Phases of (2+1)D SO(5) non-linear sigma model with topological term on a sphere

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Introduction

Novel critical phenomena beyond the Landau-Ginzburg-Wilson paradigm have been long sought after. Among many candidate scenarios, the deconfined quantum critical point (DQCP) constitutes the most fascinating one, and its lattice model realization has been debated over the past two decades. Here we apply the spherical Landau level regularization upon the exact (2+1)D SO(5) non-linear sigma model with a topological term to study the potential DQCP therein. Utilizing SU(2) DMRG, accompanied by quantum Monte Carlo simulations, we accurately obtain the comprehensive phase diagram of the model on a sphere. We find various novel quantum phases, including Néel, ferromagnet (FM), valence bond solid (VBS), valley polarized (VP) states and quantum disordered phase occupying extended area of the phase diagram. Our results show that two different symmetry-breaking phases, i.e., the SO(2)-breaking VBS and the SO(3)-breaking Néel states, are separated by either a weakly first-order transition or the disordered region with a multicritical point in between, thus opening up more interesting questions on this two-decade long debate on the nature of DQCP.

Model — SO(5) model on sphere

- **The (2+1)D SO(5) model** [2]

$$H_\Gamma = \frac{1}{2}\int d\Omega\{U_0\left[\psi^\dagger(\Omega)\psi(\Omega)-2\right]^2 - \sum_{i=1}^{5} u_i\left[\psi^\dagger(\Omega)\Gamma^i\psi(\Omega)\right]^2\}, \qquad (1)$$

where $\psi_{\tau\sigma}(\Omega)$ is the 4-component fermion annihilation operator with valley τ and spin σ indices, and
$\Gamma^i = \{\tau_x\otimes\mathbb{I}, \tau_y\otimes\mathbb{I}, \tau_z\otimes\sigma_x, \tau_z\otimes\sigma_y, \tau_z\otimes\sigma_z\}$.

- **Possible DQCP scenario**

We set $u_1 = u_2 = u_K$ and $u_3 = u_4 = u_5 = u_N$.
It is proposed, $u_K > u_N$ stabilizes VBS state, and $u_N > u_K$ stabilizes Néel state.
If a direct and continuous phase transition between VBS and Néel states arises at $u_K = u_N$, at the transition the system has an explicit SO(5) symmetry, which realizes a DQCP.

- **Spherical Landau level** [3]

Hamiltonian for electrons moving on the surface of a sphere with $4\pi s$ monopole ($2s \in Z$):

$$H_0 = \frac{1}{2M_e r^2}\Lambda_\mu^2,$$

and $\Lambda_\mu = \partial_\mu + iA_\mu$.
The eigen-energies (spherical Landau levels) are

$$E_n = [n(n+1)+(2n+1)s]/(2M_e r^2)$$

whose degeneracies are $(2s+2n+1)$-fold.
We assume all interactions are much smaller than the energy gap between Landau levels, and just consider the lowest Landau level (LLL) $n=0$, which is $(2s+1)$-fold degenerate and we denote $N = 2s+1$ as the system size of the problem.
With the gauge $\vec{A} = \frac{\hbar s}{er}\hat{\phi}\cot(\theta)$, the wave-functions of LLL orbital are monopole harmonics

$$\Phi_m(\theta,\phi) = N_m e^{im\phi}\cos^{s+m}(\tfrac{\theta}{2})\sin^{s-m}(\tfrac{\theta}{2}),$$

with $m = -s, -s+1, \cdots, s$ and $N_m = \sqrt{\frac{(2s+1)!}{4\pi(s+m)!(s-m)!}}$.

- **Projected Hamiltonian**

Via the expansion $\psi(\Omega) = \sum_m \Phi_m(\Omega)c_m$, we have

$$\hat{H}_\Gamma = U_0\hat{H}_0 - \sum_i u_i\hat{H}_i, \text{ with}$$

$$\hat{H}_i = \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}\times\left(c^\dagger_{m_1}\Gamma^i c_{m_1+m} - 2\delta_{i0}\delta_{m0}\right) \times\left(c^\dagger_{m_2}\Gamma^i c_{m_2-m} - 2\delta_{i0}\delta_{m0}\right) \qquad (2)$$

with $\Gamma^0 = \mathbb{I}\otimes\mathbb{I}$.

Order parameters

For all the ordered phases observed, the order parameters take the form of fermion bilinears: $\langle O\rangle = \int d\Omega\langle\psi^\dagger(\Omega)M\psi(\Omega)\rangle = \sum_m\langle c^\dagger_m M c_m\rangle$, where M is either a Γ-matrix or one of the SO(5) generators L^{ij}.
In our simulations, we find the following order parameters being finite in different regions of parameter space.

- **Néel order** with $m^2_{\rm Neel} = \frac{1}{3N^2}\langle(O_3^2+O_4^2+O_5^2)\rangle$
- **VBS order** with $m^2_{\rm VBS} = \frac{1}{2N^2}\langle(O_1^2+O_2^2)\rangle$
- **FM order** with $m^2_{\rm FM} = \frac{1}{N^2}\langle(\tilde{O}_{34}^2+\tilde{O}_{35}^2+\tilde{O}_{45}^2)\rangle$
- **VP order** with $m^2_{\rm VP} = \frac{1}{N^2}\langle\tilde{O}_{12}^2\rangle$.

We also consider the RG-invariant Binder ratios $\langle O_i^2\rangle^2/\langle O_i^4\rangle$ and Correlation ratios defined as $R = 1 - \langle O^2_{l=1}\rangle/\langle O^2_{l=0}\rangle$.

Results

- **Phase Diagram**

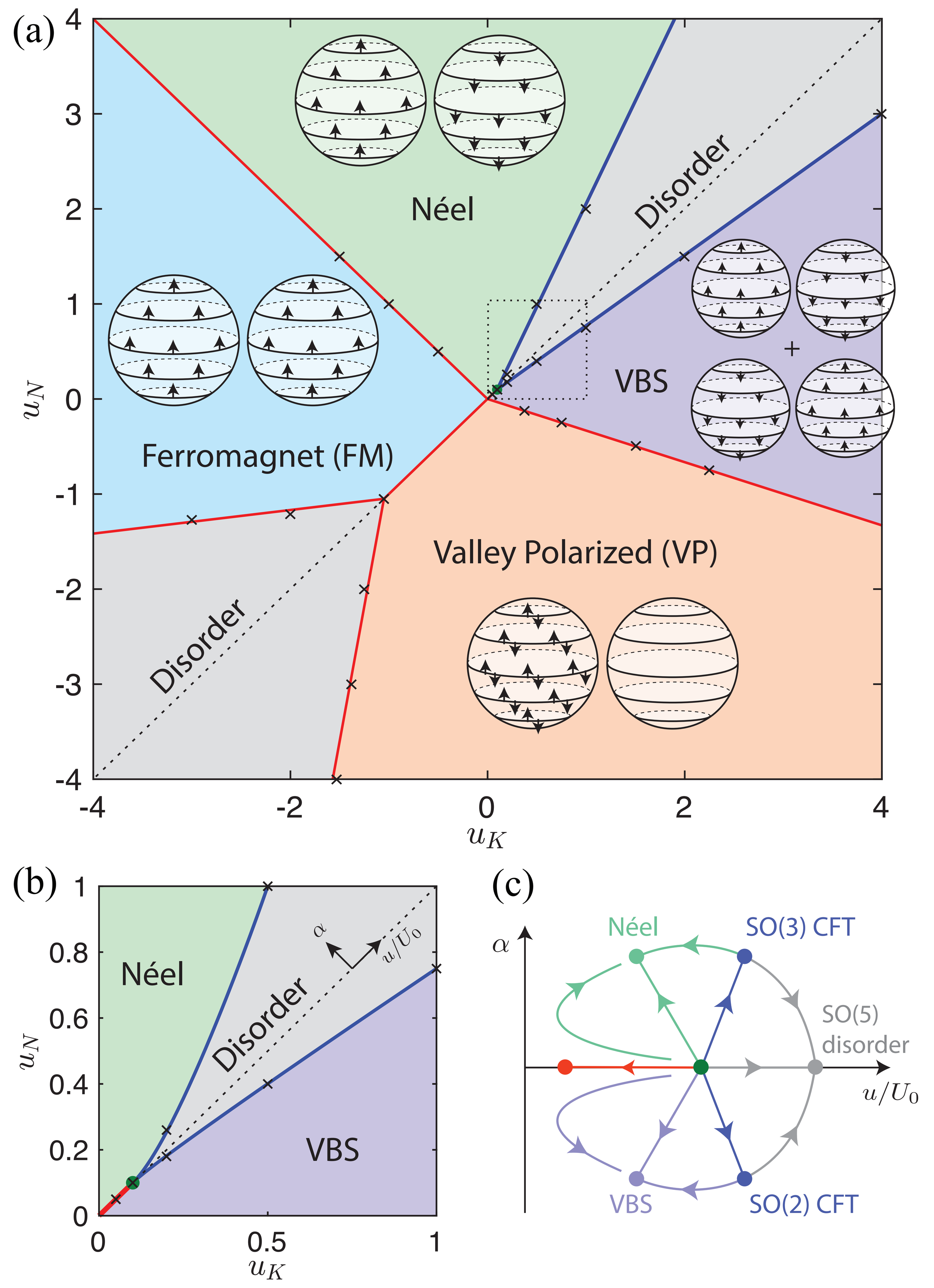

Fig 1: (a) Overall phase diagram with Néel, VBS, ferromagnet (FM), and valley polarized (VP) phases, and the disorder phase as denoted. (b) Zoomed-in phase diagram as indicated in (a). The two critical boundaries meet at a multicritical point (deep green dot) below which the SO(5) symmetry is broken. (c) Possible RG flow in the considered parameter space in (b), with multicritical point (deep green dot), SO(5) disorder (grey dot), SO(2)/SO(3) CFT (blue dots), Néel-orderd (light green dot), VBS-ordered (light purple dot), and the SO(5) breaking (red dot) fixed points.

- **O(2) and O(3) transition, and Disorder phase**

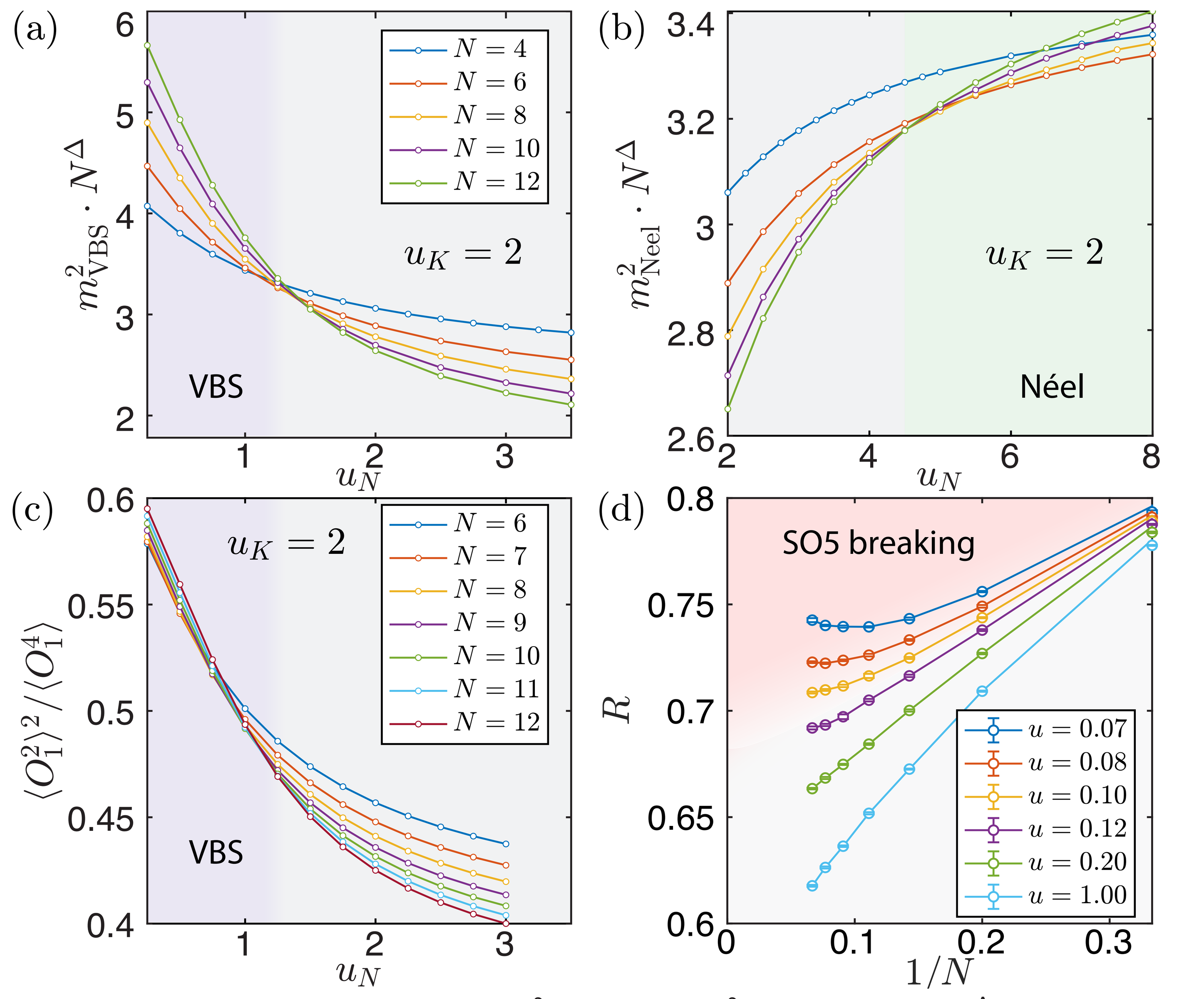

Fig 2: Along the fixed $u_K = 2$ cut, (a) $m^2_{\rm VBS}$ and (b) $m^2_{\rm Neel}$, rescaled by N^Δ with scaling dimension $\Delta = 0.519$, show converged crossing behavior at $u_N \simeq 1.5$ and $u_N \simeq 4.5$, suggesting a disorder phase in between. (c) Binder ratios for VBS order $\langle O_1^2\rangle^2/\langle O_1^4\rangle$ show crossing at $u_N \simeq 1.21$ for two largest $N = 11$ and $N = 12$. (d) Correlation ratio R (up to $N = 15$), along the SO(5) line, indicate the phase transition point near $u \simeq 0.1$

Conclusion

- Our study provides a comprehensive phase diagram for the (2+1)D SO(5) NLSM with WZW term on a sphere. It reveals novel quantum states and suggests a SO(5) disordered region separating the O(2) breaking VBS and O(3) breaking Néel phases, which terminates at a multicritical point.
- The unexpected disordered phase unlocks new insights, as it is poised to host highly unconventional quantum states: a gapless critical phase or some "hidden" symmetry-breaking order such as a chiral spin liquid.

References

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