Phase diagram of quantum loop model on triangular lattice

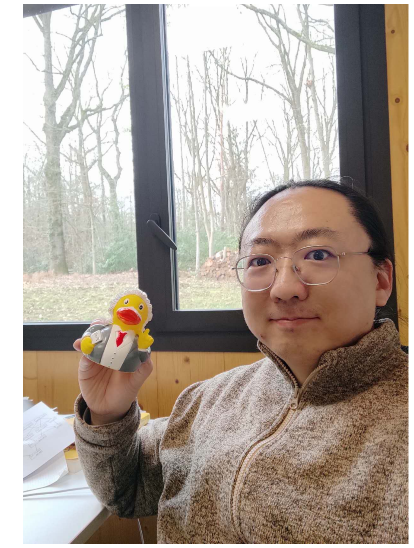
Xiaoxue Ran & Zi Yang Meng
https://quantummc.xyz/

Collaborators

Xiaoxue Ran

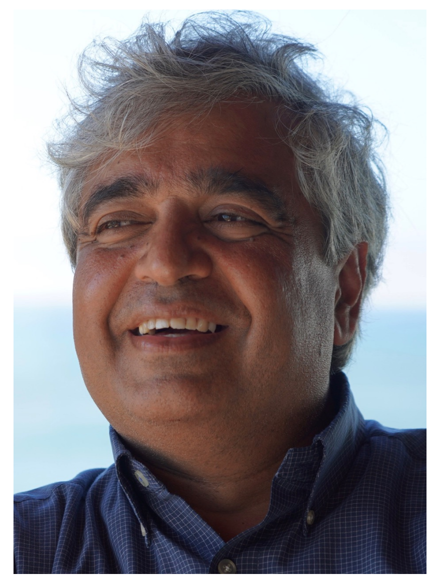
Zheng Yan

Yan-Cheng Wang



Junchen Rong

Rhine Samajdar



Subir Sachdev

Yang Qi

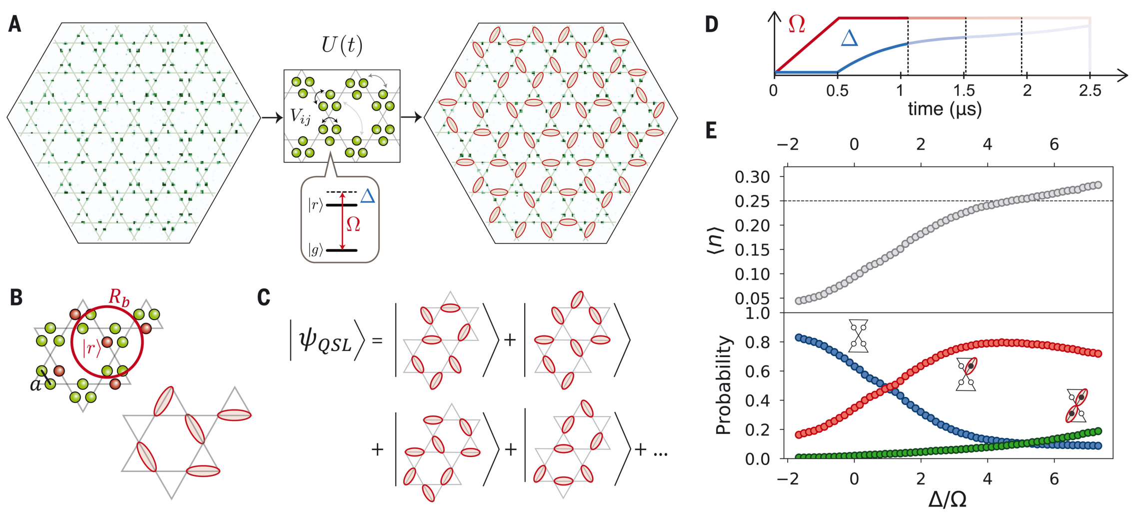
X. Ran, Z. Yan, Y-C Wang, R. Samajdar, J. Rong, S. Sachdev, Y. Qi, and Z. Y. Meng. Commun Phys 7, 207 (2024).

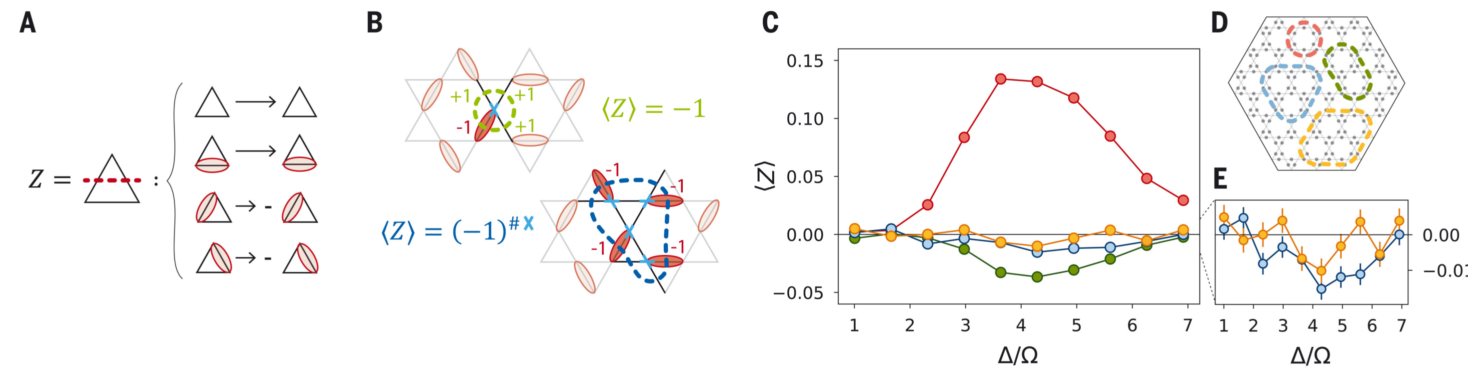
X. Ran, Z. Yan, Y-C Wang, J. Rong, Y. Qi, and Z. Y. Meng. Phys. Rev. B 109, L241109 (2024).

Z .Yan, Y-C Wang, R. Samajdar, S. Sachdev, Z. Y. Meng, Phys. Rev. Lett. 130, 206501 (2023).

Rydberg atom array

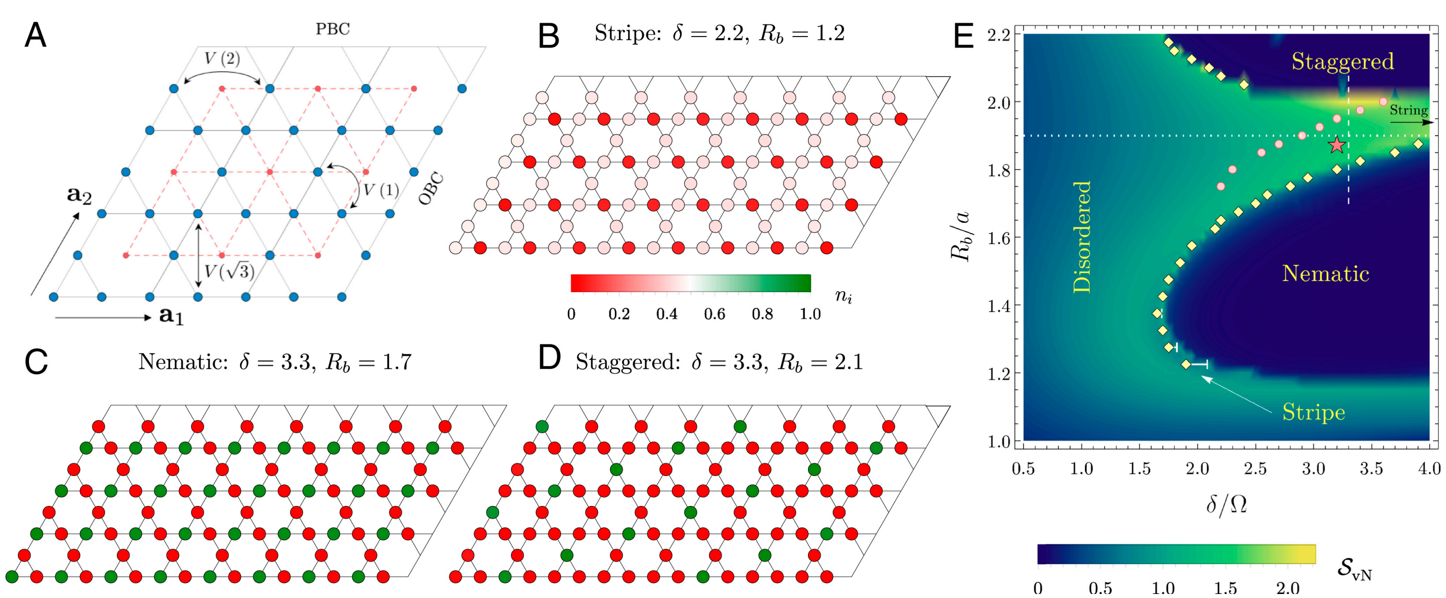
Rydberg atoms on the kagome lattice (links) maps to dimer model on the kagome lattice

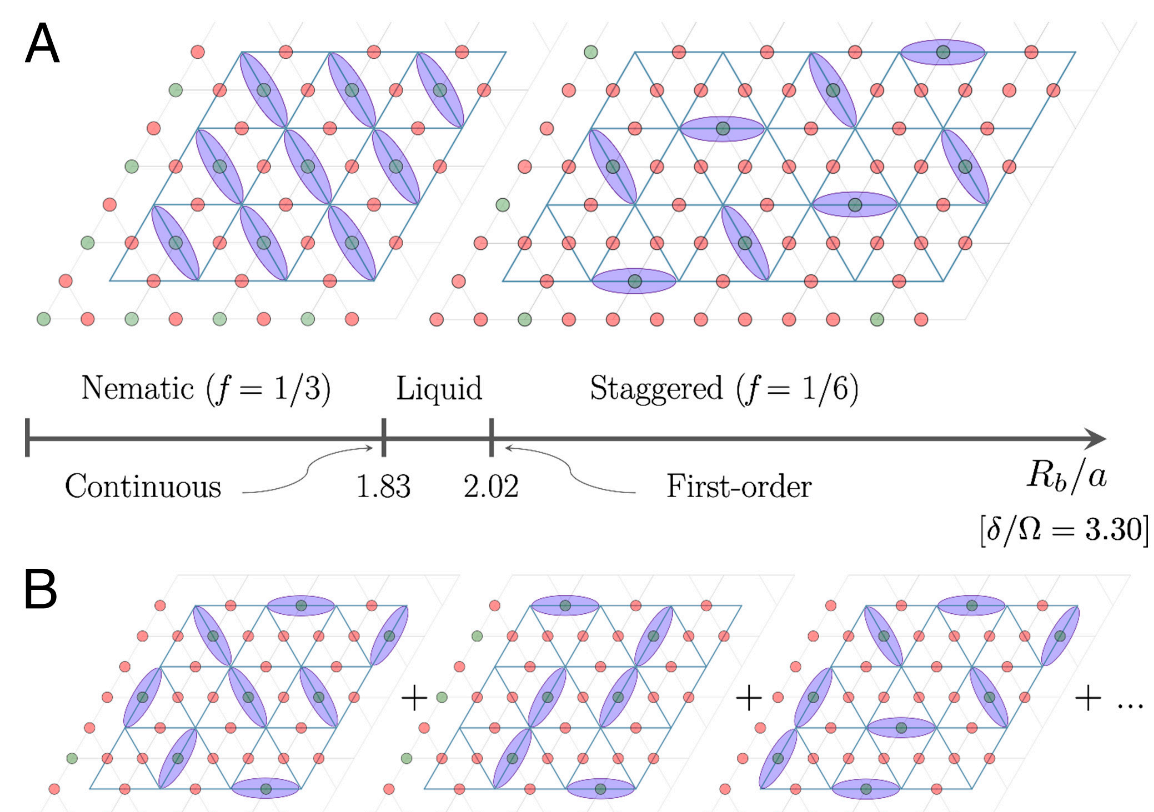




Semeghini G, Levine H, Keesling A, et al. Science, 374, 6572 (2021).

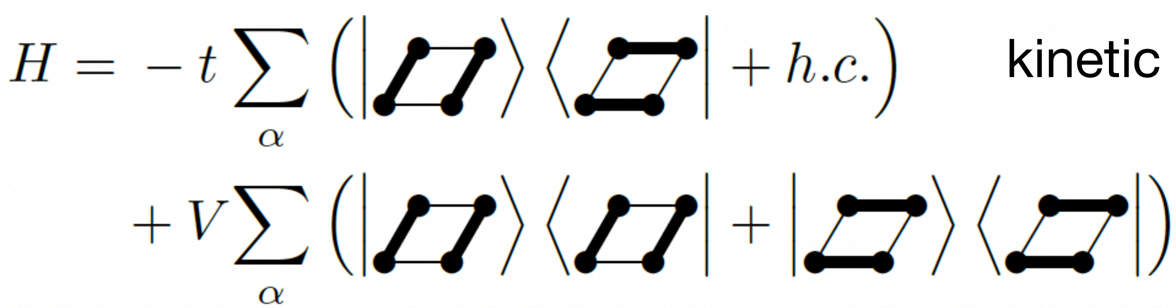
Rydberg atoms on the kagome lattice (sites) maps to dimer\loop model on the triangular lattice





Samajdar R, Ho W W, Pichler H, et al. PNAS, 118, 4 (2021).

Quantum loop model (QLM)

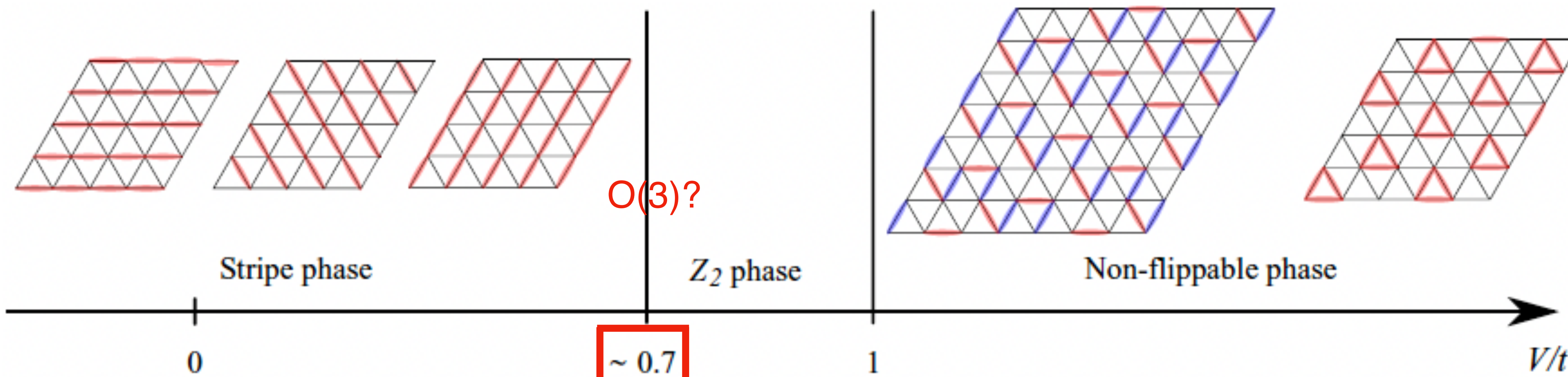


kinetic term

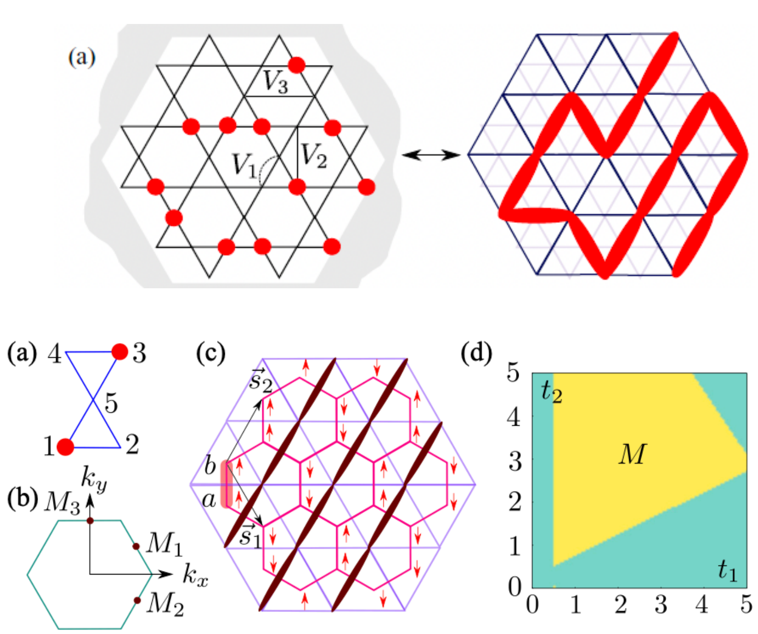
potential term

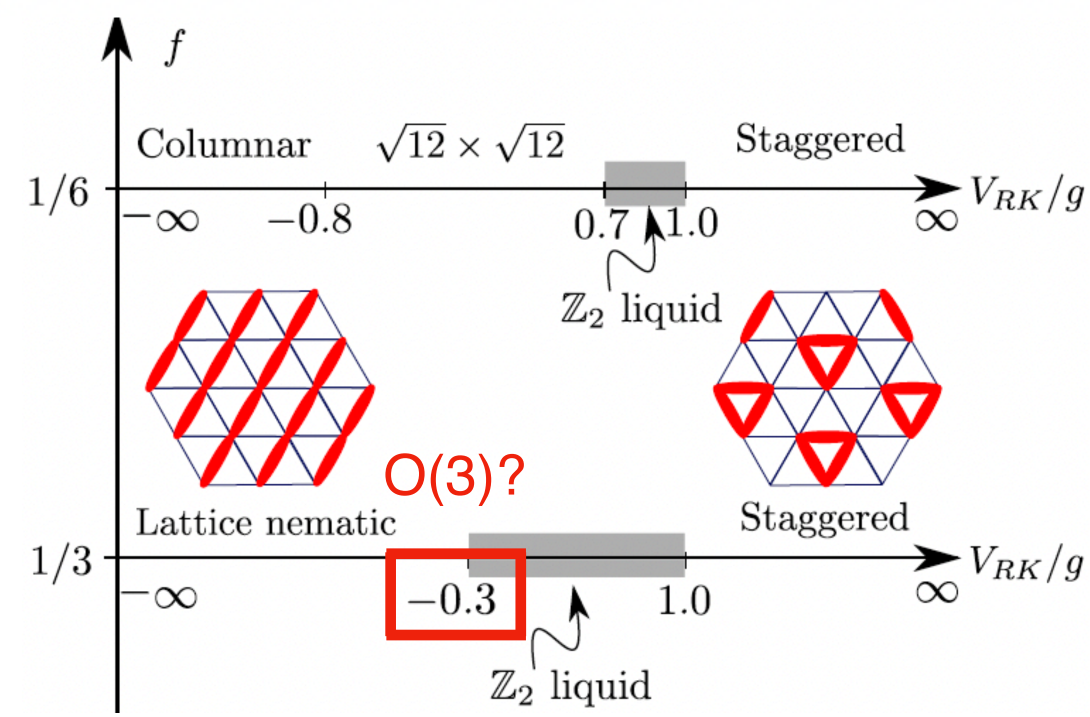
Constraint: two dimer per site

X. Plat, F. Alet, S. Capponi, and K. Totsuka, Phys. Rev. B 92, 174402 (2015).

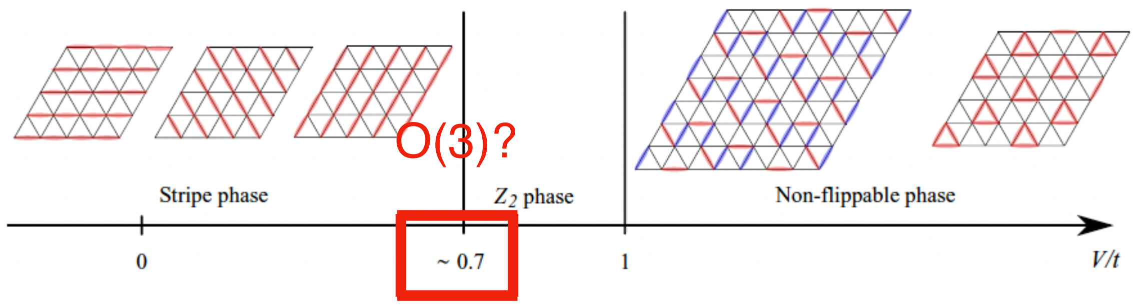


1/3 filled hard-core boson model



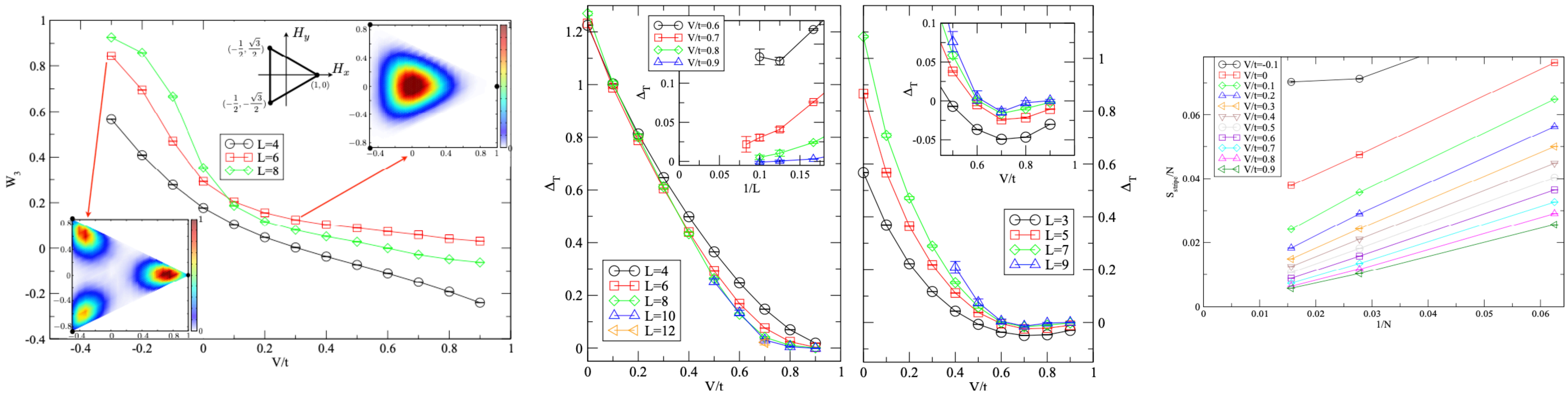


K. Roychowdhury, S. Bhattacharjee, and F. Pollmann, Phys. Rev. B 92, 075141 (2015).



X. Plat, F. Alet, S. Capponi, and K. Totsuka, Phys. Rev. B 92, 174402 (2015).

“We cannot exclude an intermediate different crystalline phase …”



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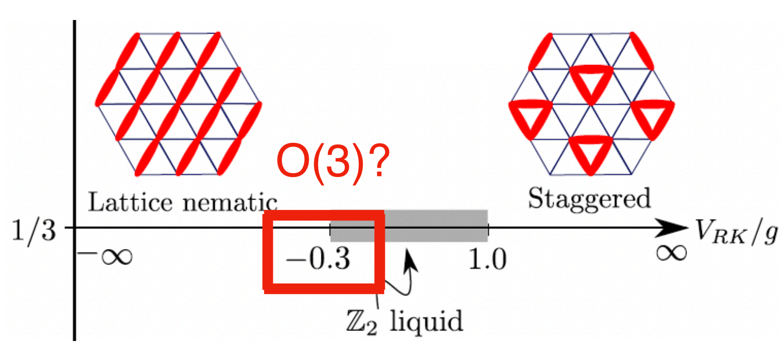
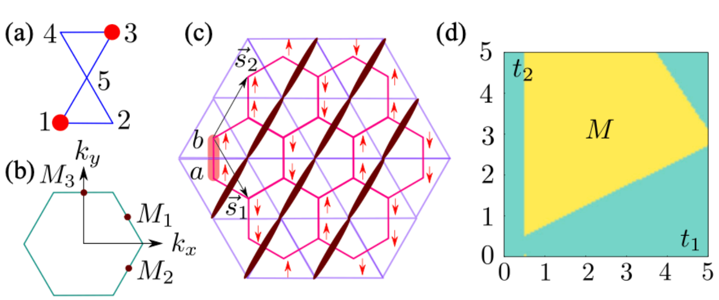
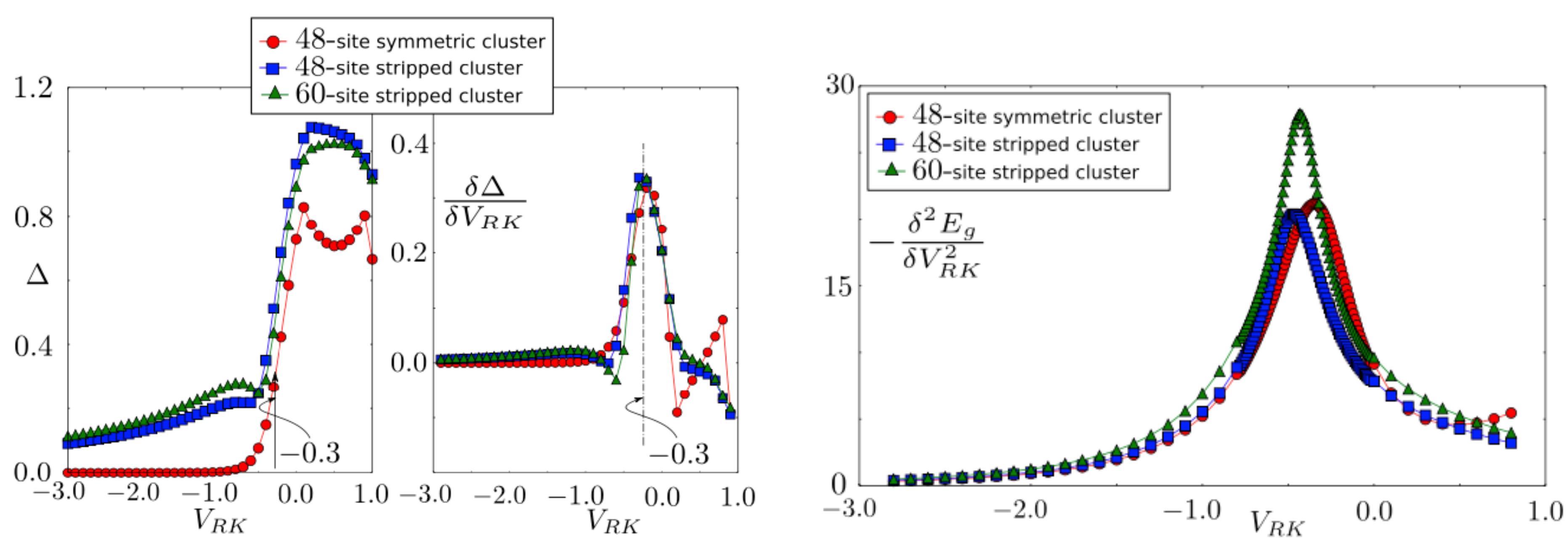






Sweeping cluster quantum Monte Carlo method

Based on stochastic series expansion (SSE) in the path integral $Z = \mathrm{Tr}\{e^{-\beta H}\}$

Plaquette operators

$$
\begin{aligned}
H_{1,p} &= -V\left(|\,\shortparallel\,\rangle\langle\,\shortparallel\,| + |=\rangle\langle=|\right) + V + C, \\
H_{2,p} &= t\left(|\,\shortparallel\,\rangle\langle=| + |=\rangle\langle\,\shortparallel\,|\right).
\end{aligned}
$$

$t = 1,\ C = 1$

Partition function

$$
Z = \sum_{\alpha}\sum_{S_M} \frac{\beta^n (M-n)!}{M!} \left\langle \alpha \left| \prod_{i=1}^{M} H_{a_i,p_i} \right| \alpha \right\rangle
$$

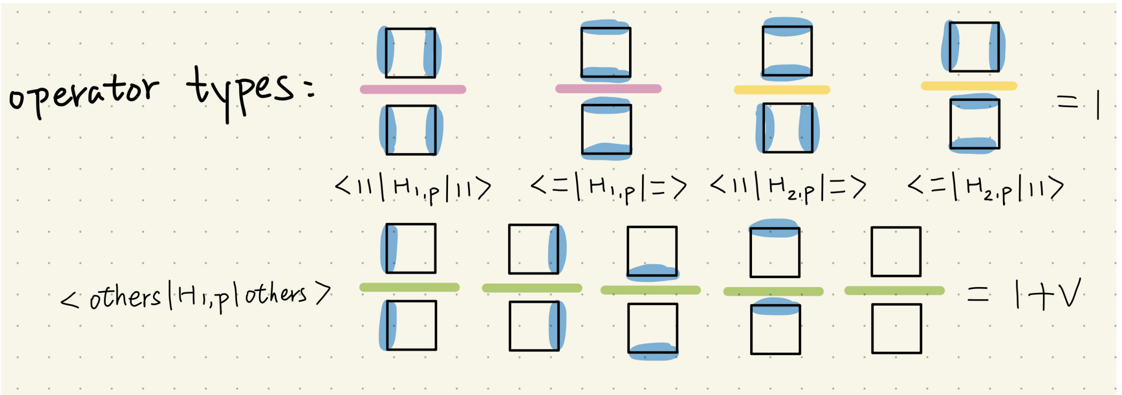
Sampling in the restricted Hilbert space

Diagonal update

$$
P_{\text{ins}} = \frac{N_p \beta \langle \alpha | H_{1,p} | \alpha \rangle}{M-n},
$$

$$
P_{\text{del}} = \frac{M-n+1}{N_p \beta \langle \alpha | H_{1,p} | \alpha \rangle}.
$$

Non-zero plaquette matrix elements



$|\text{others}\rangle$ plaquettes have 0 or 1 dimer

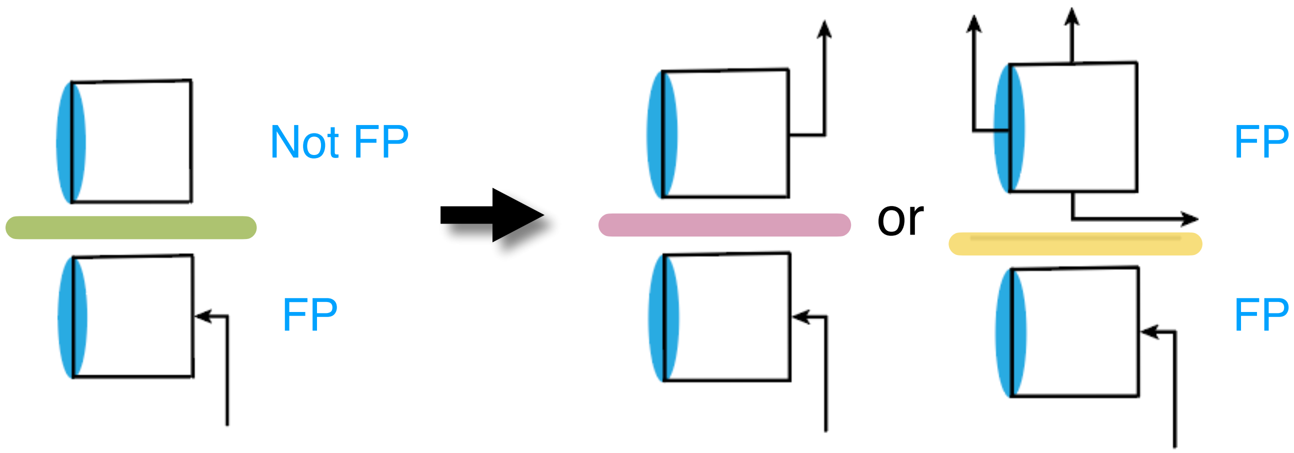
Z. Yan, Y. Wu, C. Liu, O. F. Syljuasen, J. Lou, and Y. Chen, Phys. Rev. B 99, 165135 (2019).

Off-diagonal (sweeping cluster) update

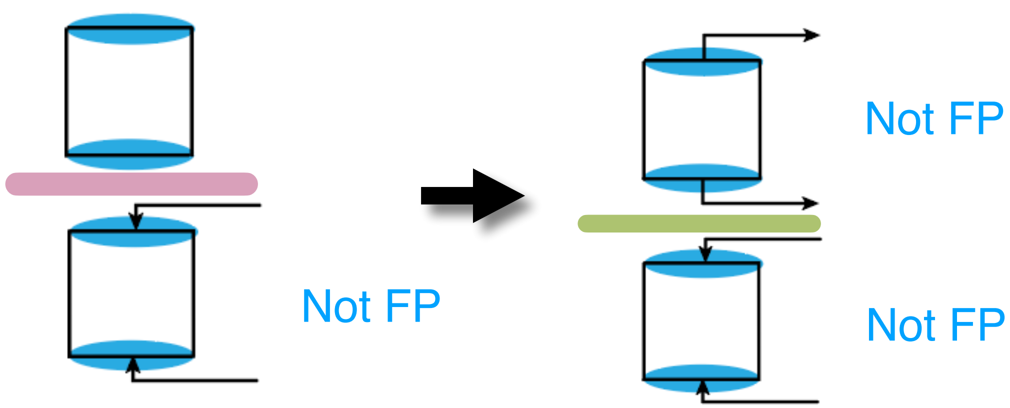
(1). Choose a flippable plaquette (FP) , create four update lines;

(2). Three conditions:

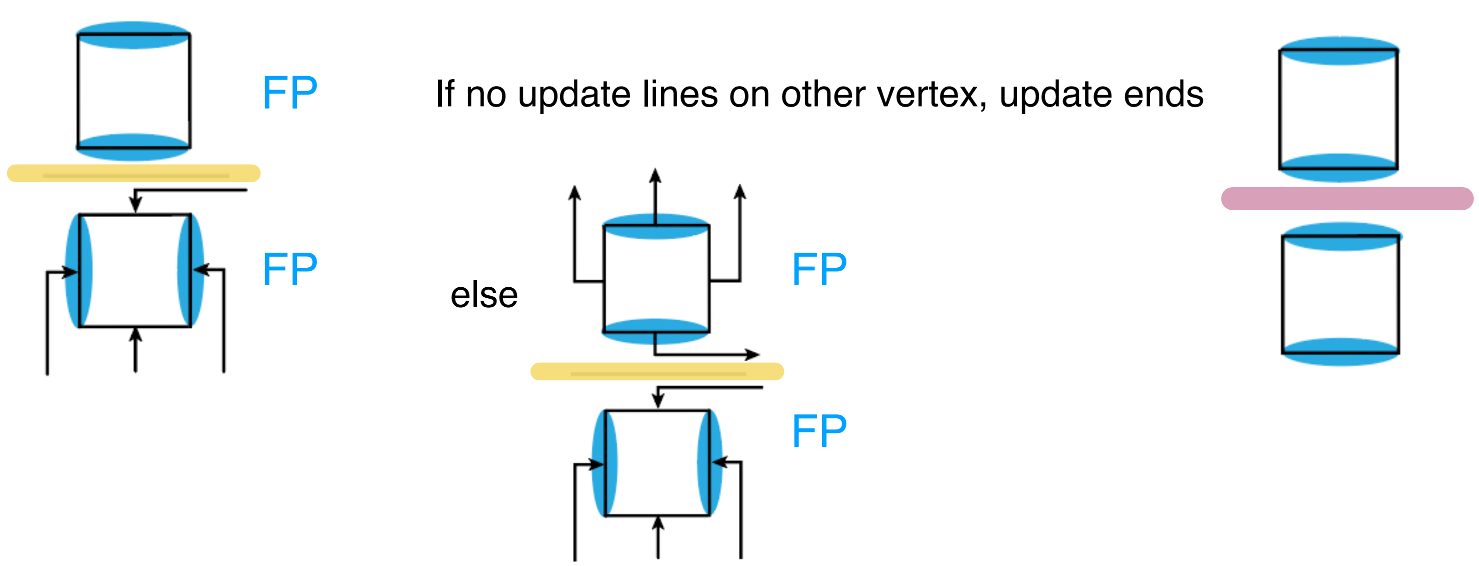
(a)



(b)



(c)



(3). Metropolis acceptance probabilities

$$P_{accept}(A \to B) = \min\left(\frac{W(B)P_{\text{select}}(B \to A)}{W(A)P_{\text{select}}(A \to B)}, 1\right)$$

$$= \min\left(\frac{N_{\text{FP}}}{N_{\text{FP}} + \Delta}\left(\frac{2}{1+V}\right)^{\Delta}, 1\right)$$

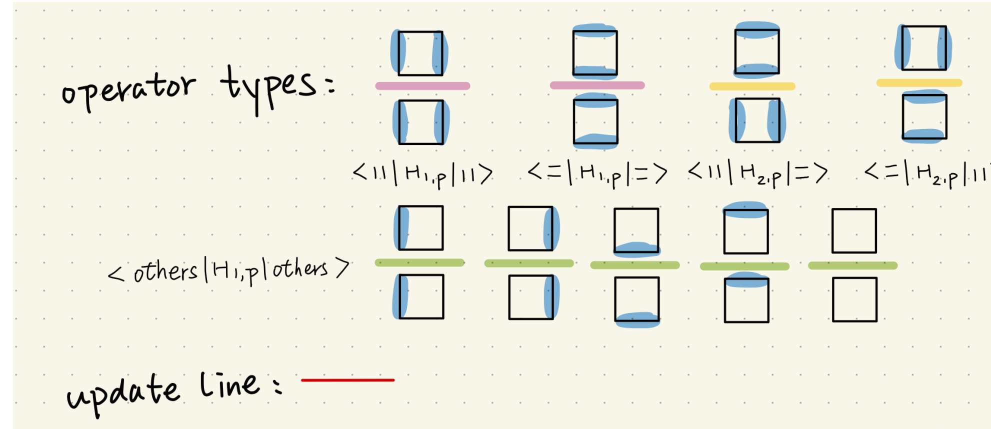
N_{FP} : No. of flippable vertex in A

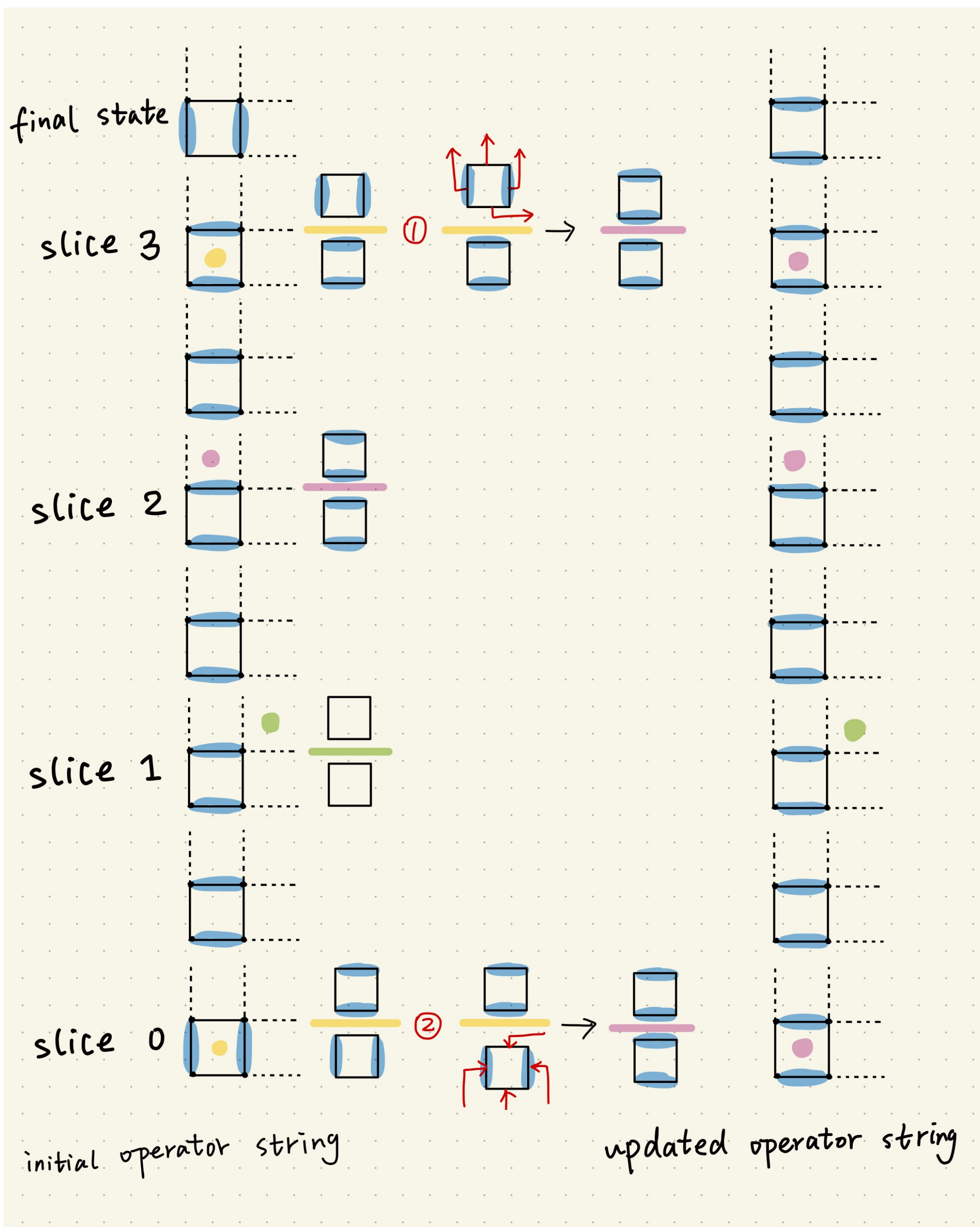
$N_{FP} + \Delta$: No. of flippable vertex in B

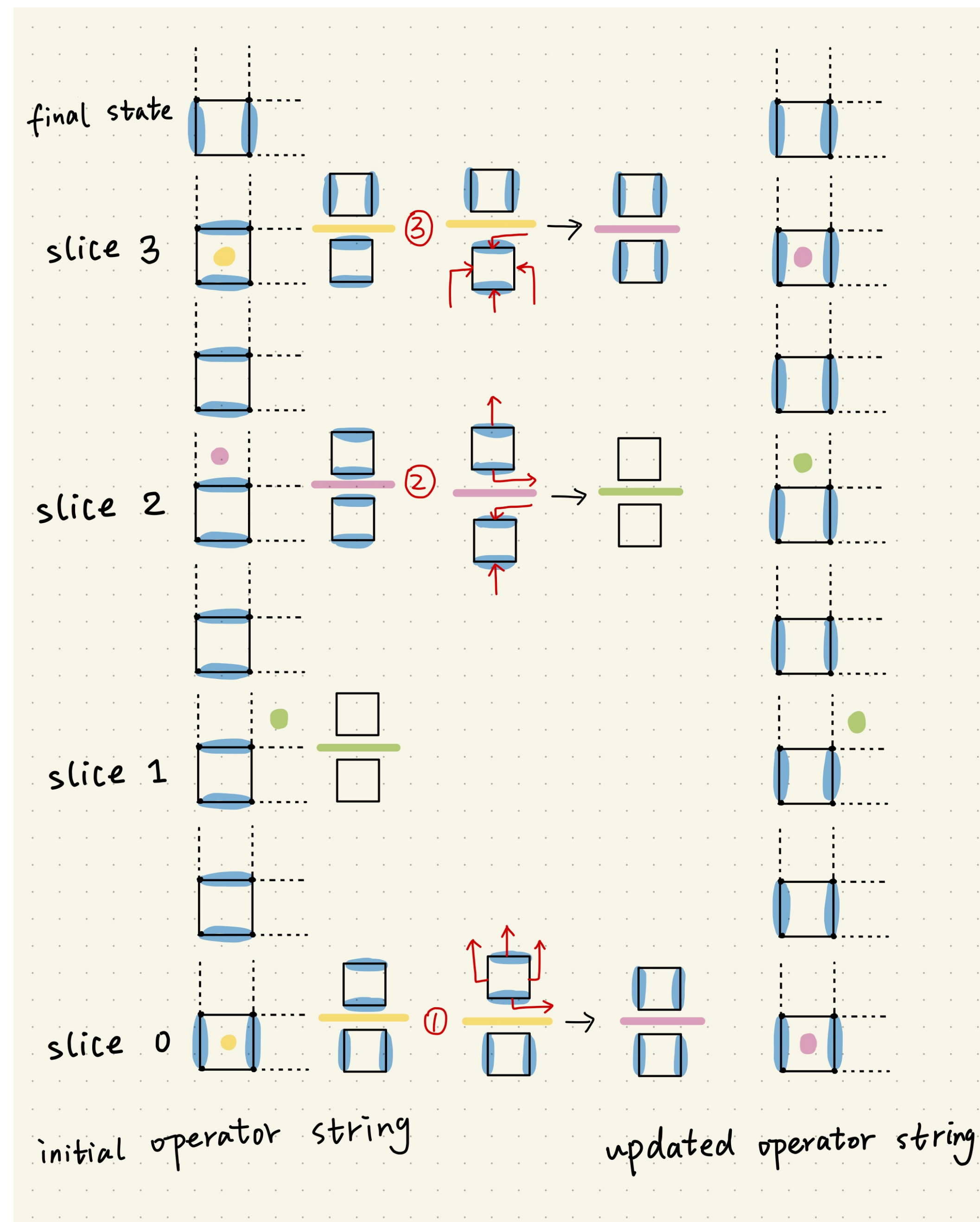
Z. Yan, Y. Wu, C. Liu, O. F. Syljuasen, J. Lou, and Y. Chen, Phys. Rev. B 99, 165135 (2019).

Sketches for two possible cluster updates for QDM:

2 x 2 square lattice

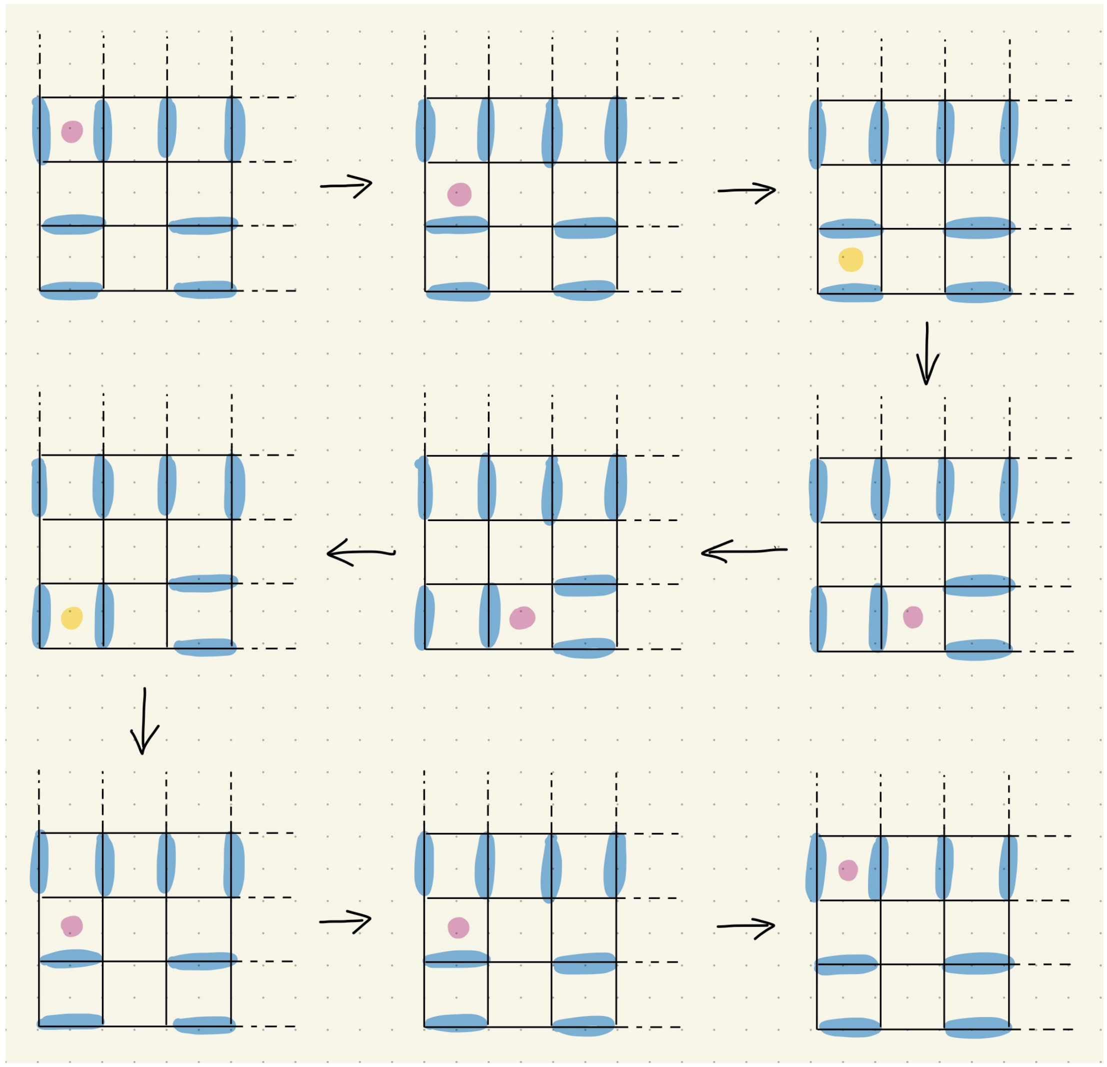




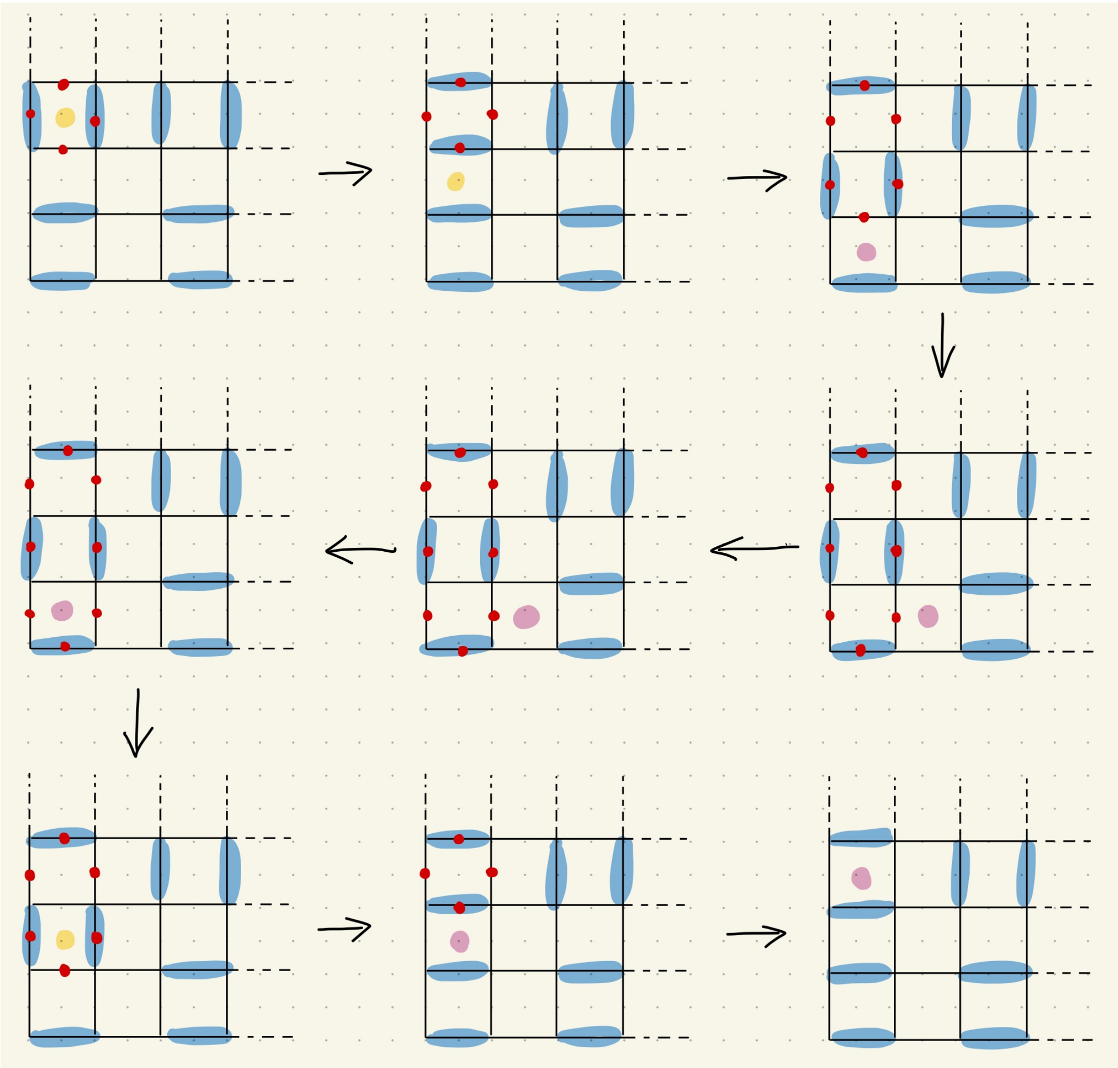


Sketches for a possible cluster updates for QDM:

3 x 3 square lattice



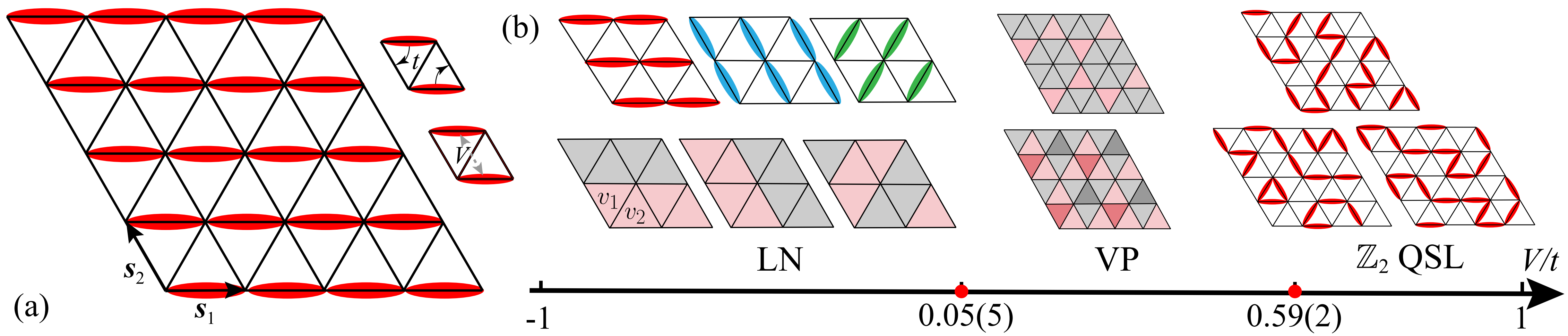
Before update



After update

Phase diagram of quantum loop model

With sweeping cluster QMC up to $L = 20,\ \beta \propto L$

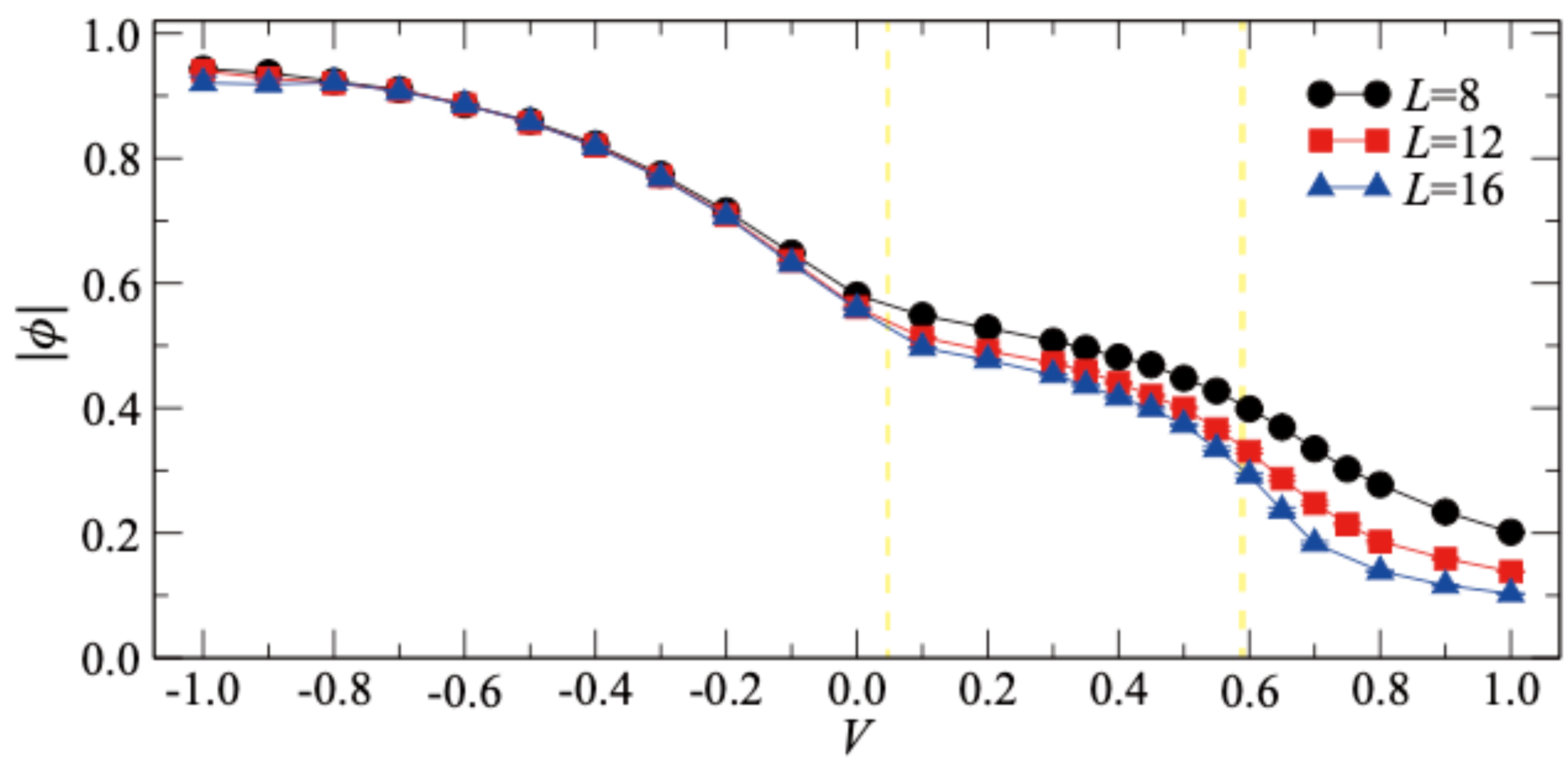


O(3) order parameter

$$\phi_j = \sum_{\mathbf{r}} (v_{1,\mathbf{r}}, v_{2,\mathbf{r}}) \cdot \mathbf{u}_j e^{i\mathbf{M}_j \cdot \mathbf{r}}, \quad j = 1,2,3$$

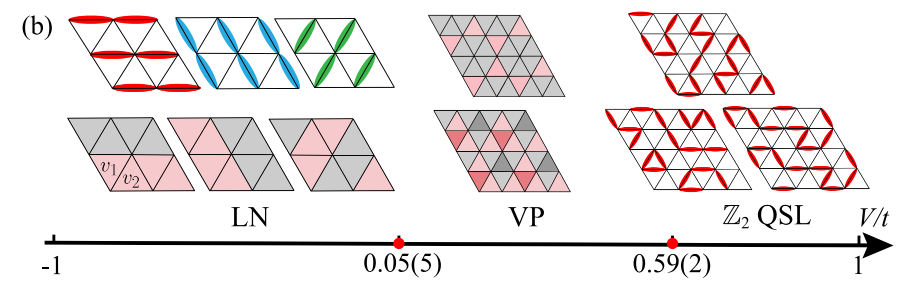
$$u_{1,2} = (1,1)^T \quad u_3 = (1,-1)^T$$

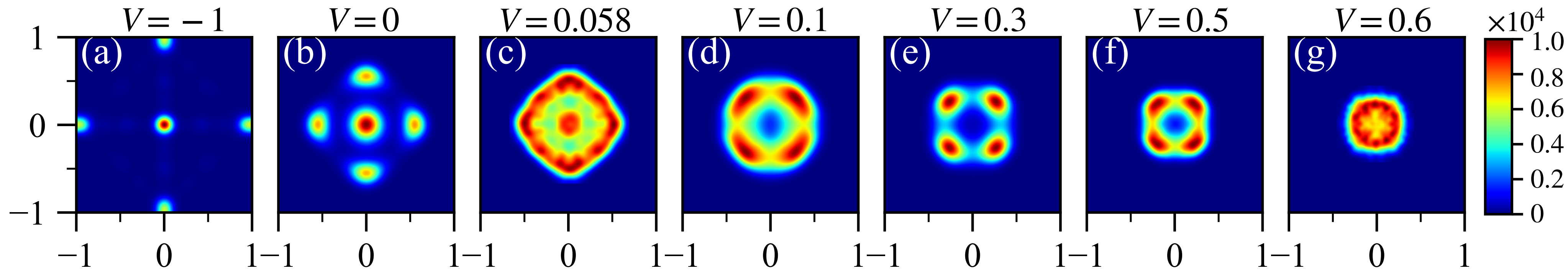
$$|\phi| = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2}$$



X. Ran, Z. Yan, Y-C Wang, R. Samajdar, J. Rong, S. Sachdev, Y. Qi, and Z. Y. Meng. Commun Phys 7, 207 (2024).

LN-VP first-order transition





face-cubic Coexist phase corner-cubic

LN phase:

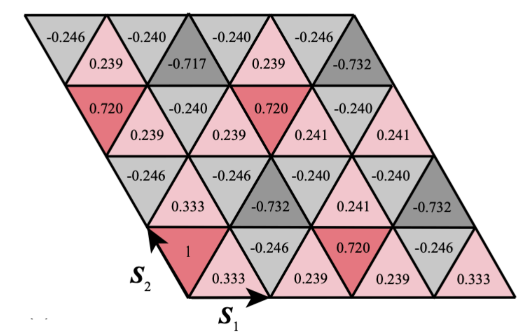
Breaking C_3 symmetry

Preserving translational symmetry

VP phase: Hidden order

Breaking translational symmetry

Preserving C_3 symmetry



Real-space vison correlation at $V = 0.3$

Renormalization-group (RG) analysis

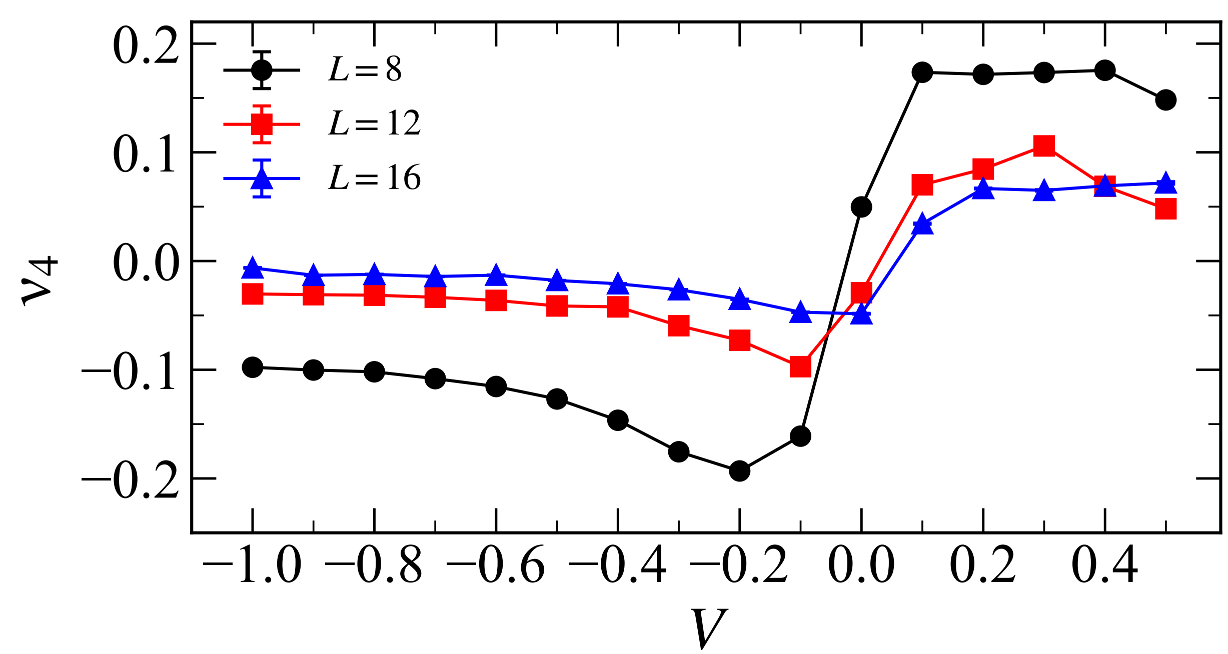
3D Cubic fixed point

$$S = \int dtdx^2 \sum_{i=1}^{3} (\partial_\mu \phi_i)^2 + r\sum_{i=1}^{3} \phi_i^2 + \mu(\sum_{i=1}^{3} \phi_i\phi_i)^2 + \nu_4 \sum_{i=1}^{3} (\phi_i)^4 \quad + \ldots$$

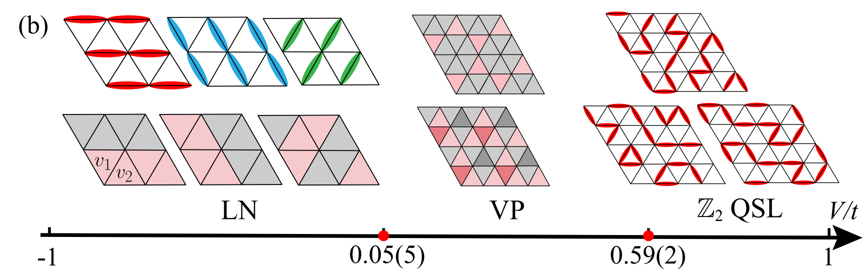
$r < 0,\ \nu_4 < 0 \qquad \langle\phi_1\rangle = \pm v', \quad \langle\phi_2\rangle = \langle\phi_3\rangle = 0$

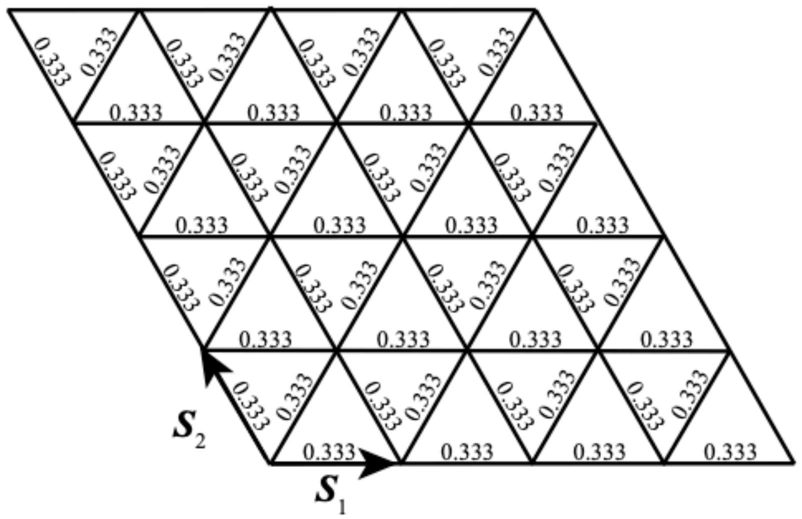
$r < 0,\ \nu_4 > 0 \qquad \langle\phi_1\rangle = \pm v, \quad \langle\phi_2\rangle = \pm v, \quad \langle\phi_3\rangle = \pm v.$

$$\nu_4 = -\frac{1}{vol}\frac{15\sqrt{\pi}}{\langle\phi^4\rangle}\left(\langle Y_4^0\rangle + \frac{\langle\phi^2 Y_4^0\rangle\langle\phi^4\rangle\langle\phi^6\rangle - \langle Y_4^0\rangle\langle\phi^6\rangle^2}{\langle\phi^6\rangle^2 - \langle\phi^4\rangle\langle\phi^8\rangle}\right)$$

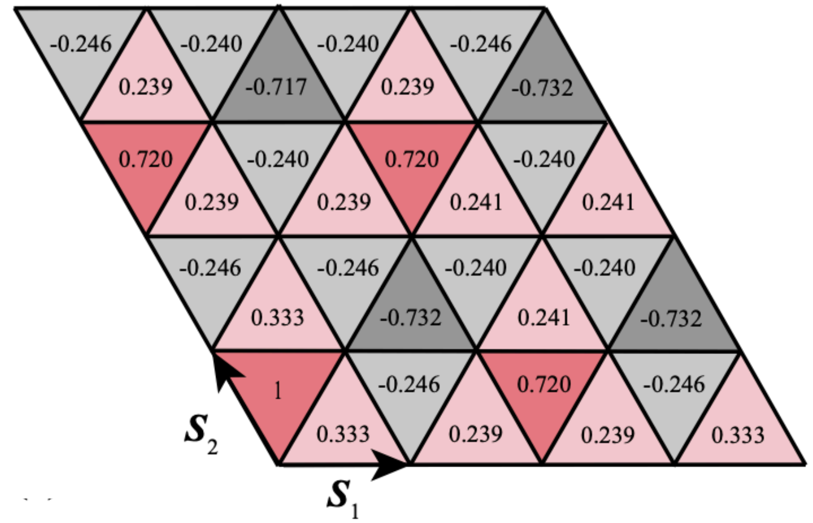


VP phase invisible to diagonal measurements

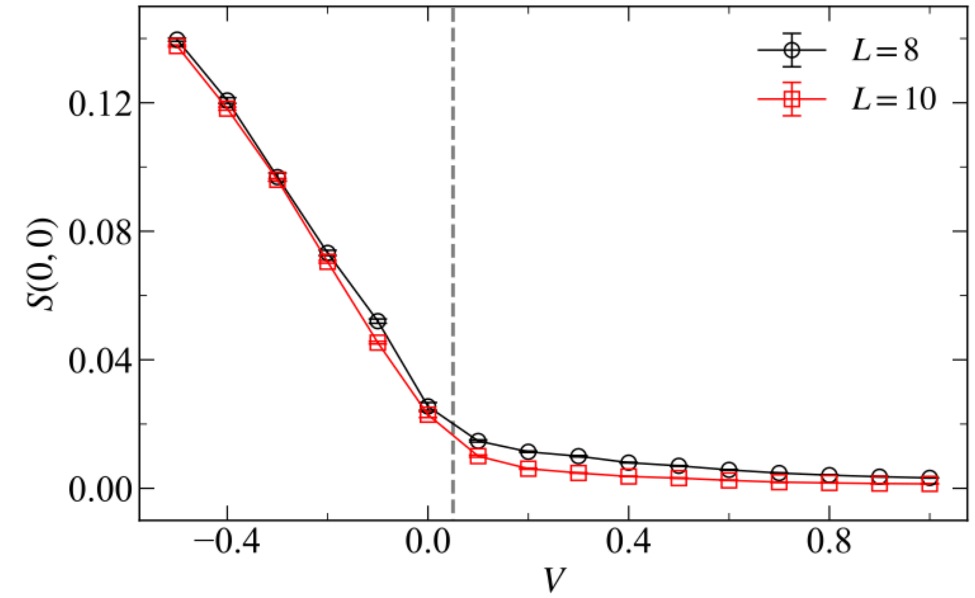




Dimer density at V=0.3

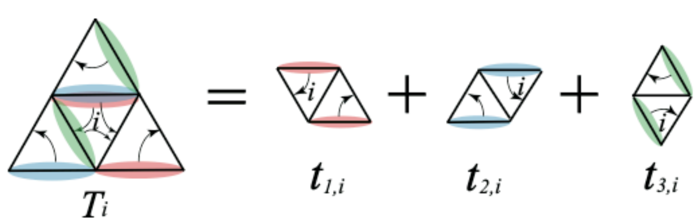


Vison correlation at V=0.3



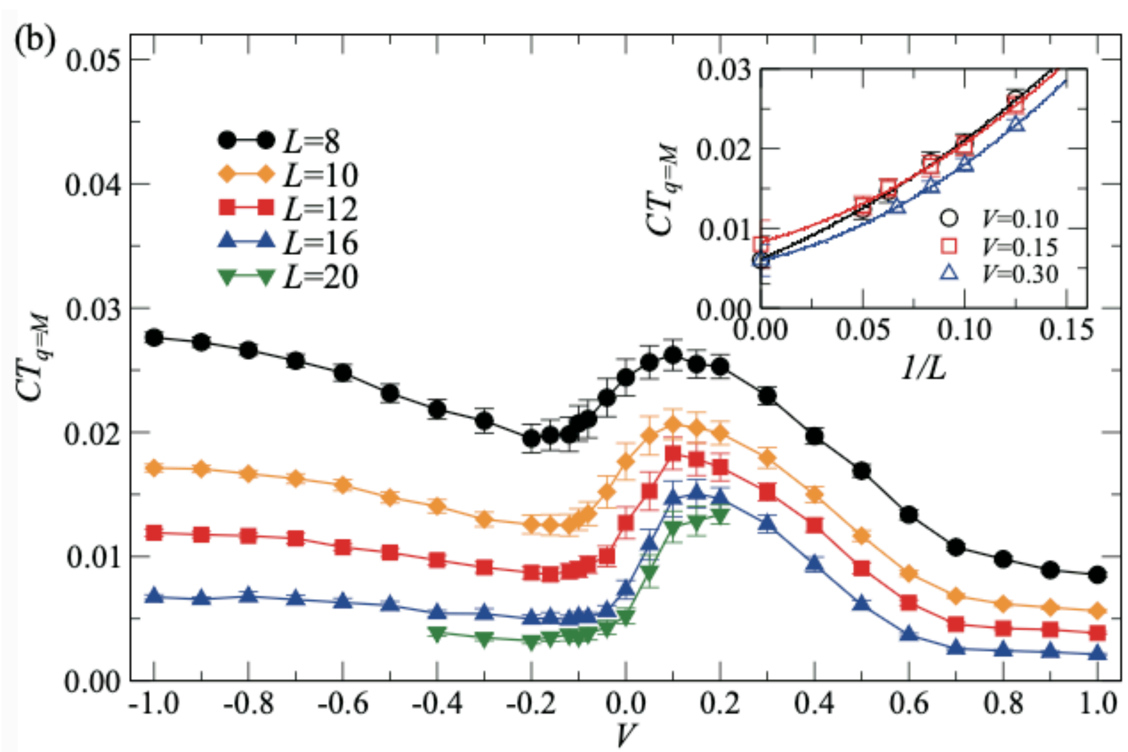
The structure factor of the dimer correlation functions

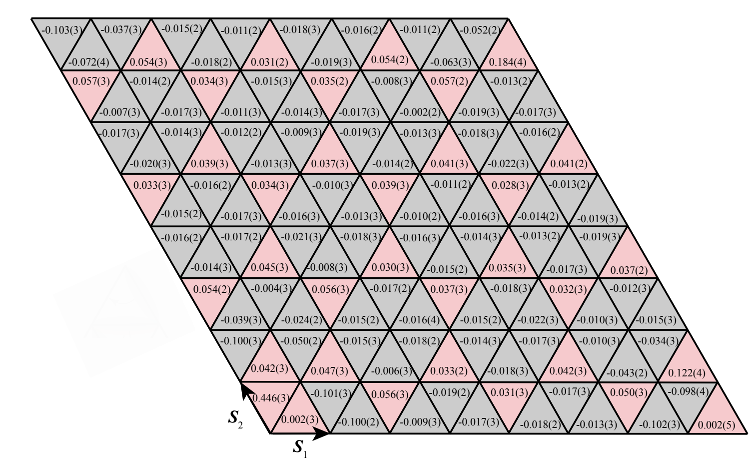
T-operator correlations



$$CT = \langle T_i T_j \rangle \quad T_i = t_{1,i} + t_{2,i} + t_{3,i}$$

$$\{t_1, t_2, t_3\} \sim \{\phi_1\phi_2, \phi_2\phi_3, -\phi_1\phi_3\}$$

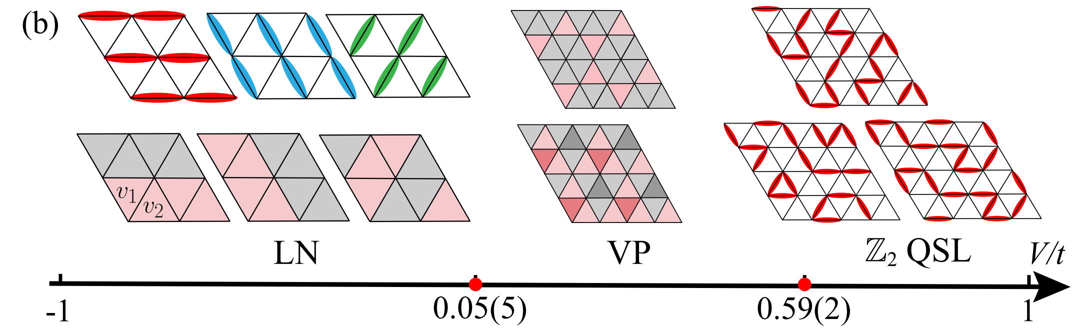




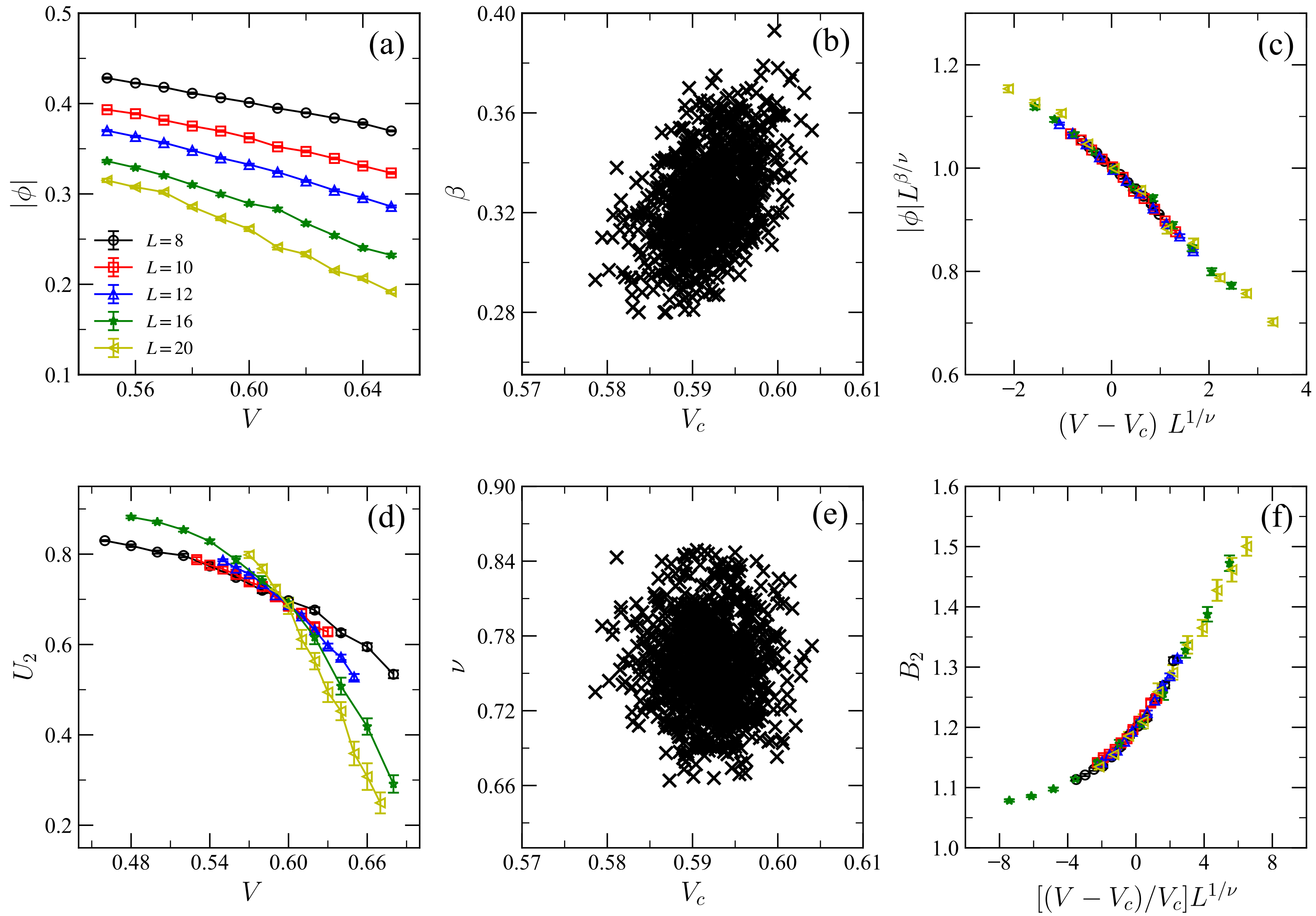
Real-space T-operator correlations at V = 0.3

rank-2 tensor (or tensorial magnetization) of the (2+1)D O(3)/cubic universality

VP-QSL continuous transition



Cubic* fixed point



Independence fits give:

$$V_c = 0.59(2) \quad \beta = 0.33(5) \quad \nu = 0.73(3)$$

O(3): $\beta = 0.3689(3) \qquad \nu = 0.7112(5)$

3D Cubic fixed point

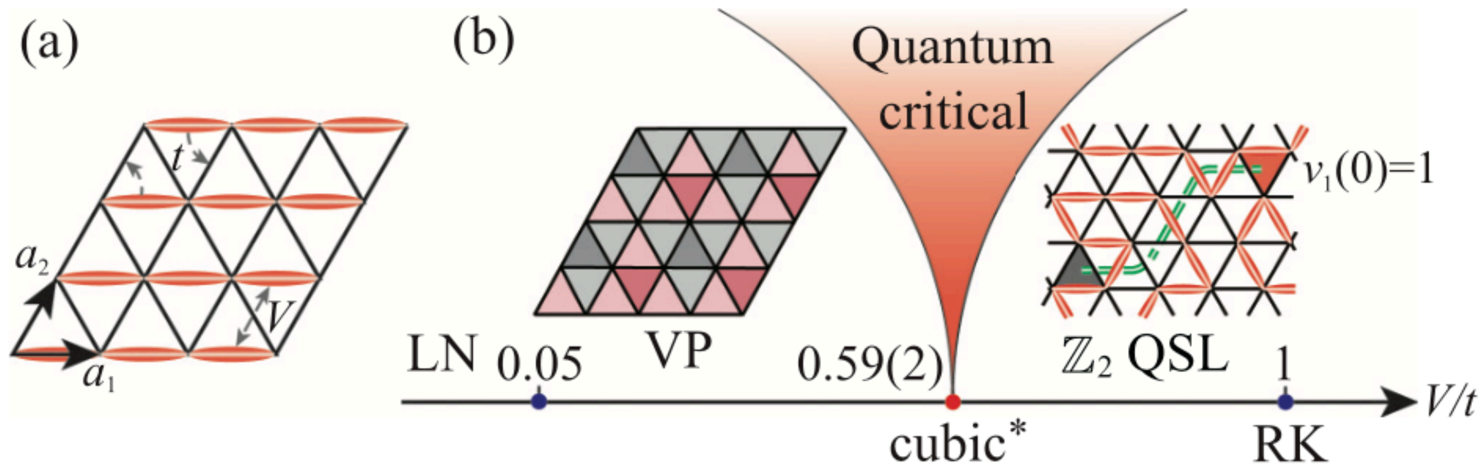
$$S = \int dt dx^2 \sum_{i=1}^{3} (\partial_\mu \phi_i)^2 + r \sum_{i=1}^{3} \phi_i^2 + \mu(\sum_{i=1}^{3} \phi_i \phi_i)^2 + \nu_4 \sum_{i=1}^{3} (\phi_i)^4 \quad + \dots$$

$r < 0,\ \nu_4 < 0 \qquad \langle \phi_1 \rangle = \pm v', \quad \langle \phi_2 \rangle = \langle \phi_3 \rangle = 0$

$r < 0,\ \nu_4 > 0 \qquad \langle \phi_1 \rangle = \pm v, \quad \langle \phi_2 \rangle = \pm v, \quad \langle \phi_3 \rangle = \pm v.$

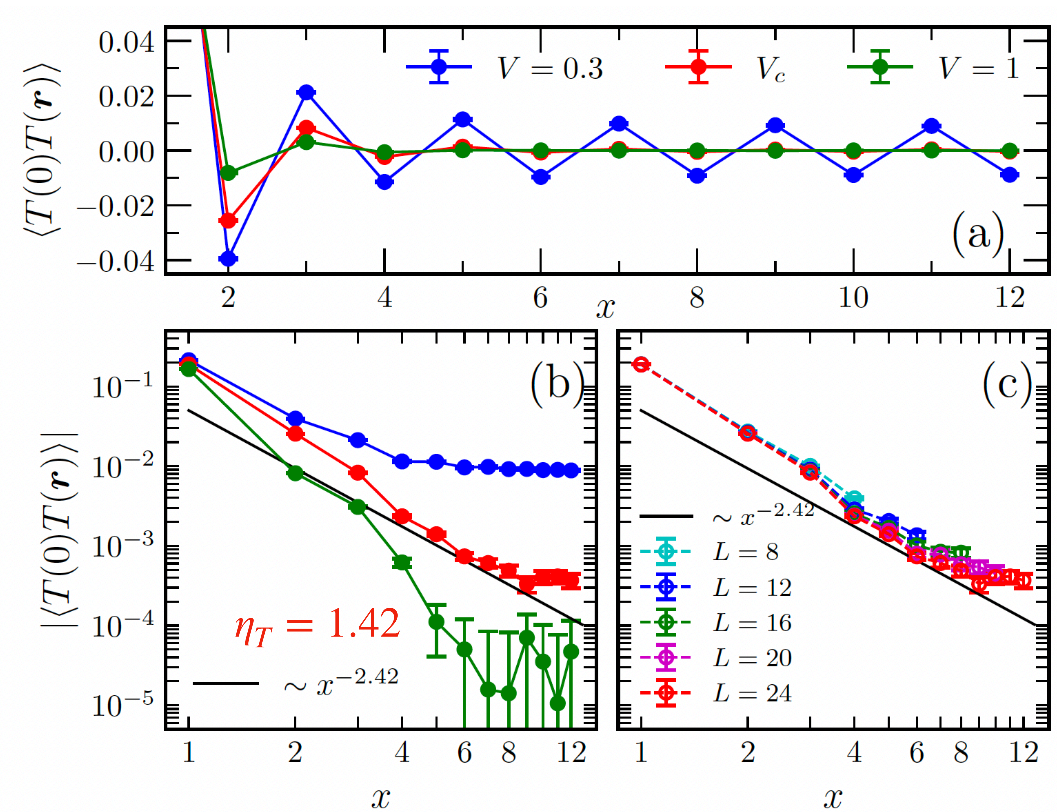
Cubic* criticality

With sweeping cluster QMC up to L=24



t-term correlator

$$\langle T(0)T(\mathbf{r})\rangle = \frac{1}{3}[\langle t_1(0)t_1(\mathbf{r})\rangle + \langle t_2(0)t_2(\mathbf{r})\rangle + \langle t_3(0)t_3(\mathbf{r})\rangle] \sim 1/x^{1+\eta_T}$$

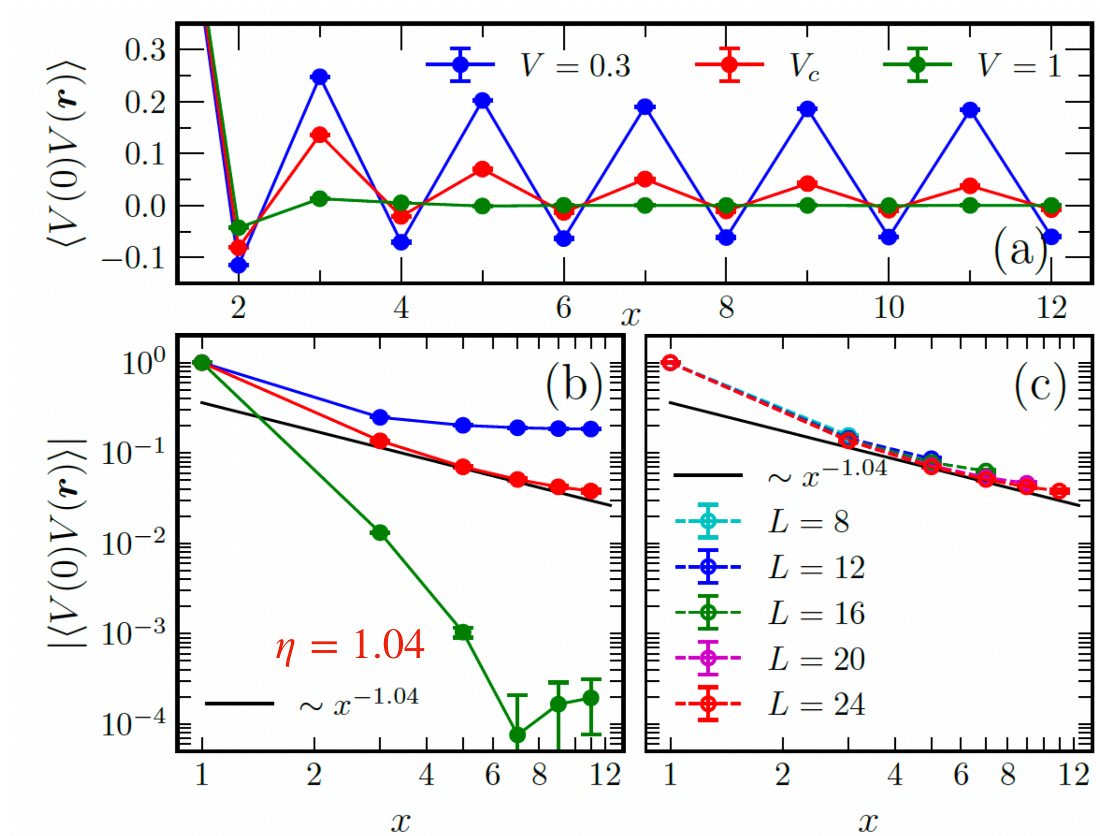


$$\{t_1, t_2, t_3\} \sim \{\phi_1\phi_2, \phi_2\phi_3, -\phi_1\phi_3\}$$

rank-2 tensor (or tensorial magnetization) of the (2+1)D O(3)/cubic universality

Vison correlator

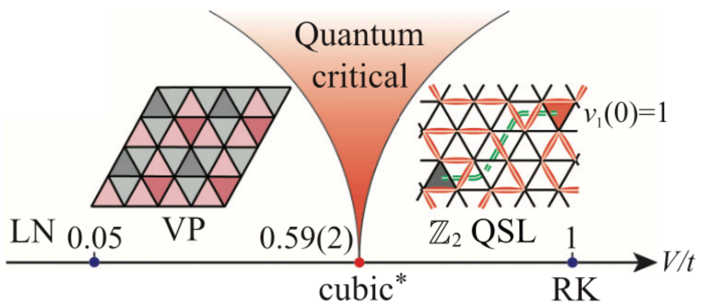
$$\langle V(0)V(\mathbf{r})\rangle = \frac{1}{2}[\langle v_1(0)v_1(\mathbf{r})\rangle + \langle v_2(0)v_2(\mathbf{r})\rangle] \sim 1/x^{1+\eta}$$



$$\{\phi_1, \phi_2, \phi_3\}$$

scaler of the (2+1)D O(3)/cubic universality

X. Ran, Z. Yan, Y-C Wang, J. Rong, Y. Qi, and Z. Y. Meng. Phys. Rev. B 109, L241109 (2024).



$$L = 12,\ \beta = 200$$

At the cubic* QCP,

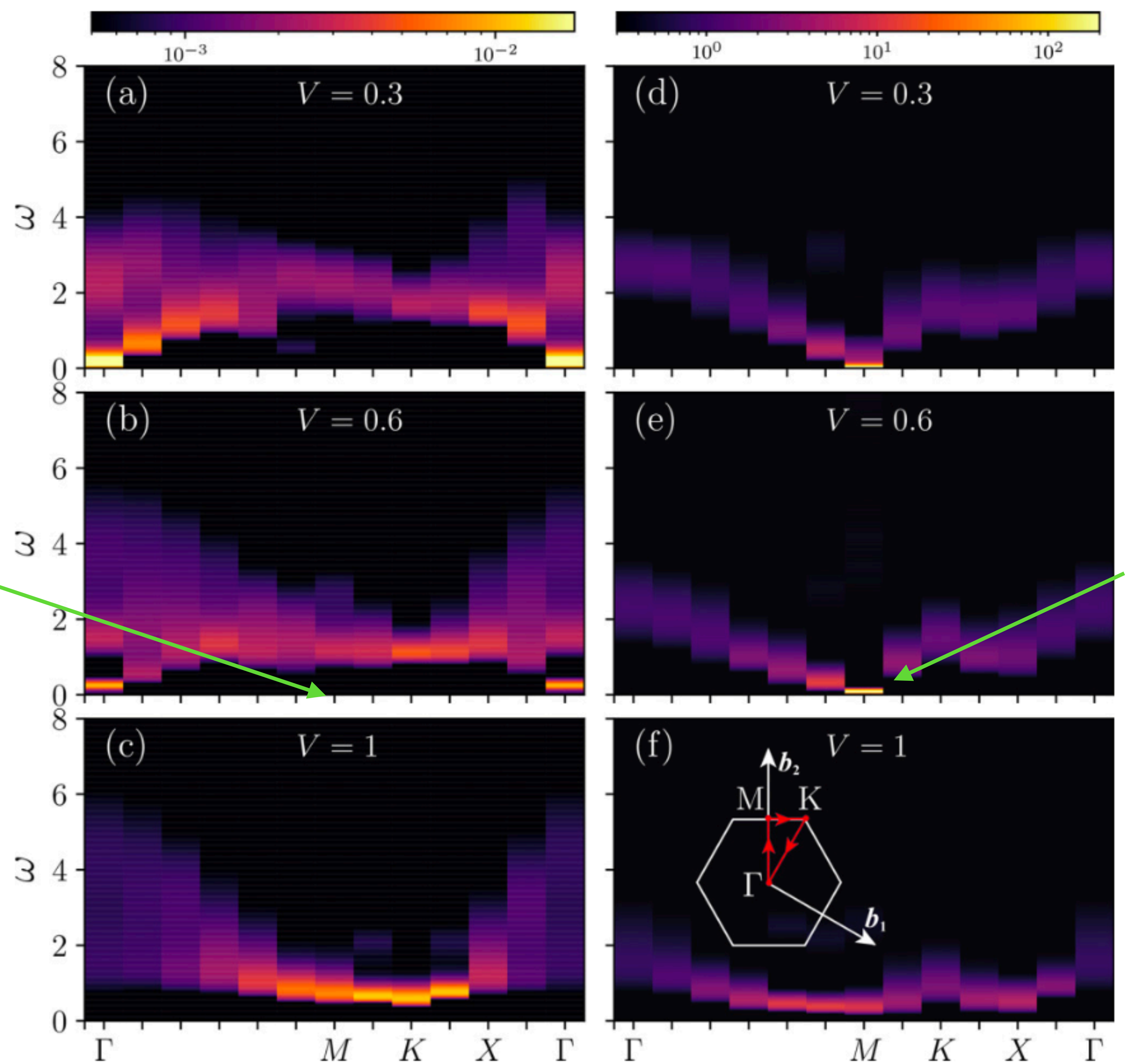
$$t_{1,2,3} \sim \phi\phi$$

is a composite of the fractionalized visons

$$\nu_{1,2} \sim \phi$$

$$D(\mathbf{k},\tau) = \frac{1}{3N}\sum_{\substack{i,j \\ \alpha=1,2,3}}^{L^2} e^{i\mathbf{k}\cdot\mathbf{r}_{ij}} \left(\langle n_{i,\alpha}(\tau) n_{j,\alpha}(0)\rangle - \langle n_{i,\alpha}\rangle\langle n_{j,\alpha}\rangle\right)$$

$$v(\mathbf{k},\tau) = \frac{1}{2N}\sum_{\substack{i,j \\ \gamma=1,2}}^{L^2} e^{i\mathbf{k}\cdot\mathbf{r}_{ij}} \left(\langle v_{i,\gamma}(\tau) v_{j,\gamma}(0)\rangle - \langle v_{i,\gamma}\rangle\langle v_{j,\gamma}\rangle\right)$$

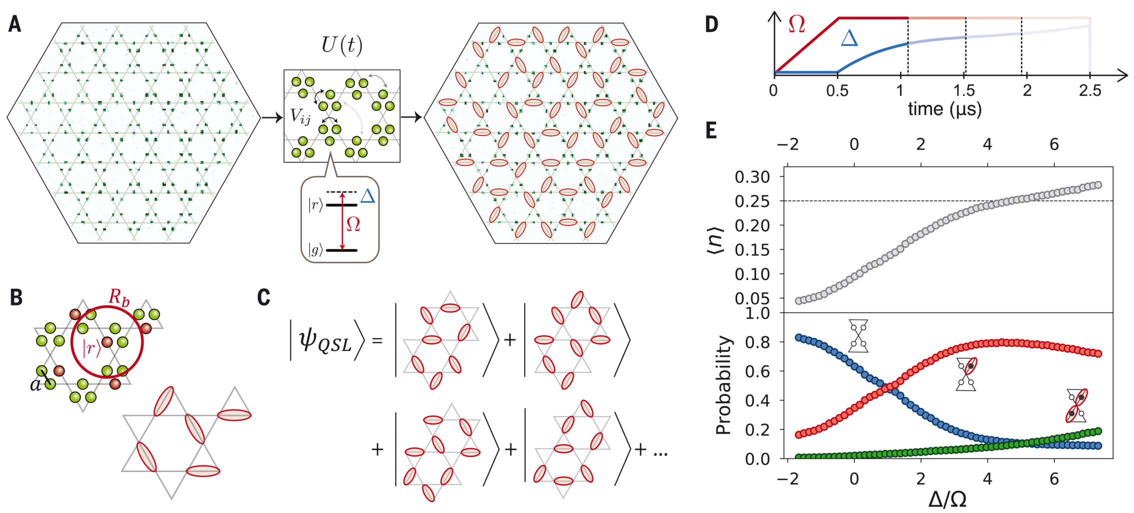


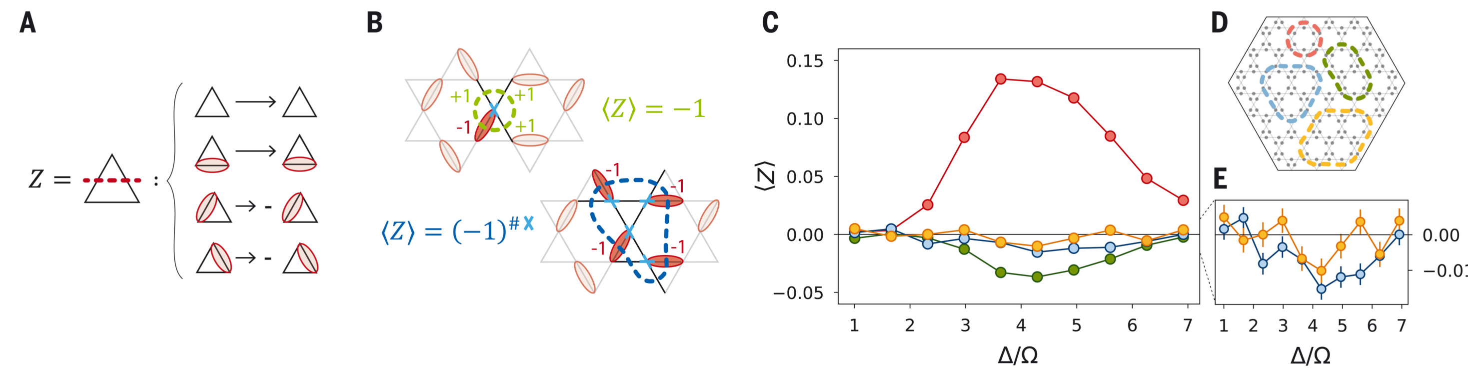
Gapped at M

Gapless at M

Rydberg atom array

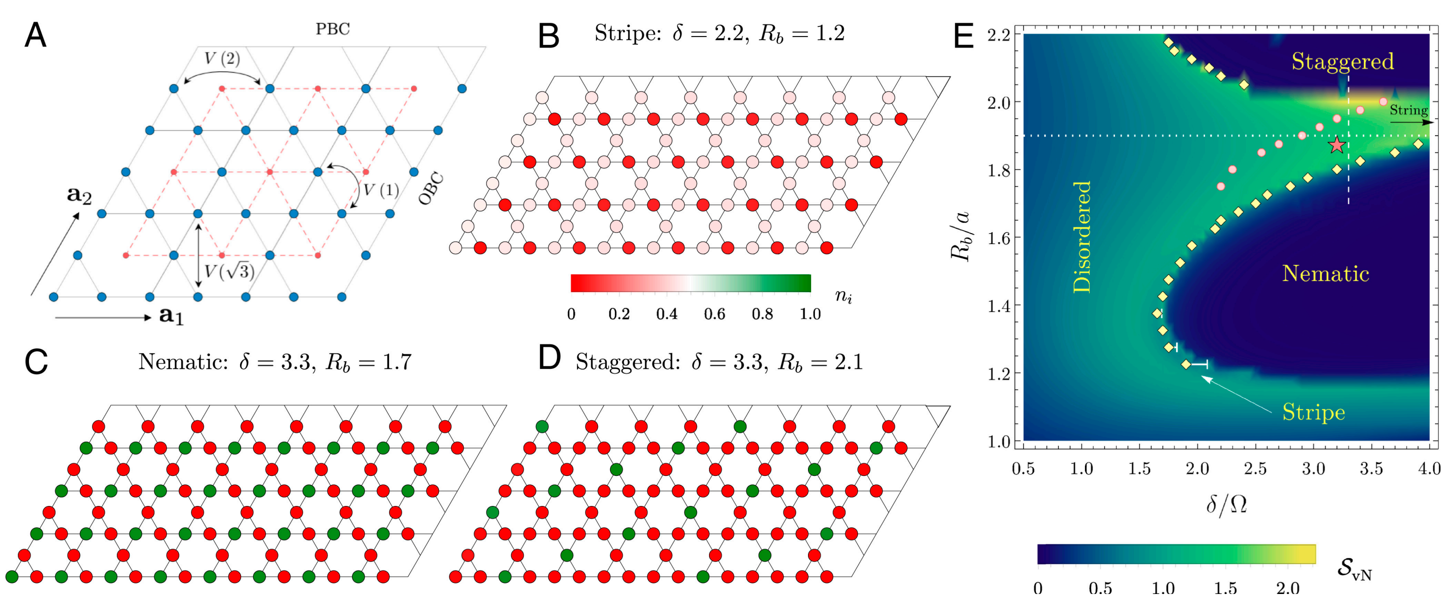
Rydberg atoms on the kagome lattice (links) maps to dimer model on the kagome lattice

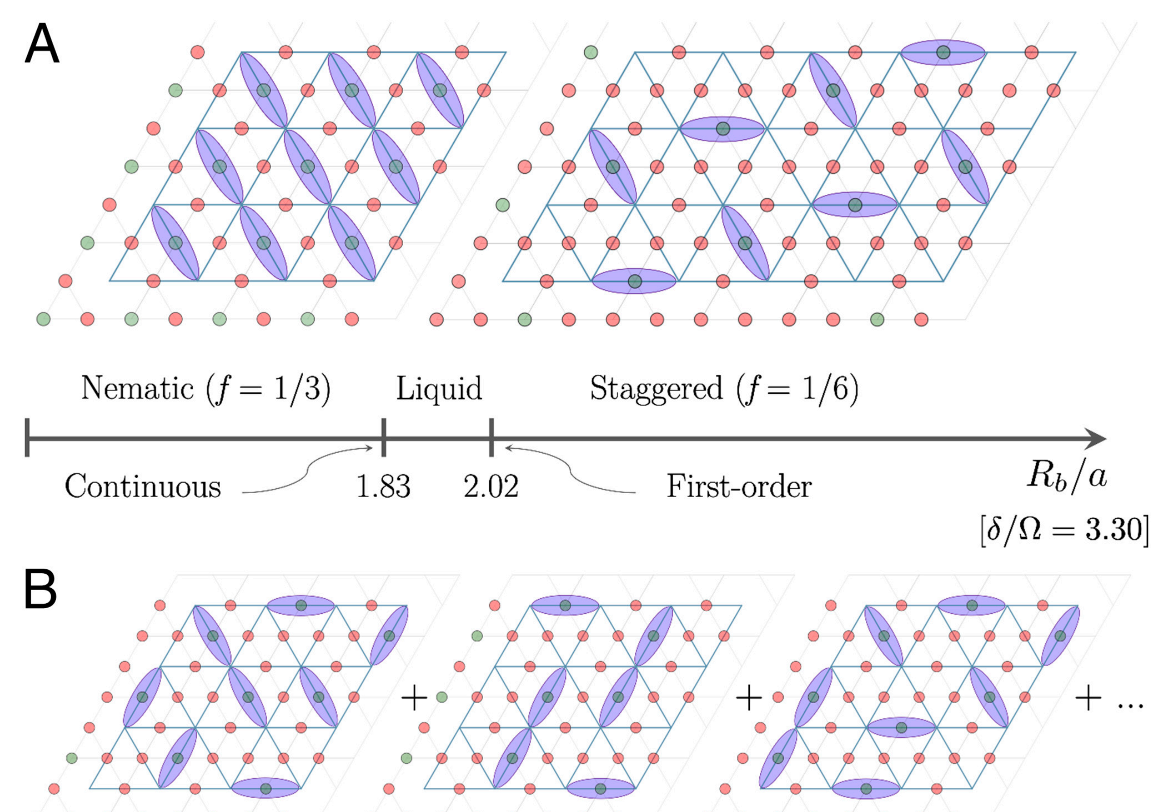




Semeghini G, Levine H, Keesling A, et al. Science, 374, 6572 (2021).

Rydberg atoms on the kagome lattice (sites) maps to dimer\loop model on the triangular lattice





Samajdar R, Ho W W, Pichler H, et al. PNAS, 118, 4 (2021).

Emergent glassy behavior in a kagome Rydberg atom array

Z .Yan, Y.-C. Wang, R. Samajdar, S. Sachdev, Z. Y. Meng, Phys. Rev. Lett. 130, 206501 (2023).

Phase diagram

$$H=\sum_{i=1}^{N}\left[\frac{\Omega}{2}\left(|g\rangle_i\langle r|+|r\rangle_i\langle g|\right)-\delta\,|r\rangle_i\langle r|\right]$$
$$+\sum_{i,j=1}^{N}\frac{V_{ij}}{2}\left(|r\rangle_i\langle r|\otimes|r\rangle_j\langle r|\right),$$

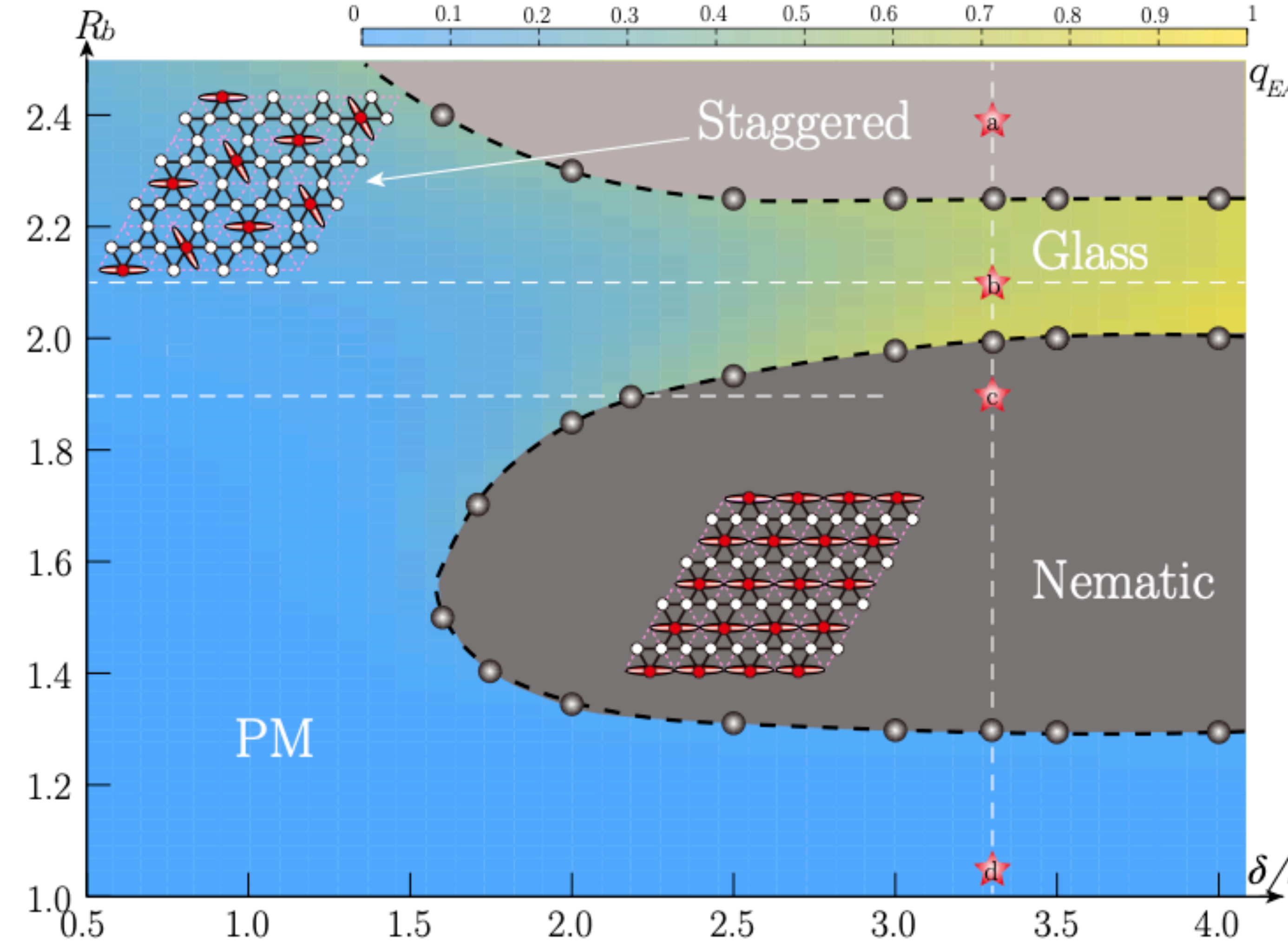
Repulsive interaction $V_{ij}=\Omega R_b^6/R_{ij}^6 \quad \Omega=1$

Ω : Rabi frequency, transverse field

δ : detuning, longitudinal field

R_b : Rydberg blockade radius

$R_{i,j}$: distance between the sites *i* and *j*



Equal-time dimer-structure factor

$$S(\mathbf{k})=\frac{1}{N}\sum_{i,j,\alpha=1,2,3}^{N}e^{i\mathbf{k}\cdot\mathbf{r}_{ij}}\left(\langle n_{i,\alpha}n_{j,\beta}\rangle-\langle n_{i,\alpha}\rangle\langle n_{j,\beta}\rangle\right)$$

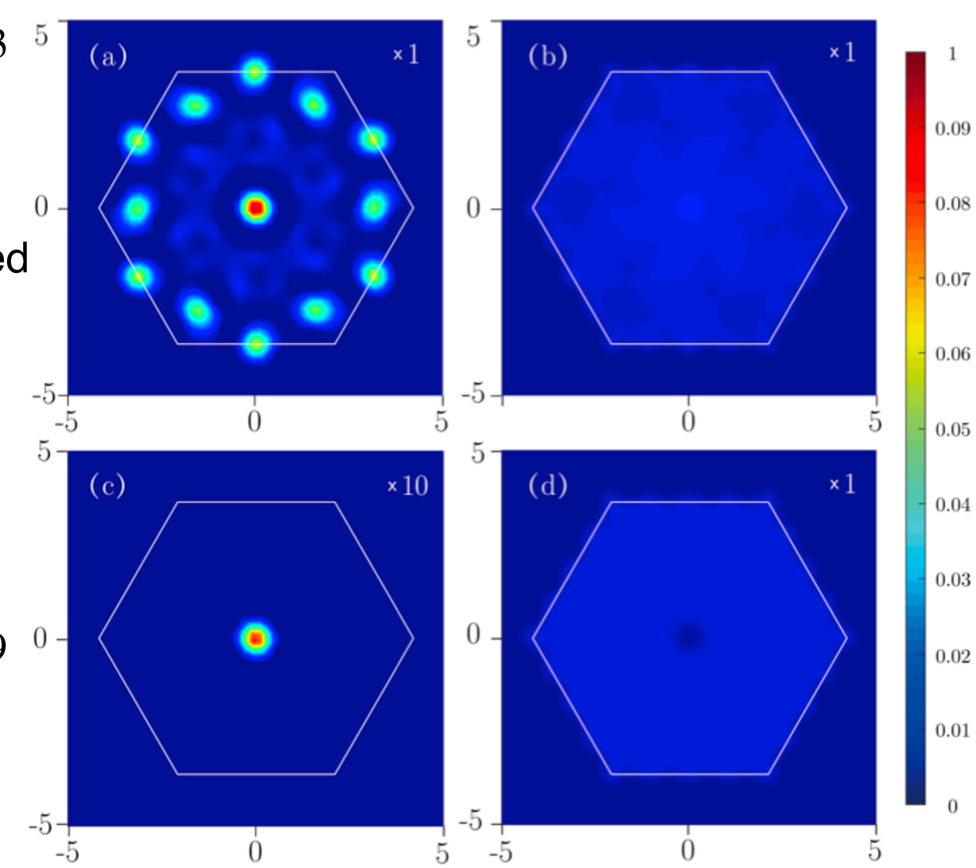
$\delta/\Omega=3.3$

1/6 staggered $R_b=2.3$

glass $R_b=2.1$

nematic $R_b=1.9$

PM $R_b=1.05$



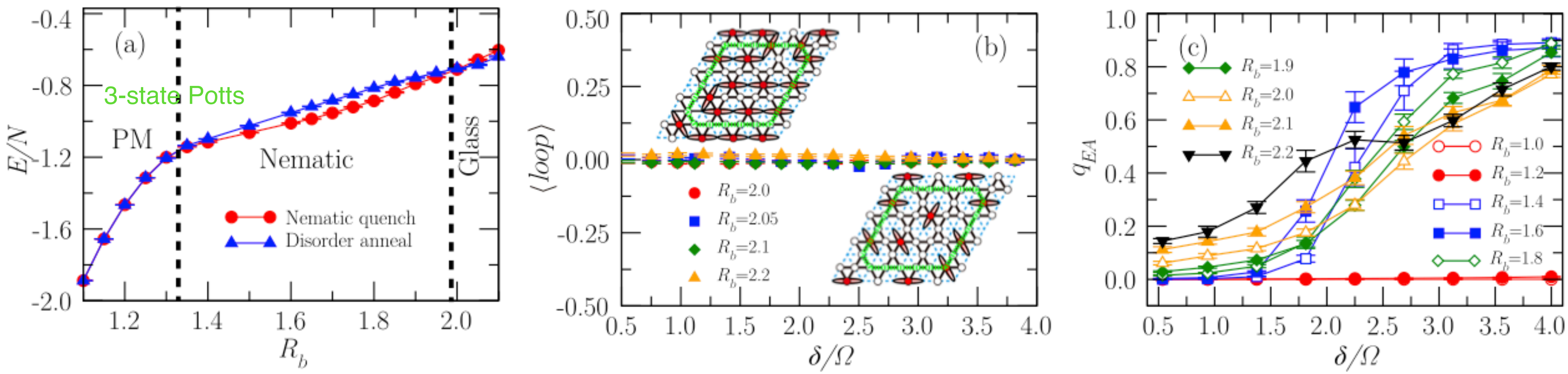
Z Yan, YC Wang, R Samajdar, S Sachdev, ZY Meng, Phys. Rev. Lett. 130, 206501 (2023).

Phase transition

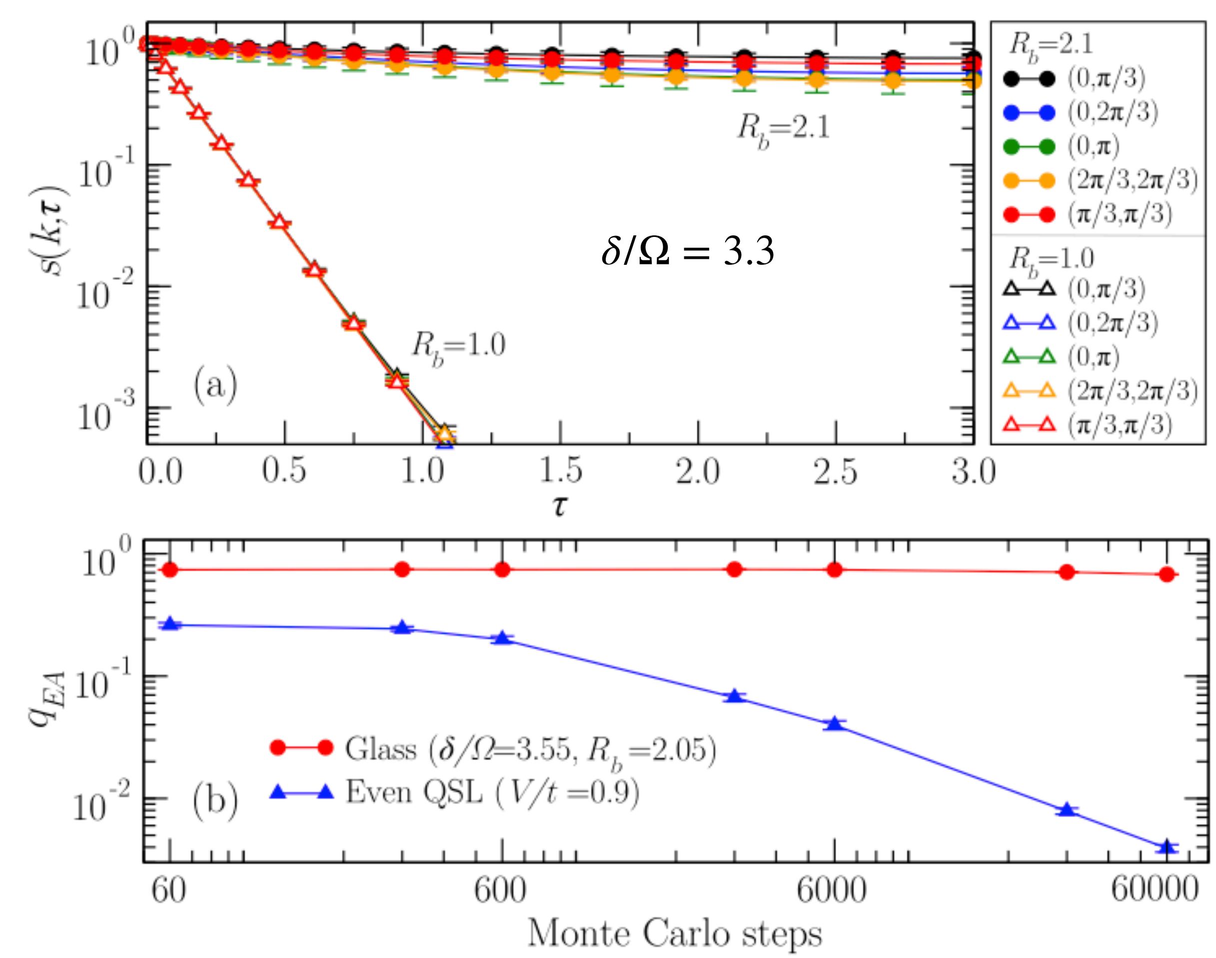
Nonlocal loop operator $< loop > = (-1)^{N_{P_{ij}}}$ $N_{P_{ij}}$: #cut dimers

Edwards-Anderson order parameter $q_{\rm EA} = \sum_{i=1}^{N} \langle n_i - \rho \rangle^2 / [N\rho(1-\rho)]$ $n_i \equiv |r\rangle_i \langle r|$ $\rho \equiv \sum_{i=1}^{N} \langle n_i \rangle / N$

$\delta/\Omega = 3.3$



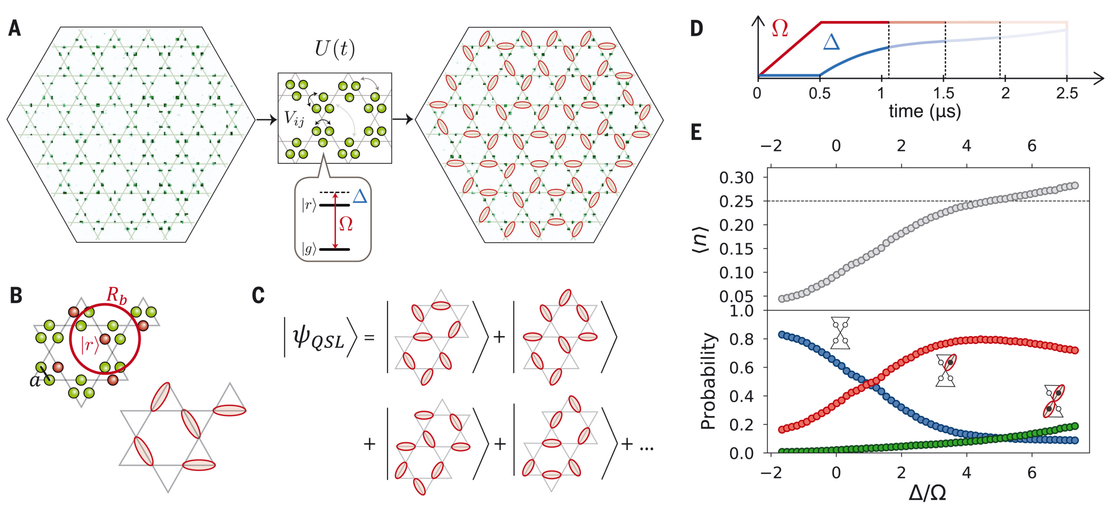
Glassy dynamics

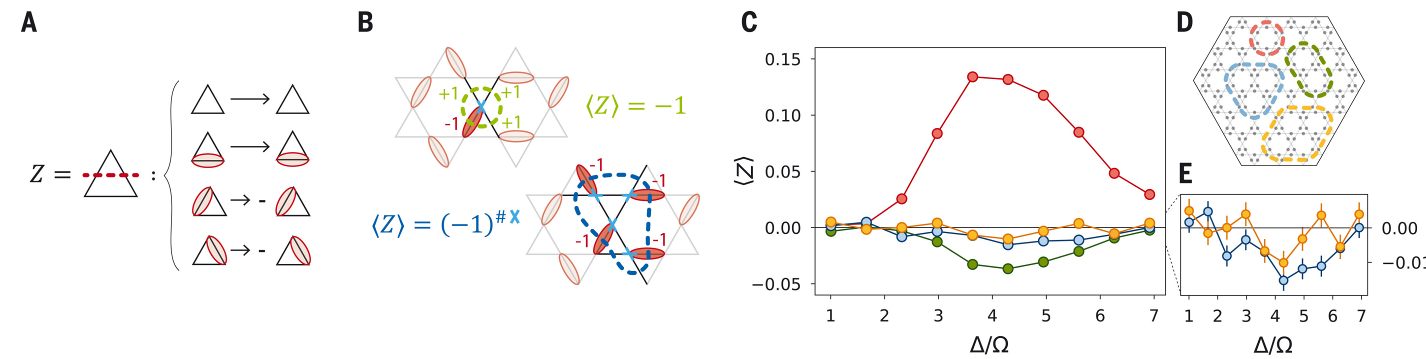


Triangular lattice quantum dimer model with variable dimer density

Rydberg atom array

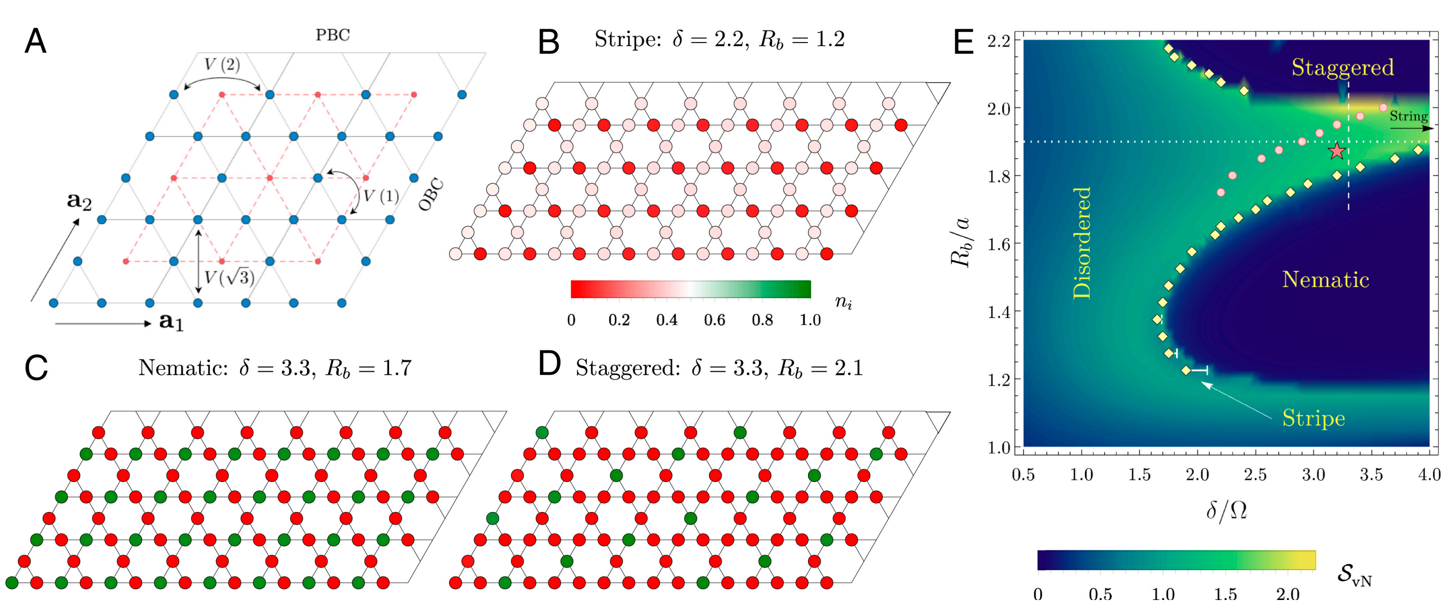
Rydberg atoms on the kagome lattice (links) maps to dimer model on the kagome lattice

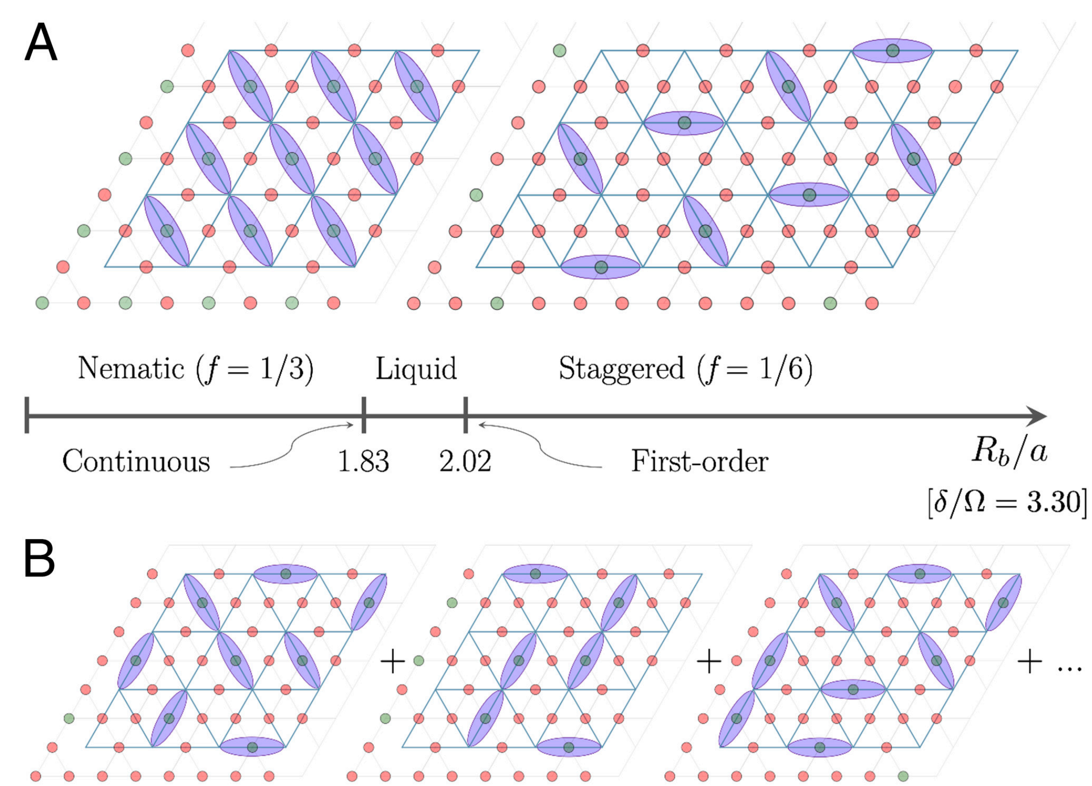




Semeghini G, Levine H, Keesling A, et al. Science, 374, 6572 (2021).

Rydberg atoms on the kagome lattice (sites) maps to dimer\loop model on the triangular lattice

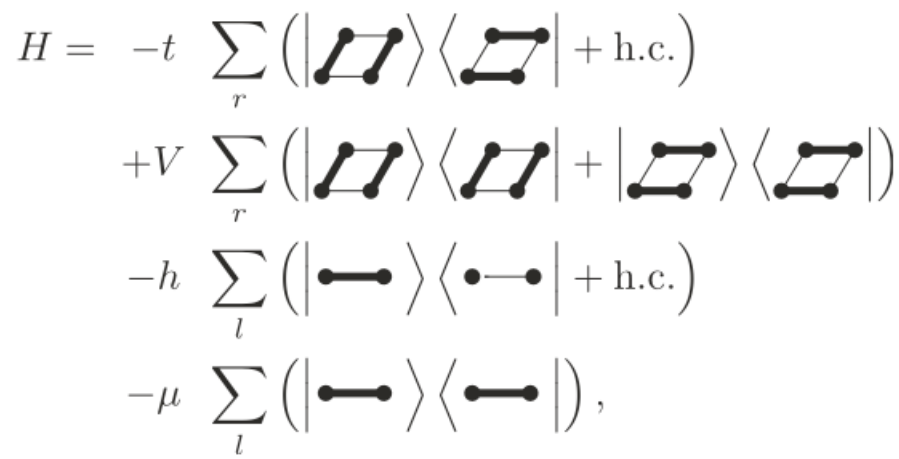




Samajdar R, Ho W W, Pichler H, et al. PNAS, 118, 4 (2021).

Phase diagram

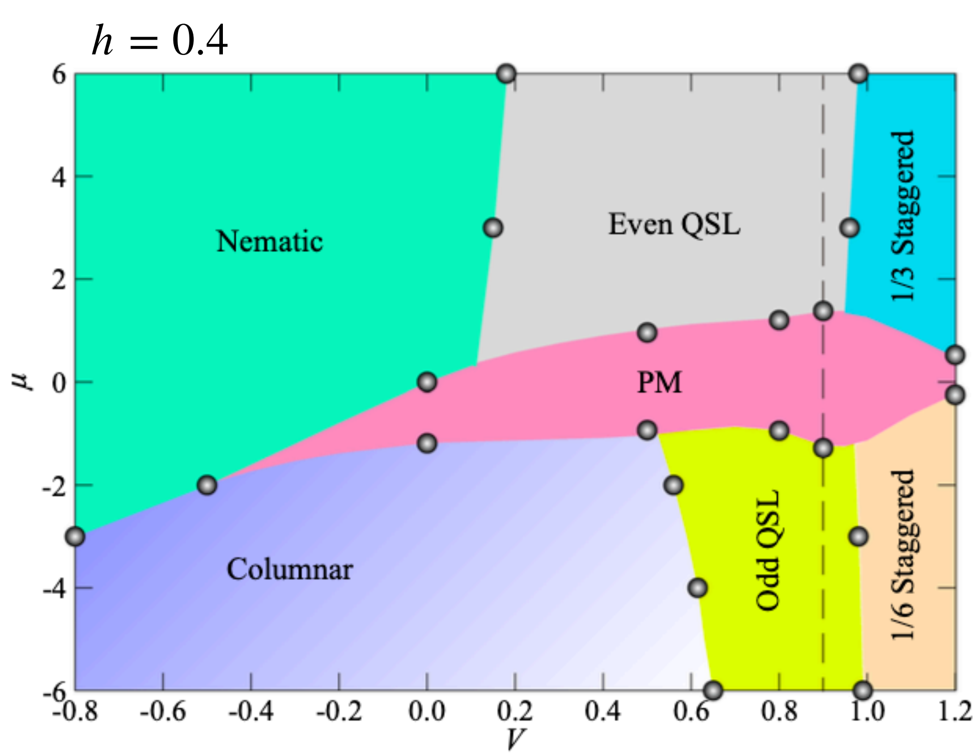
Soft constraint: one or two dimer(s) per site

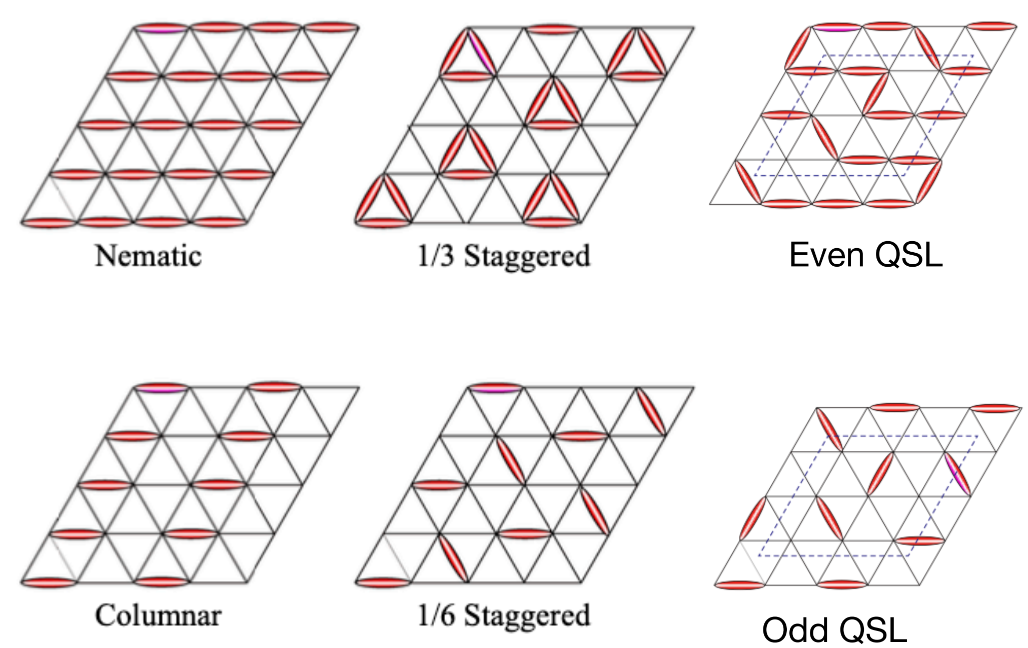


$t = 1$

h : transverse- field term of strength

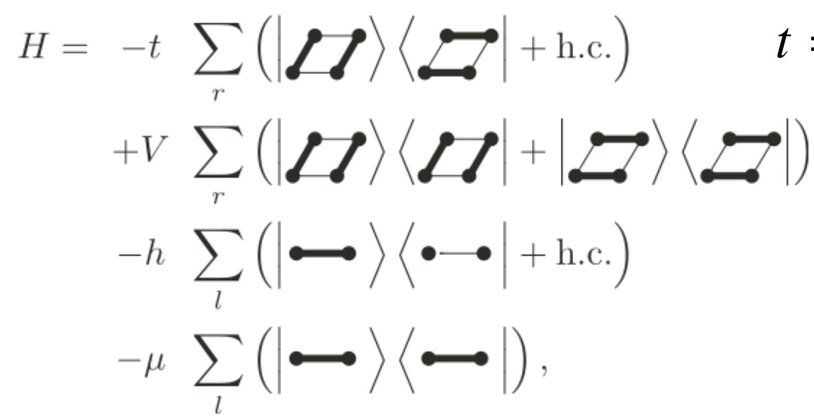
μ : chemical potential





Z Yan, R Samajdar, YC Wang, S Sachdev, ZY Meng, Nat Commun 13, 5799 (2022).

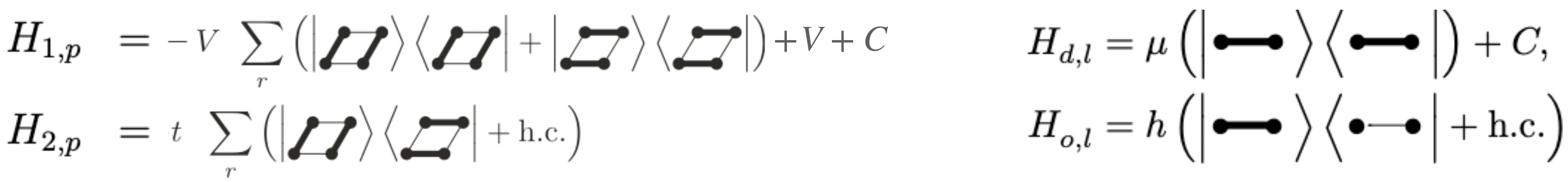
Phase diagram

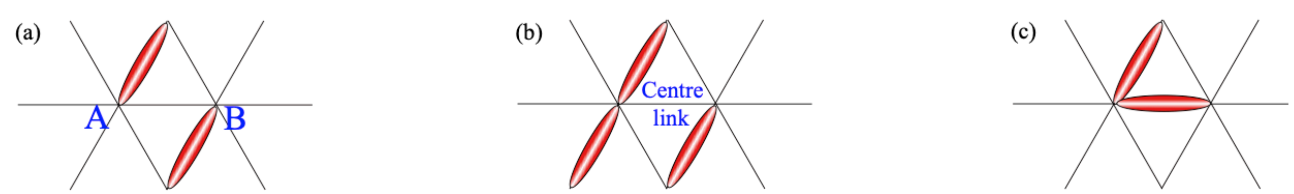


$t = 1$

h : transverse- field term of strength

μ : chemical potential



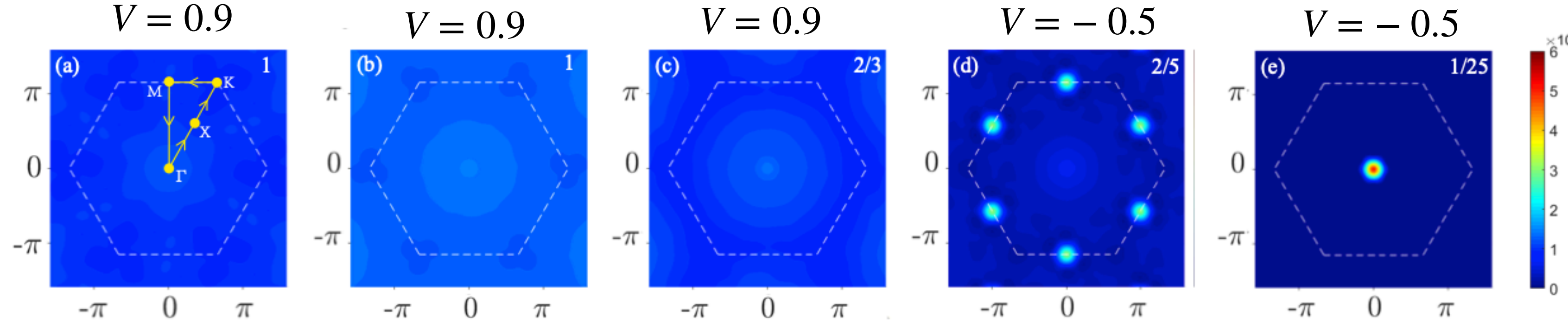


Only (a) allowed to create/annihilate a dimer on the centre link

Equal-time dimer-structure factor $\tau = 0$

$$D(\mathbf{k},\tau) = \frac{1}{N}\sum_{\substack{i,j \\ \alpha=1,2,3}}^{L^3} e^{i\mathbf{k}\cdot\mathbf{r}_{ij}} \left(\langle n_{i,\alpha}(\tau)n_{j,\alpha}(0)\rangle - \langle n_{i,\alpha}\rangle\langle n_{j,\alpha}\rangle\right)$$

$\mu = -3$, $V = 0.9$	$\mu = 0$, $V = 0.9$	$\mu = 3$, $V = 0.9$	$\mu = -3$, $V = -0.5$	$\mu = 3$, $V = -0.5$



Odd QSL | PM | Even QSL | Columnar | Nematic

Phase transitions between QSLs and the PM phase $V = 0.9 \quad h = 0.4$

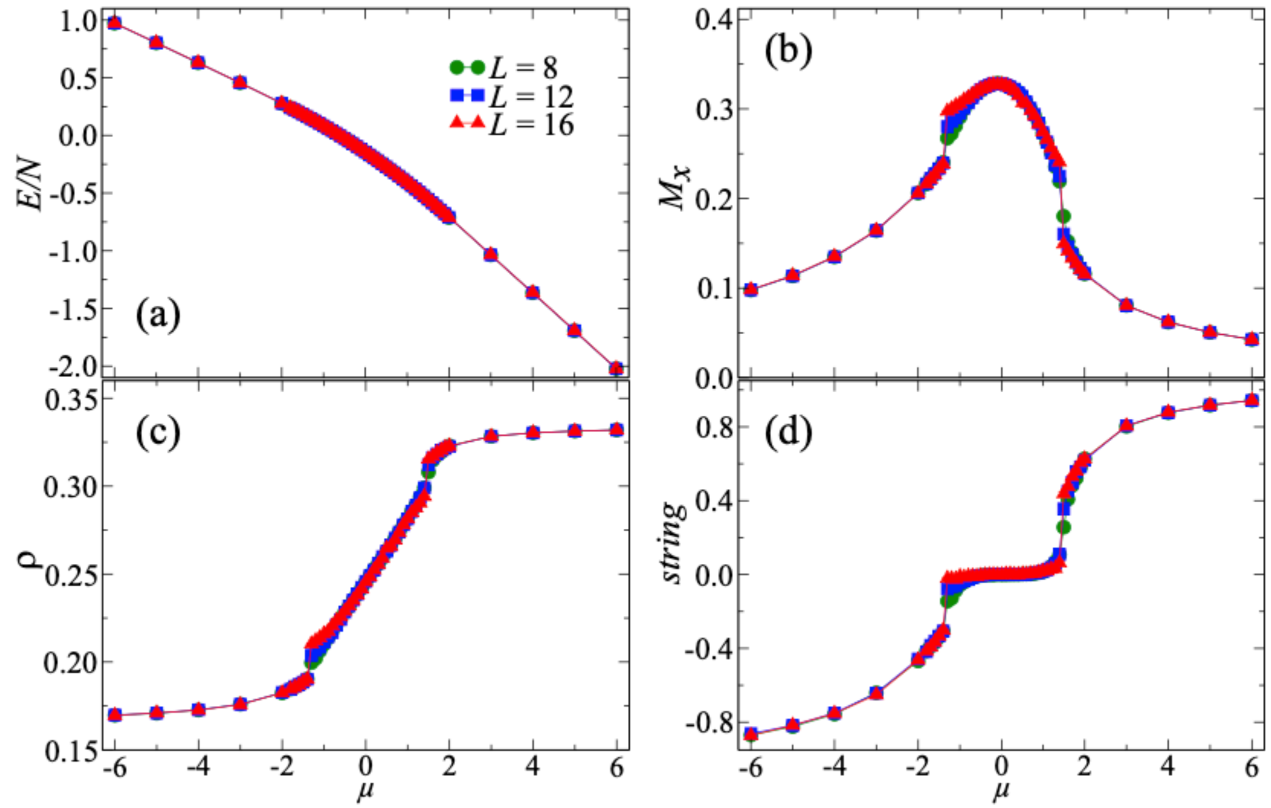
Polarization

$$M_x = \frac{1}{N}\sum_l \left(\left| \bullet\!\!-\!\!\bullet \right\rangle \left\langle \bullet\!-\!\bullet \right| + \text{h.c.} \right) \sim \frac{1}{N}\sum_l S_l^x$$

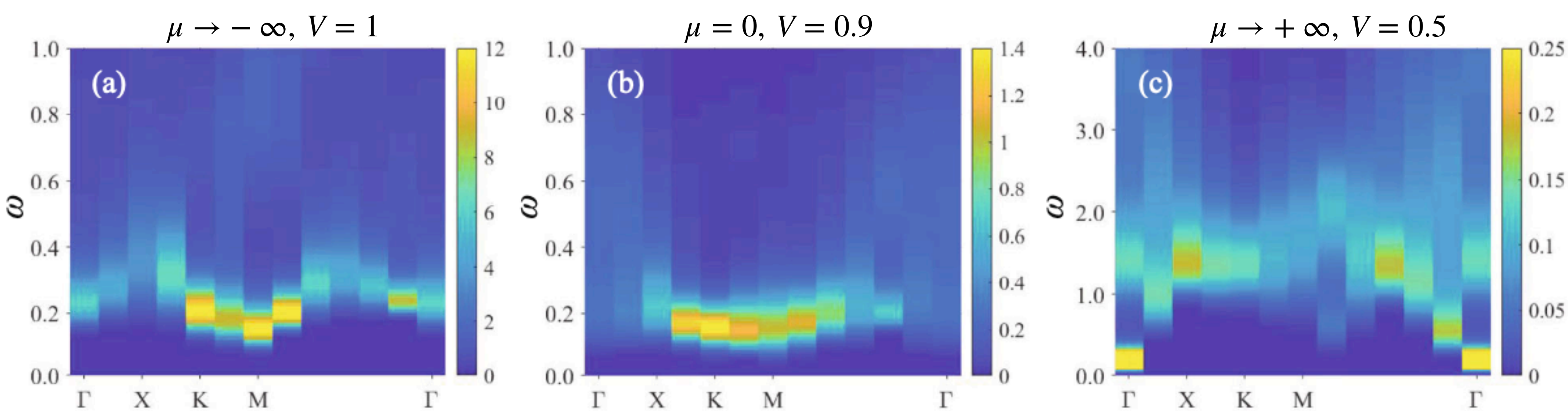
String operator

$$< string > = (-1)^{N_{P_{ij}}} \quad N_{P_{ij}} : \#\text{cut dimers}$$

Dimer filling



Dynamical dimer spectra



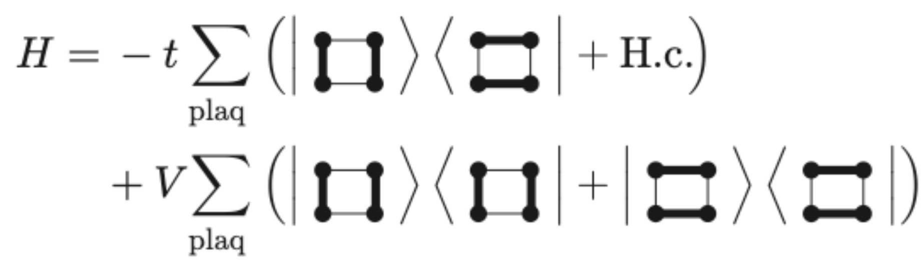
Odd QSL

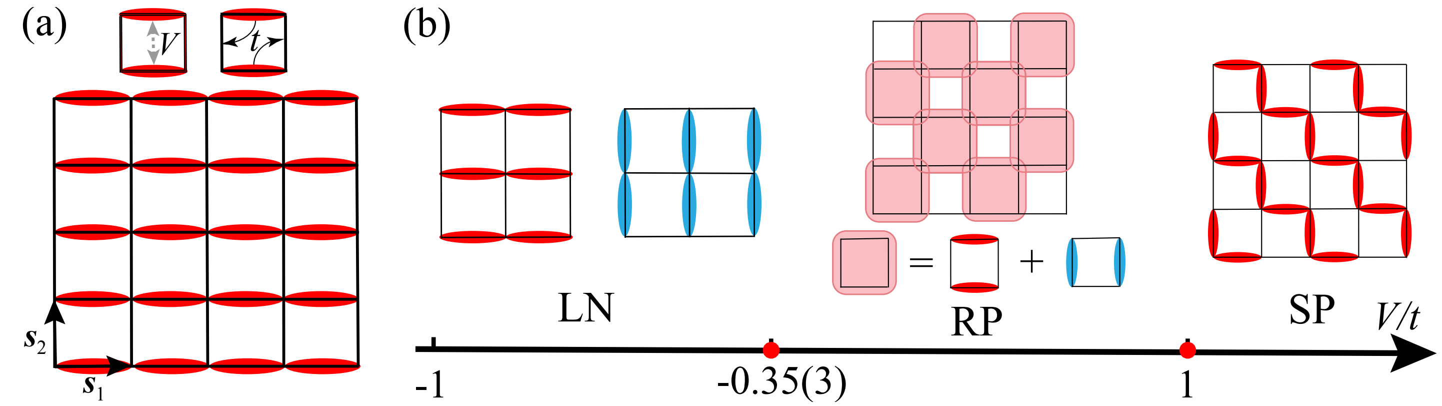
PM

Even QSL

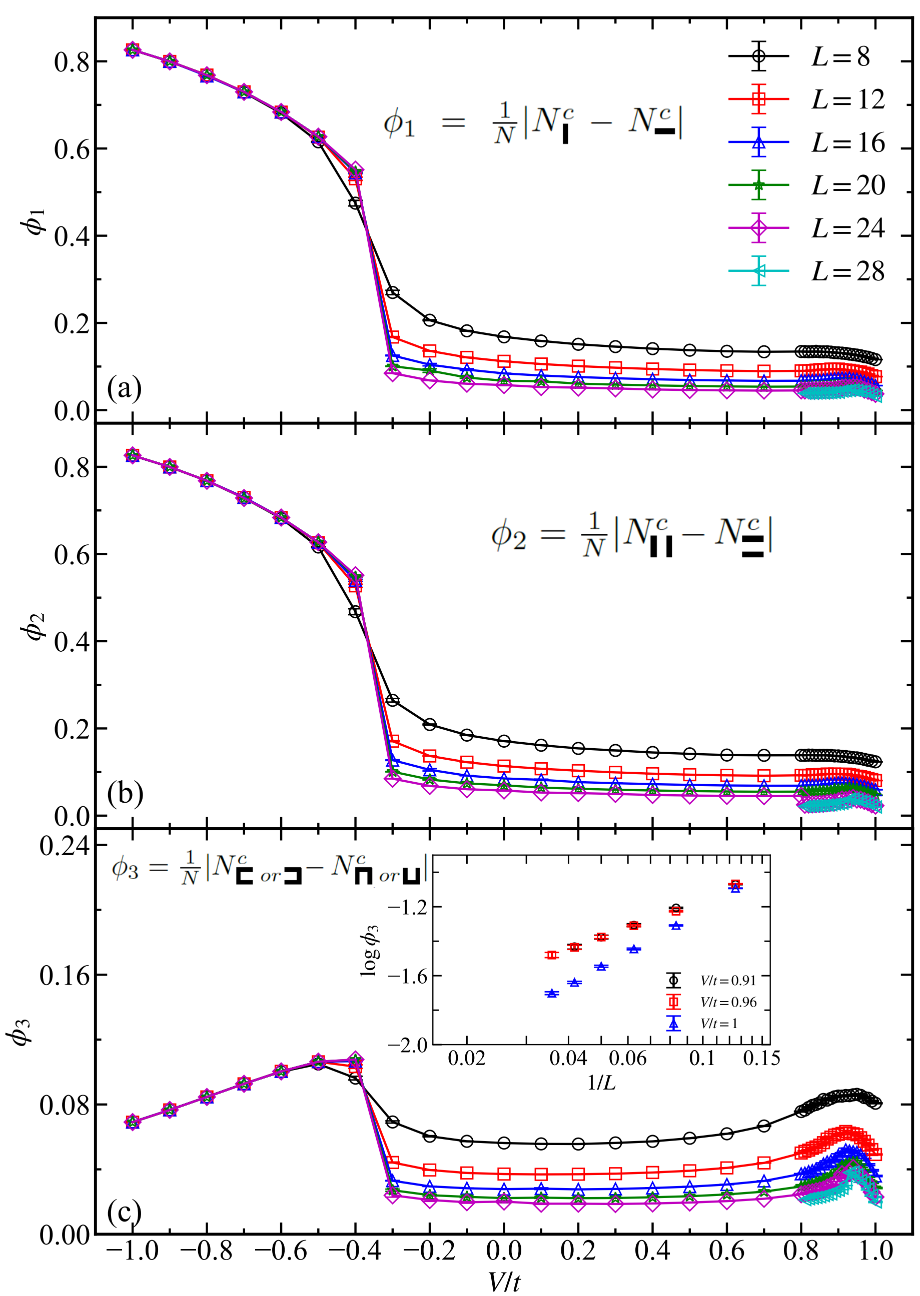
Phase diagram of quantum loop model on the square lattice

Phase diagram

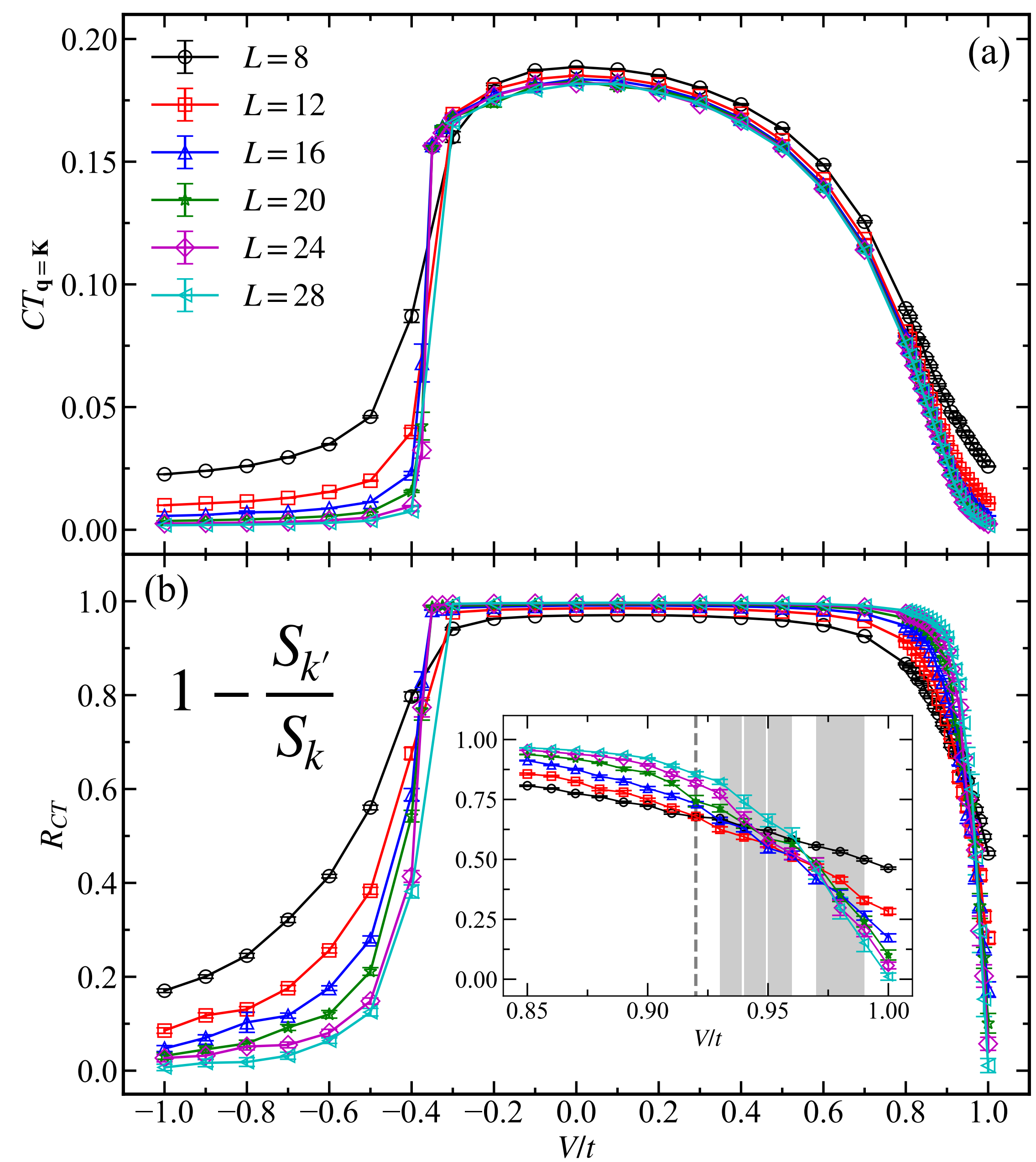




Order parameter for rotational symmetry breaking



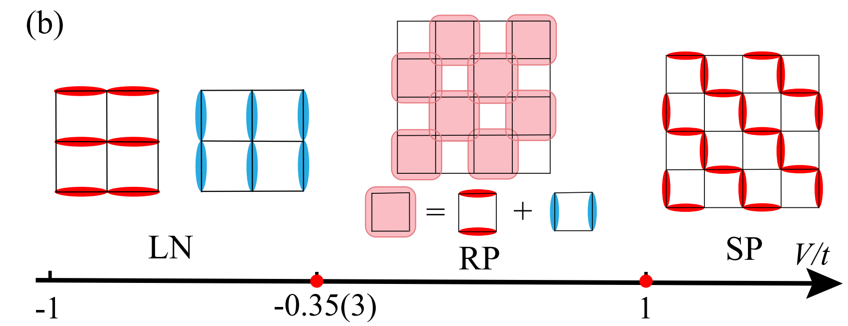
Translational symmetry breaking



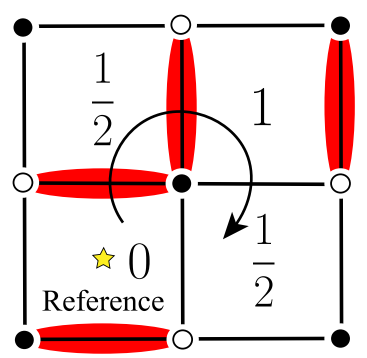
X. Ran, Z. Yan, Y.-C. Wang, J. Rong, Y. Qi, and Z. Y. Meng, Phys. Rev. B 107, 125134 (2023).

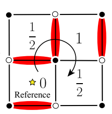
Effective height field action

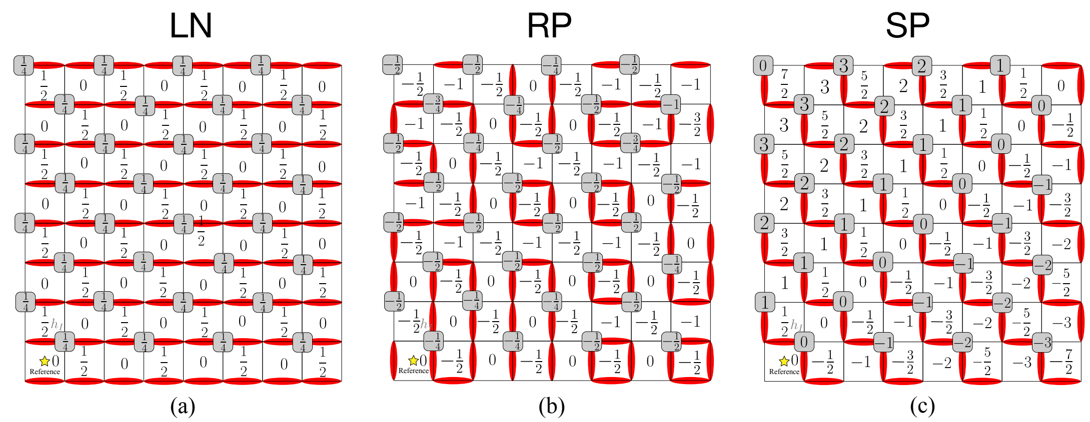
$$\mathcal{L} = \frac{1}{2}(\partial_\tau h)^2 + \frac{1}{2}\rho_2(\nabla h)^2 + \frac{\kappa^2}{2}(\nabla^2 h)^2 + \lambda\cos(4\pi h)$$



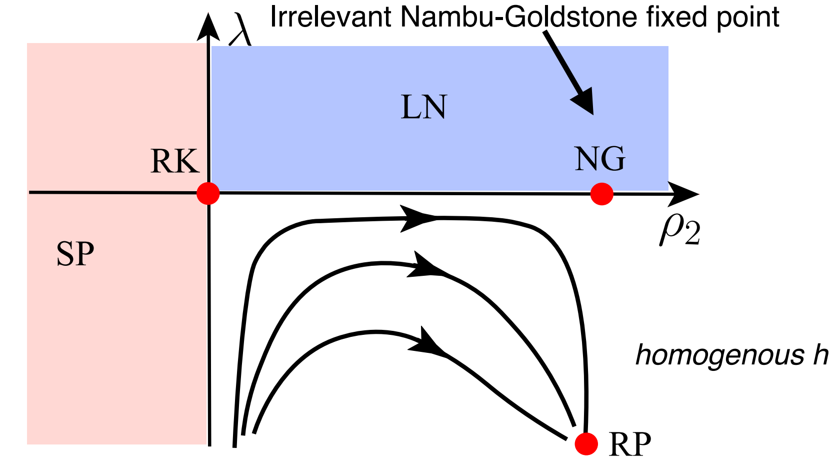
Height representation $\{h\}$



$h_I = average$ 



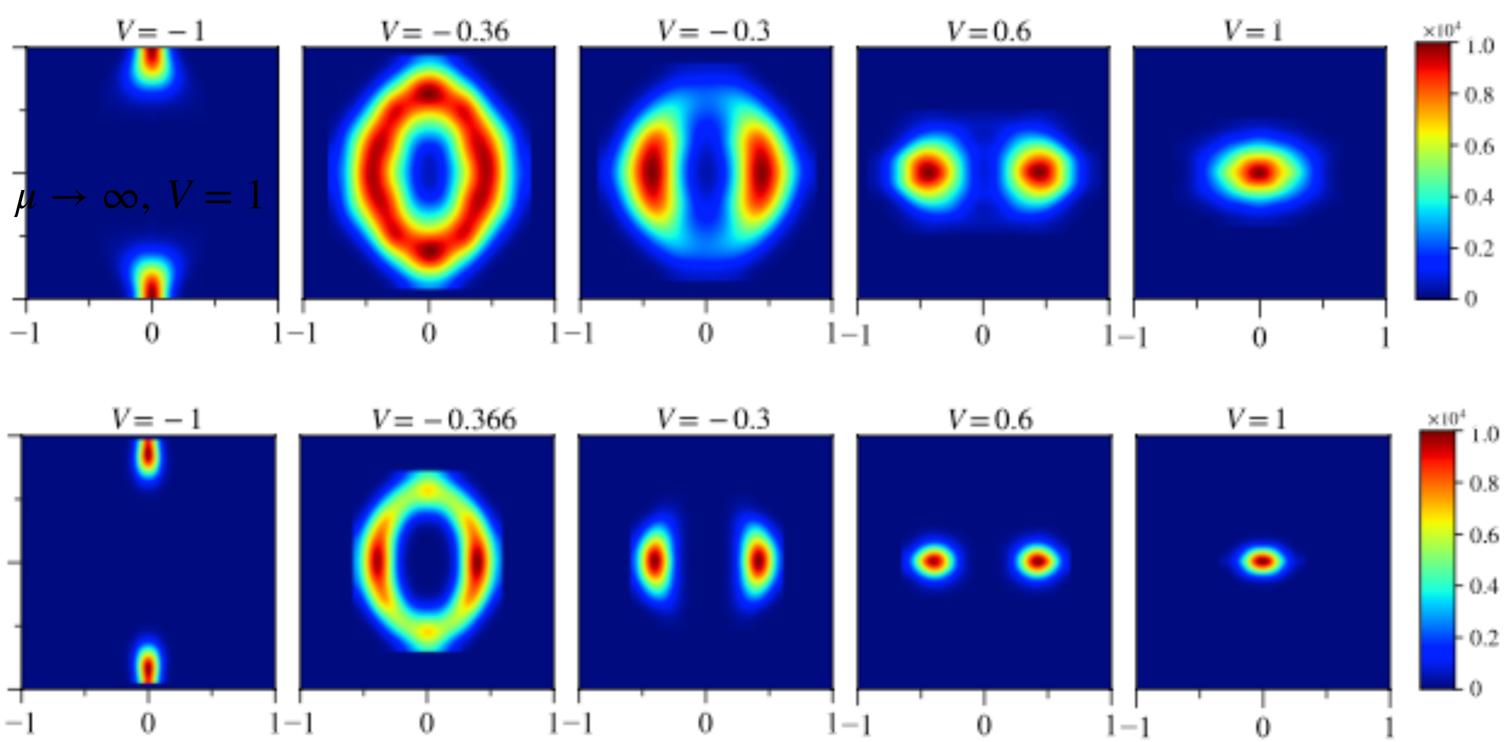
$h_I = \pm 1/4$ $h_I = \pm 1/2, 0$ $|\nabla h_I| = 1$



Histograms of height variable order parameter $\langle \frac{4}{N}\sum_I \cos(2\pi h_I), \frac{4}{N}\sum_I \sin(2\pi h_I)\rangle$

$L = 8$

$L = 16$



Coexist phase