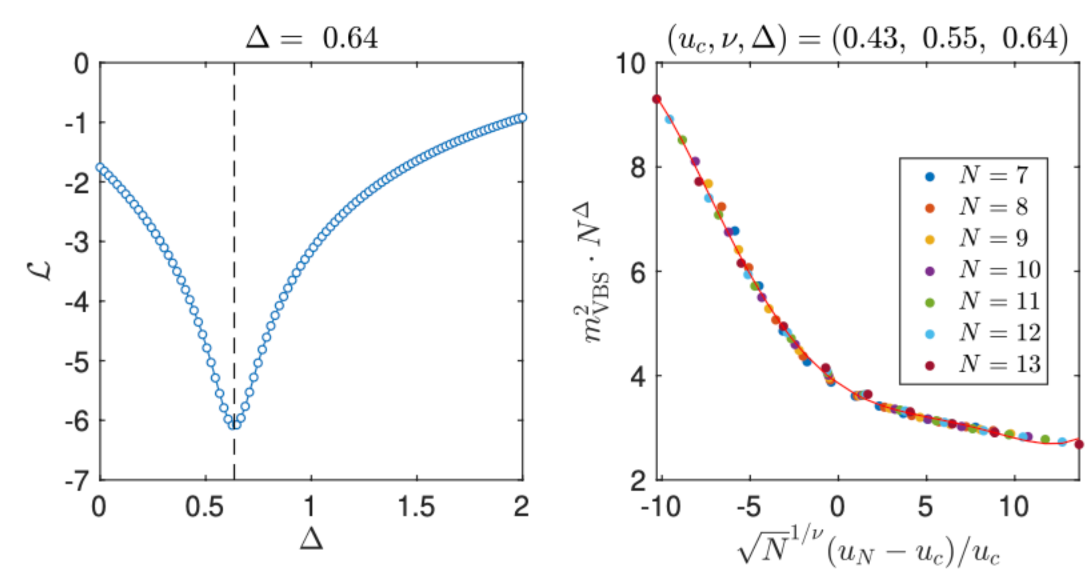
$u_K = 4$

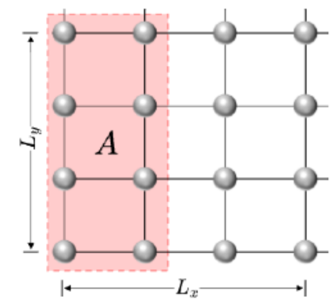


$u_K = 0.5$

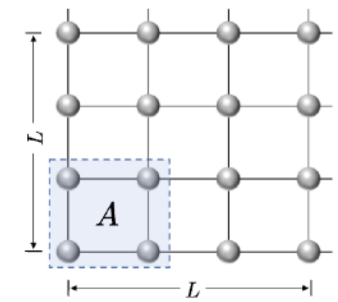


$$S_A(l) = al - s\ln(\frac{l}{\epsilon}) - \gamma + O(1/l)$$

Smooth boundary, no log



Corner, log



d=1 CFT	$S \sim c\ln(l)$	Heisenberg chain, Luttinger liquid	DMRG
d=2 QCP	$S \sim al - s_C\ln(l) - \gamma$	Wilson-Fisher O(N), GNY	QMC
SSB	$S \sim al - (s_G + s_C)\ln(l) - \gamma$	Antiferromagnet, Superfluid	QMC
Topological order	$S \sim al - \gamma_{top}$	Z2 top ord, Kitaev QSL	QMC
Fermi surface	$S \sim l\ln(l) + al - \cdots$	free fermion, interaction ?	

Quantum Entanglement & Fractional Chern Insulator

Recent Topics in

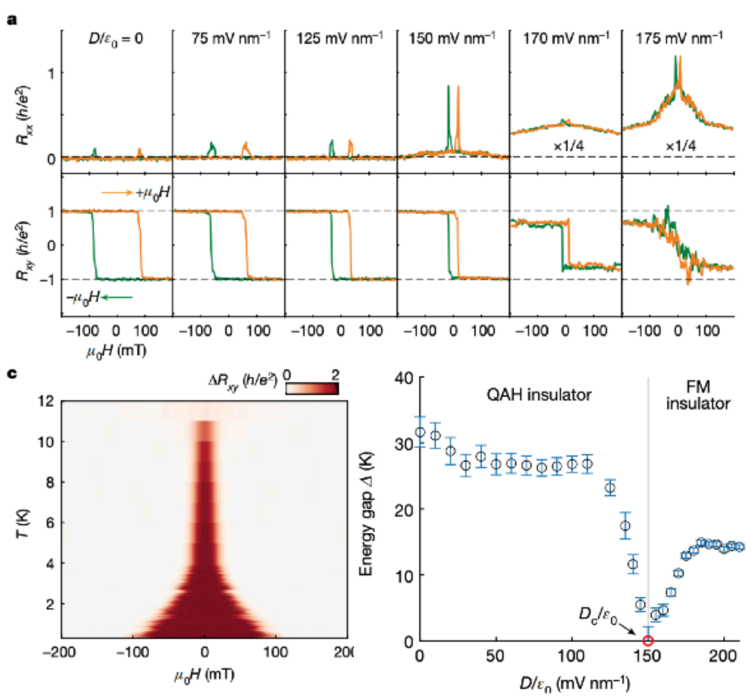
Quantum Many-Body Computation

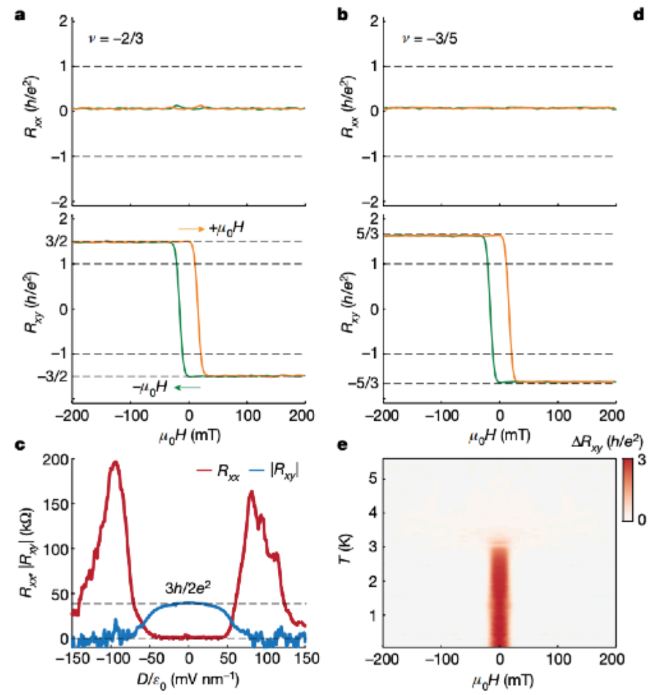
ZI YANG MENG
孟子楊
https://quantummc.xyz/

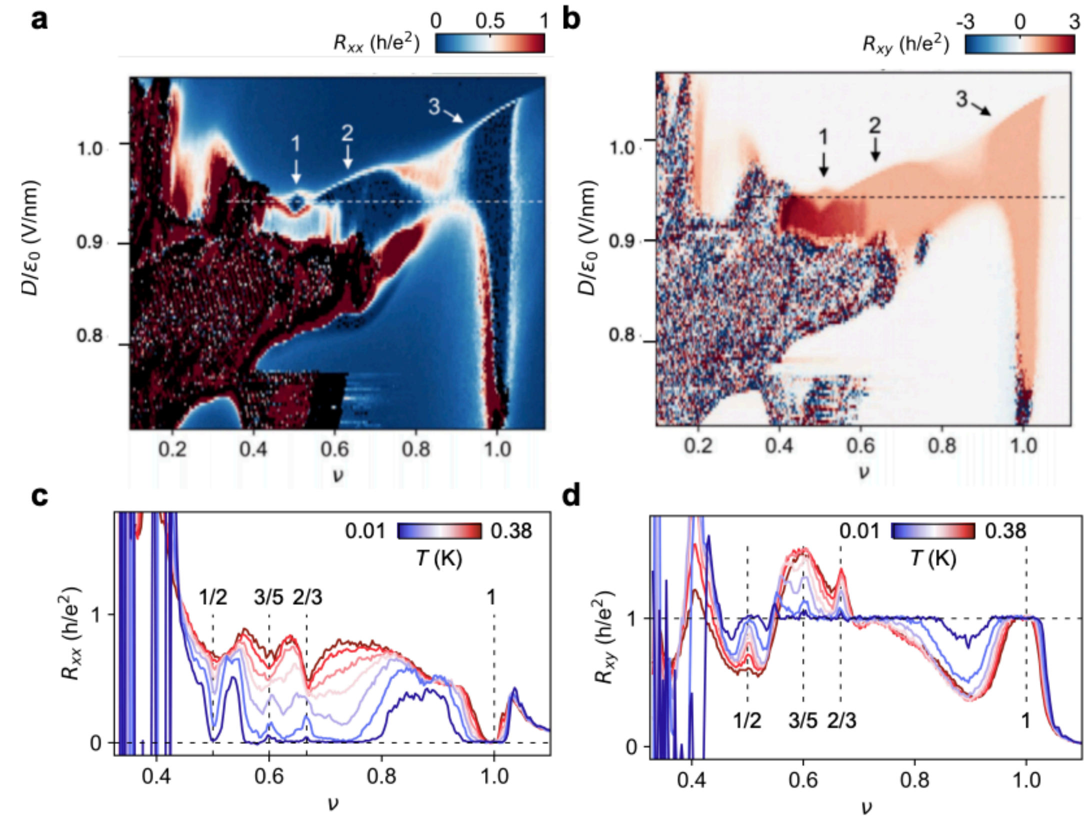
Integer and Fractional Quantum Anomalous Hall Effects

twisted bilayer MoTe2

rhombohedral pentalayer graphene/hBN







difference in temperature/energy scales

energy gap ~ 20 K

FQAH ~ 0.4 K
FQAH - QAH 0.04 K
FQAH - QAH - CDW - FL transitions

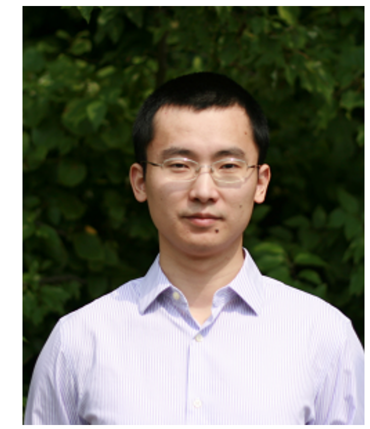
- H. Park et al., Observation of fractionally quantized anomalous Hall effect, Nature 622, 74 (2023)
- J. Cai et al., Signatures of fractional quantum anomalous Hall states in twisted MoTe2, Nature 622, 63 (2023)
- Y. Zeng et al., Thermodynamic evidence of fractional Chern insulator in moiré MoTe2, Nature 622, 69 (2023)
- K. Kang et al., Evidence of the fractional quantum spin Hall effect in moiré MoTe2, Nature 628, 522 (2024)
- Z. Lu et al., Fractional quantum anomalous Hall effect in multilayer Graphene, Nature 626, 759 (2024)
- Z. Lu et al., Extended Quantum Anomalous Hall States in Graphene/hBN moiré superlattices, arXiv:2408.10203
- ......

Rep. Prog. Phys. **87** (2024) 108003

From Fractional Quantum Anomalous Hall Smectics to Polar Smectic Metals: Nontrivial Interplay Between Electronic Liquid Crystal Order and Topological Order in Correlated Topological Flat Bands

Hongyu Lu[1,4], Han-Qing Wu[2,4], Bin-Bin Chen[1,*], Kai Sun[3,*] and Zi Yang Meng[1,*]



arXiv:2408.07111

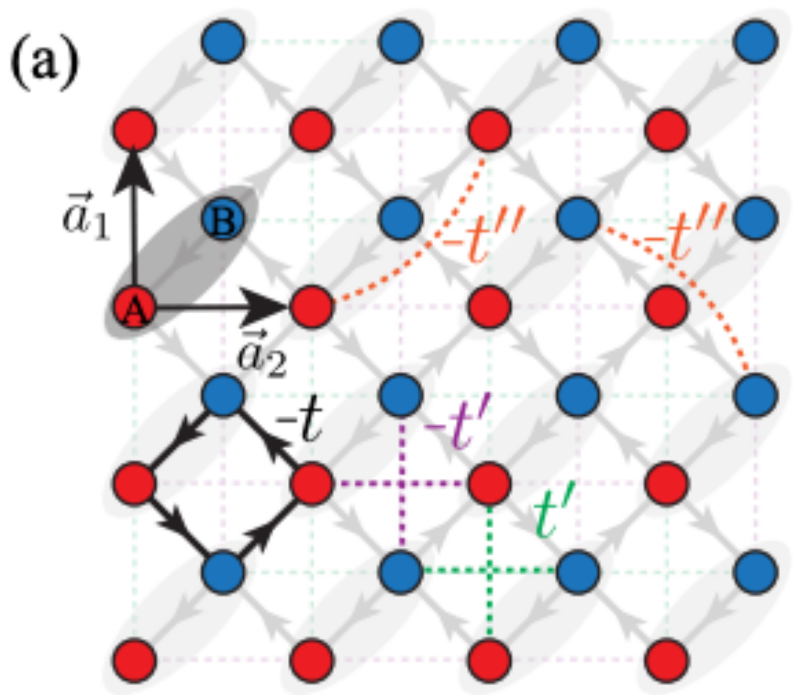
Vestigial Gapless Boson Density Wave Emerging between $\nu = 1/2$ Fractional Chern Insulator and Finite-Momentum Supersolid

Hongyu Lu,[1] Han-Qing Wu,[2] Bin-Bin Chen,[1] and Zi Yang Meng[1,*]

PHYSICAL REVIEW LETTERS **132,** 236502 (2024)

Thermodynamic Response and Neutral Excitations in Integer and Fractional Quantum Anomalous Hall States Emerging from Correlated Flat Bands

Hongyu Lu,[1] Bin-Bin Chen,[1] Han-Qing Wu,[2] Kai Sun,[3,*] and Zi Yang Meng[1,†]



$$H = H_0 + H_I$$

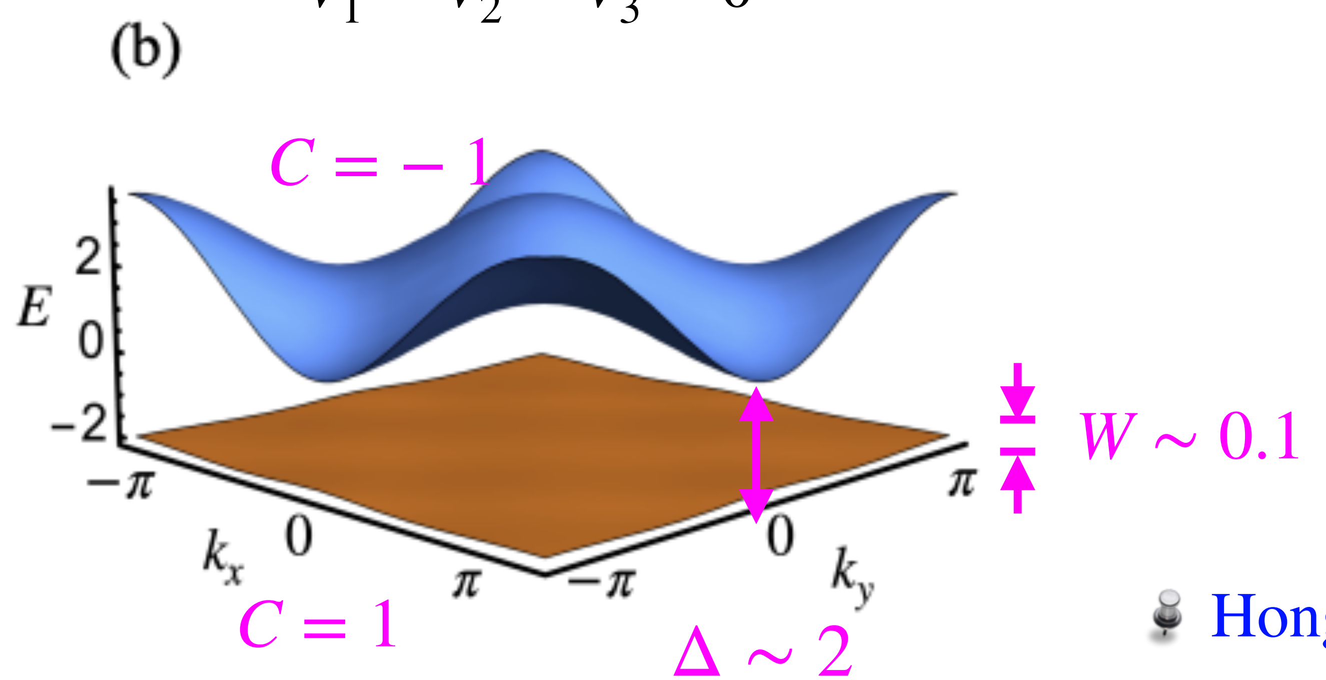
$$H_0 = -t \sum_{\langle i,j \rangle} e^{i\phi_{ij}} (c_i^\dagger c_j + h.c.) - \sum_{\langle\langle i,j \rangle\rangle} t'_{ij} (c_i^\dagger c_j + h.c.) - t'' \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} (c_i^\dagger c_j + h.c.)$$

$$t = 1 \qquad t' = \pm \frac{1}{2+\sqrt{2}} \qquad t'' = -\frac{1}{2+2\sqrt{2}}$$

$$\phi_{ij} = \frac{\pi}{4}$$

$$H_I = V_1 \sum_{\langle i,j \rangle} n_i n_j + V_2 \sum_{\langle\langle i,j \rangle\rangle} n_i n_j + V_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} n_i n_j + \mu \sum_i n_i$$

$$V_1 = V_2 = V_3 = 0$$

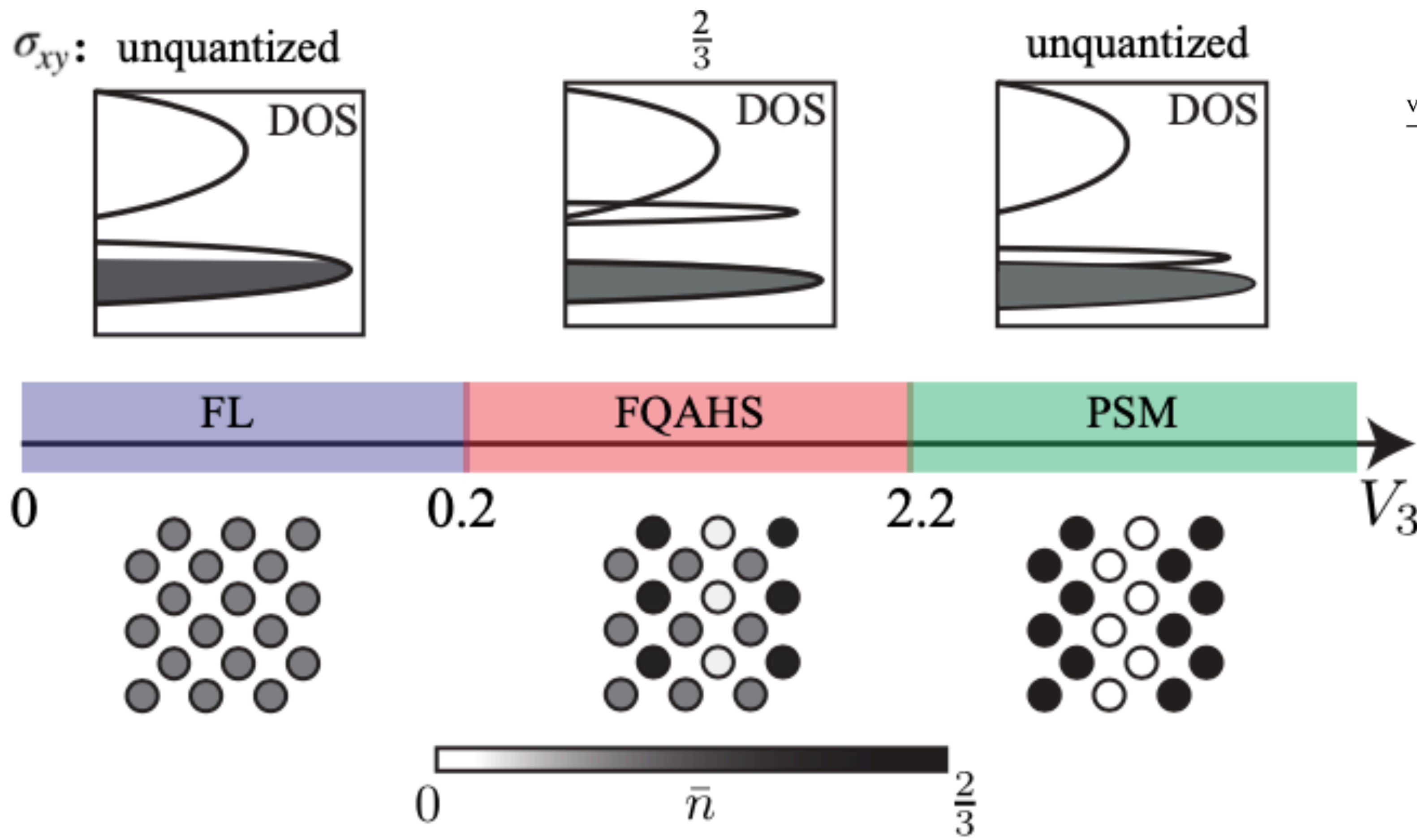


Consider filling factor of the flat band

$$\nu = \frac{2}{3}$$

and consider the NNN interaction V_3

Hongyu Lu et al., Rep. Prog. Phys. 87 (2024) 108003



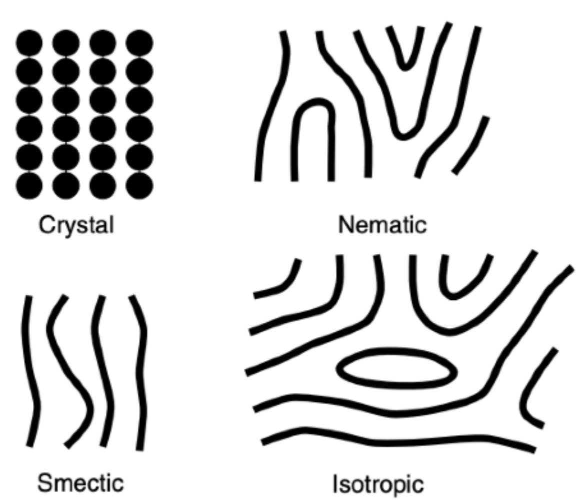
FL — Fermi liquid
FQAHS — Fractional quantum anomalous Hall smectic state
PSM — Polar smectic metal

Intertwinement of topological order and Landau order

Electronic liquid-crystal phases of a doped Mott insulator

S. A. Kivelson*, E. Fradkin† & V. J. Emery‡

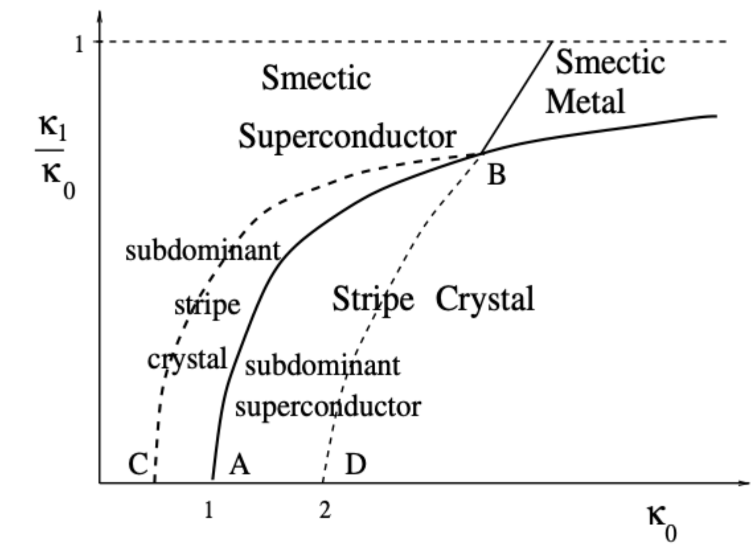
Nature 393, 550 (1998)



VOLUME 85, NUMBER 10 PHYSICAL REVIEW LETTERS 4 SEPTEMBER 2000

Quantum Theory of the Smectic Metal State in Stripe Phases

V. J. Emery,[1] E. Fradkin,[2] S. A. Kivelson,[3,4] and T. C. Lubensky[5]

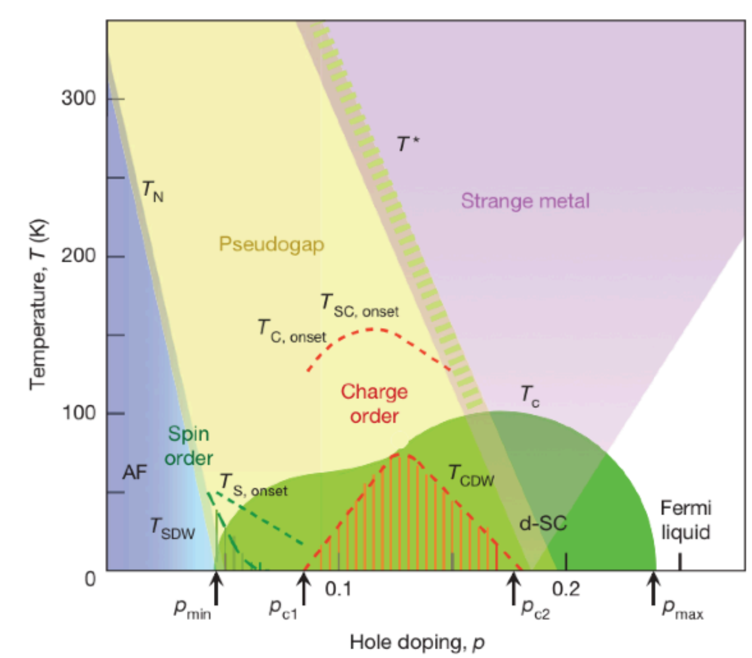


REVIEW

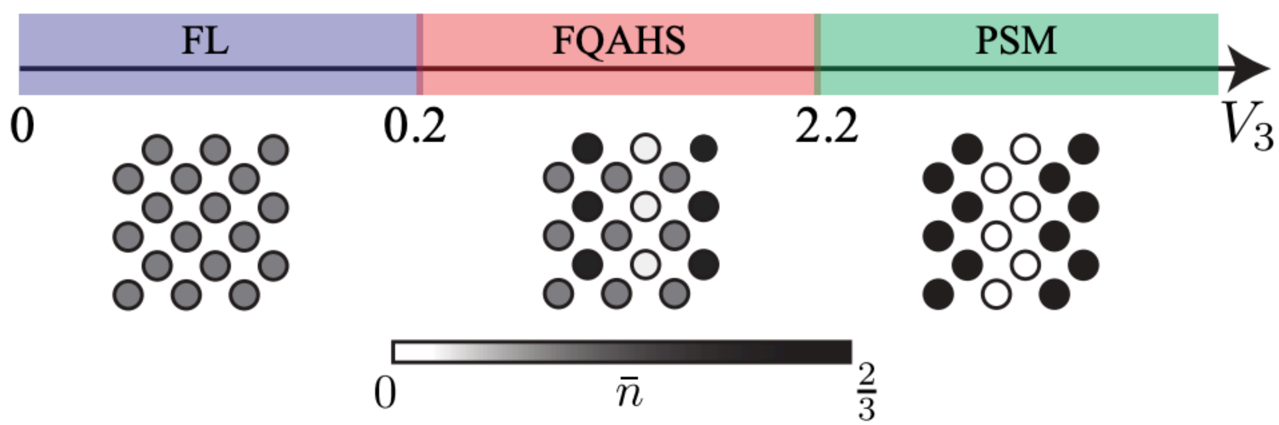
doi:10.1038/nature14165

From quantum matter to high-temperature superconductivity in copper oxides

B. Keimer[1], S. A. Kivelson[2], M. R. Norman[3], S. Uchida[4] & J. Zaanen[5]



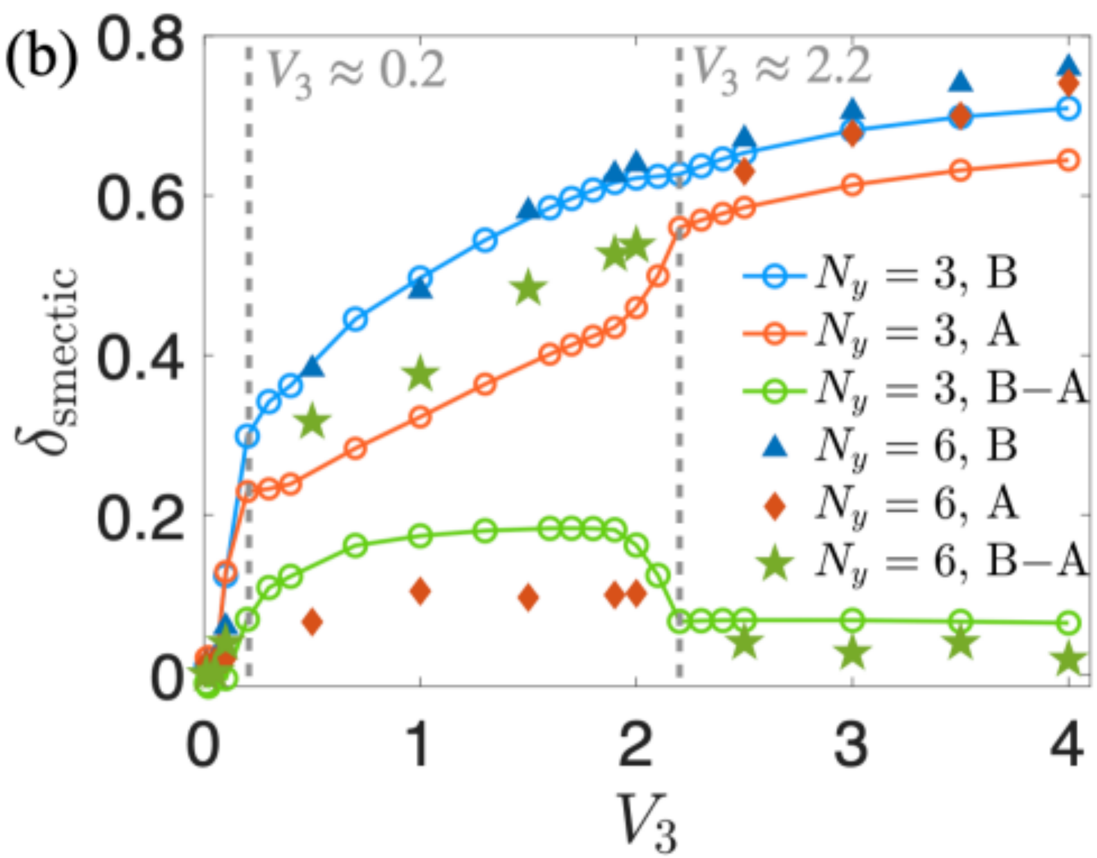
Hongyu Lu et al., Rep. Prog. Phys. 87 (2024) 108003

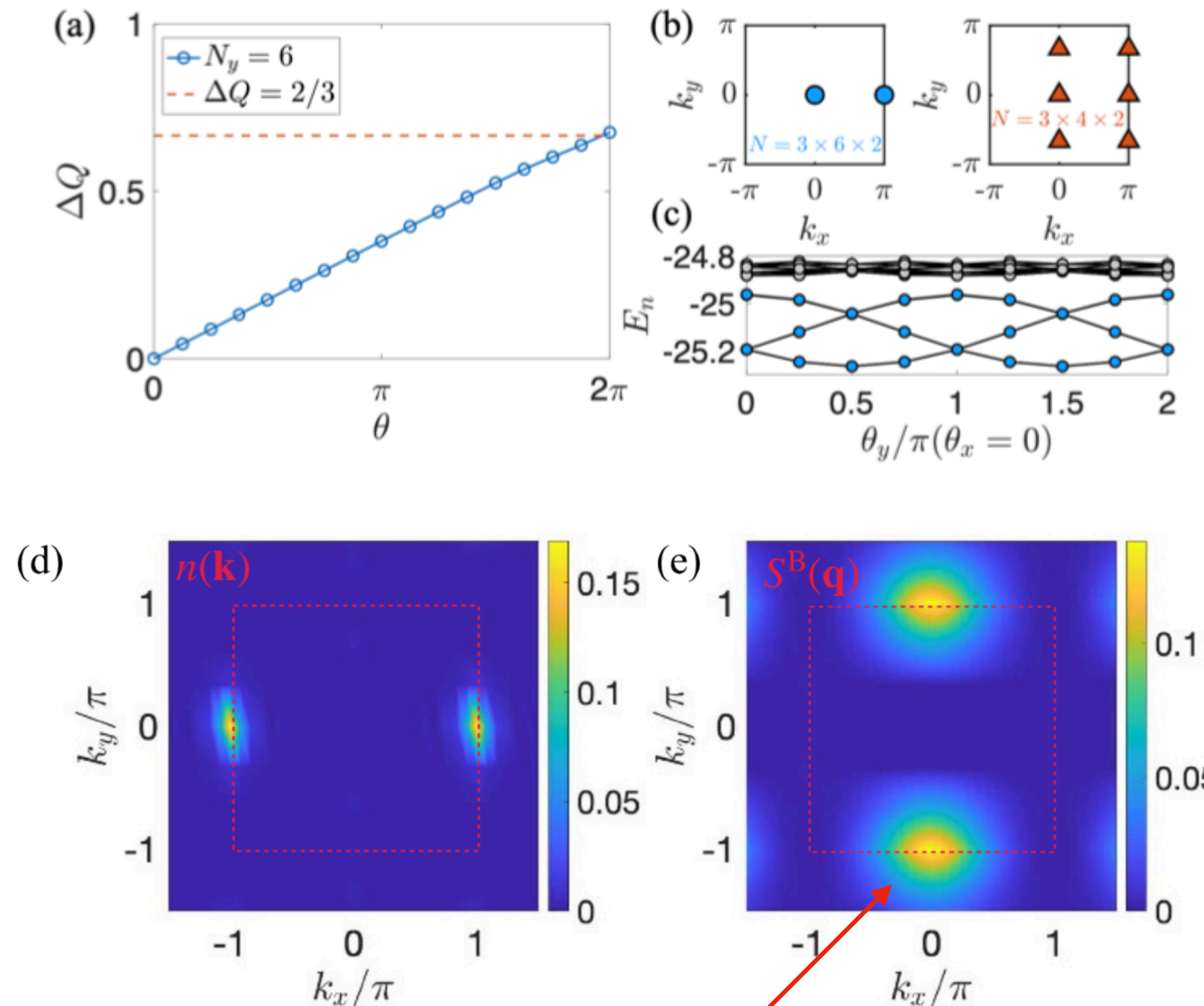


Charge pumping

$$\sigma_{xy} = \frac{2}{3}\frac{e^2}{h}$$

6-fold degeneracy



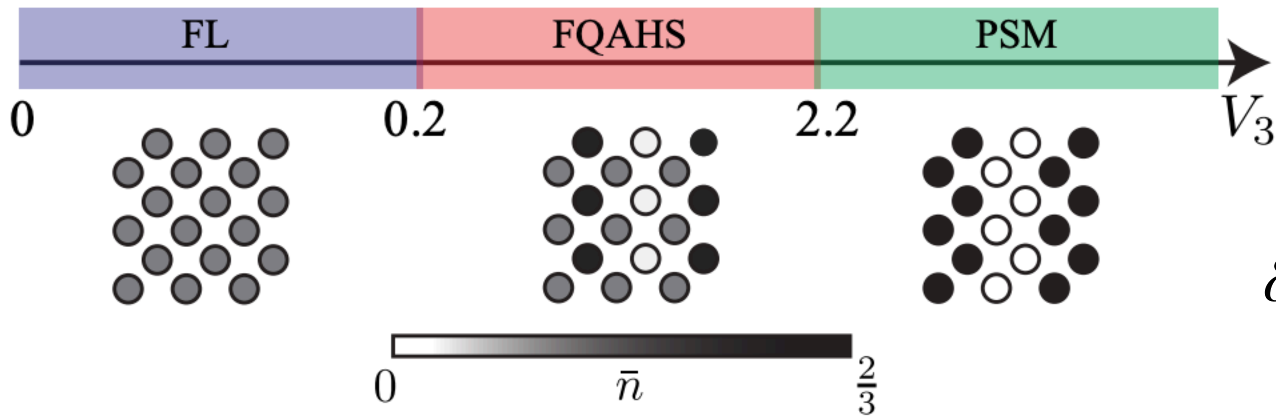


$$\delta^{A/B}_{smectic} = \frac{1}{N}\sum_i (-1)^{x_i}\, n_i^{A/B} \qquad i \text{ unit cell}$$

$$S^{A/B}(\mathbf{q}) = \sum_i e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_0})}(\langle n_0^{A/B} n_i^{A/B}\rangle - \langle n_0^{A/B}\rangle\langle n_i^{A/B}\rangle)$$

Charge-neutral magnetoroton

Hongyu Lu et al., Rep. Prog. Phys. 87 (2024) 108003



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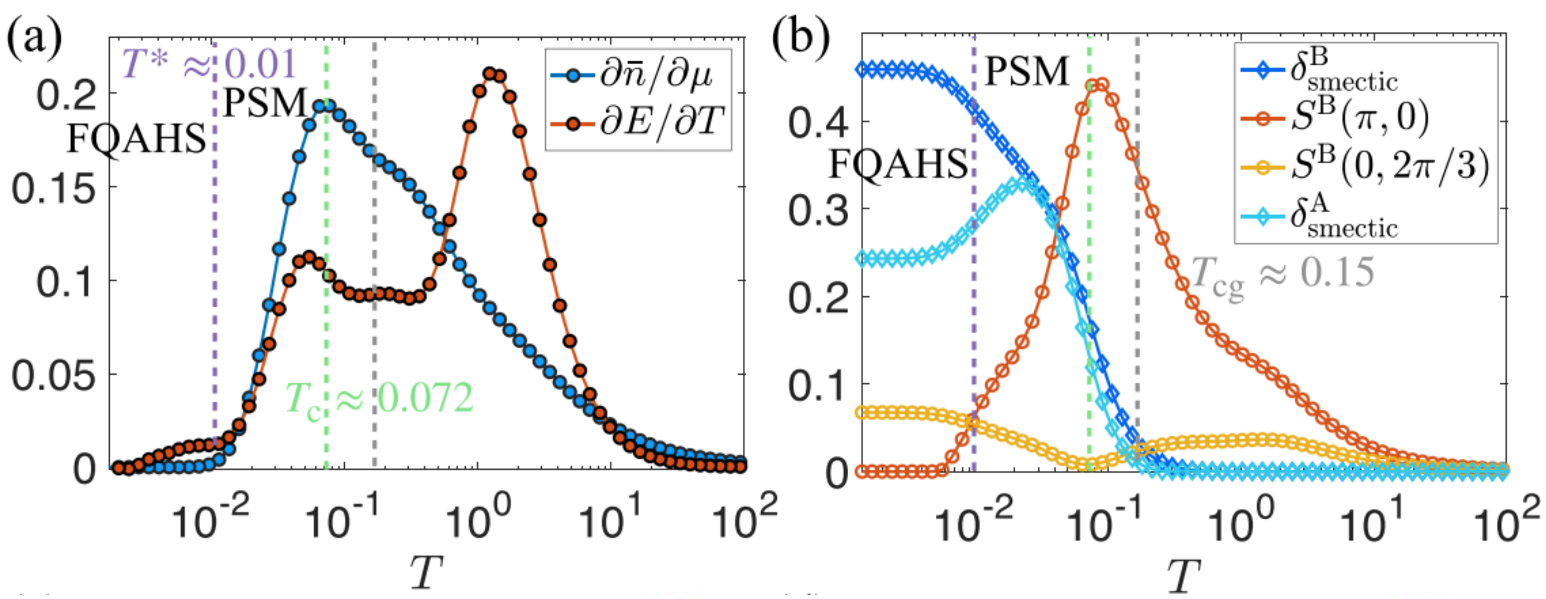
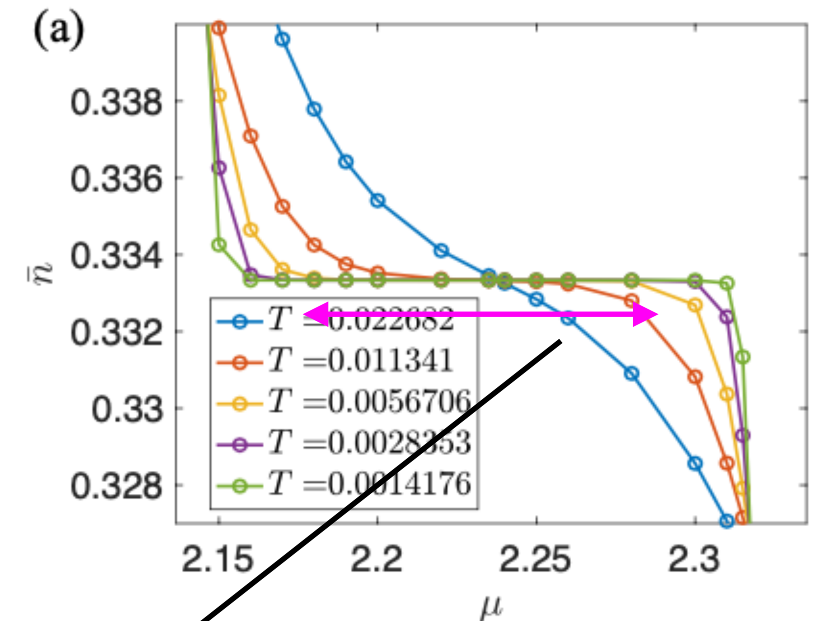
$$S^{A/B}(\mathbf{q}) = \sum_i e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_0})}(\langle n_0^{A/B} n_i^{A/B}\rangle - \langle n_0^{A/B}\rangle\langle n_i^{A/B}\rangle)$$

$V_3 = 1$

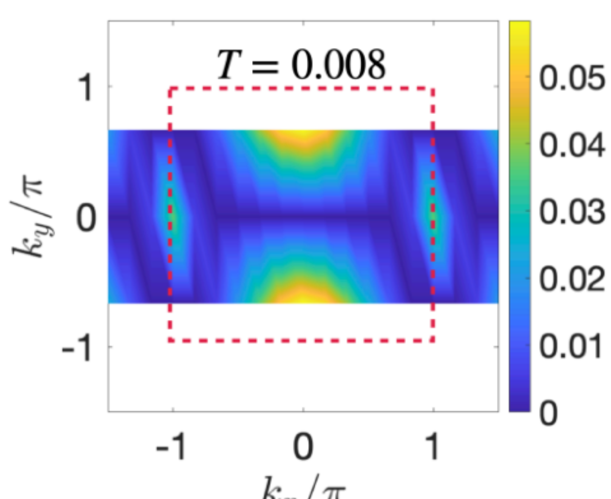


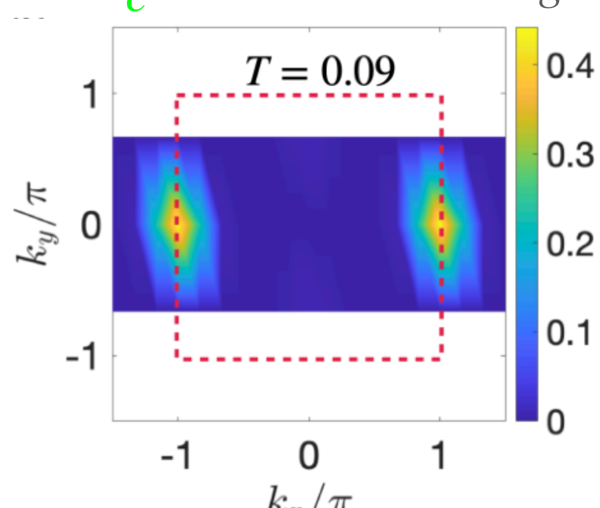


Magnetoroton Smectic order

T

T^* T_c $T_{cg} \sim \Delta_{cg} \sim 0.15$





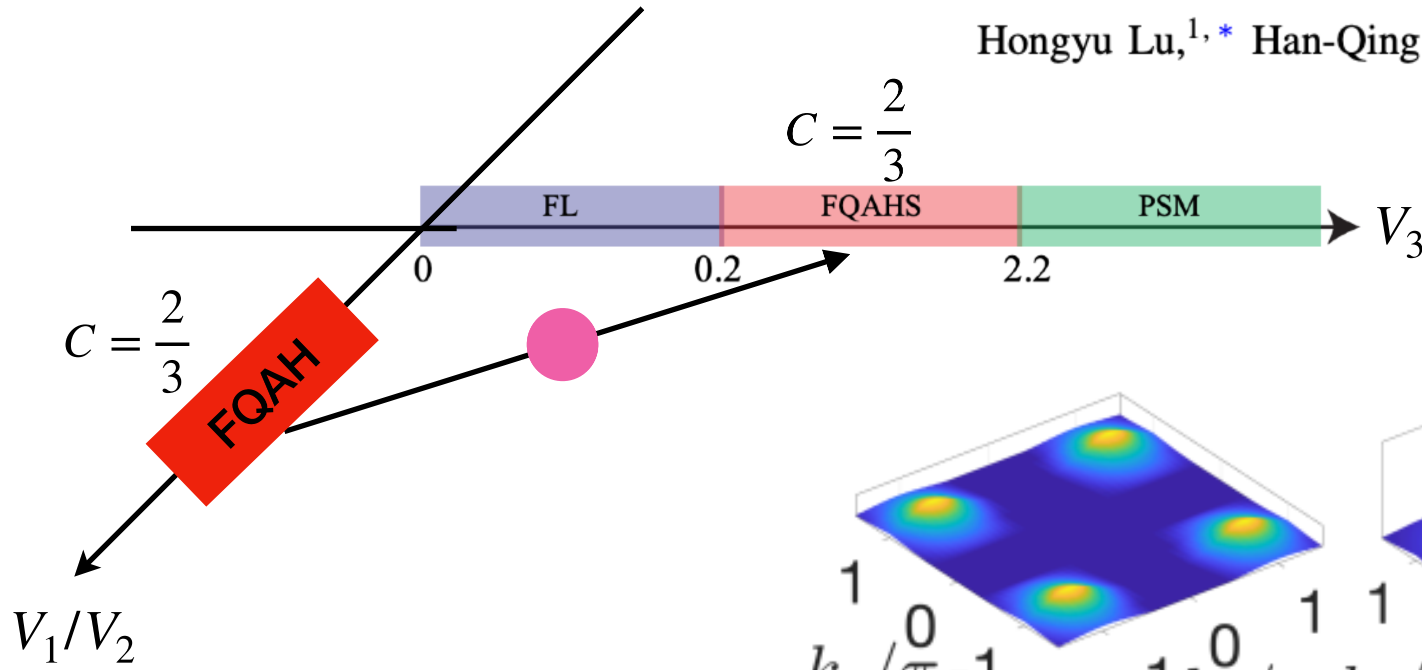
Roton gap determines the onset temperature of FCI

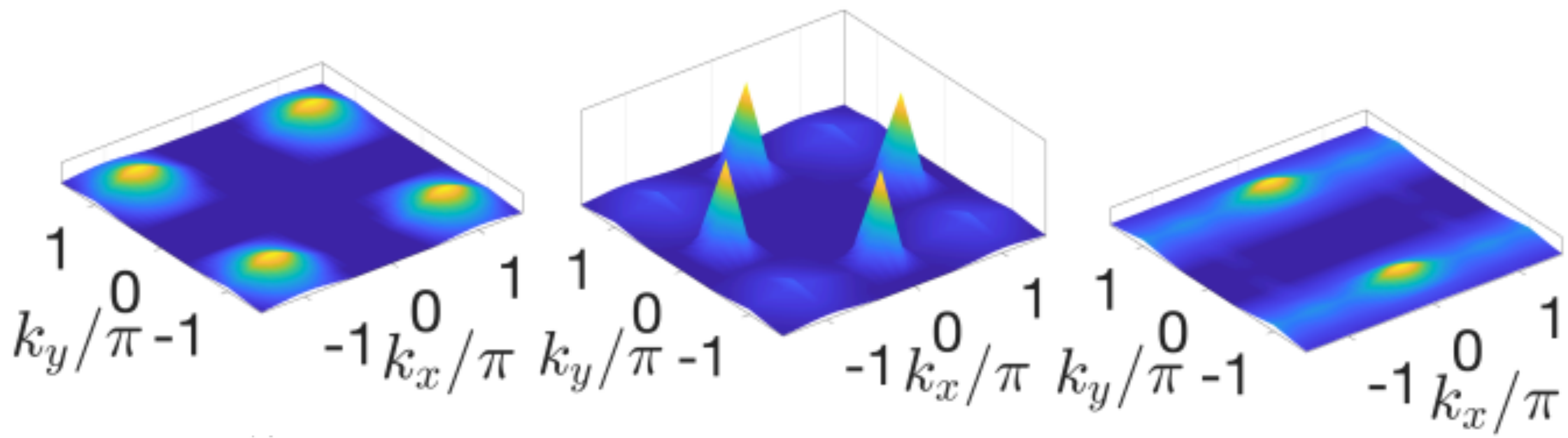
Hongyu Lu et al., PRL 132, 236502 (2024)

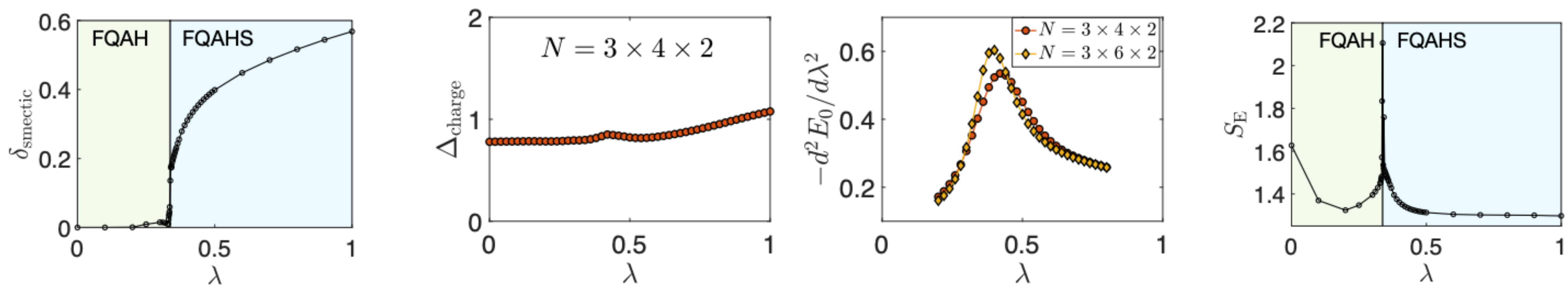
Hongyu Lu et al., arXiv:2404.06745

From a fractional quantum anomalous Hall state to a smectic state with equal Hall conductance

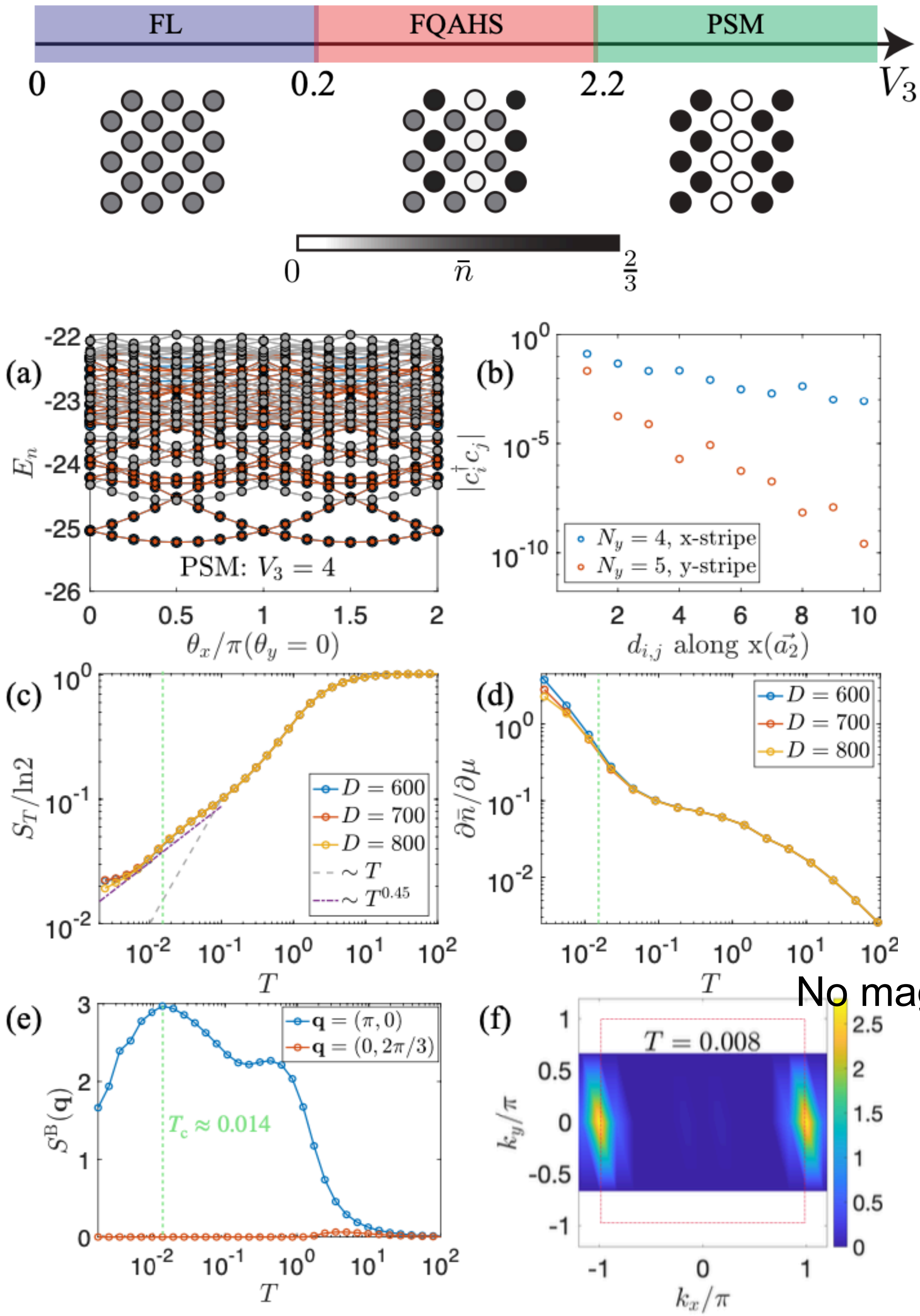
Hongyu Lu,[1,*] Han-Qing Wu,[2,*] Bin-Bin Chen,[1] and Zi Yang Meng[1,†]







Hongyu Lu et al., Rep. Prog. Phys. 87 (2024) 108003

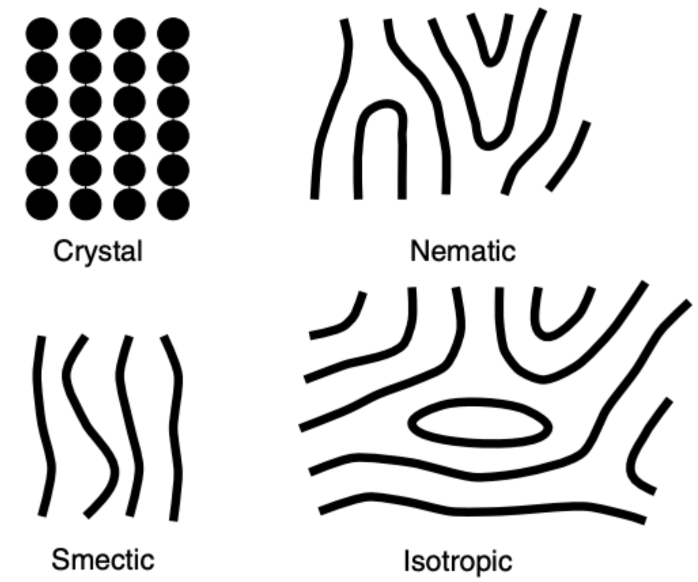


No magnetoroton

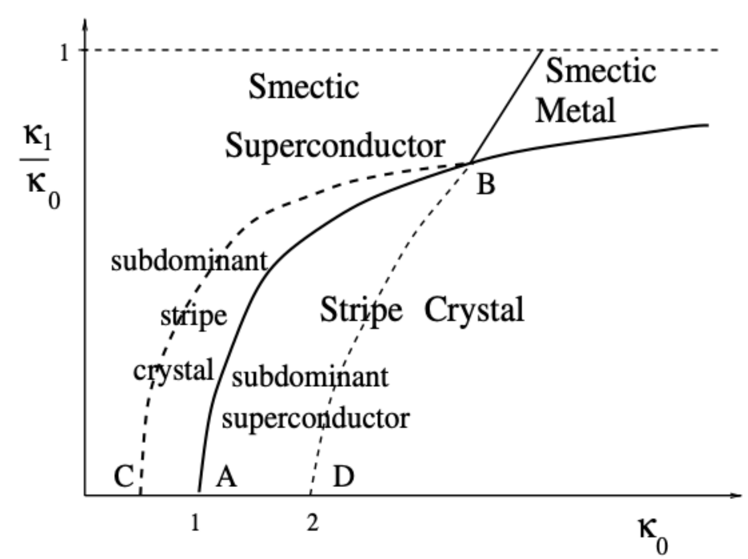
Electronic liquid-crystal phases of a doped Mott insulator

Nature 393, 550 (1998)

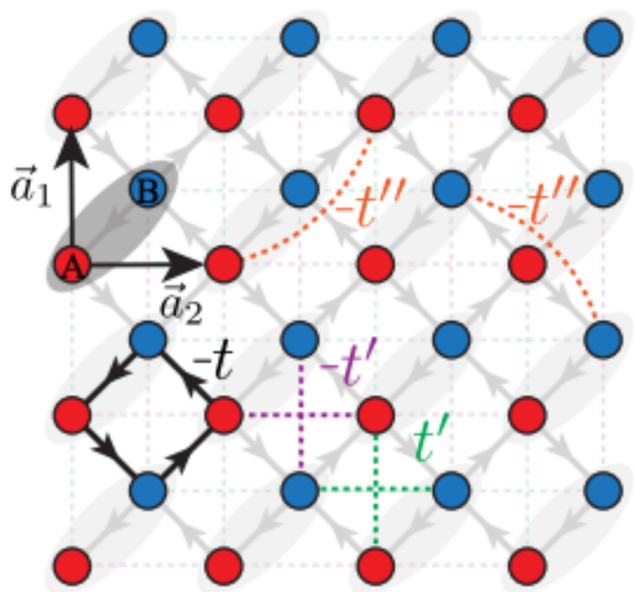
S. A. Kivelson*, E. Fradkin† & V. J. Emery‡

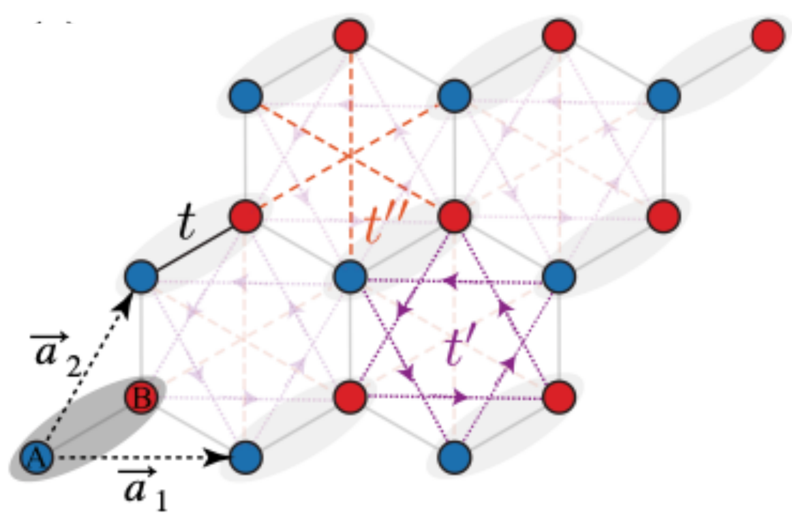


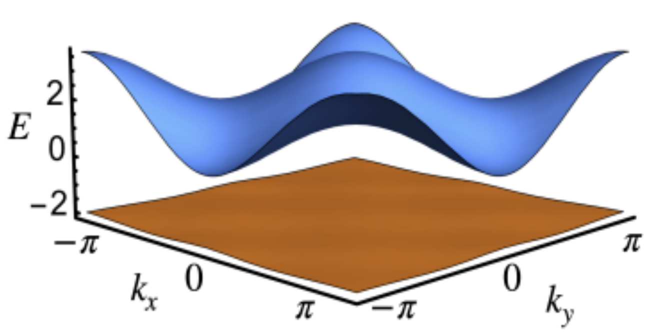
Coupled Luttinger liquid (non-Fermi liquid)

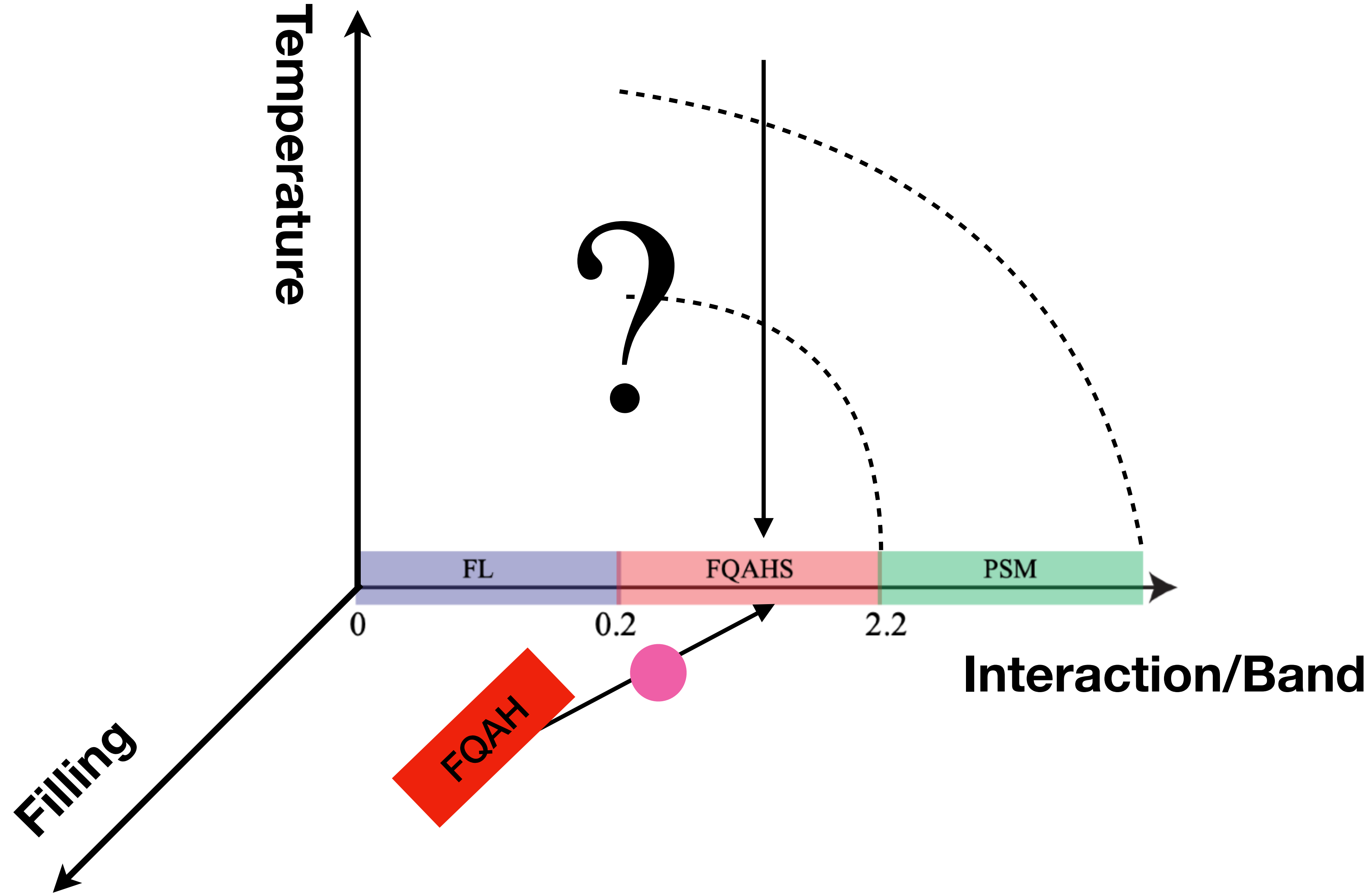


Emery, Fradkin, Kivelson, Lubensky, PRL 85, 2160 (2000)









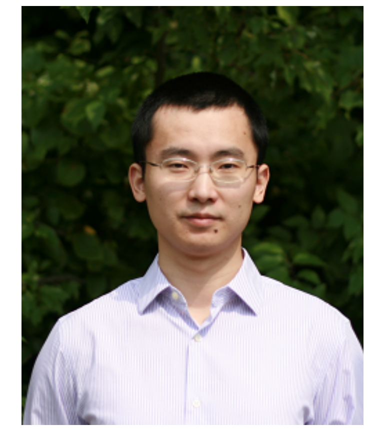
More questions:

1. Different fractional fillings of correlated flat bands
2. Different lattice geometries
3. Different excitations: magnetoroton, geometric graviton
4. Different techniques: ED, DMRG, Tensor, QMC
5. Different experiments: transport, STM, thermal measurements
6. Different communities: FQH, Stripe, FCI, Intertwinement, Vestigial
7. ……

Rep. Prog. Phys. **87** (2024) 108003 (11pp)

From Fractional Quantum Anomalous Hall Smectics to Polar Smectic Metals: Nontrivial Interplay Between Electronic Liquid Crystal Order and Topological Order in Correlated Topological Flat Bands

Hongyu Lu[1,4], Han-Qing Wu[2,4], Bin-Bin Chen[1,*], Kai Sun[3,*] and Zi Yang Meng[1,*]

arXiv:2408.07111

Vestigial Gapless Boson Density Wave Emerging between $\nu = 1/2$ Fractional Chern Insulator and Finite-Momentum Supersolid

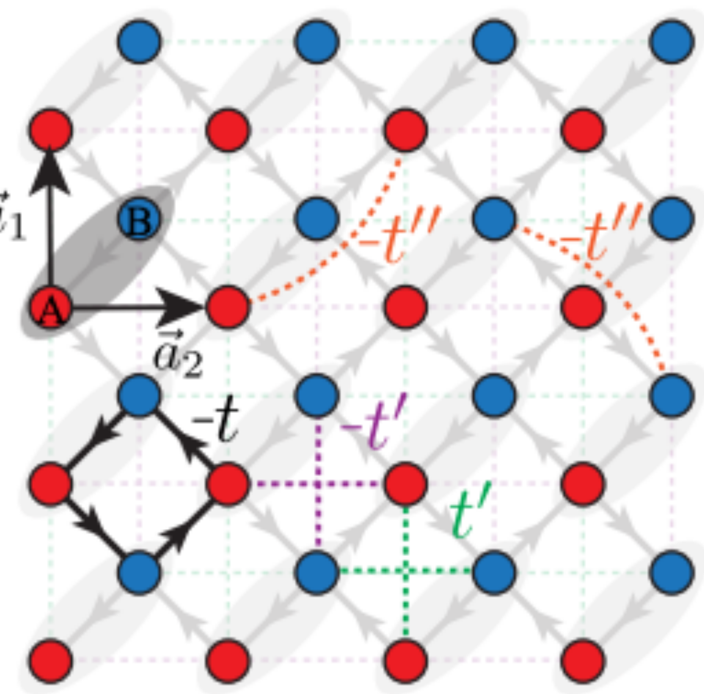
Hongyu Lu,[1] Han-Qing Wu,[2] Bin-Bin Chen,[1] and Zi Yang Meng[1,*]

PHYSICAL REVIEW LETTERS **132,** 236502 (2024)

Thermodynamic Response and Neutral Excitations in Integer and Fractional Quantum Anomalous Hall States Emerging from Correlated Flat Bands

Hongyu Lu,[1] Bin-Bin Chen,[1] Han-Qing Wu,[2] Kai Sun,[3,*] and Zi Yang Meng[1,†]

(a)



$$H = H_0 + H_I$$

$$H_0 = -t\sum_{\langle i,j\rangle} e^{i\phi_{ij}}(b_i^\dagger b_j + h.c.) - \sum_{\langle\langle i,j\rangle\rangle} t'_{ij}(b_i^\dagger b_j + h.c.) - t''\sum_{\langle\langle\langle i,j\rangle\rangle\rangle}(b_i^\dagger b_j + h.c.)$$

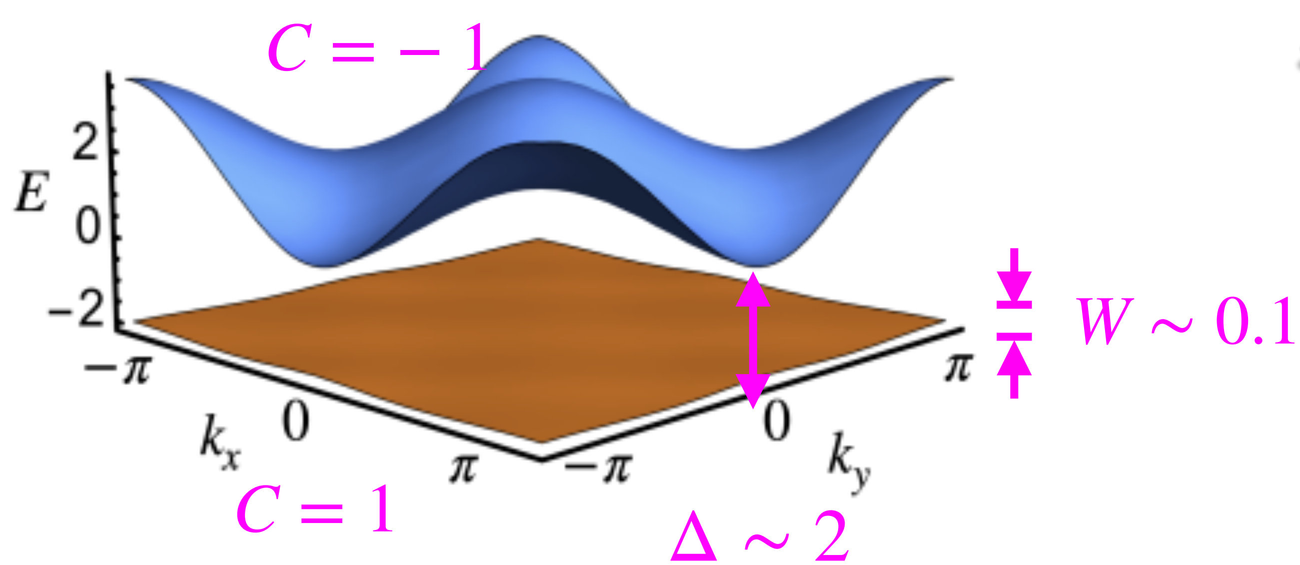
$$t = 1 \qquad t' = \pm\frac{1}{2+\sqrt{2}} \qquad t'' = -\frac{1}{2+2\sqrt{2}}$$

$$\phi_{ij} = \frac{\pi}{4}$$

$$H_I = V_1\sum_{\langle i,j\rangle} n_i n_j + V_2\sum_{\langle\langle i,j\rangle\rangle} n_i n_j$$

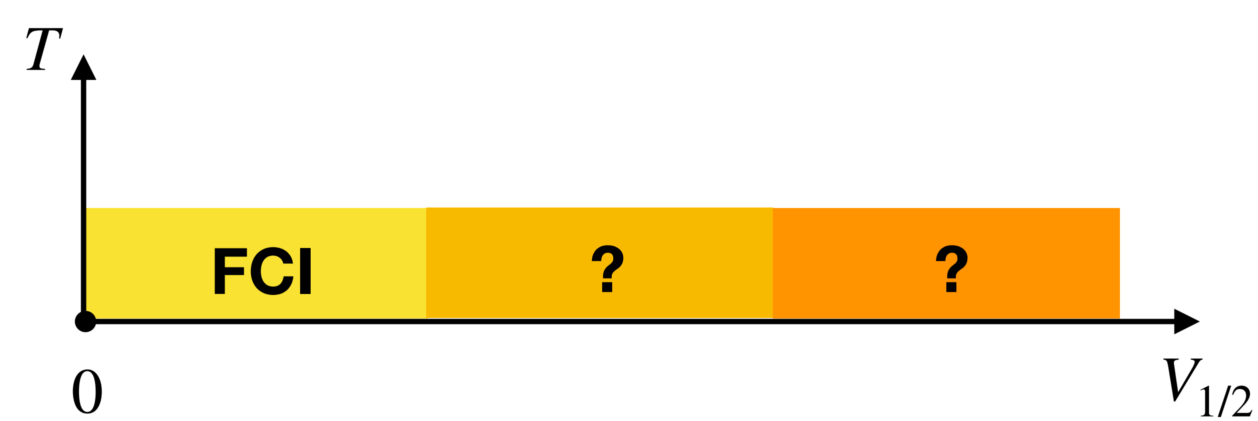
Hard-core boson and Consider filling factor of the flat band $\nu = 1/2$

(b)



Bosonic FCI at $V_1 = V_2 = 0$ limit

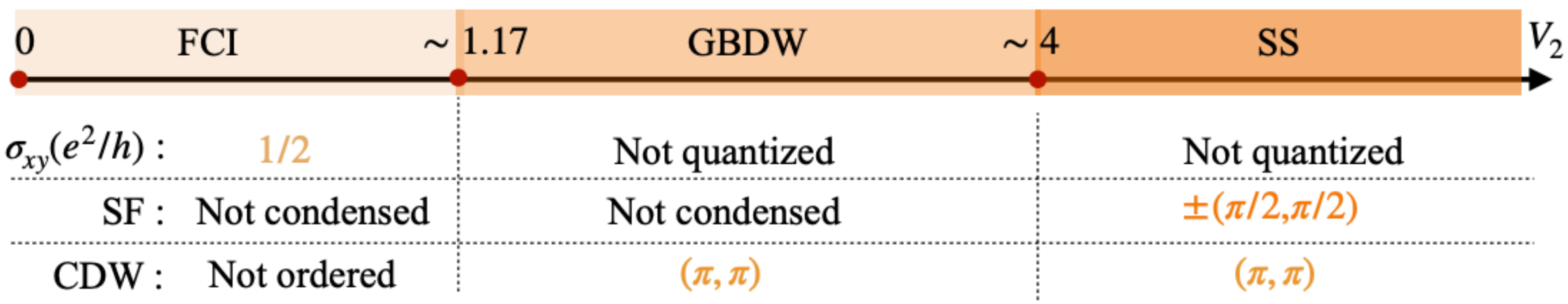
Y.-F. Wang, …, and D. N. Sheng, PRL 107, 146803 (2011)



Hongyu Lu et al., arXiv:2408.07111

$(V_1 = 1)$

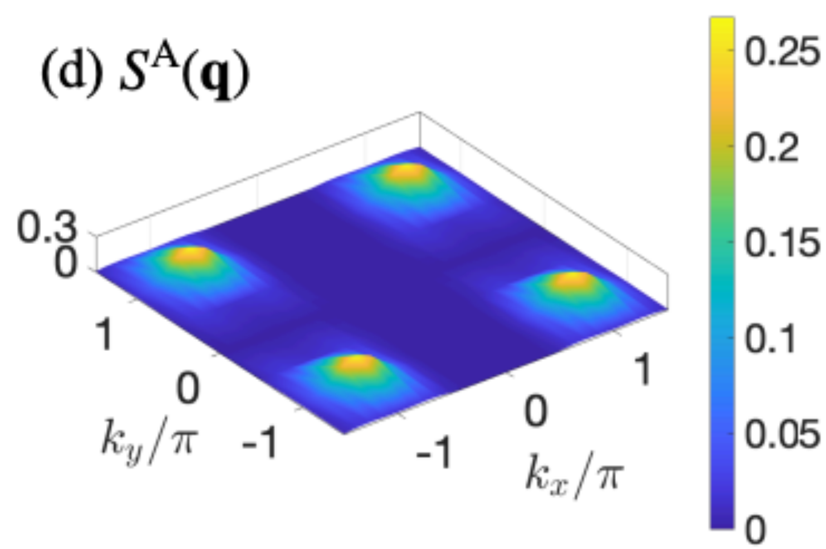
Hongyu Lu et al., arXiv:2408.07111

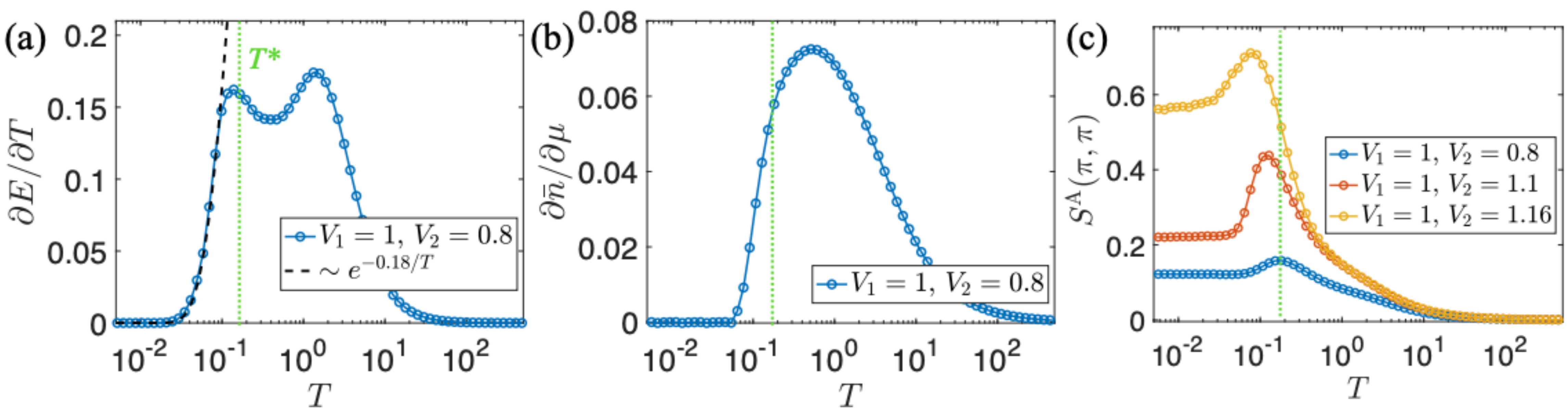


Boson FCI

$$S^{\alpha}(\mathbf{q}) = \frac{1}{N}\sum_{i,j} e^{-i\mathbf{q}\cdot\mathbf{r}_{i,j}}(\langle n_{i,\alpha}n_{j,\alpha}\rangle - \langle n_{i,\alpha}\rangle\langle n_{j,\alpha}\rangle)$$

Magnetoroton at (π, π)





$(V_1 = 1)$

Hongyu Lu et al., arXiv:2408.07111

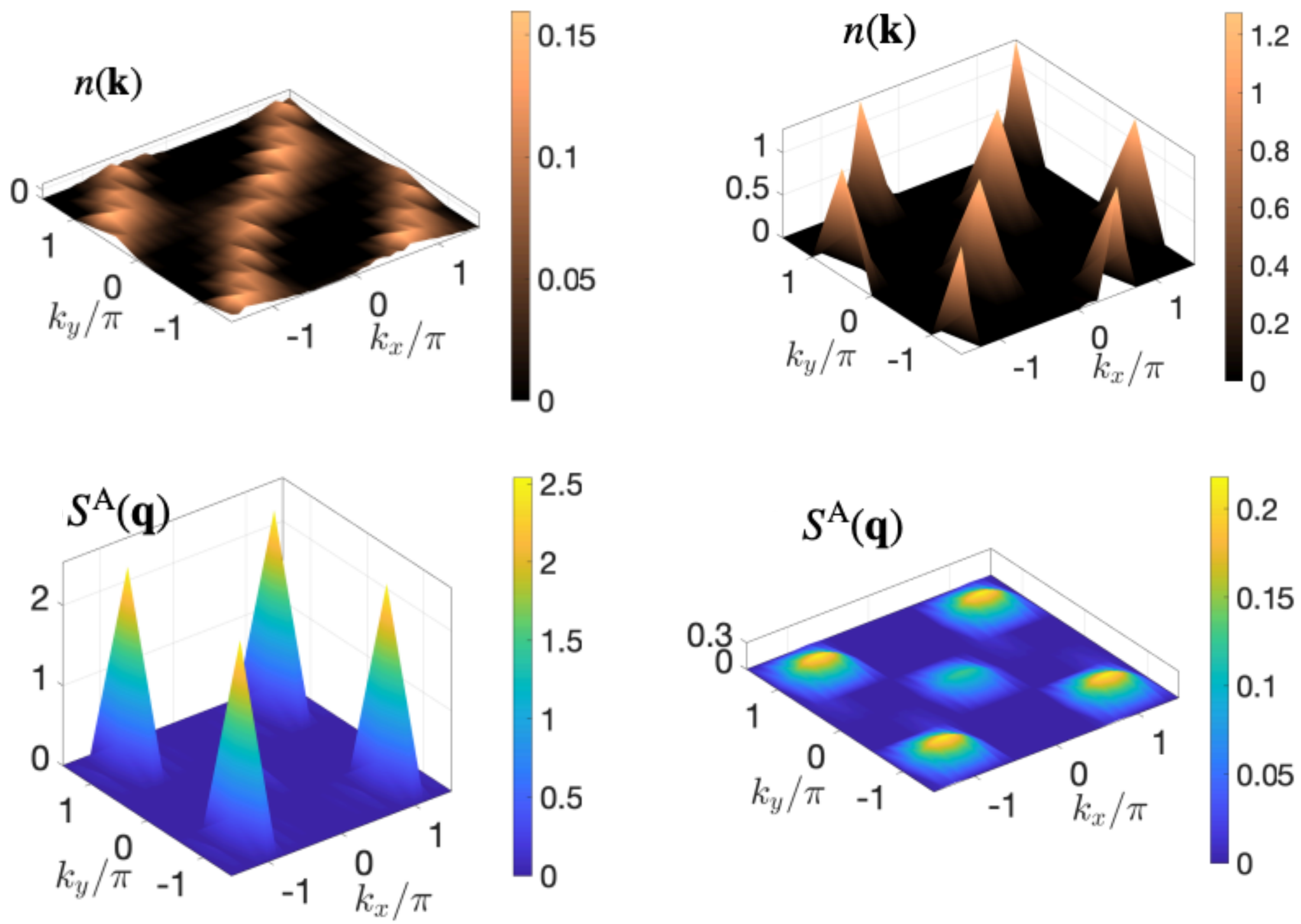
$\sigma_{xy}(e^2/h)$:	1/2	Not quantized	Not quantized
SF :	Not condensed	Not condensed	$\pm(\pi/2, \pi/2)$
CDW :	Not ordered	(π, π)	(π, π)

$$n(\mathbf{k}) = \frac{1}{2}\sum_{\alpha=A,B} n_\alpha(\mathbf{k}) = \frac{1}{N}\sum_{i,j} e^{-i\mathbf{k}\cdot\mathbf{r}_{i,j}}\langle b^\dagger_{i,\alpha} b_{j,\alpha}\rangle$$

$$S^\alpha(\mathbf{q}) = \frac{1}{N}\sum_{i,j} e^{-i\mathbf{q}\cdot\mathbf{r}_{i,j}}(\langle n_{i,\alpha} n_{j,\alpha}\rangle - \langle n_{i,\alpha}\rangle\langle n_{j,\alpha}\rangle)$$

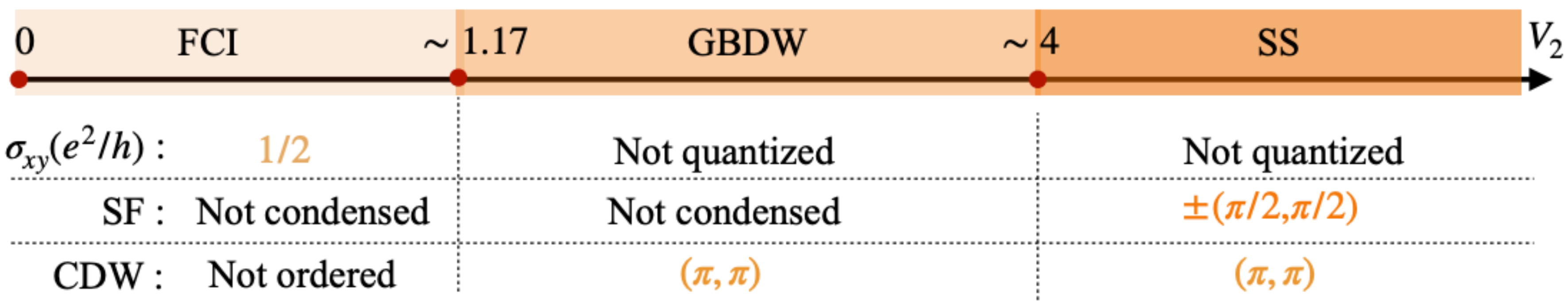
Gapless boson density wave

Supersolid

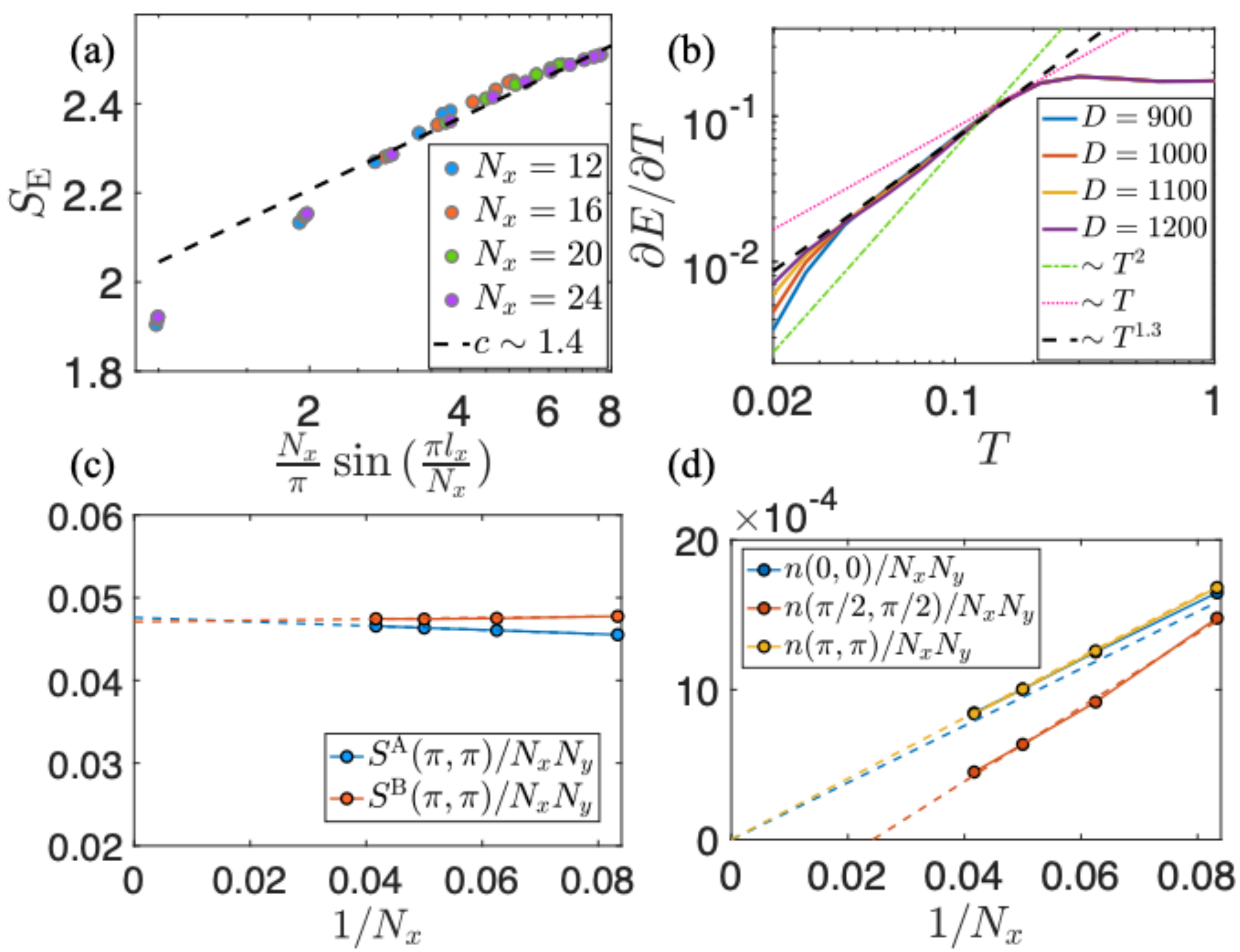


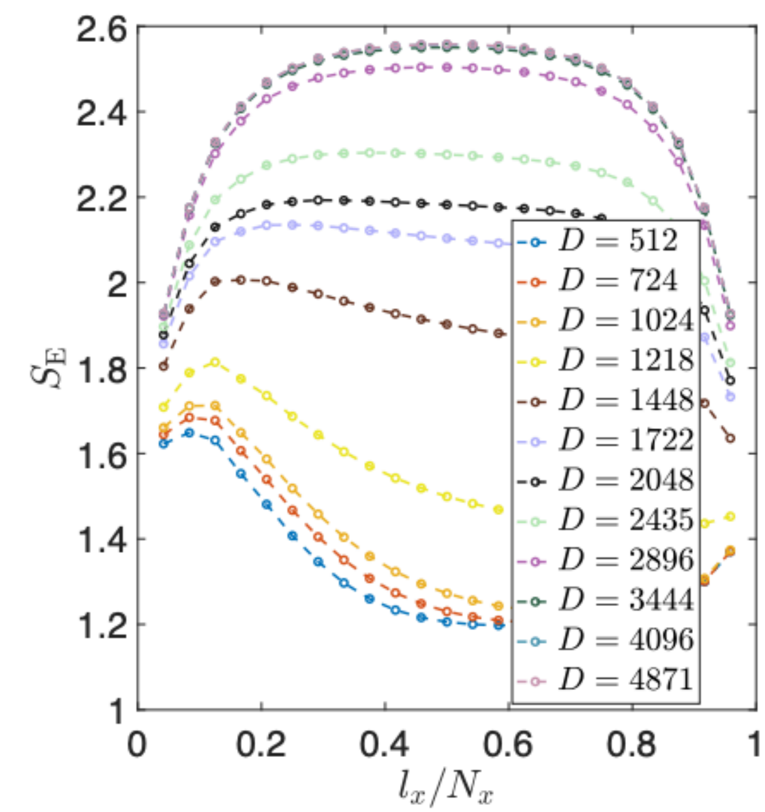
$(V_1 = 1)$

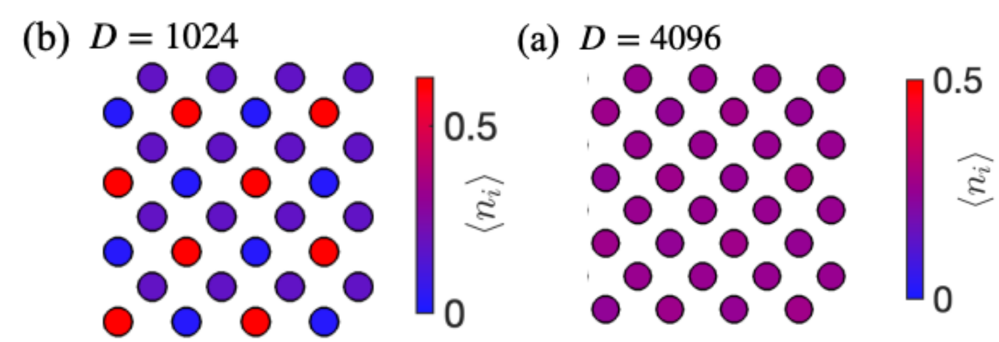
Hongyu Lu et al., arXiv:2408.07111



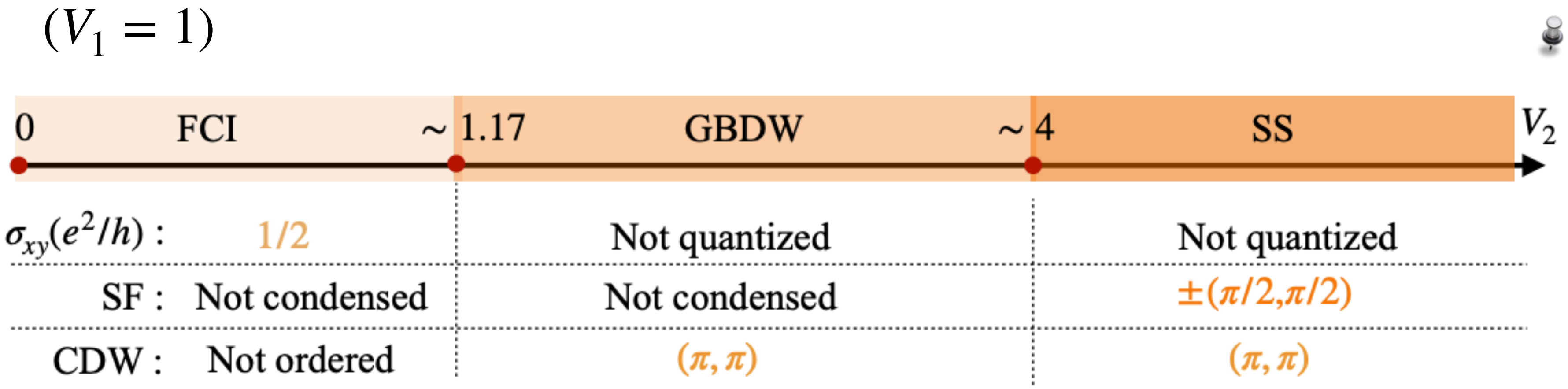
Gapless boson density wave



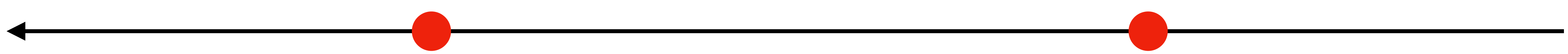




Hongyu Lu et al., arXiv:2408.07111



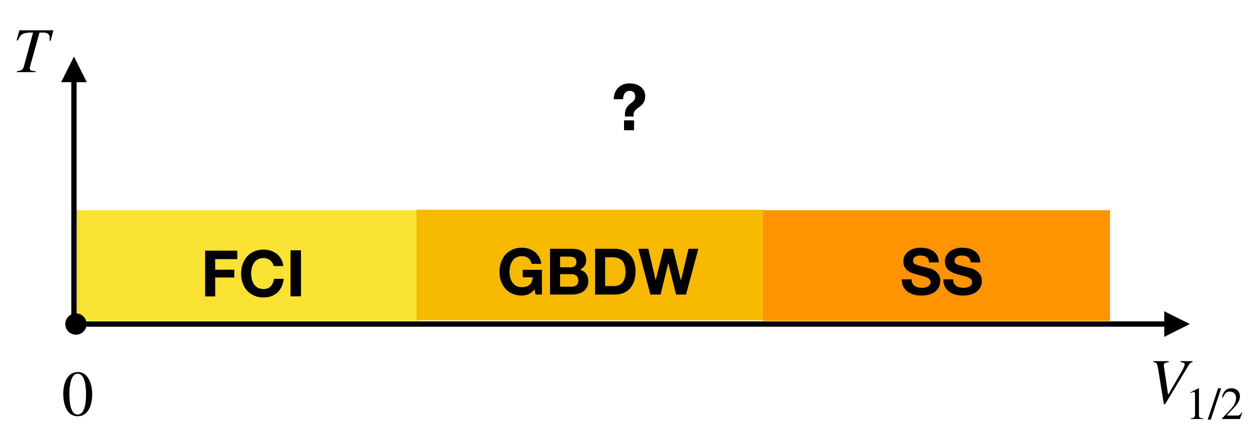
FCI | Gapless boson density wave | Supersolid

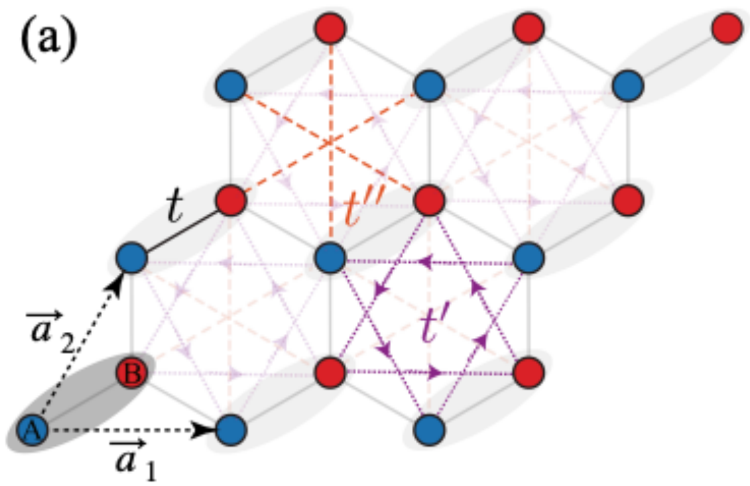


"Bose metal" | $\langle b_{\pm(\frac{\pi}{2},\frac{\pi}{2})} \rangle$

Topological order | $\langle \rho_{(\pi,\pi)} \propto b^\dagger_{(\frac{\pi}{2},\frac{\pi}{2})} b_{-(\frac{\pi}{2},\frac{\pi}{2})} \rangle$ | $\langle \rho_{(\pi,\pi)} \propto b^\dagger_{(\frac{\pi}{2},\frac{\pi}{2})} b_{-(\frac{\pi}{2},\frac{\pi}{2})} \rangle$

Roton softening **Vestigial / pair density wave transition**





$$H = H_0 + H_I$$

$$H_0 = -t \sum_{\langle i,j \rangle} (b_i^\dagger b_j + h.c.) - t' \sum_{\langle\langle i,j \rangle\rangle} (e^{i\phi} b_i^\dagger b_j + h.c.) - t'' \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} (b_i^\dagger b_j + h.c.)$$

$t = 1$ $\quad t' = 0.6$ $\quad t'' = -0.58$

$\phi = 0.4\pi$

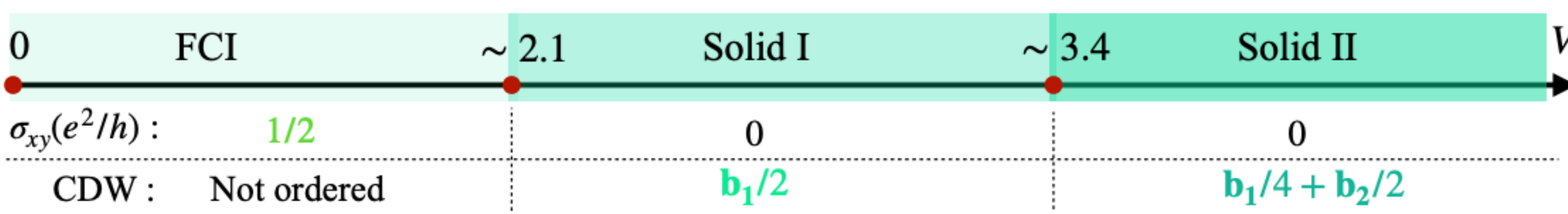
$$H_I = V_1 \sum_{\langle i,j \rangle} n_i n_j + V_2 \sum_{\langle\langle i,j \rangle\rangle} n_i n_j$$

Hard-core boson and Consider filling factor of the flat band $\nu = 1/2$

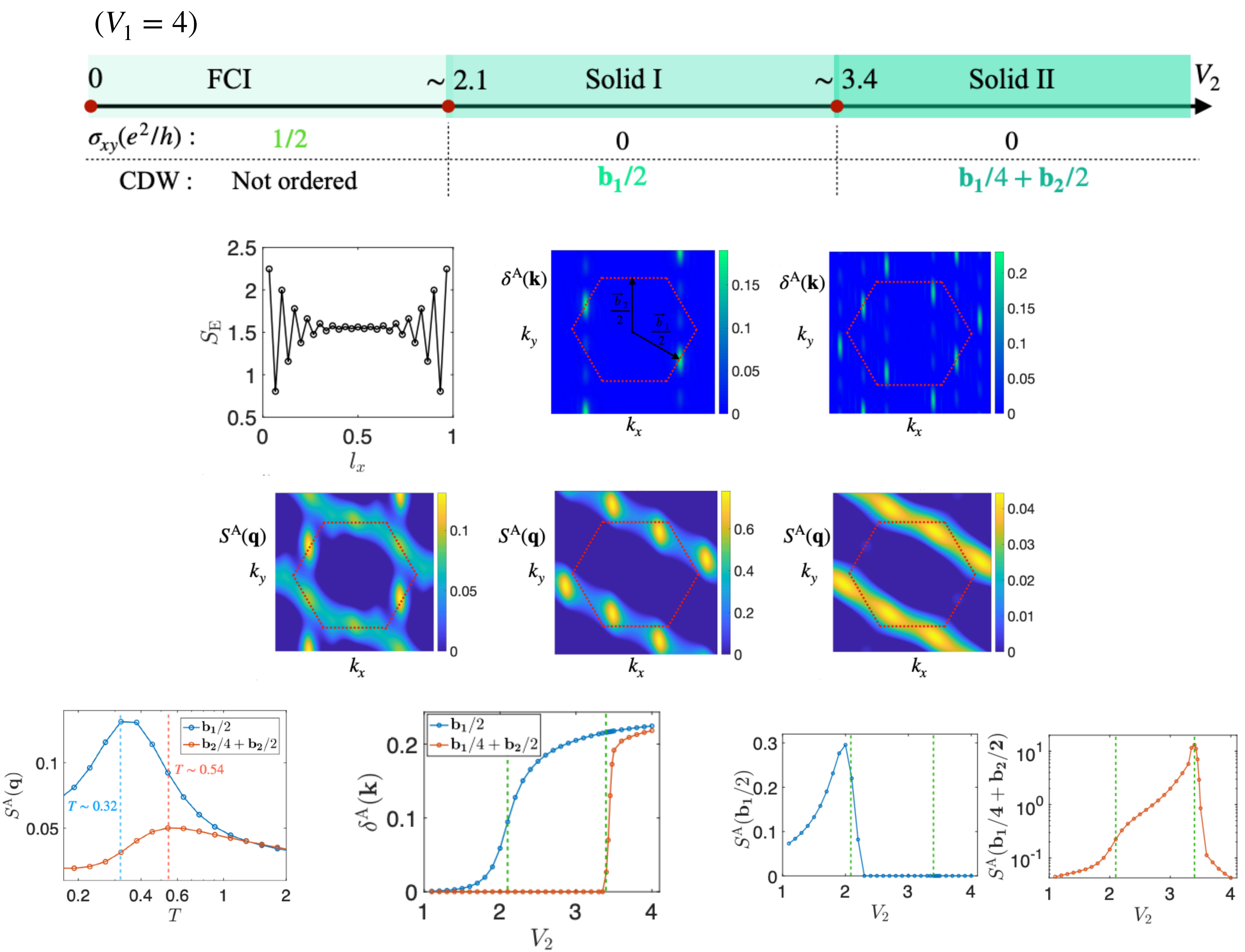
Bosonic FCI at $V_1 = V_2 = 0$ limit

- Y.-F. Wang, …, and D. N. Sheng, PRL 107, 146803 (2011)
- W.-W. Luo, …, and C.-D. Gong, PRB 102, 155120 (2020)

$(V_1 = 4)$



- Hongyu Lu et al., arXiv:2408.07111

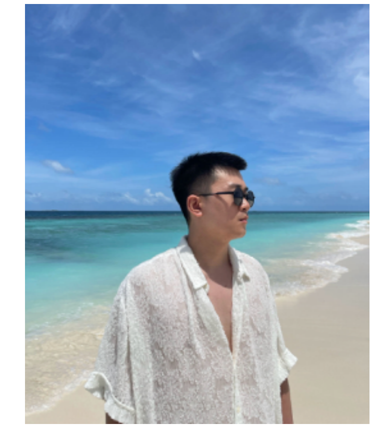


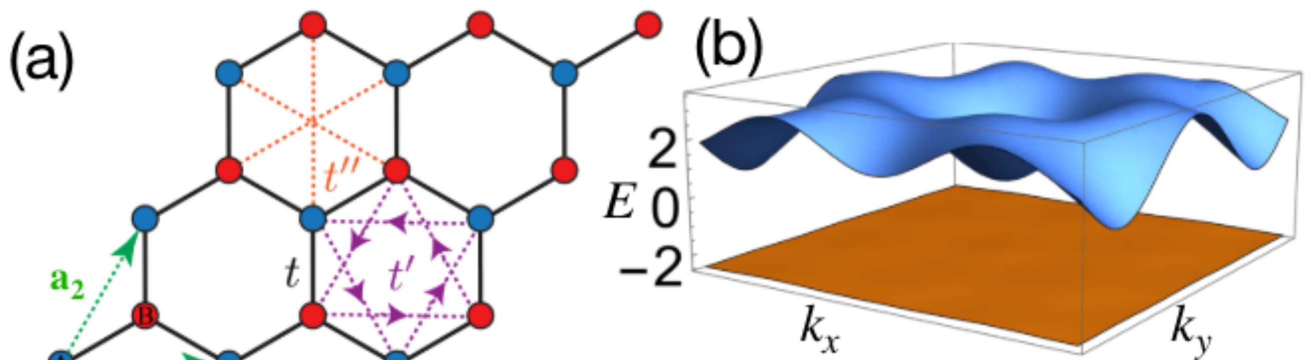
$(V_1 = 4)$
0
FCI
~ 2.1
Solid I
~ 3.4
Solid II
V_2
$\sigma_{xy}(e^2/h)$:
1/2
0
0
CDW :
Not ordered
$\mathbf{b}_1/2$
$\mathbf{b}_1/4 + \mathbf{b}_2/2$
S_E
l_x
$\delta^A(\mathbf{k})$
k_y
k_x
$S^A(\mathbf{q})$
$T \sim 0.32$
$T \sim 0.54$
T
V_2
$S^A(\mathbf{b}_1/2)$
$S^A(\mathbf{b}_1/4 + \mathbf{b}_2/2)$

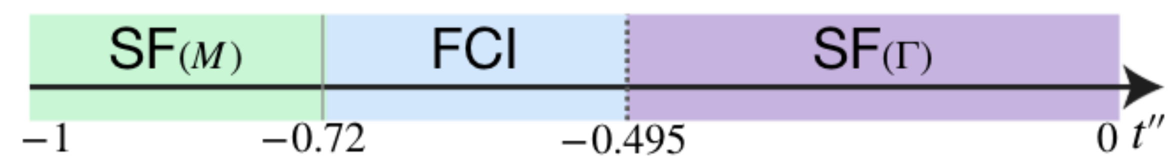
arXiv:2405.18269

Continuous Transition between Bosonic Fractional Chern Insulator and Superfluid

Hongyu Lu,[1] Han-Qing Wu,[2] Bin-Bin Chen,[1,*] and Zi Yang Meng[1,†]





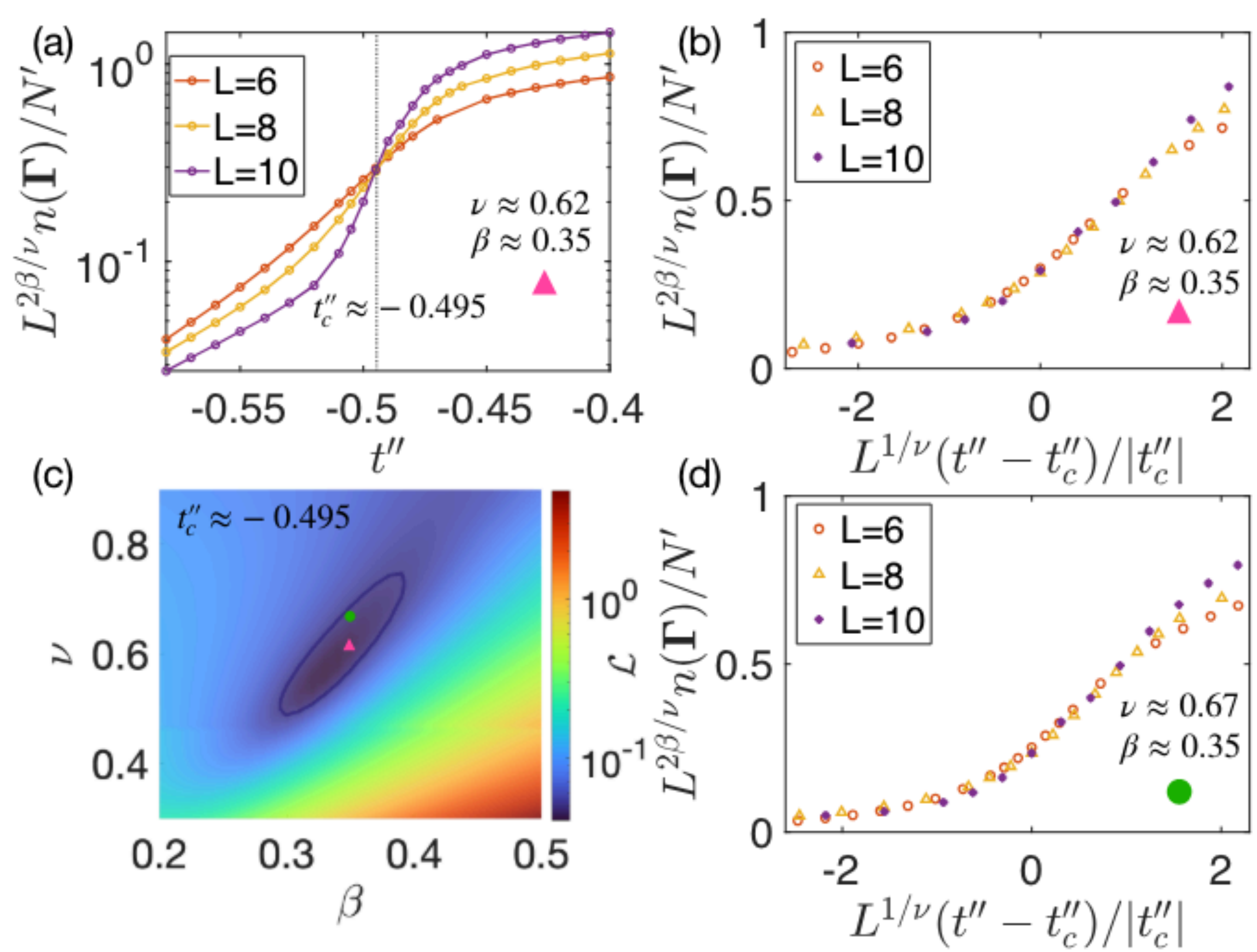


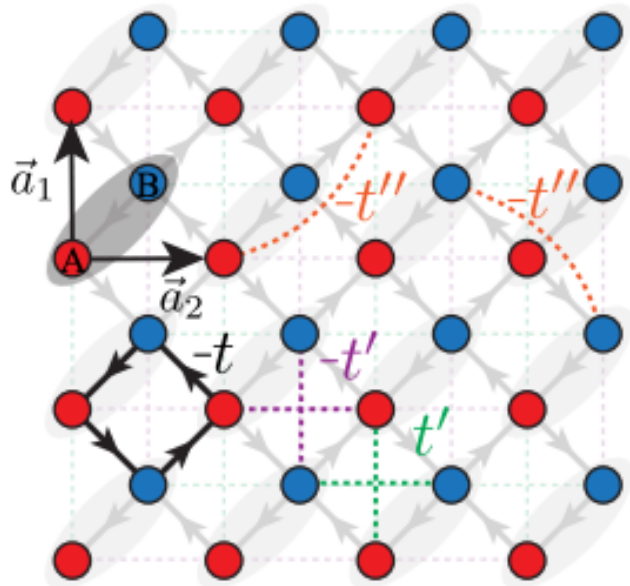
$$H = -\sum_{\langle i,j\rangle} t(b_i^\dagger b_j + \text{H.c.}) - \sum_{\langle\langle i,j\rangle\rangle} t'(e^{i\phi} b_i^\dagger b_j + \text{H.c.})$$

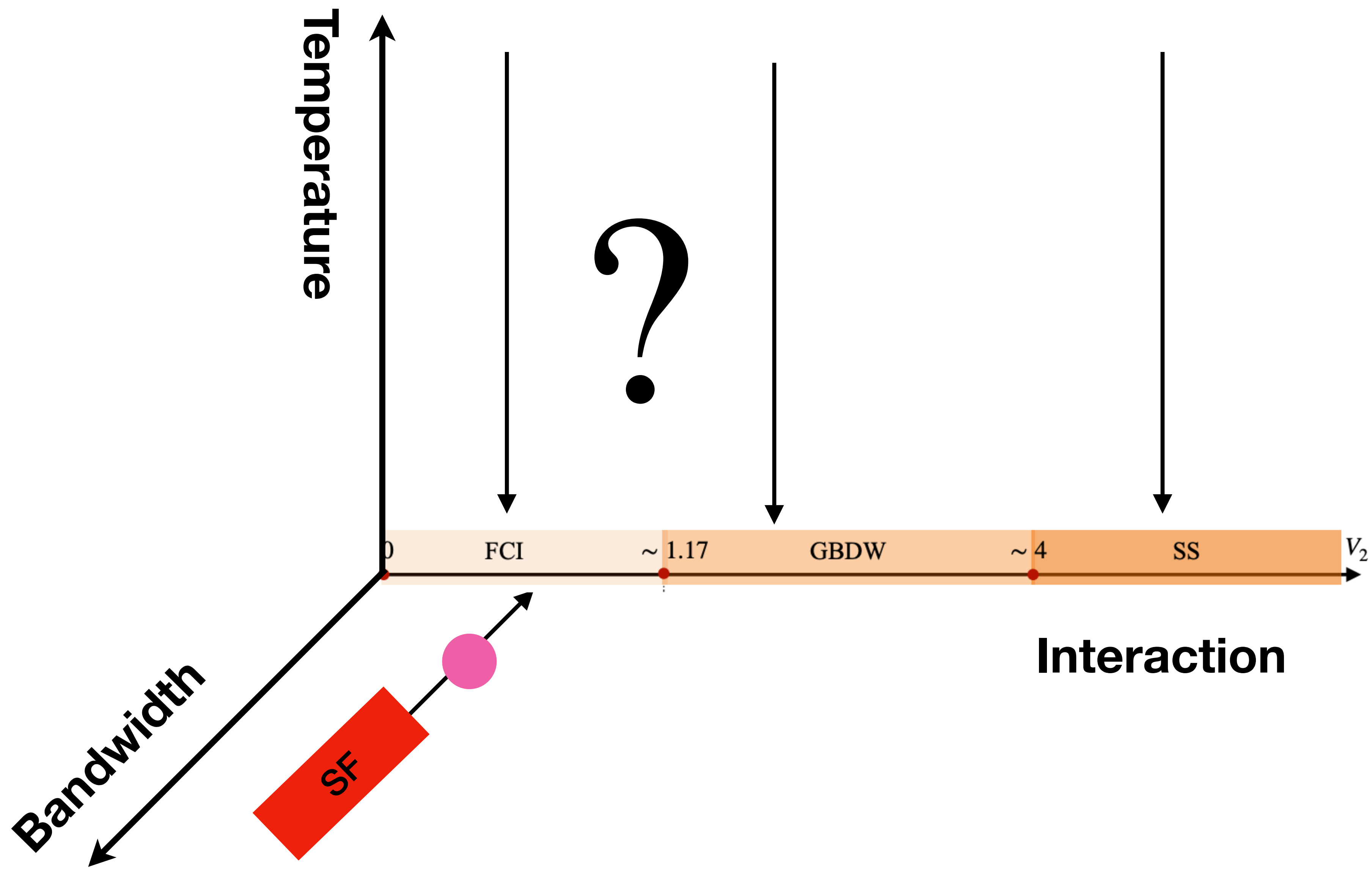
$$- \sum_{\langle\langle\langle i,j\rangle\rangle\rangle} t''(b_i^\dagger b_j + \text{H.c.}) + V_1 \sum_{\langle i,j\rangle} n_i n_j + V_2 \sum_{\langle\langle i,j\rangle\rangle} n_i n_j$$

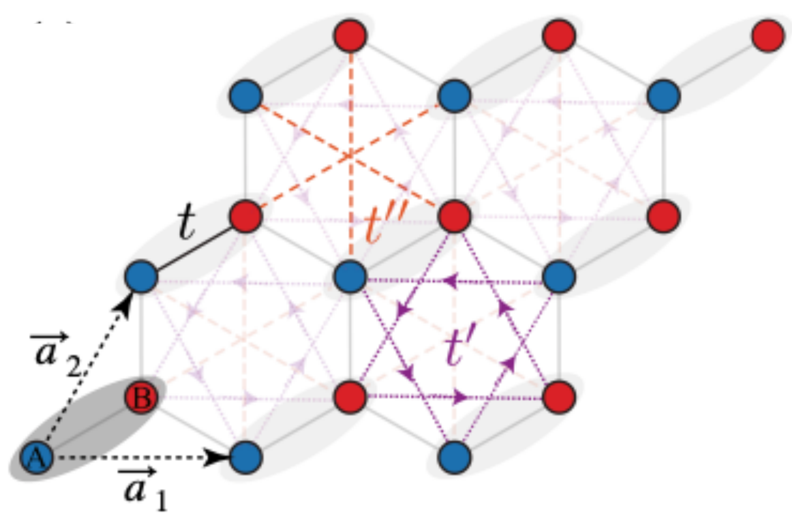
$t = 1 \quad t' = 0.6 \quad t'' = -0.58 \quad \phi = 0.4\pi$

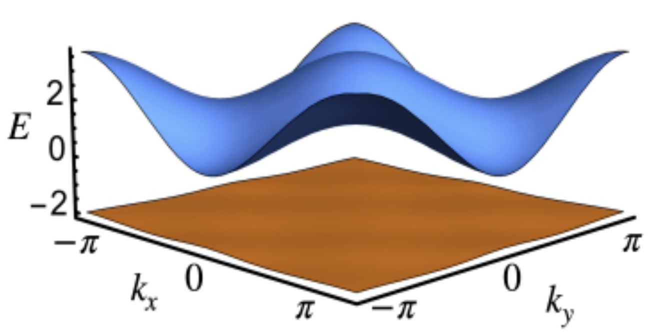
Hard-core boson $\quad V_1 = V_2 = 0$











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7. ……