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Content



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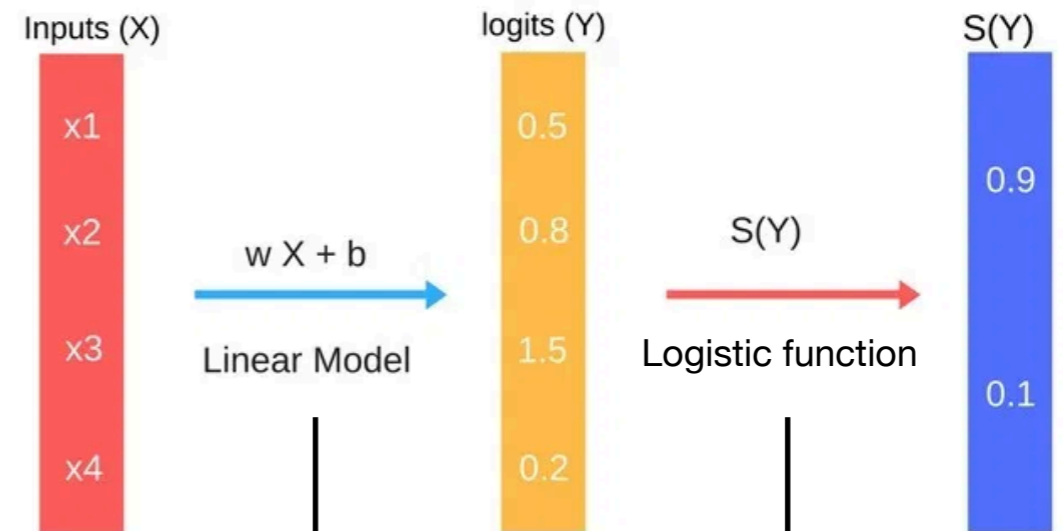
Logistic Regression Model



Used when the output is categorical;
 Classification;
 Bounded output;

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N	Penguin Activity	Penguin Activity Description	How Penguin felt (Target)
1	X1	Eating squids	Happy
2	X2	Eating small Fishes	Happy
3	X3	Hit by other Penguin	Sad
4	X4	Eating Crabs	Sad



Logistic Regression for Binary Classification
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$$\theta^T \cdot x$$

logit

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T \cdot x}}$$

hypothesis/activation/sigmoid

Logistic Regression — Detailed Overview

<https://towardsdatascience.com/logistic-regression-detailed-overview-46c4da4303bc>

Logistic Regression

$$\{(x_j^{(i)}, y^{(i)}), \theta_j\}; j = 1, 2, \dots, N; i = 1, 2, \dots, M; N < M$$

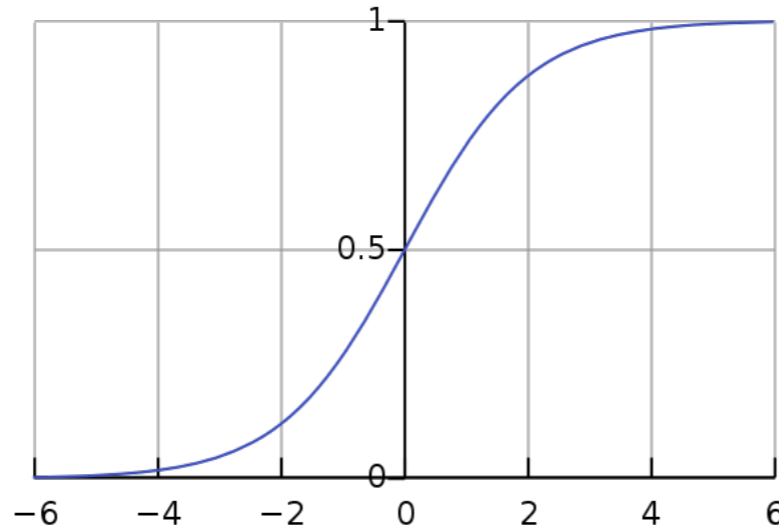
$$\begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(M)} \end{bmatrix}$$

$$y = h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T \cdot x}}$$

$$\begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & \dots & x_N^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & \dots & x_N^{(2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_1^{(M)} & x_2^{(M)} & \dots & x_N^{(M)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_N \end{bmatrix}$$

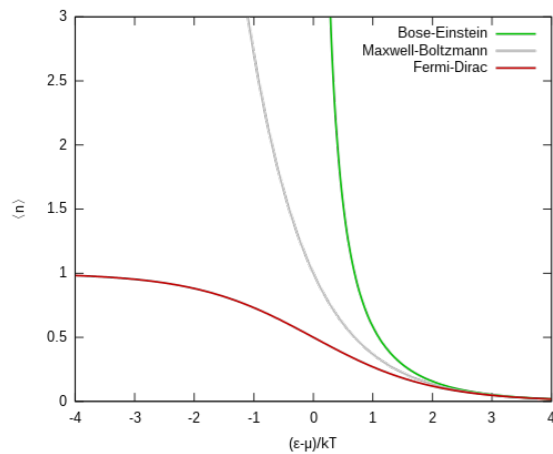
Discrete/binary Output

$$y^{(i)} = 0 \text{ or } 1$$



logistic unit (logit): $\theta^T \cdot x$

hypothesis/activation/sigmoid: $h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T \cdot x}}$

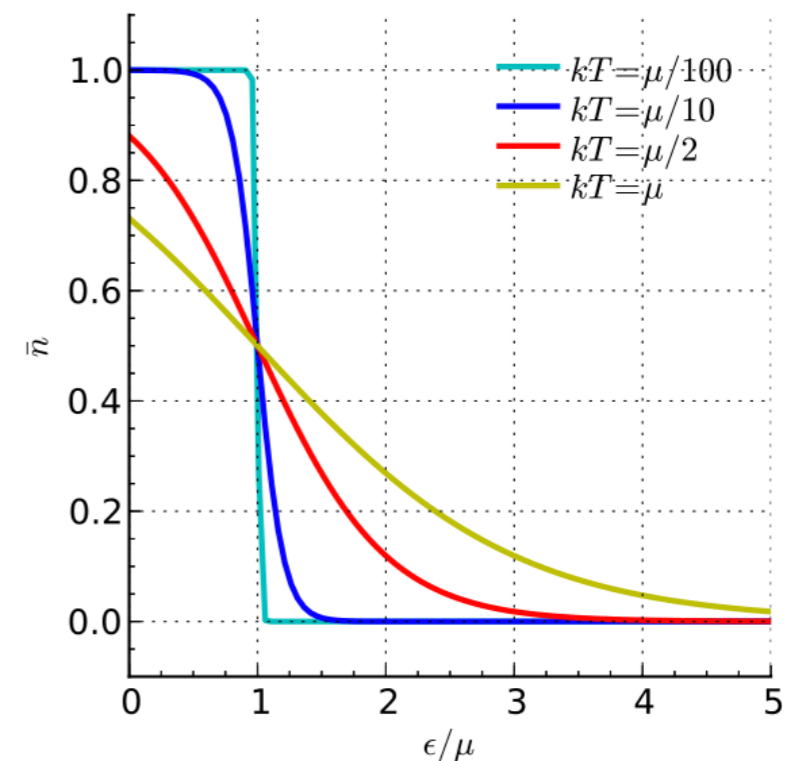


Fermi-Dirac distribution

$$\bar{n}_i = \frac{1}{1 + e^{(\epsilon_i - \mu)/k_B T}}$$

$$\bar{n}_i = \frac{g_i}{e^{(\epsilon_i - \mu)/k_B T} - 1}$$

Bose-Einstein distribution



Logistic Regression

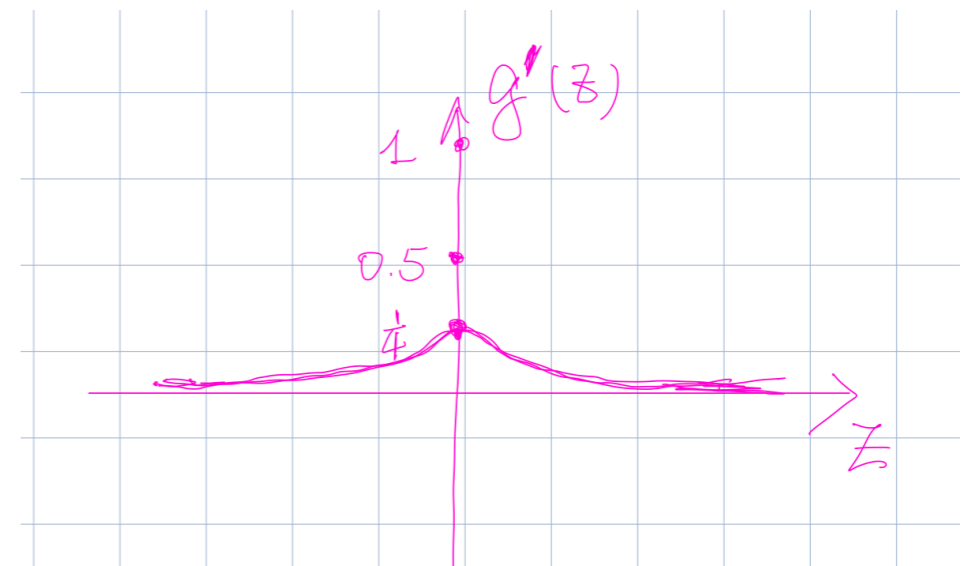
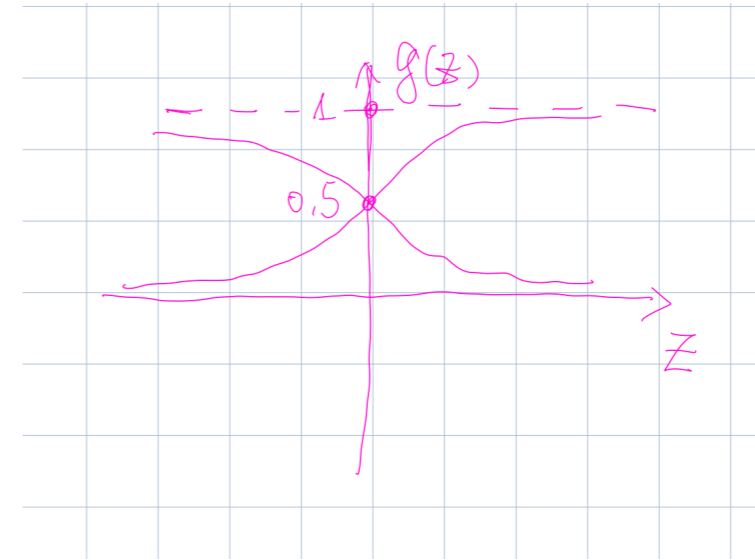
$$z = \theta^T \cdot x \quad h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T \cdot x}} \rightarrow g(z) = \frac{1}{1 + e^{-z}}$$

Several important properties of logistic function

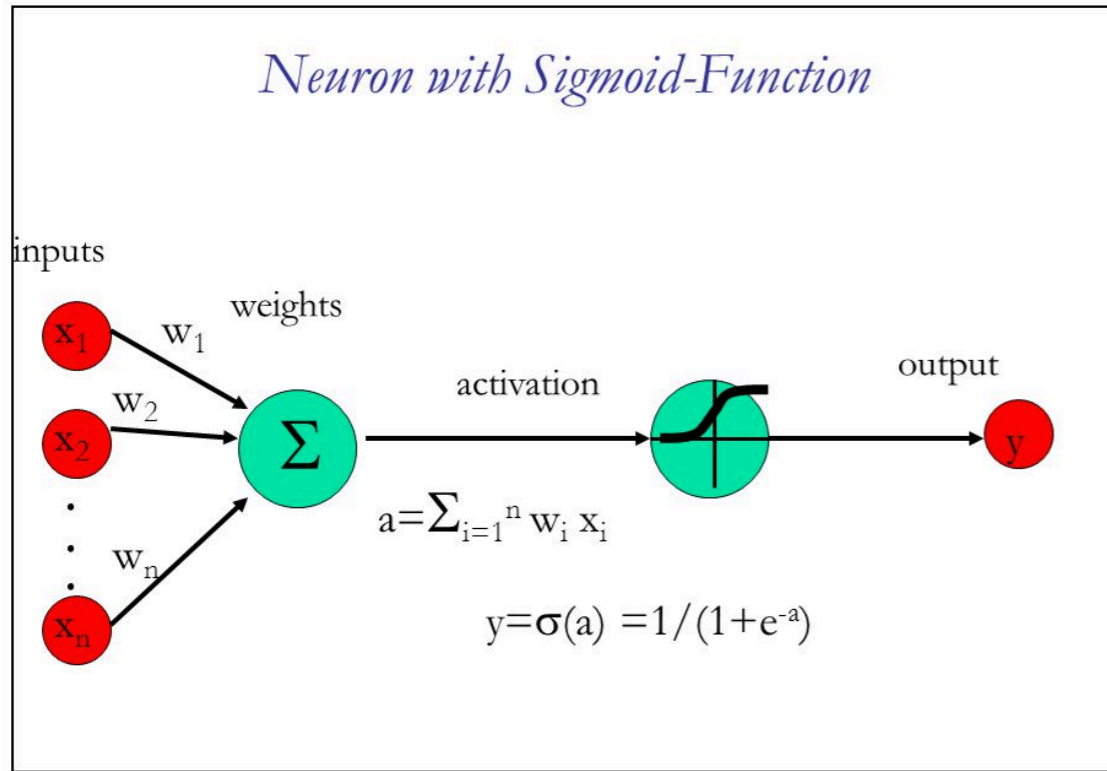
$$g(z) + g(-z) = 1$$

$$g'(z) = \frac{e^{-z}}{(1 + e^{-z})^2} = \dots = g(z)g(-z)$$

$$g'(z) = \dots = g'(-z)$$

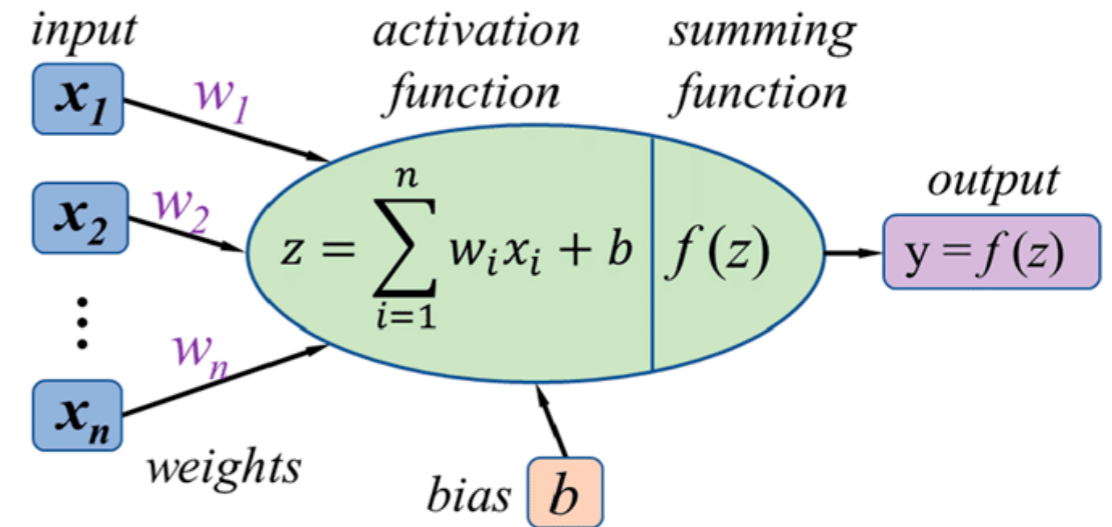


Perceptron

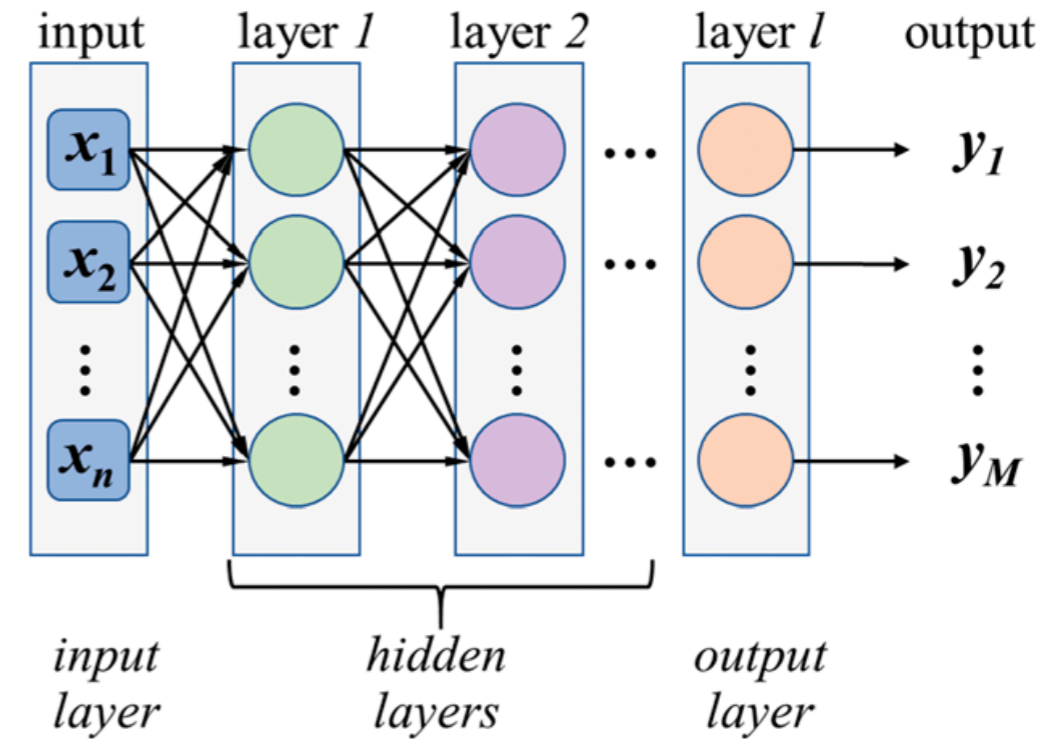


Neural network

(A) a neuron of an artificial neural network



(B) deep neural network



Human Body: Nervous System

The nervous system is made up of the central nervous system and peripheral nervous system. These systems work together to collect and interpret data from the body's internal and external environment and control responses.

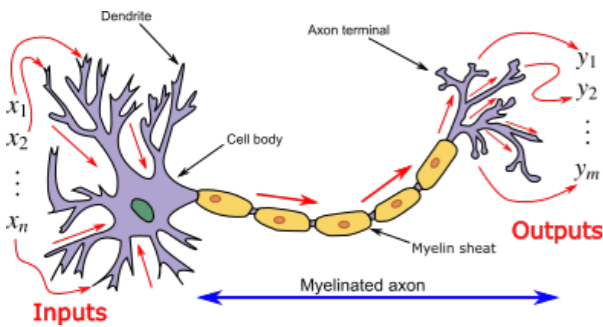
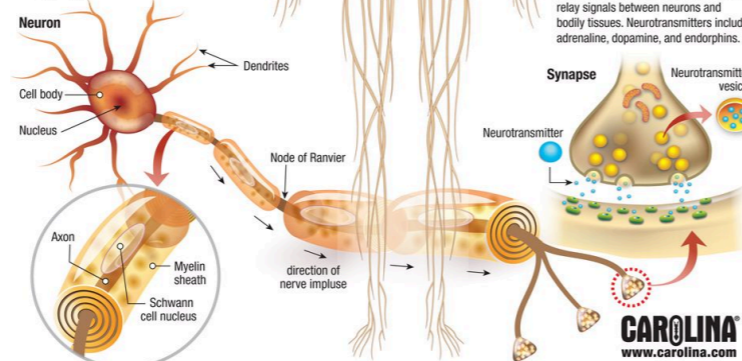
Central Nervous System

The central nervous system (CNS) manages the body's essential functions. Made up of the brain and spinal cord, the CNS receives sensory information and coordinates an appropriate response.

Peripheral Nervous System

The peripheral nervous system (PNS) connects the CNS to the rest of the body. Nerves branch out from the brain and spinal cord, extending to the organs, muscles, and other parts of the body.

Neurons are highly specialized cells that transmit chemical and electrical information in the body. Neurons use short, branched extensions called dendrites to receive nerve impulses from surrounding cells. These messages then travel through the cell body to the axon, a threadlike structure. The impulse moves through the axon and is transmitted via chemical or electrical signals that pass through a synapse.

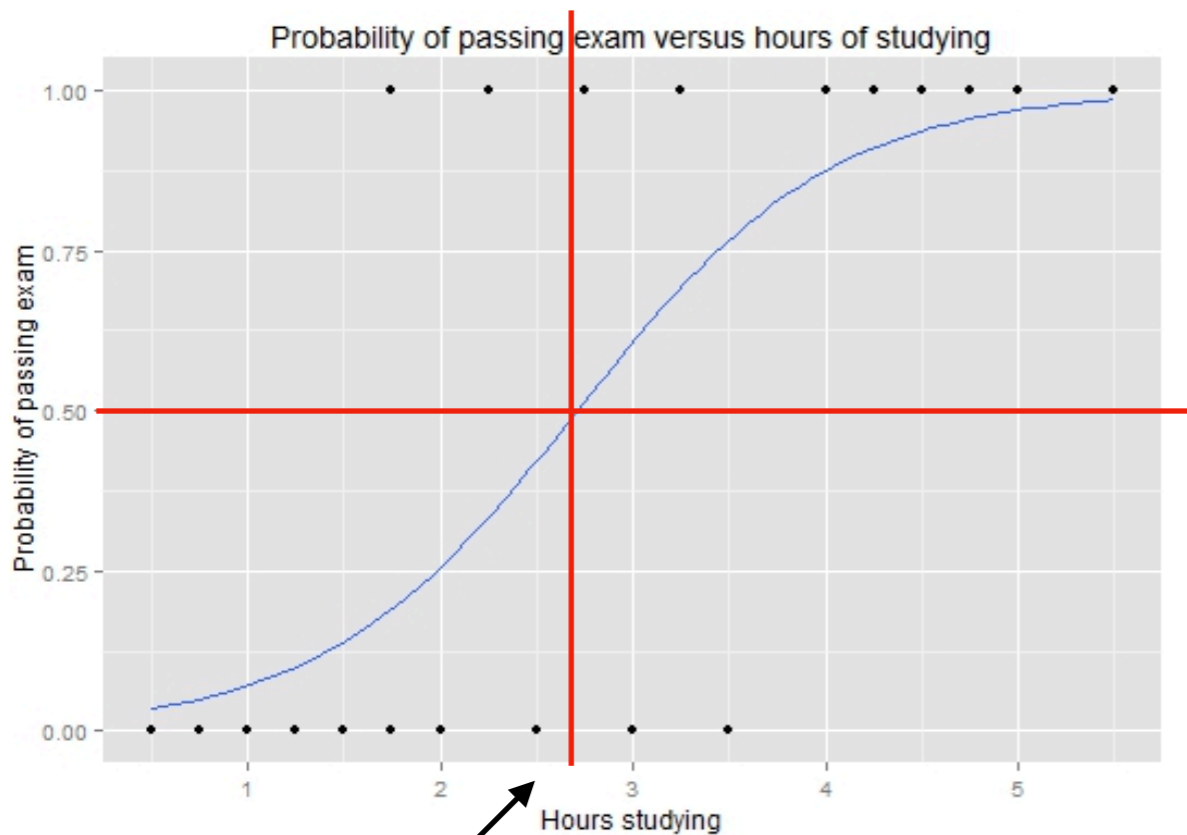


Γ

Logistic Regression – it is very logical !

A group of students spends between 0 and 6 hours studying for an exam. How does the number of hours spent affect the probability of the student passing the exam?

Hours	0.50	0.75	1.00	1.25	1.50	1.75	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.25	4.50	4.75	5.00	5.50
Pass	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	1	1	1



$$h_{\theta}(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}} = \frac{1}{1 + e^{-(-4.0777 + 1.5046 \cdot \text{Hours})}}$$

$$h_{\theta}(x) = 0.5 \quad \text{happens at} \quad x = 2.71$$

$$\text{Probability of passing exam} = \frac{1}{1 + \exp(-(1.5046 \cdot 2 - 4.0777))} = 0.26$$

$$\text{Probability of passing exam} = \frac{1}{1 + \exp(-(1.5046 \cdot 4 - 4.0777))} = 0.87$$

Decision boundary

Logistic Regression – it is very logical !

Model in learning

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T \cdot x}}$$

Likelihood function

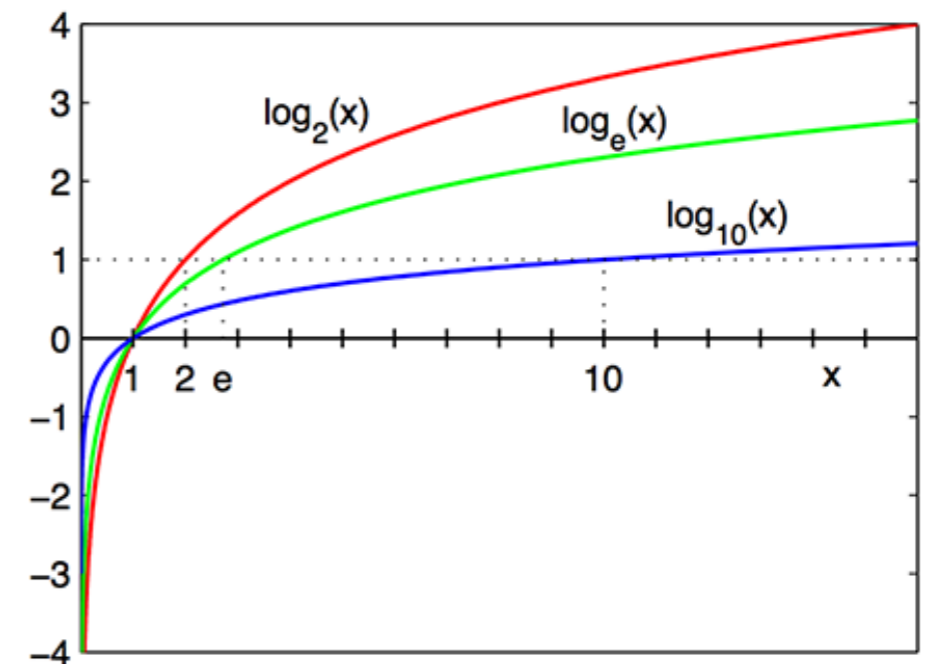
$$L(\theta | \dots (x^{(i)}, y^{(i)}) \dots) \stackrel{\text{i.i.d.}}{=} \prod_{i=1}^M h_{\theta}(x^{(i)})^{y^{(i)}} (1 - h_{\theta}(x^{(i)}))^{1-y^{(i)}}$$

Cost function

$$J(\theta) = -\frac{1}{M} \ln(L(\theta)) = -\frac{1}{M} \sum_{i=1}^M [y^{(i)} \ln(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \ln(1 - h_{\theta}(x^{(i)}))]$$

Optimization procedure

$$\theta^* = \arg \min_{\theta} J(\theta)$$

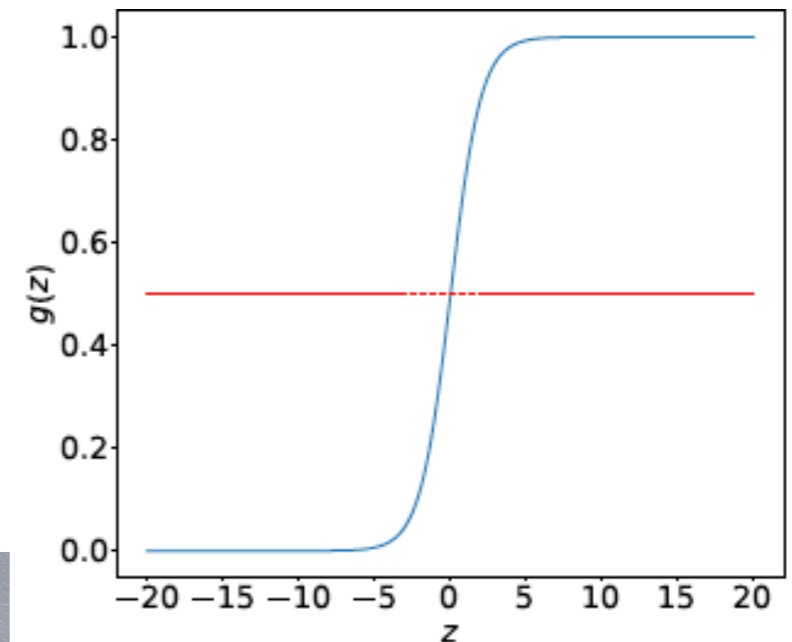


Logistic Regression – it is very logical !

$$z = \theta^T \cdot x \quad h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T \cdot x}} \rightarrow g(z) = \frac{1}{1 + e^{-z}}$$

Several important properties of logistic function

1. $g(z) + g(-z) = 1$
2. $g'(z) = g(z)(1 - g(z)) = g(z)g(-z)$
3. $g'(-z) = g'(z)$



cost function $J(\theta) = -\frac{1}{M} \sum_{i=1}^M [y^{(i)} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))]$

$y=1$
 $M=1$
 $J(\theta) = -\log\left(\frac{1}{1 + e^{-\theta x}}\right) = \log(1 + e^{-\theta x})$

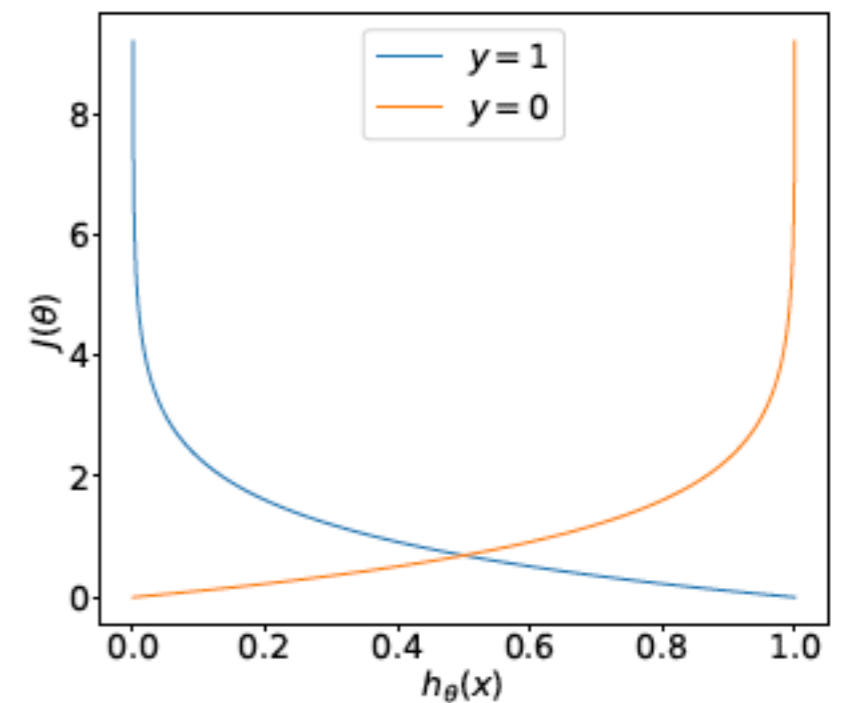
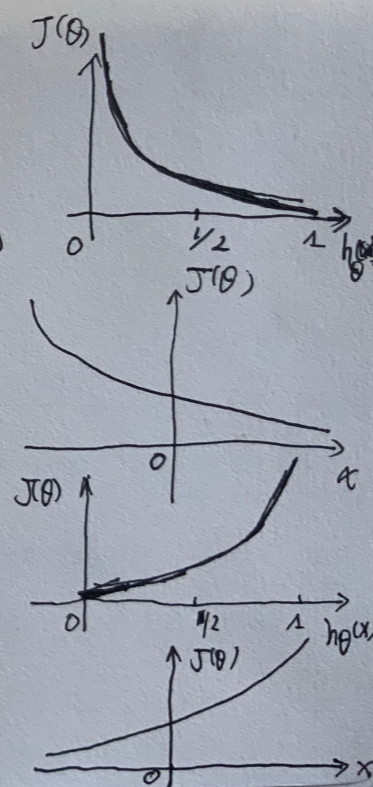
$x \rightarrow \infty$, $h_{\theta}(x) = \frac{1}{1 + e^{-\theta x}} \rightarrow 1$, $J(\theta) \rightarrow 0$

$x = 0$, $h_{\theta}(x) = \frac{1}{2}$, $J(\theta) = \log(2)$

$x \rightarrow -\infty$, $h_{\theta}(x) = \frac{1}{1 + e^{-\theta x}} \rightarrow 0$, $J(\theta) \rightarrow \infty$

$y=0$
 $M=1$

the cost function of logistic regression is cross entropy



Logistic Regression – it is very logical !

Gradient descent $\theta := \theta - \alpha \nabla_{\theta} J(\theta)$

$$= \theta - \alpha \left[- \langle xy \rangle + \frac{1}{M} \sum_{i=1}^M x^{(i)} h_{\theta}(x^{(i)}) \right]$$

1. $g(z) + g(-z) = 1$
2. $g'(z) = g(z)(1 - g(z))$
3. $g'(-z) = g'(z)$

$$\begin{aligned} \nabla_{\theta} J(\theta) &= -\frac{1}{M} \sum_{i=1}^M \left[x^{(i)} y^{(i)} \frac{g'(\theta^T x^{(i)})}{g(\theta^T x^{(i)})} - \right. \\ &\quad \left. x^{(i)} (1 - y^{(i)}) \frac{g'(-\theta^T x^{(i)})}{g(-\theta^T x^{(i)})} \right] \\ &= -\frac{1}{M} \sum_{i=1}^M \left[x^{(i)} y^{(i)} g(-\theta^T x^{(i)}) - \right. \\ &\quad \left. x^{(i)} (1 - y^{(i)}) g(\theta^T x^{(i)}) \right] \\ &= -\frac{1}{M} \sum_{i=1}^M \left[x^{(i)} y^{(i)} - x^{(i)} g(\theta^T x^{(i)}) \right] \\ &= -\langle xy \rangle + \frac{1}{M} \sum_{i=1}^M x^{(i)} g(\theta^T x^{(i)}) \end{aligned}$$

the same holds for linear reg.

$$\begin{aligned} \nabla_{\theta} J(\theta) &= \frac{1}{2M} \nabla_{\theta} \sum_{i=1}^M (\theta^T x^{(i)} - y^{(i)})^2 \\ &= \frac{1}{M} \sum_{i=1}^M x^{(i)} (\theta^T x^{(i)} - y^{(i)}) \\ &= -\frac{1}{M} \sum_{i=1}^M x^{(i)} y^{(i)} + \frac{1}{M} \sum_{i=1}^M x^{(i)} \theta^T x^{(i)} \\ &= -\langle xy \rangle + \frac{1}{M} \sum_{i=1}^M x^{(i)} g(\theta^T x^{(i)}) \end{aligned}$$

Logistic Regression — it is very logical !

$$J(\theta) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x))$$

$$h_{\theta} = \frac{1}{1 + e^{-z}}, \quad z = \theta_1 x_1 + \theta_2 x_2 + \theta_0$$

$$\frac{\partial J(\theta)}{\partial \theta_1} = -y \frac{1}{h_{\theta}(z)} h_{\theta}(z) h_{\theta}(-z) x_1 + (1-y) \frac{1}{1-h_{\theta}(z)} h_{\theta}(z) h_{\theta}(-z) x_1$$

$$= -y h_{\theta}(-z) x_1 + (1-y) h_{\theta}(z) x_1$$

$$= -x_1 y + x_1 h_{\theta}(z)$$

$$\frac{\partial J(\theta)}{\partial \theta_2} = -x_2 y + x_2 h_{\theta}(z)$$

$$\frac{\partial J(\theta)}{\partial \theta_0} = -y + h_{\theta}(z)$$

$$\nabla_{\theta} J(\theta) = -\langle x y \rangle + \frac{1}{M} \sum_{i=1}^M x^{(i)} g(\theta^T x^{(i)})$$