Continuous Transition between Bosonic Fractional Chern Insulator and Superfluid

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Introduction

The continuous transition between fractional Chern insulator (FCI) and superfluid (SF) has been predicted by field theoretical studies, and it could be an example of transitions beyond the Landau-Ginzburg paradigm. Experimentally, it has been proposed as a scheme to (quasi)adiabatically prepare the FCI from SF in such as cold-atom experiments.

However, the existing numerical results of FCI-SF transition are either indirect or clearly first-order. Further, recent experimentally realized FCIs are only prepared from localized states instead of SF.

Extended Haldane model

- **Hard-core bosons on honeycomb lattice**

$$H = -\sum_{\langle i,j\rangle} t(b_i^\dagger b_j + \text{H.c.}) - \sum_{\langle\langle i,j\rangle\rangle} t'(e^{i\phi} b_i^\dagger b_j + \text{H.c.}) - \sum_{\langle\langle\langle i,j\rangle\rangle\rangle} t''(b_i^\dagger b_j + \text{H.c.}) + V_1 \sum_{\langle i,j\rangle} n_i n_j + V_2 \sum_{\langle\langle i,j\rangle\rangle} n_i n_j, \tag{1}$$

where $b_i^\dagger$ (b_i) creates (annihilates) a hard-core boson at the i-th site. We consider a zigzag geometry as shown in Fig. 1 (a) and set nearest-neighbor (NN) $t = 1$, next-nearest-neighbor (NNN) $t' = 0.6$, next-next-nearest-neighbor (NNNN) $t'' = -0.58$ and $\phi = 0.4\pi$, which are found as optimal flat-band parameters in this model and the Chern bands at this setting are shown in Fig. 1 (b). In this work, we focus on fixed t and t' while tune only t'' to control the bandwidth of the lower band. V_1 (V_2) refers to the amplitude of NN (NNN) repulsive interactions.

- **Method**

We mainly use the density matrix renormalization group (DMRG) method for simulations. We also conduct exact diagonalization (ED) simulations in this work and the results are in agreement with those of DMRG.

Results

- **Phase diagram with $V_1 = V_2 = 0$**

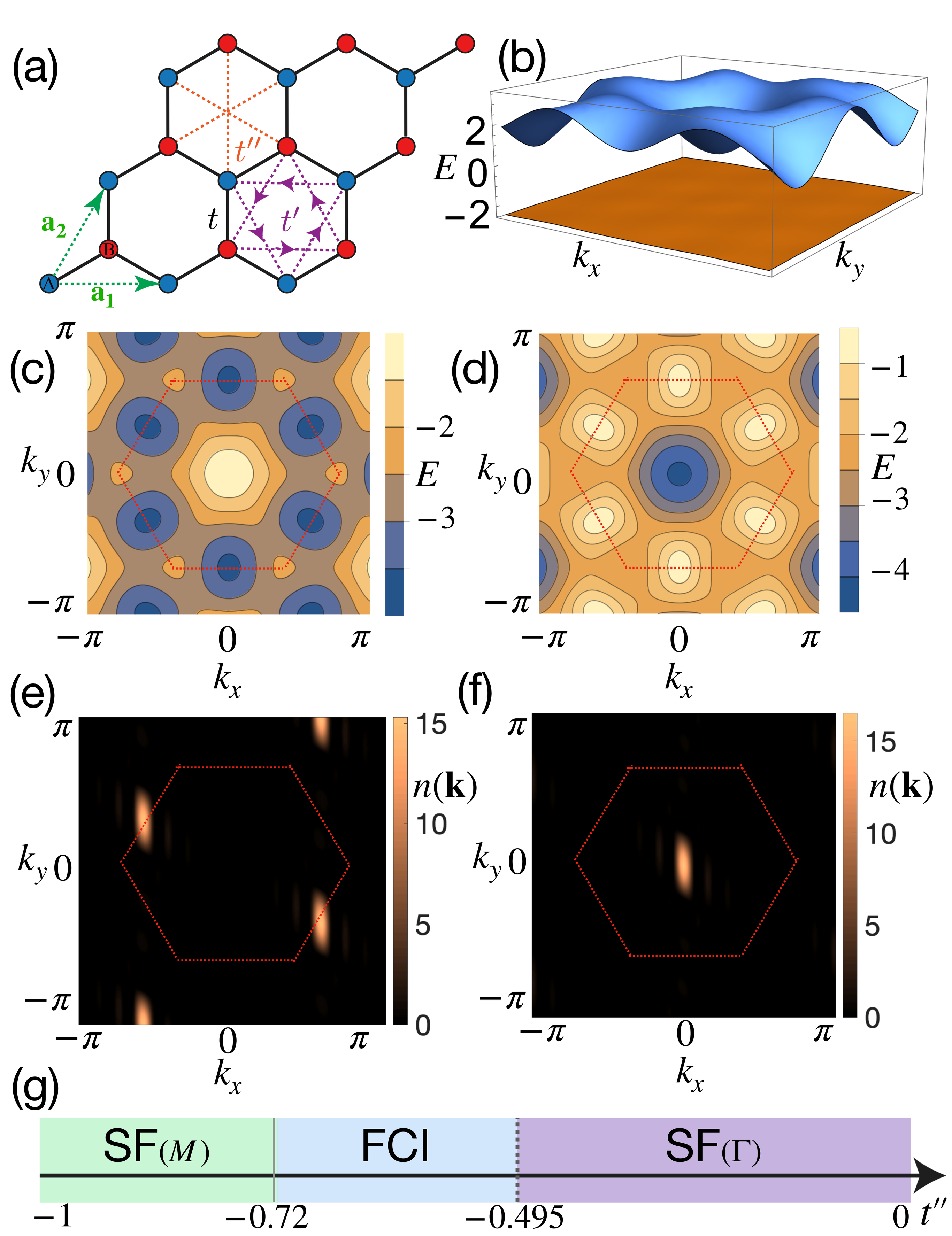

Fig 1: **Model and phase diagram.** (a) The honeycomb lattice with zigzag geometry ($N = 3 \times 3 \times 2$ for example here), where blue(red) sites refer to A(B) sublattice respectively. (b) The single-particle energy bands with the flat-band parameters in Eq. (1). We show the contour plot of the lower single-particle Chern band with (c) $t'' = -1$ and (d) $t'' = 0$, which are dispersive. The energy minimum is at (c) M point and (d) Γ point respectively. Boson occupation $n(\mathbf{k})$ in the Brillouin zone (BZ) with (e) $t'' = -1$ and (f) $t'' = 0$ from DMRG simulations of a $6 \times 18 \times 2$ cylinder. In the two SF phases, the condensed momenta are in agreement with the single-particle energy minima. The red dotted lines in (c-f) represent the first BZ. (g) Phase diagram when the lower Chern band is half filled with hard-core bosons when tuning t'' with $V_1 = V_2 = 0$. The gray solid line represents a first-order FCI-SF(M) transition, while the gray dashed line represents the continuous FCI-SF(Γ) transition.

With more supporting data in Fig. 2, we have found a continuous FCI-SF(Γ) transition by tuning the band dispersion, while the FCI-SF(M) transition turns out to be first-order. The difference of the two transitions by tuning the same parameter might be that the SF(M) state breaks not only the $U(1)$ symmetry but also the $C3$ rotation symmetry.

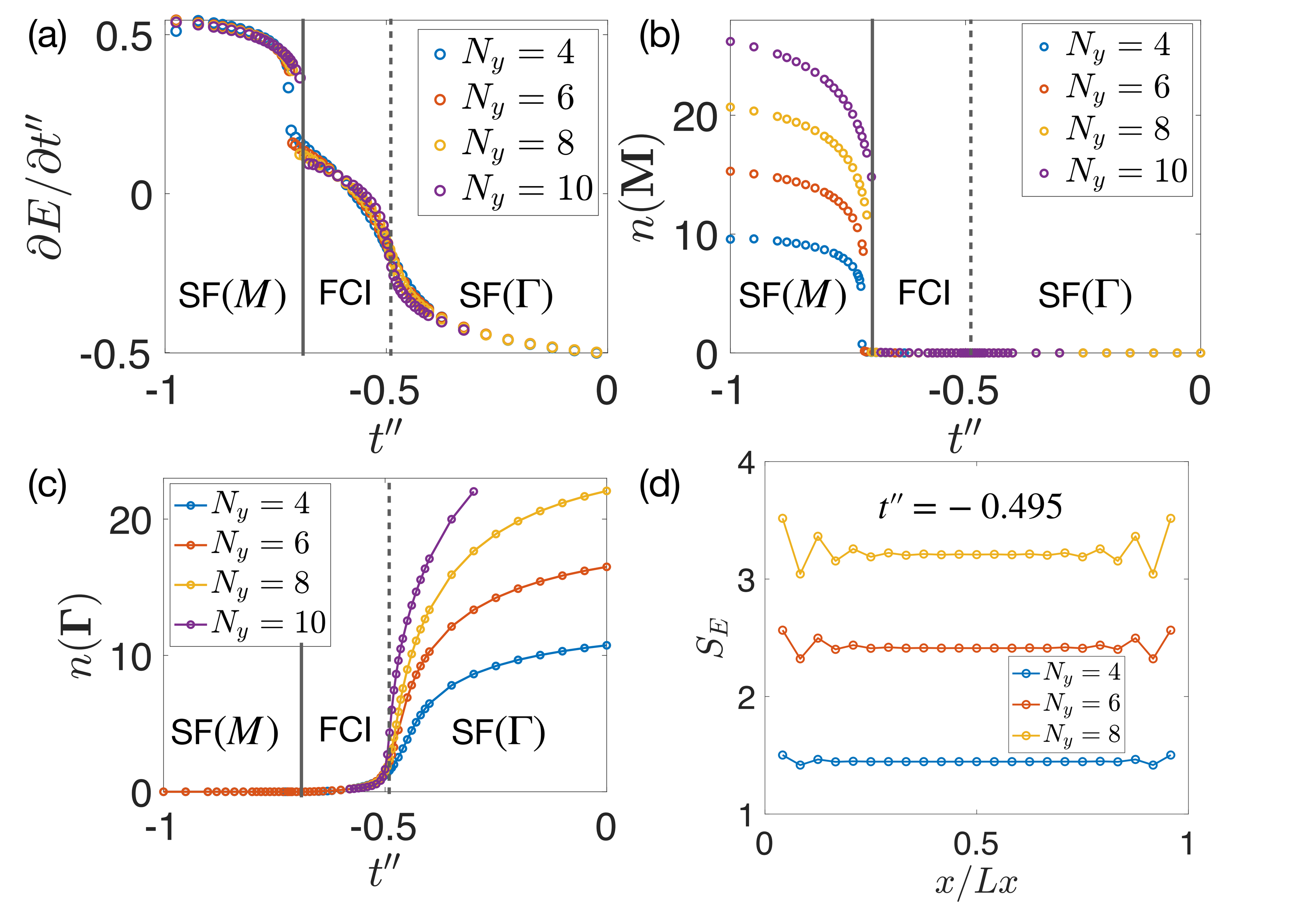

Fig 2: **Nature of the quantum phase transitions.** The DMRG results of (a) per-site energy derivative $\frac{\partial E}{\partial t''}$, boson occupations at the (b) M point and (c) Γ point in the BZ, as a function of t'' from cylinders up to $N_y = 10$, are shown. We use the grey solid and dashed lines determined from DMRG results to label the first-order FCI-SF(M) transition and the continuous FCI-SF(Γ) transition respectively as in Fig.1(g). (d) The entanglement entropy at the critical point $t'' = -0.495$.

The entanglement entropy from DMRG simulations at the critical point exhibits the area law scaling without the $\log(x)$ term, which is in agreement with the 2D critical boson nature.

- **Criticality analysis**

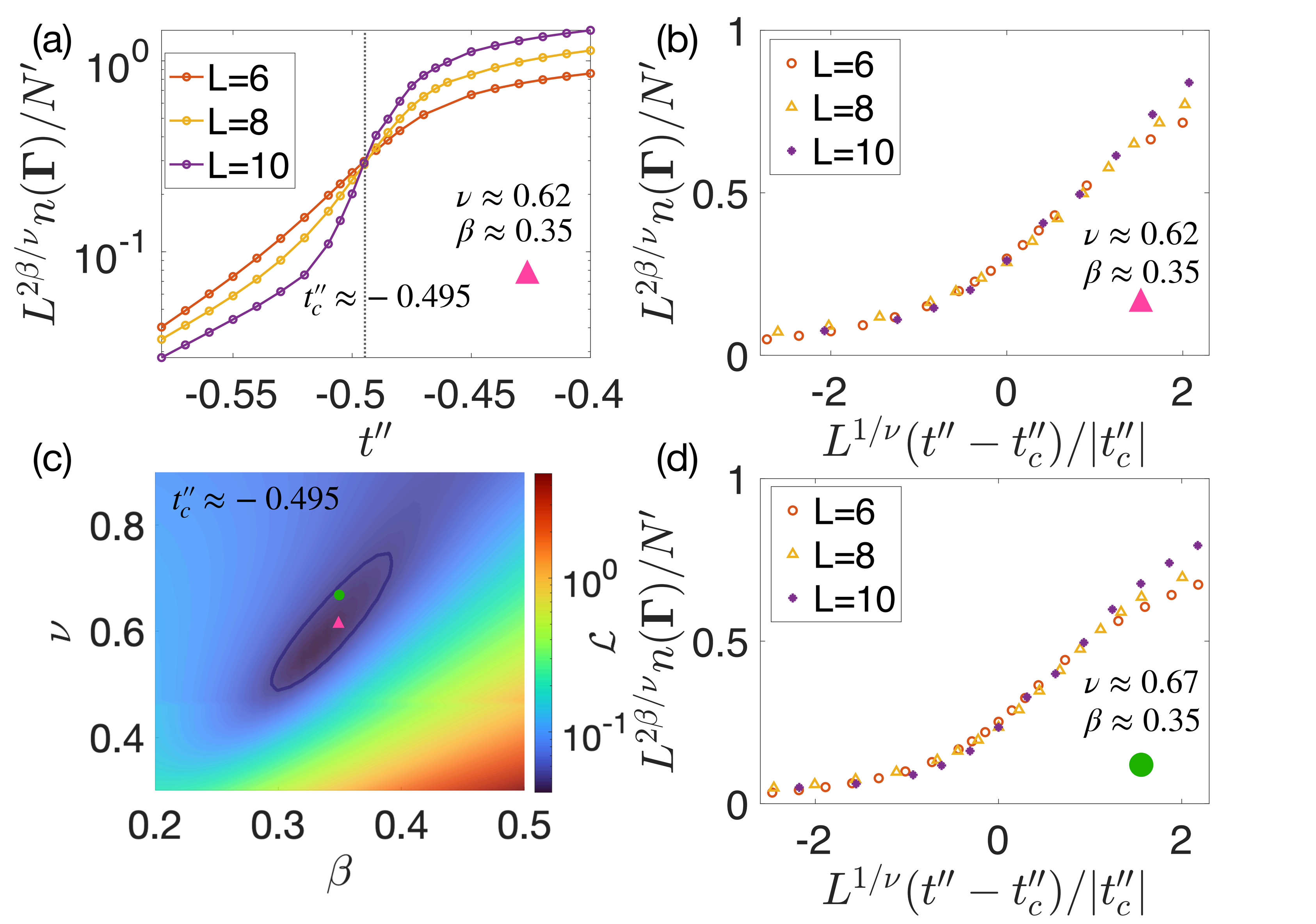

Fig 3: **Finite-size criticality analysis of FCI-SF(Γ) transition.** (a) Rescaled per-site occupation at Γ point of the BZ $n(\Gamma)/N'$ as a function of t'', which show good crossing at $t''_c \approx -0.495$. (b) Scaling collapse obtained by plotting $L^{2\beta/\nu} n(\Gamma)/N'$ as a function of $L^{1/\nu}(t'' - t''_c)/|t''_c|$. The critical exponents in (a) and (b) are from (c), which shows the loss function of data collapse with changing β and ν, defined as the squared deviation of the fitted scaling function away from the data points. With $t''_c \approx 0.495$, the optimal critical exponents exist in a range of parameter space with $\beta \approx 0.35(5)$ and $\nu \approx 0.62(12)$, denoted by the deep blue contour, which is from the nearest extremum of the loss function gradient. The pink triangle labels the exponents used in (a) and (b), while the green circle labels the critical exponents of the 3D XY universality class, with which we further show the data collapse in (d).

Limited by the finite size and cylinder geometry in our DMRG simulations, whether the exact values of the exotic FCI-SF(Γ) transition should deviate from those of the 3D XY universality class might need more accuarate simulations.

- **Continuous FCI-SF(Γ) transition with neighboring interactions**

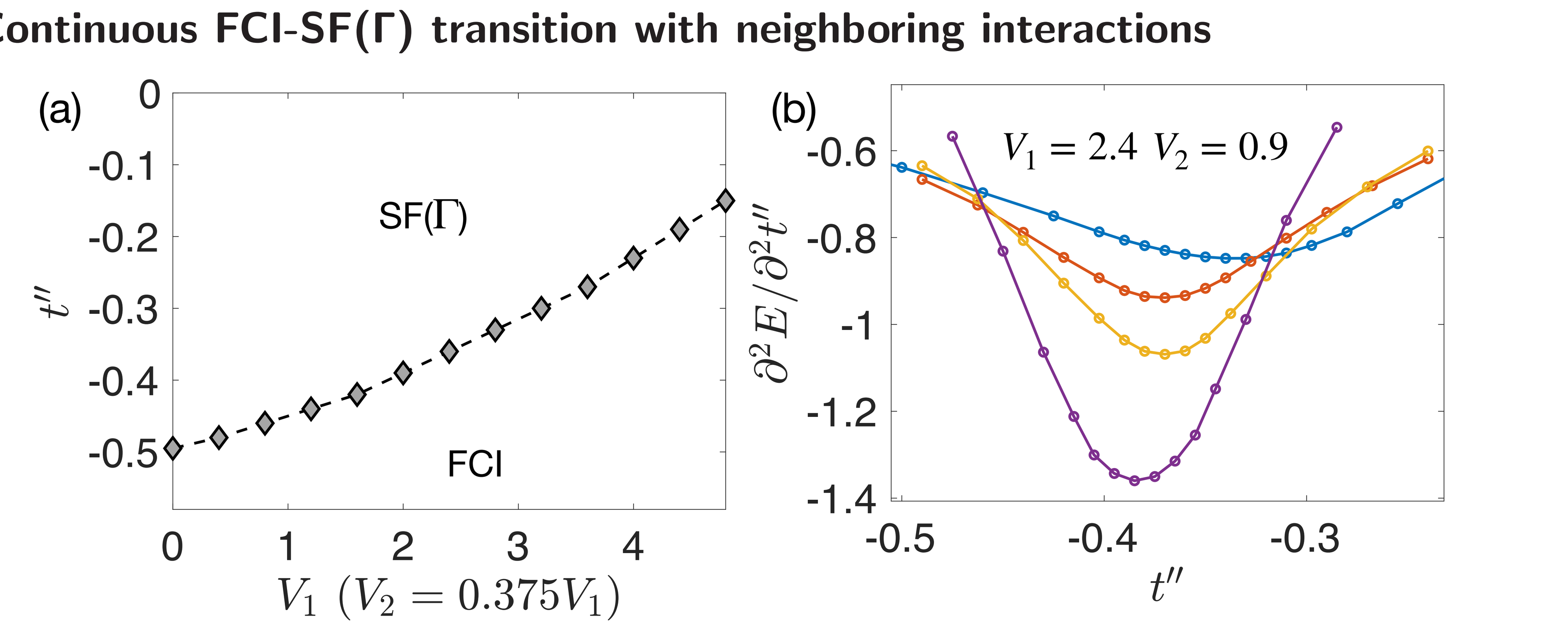

Fig 4: (a) The $V_1(V_2)$-t'' phase diagram with a fixed $V_2 = 0.375V_1$ and the continuous transition is robust against V_1/V_2 (further increasing interaction would lead to phase transitions even at the flat-band parameter). (b) The second-order energy derivative at $V_1 = 2.4$ and $V_2 = 0.9$ as an example (different colors represent N_y frm 4 to 10).

Discussions

- It is interesting to further investigate the continuous FCI-SF(Γ) transition, including the more precise critical exponents and possible emergent symmetry at critical point.
- As the first numerical discovery of this continuous transition, this might provide more perspectives for preparing FCIs in ultra-cold experiments.

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