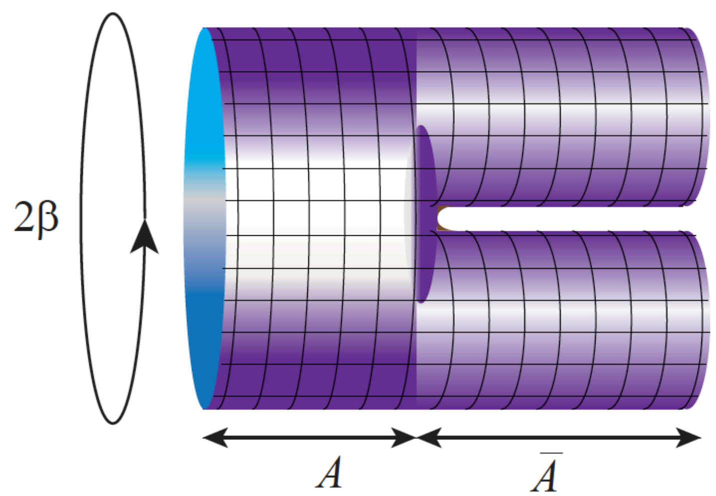
# The learning of entanglement on quantum criticalities





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https://quantummc.xyz/

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Chaoming Jian (Cornell)

Yi-Zhuang You (UCSD)

Cenke Xu (UCSB)

Mark Kac, Polish American mathematician 1914 - 1984

"My presentation will be more in the nature of a leisurely excursion than of an organised tour. It will not be my purpose to reach a specific destination at a scheduled time. Rather I would like to allow myself on many occasions the luxury of stopping and looking around."

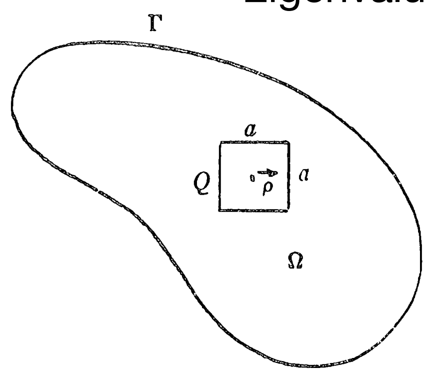
# CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

To George Eugene Uhlenbeck on the occasion of his sixty-fifth birthday


Eigenvalues of Dirichlet problem for Laplacian

Am. Math. Mon. 73, 1 (1966)




$$\tfrac{1}{2}\nabla^2 U + \lambda U = 0 \quad \text{in} \quad \Omega,$$
$$U = 0 \quad \text{on} \quad \Gamma.$$

Volume

Length of circumference

Number of holes

$$\sum_{n=1}^{\infty} e^{-\lambda_n t} \sim \frac{|\Omega|}{2\pi t} - \frac{L}{4}\frac{1}{\sqrt{2\pi t}} + (1-r)\,\frac{1}{6}$$

non-local measurement

**FINITE-SIZE DEPENDENCE OF THE FREE ENERGY IN TWO-DIMENSIONAL CRITICAL SYSTEMS**

Nucl. Phys. B 300, 377 (1988)

John L. CARDY and Ingo PESCHEL*

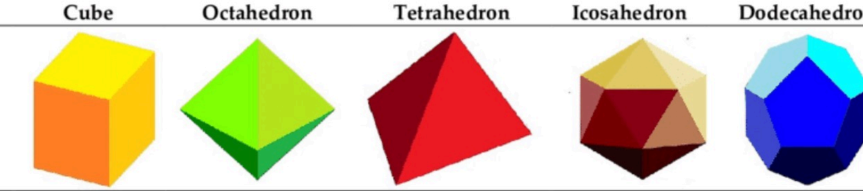
Conformal anomaly number (central charge)

$$F = f_b|A| + f_s L - \frac{1}{6}c\chi \ln L + O(1)$$

Euler characteristic

Platonic solids: homeomorphic to sphere

$$\chi = V - E + F = 2$$

| Name | Cube | Octahedron | Tetrahedron | Icosahedron | Dodecahedron |
|---|---|---|---|---|---|
| Shape |  | | | | |
| Features | 6 faces<br>8 vertices<br>12 edges | 8 faces<br>6 vertices<br>12 edges | 4 faces<br>4 vertices<br>6 edges | 20 faces<br>12 vertices<br>30 edges | 12 faces<br>20 vertices<br>30 edges |
| Facets | Squares | Equilateral triangles | Equilateral triangles | Equilateral triangles | Pentagons |

torus / cylinder / annuls

$\chi = 2 - 2g = 0$

Klein bottle / moebius

$\chi = 0$

Projective plane / disc

$\chi = 1$

sphere / polyhedron

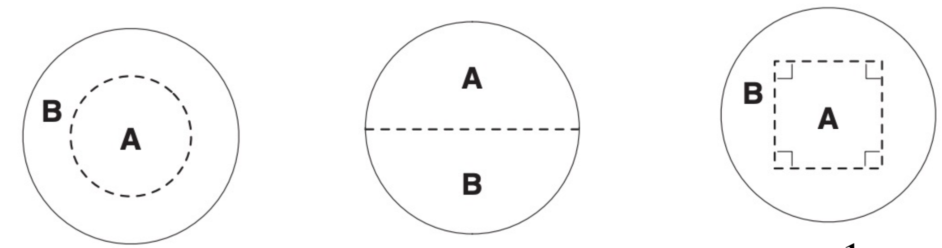
$\chi = 2$

**Entanglement Entropy of 2D Conformal Quantum Critical Points: Hearing the Shape of a Quantum Drum**

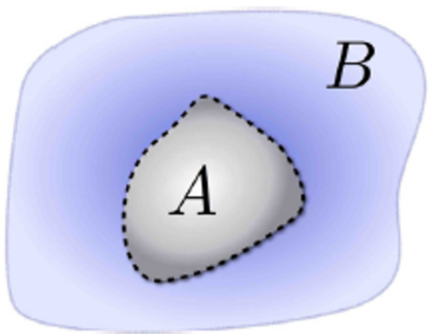
Eduardo Fradkin[1] and Joel E. Moore[2,3]

- Phys. Rev. Lett. 97, 050404 (2006)

$$\Delta F = \frac{c\gamma}{24\pi}(1-(\frac{\pi}{\gamma})^2)\ln L$$




$S_{\ln} = 0$ $\quad$ $S_{\ln} = -\frac{1}{4}c\ln(L)$ $\quad$ $S_{\ln} = -\frac{1}{9}c\ln(L)$




$$S = F_A + F_B - F_{A\cup B}$$

central charge

$$S = 2f_s\, L - \frac{1}{6}c\underbrace{(\chi_A + \chi_B - \chi_{A\cup B})}\ln(L) + O(1)$$

Geometric properties of the partition

$$S_A(l) = al - s\ln l - b$$

| | | | |
|---|---|---|---|
| d=1 CFT | $S \sim c\ln(l)$ | Heisenberg chain, Luttinger liquid | DMRG |
| d=2 QCP | $S \sim al - s(c)\ln(l) - b$ | Wilson-Fisher O(N), SC-Mott, GNY | QMC |
| SSB | $S \sim al - s(n_G)\ln(l)$ | Antiferromagnet, SC, Superfluid | QMC |
| Topological order | $S \sim al - \gamma_{top}$ | Z2 top ord, Kitaev QSL | Toy model, QMC |
| Fermi surface | $S \sim l\ln(l) + al - \cdots$ | free fermion,  interaction ? | not even QMC |

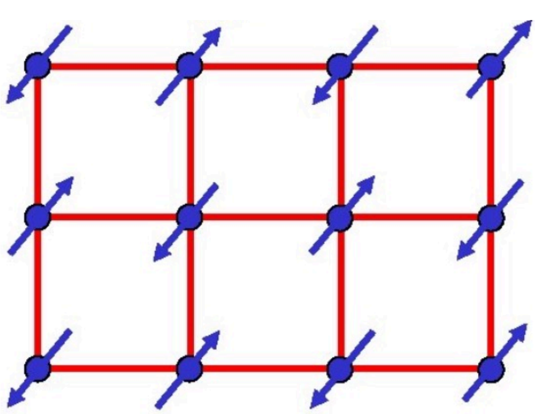
# Entanglement entropy with incremental (Qiu Ku) method

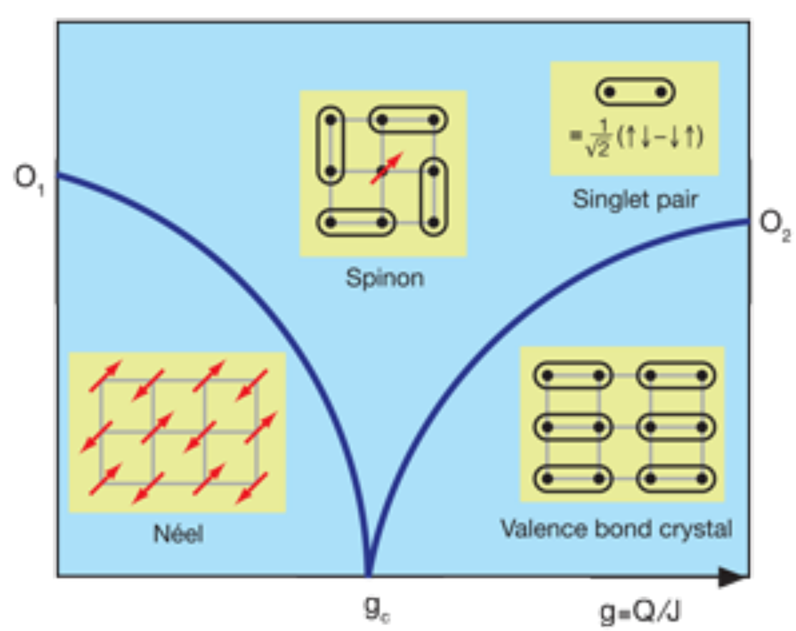
$$S_A^{(2)}(l) = al - s\ln l - b$$

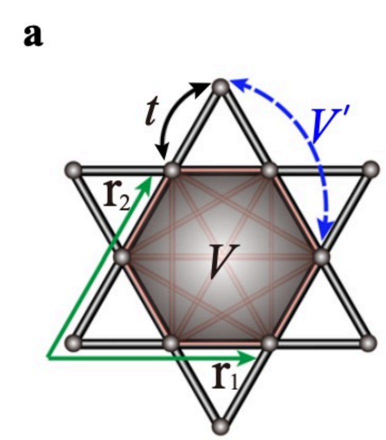
- Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)
- Jiarui Zhao, Bin-Bin Chen, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- Menghan Song, Jiarui Zhao, Lukas Janssen, Michael Scherer, ZYM, arXiv: 2307.02547
- Bin-Bin Chen, Xu Zhang, Yuxuan Wang, Kai Sun, ZYM, arXiv:2307.05307
- Zi Hong Liu, Yuan Da Liao, Gaopei Pan, …, Yi-Zhuang You, F. Assaad, ZYM, Cenke Xu, arXiv:2308.07380

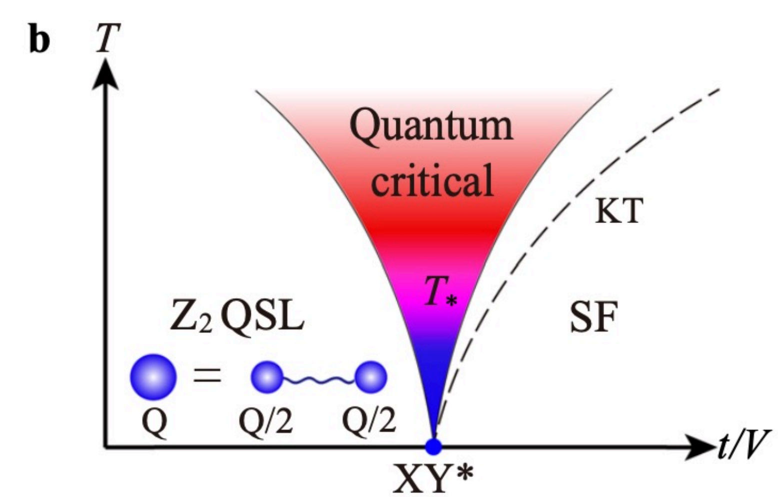
$$-\ln|\langle X_M\rangle| = al - s\ln l - b$$

- Yan-Cheng Wang, Meng Cheng, William Witczak-Krempa, ZYM, Nat. Commun. 12, 5347 (2021)
- Yan-Cheng Wang, Nvsen Ma, Meng Cheng, ZYM, SciPost Phys. 13, 123 (2022)
- Weilun Jiang, Bin-Bin Chen, Zi Hong Liu, Junchen Rong, F. Assaad, Meng Cheng, Kai Sun, ZYM, SciPost Phys. (2023)
- Zi Hong Liu, Weilun Jiang, Bin-Bin Chen, Junchen Rong, Meng Cheng, Kai Sun, ZYM, F. Assaad, PRL 130, 266501 (2023)






(2+1)d SSB, O(3), Topological order Z2 QSL, GNY, FL, nFL, DQCP, SMG, …

# What is Qiu Ku (秋裤)

How can you tell winter is coming?

In Chinese: I need to put my Qiu Ku on.

- long underwear, looks similar to leggings and **Yoga pants**
- normally made of cotton
- most popular colors are grey, blue, white and beige
- nothing to do with fashion or style
- the only reason for its existence is to keep you warm. When jeans can no longer resist the freezing air, just wear Qiu Ku under your jeans. Problem solved!

A pair of (stretchy) pants

long johns

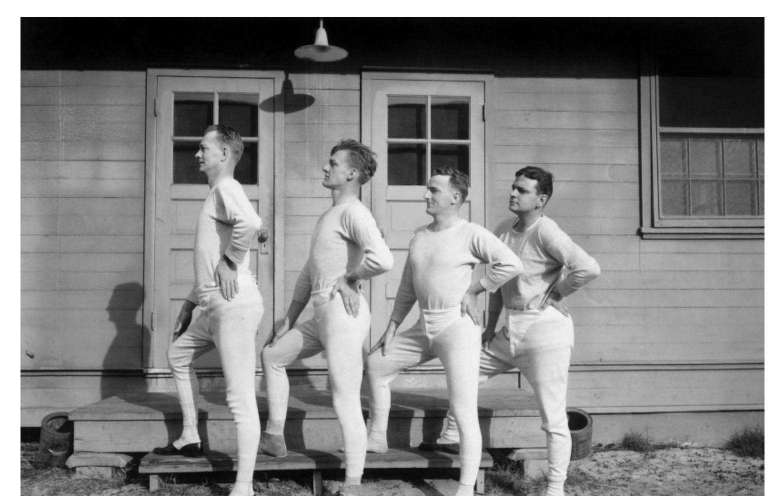


Photo: Hulton Archive/Getty Images

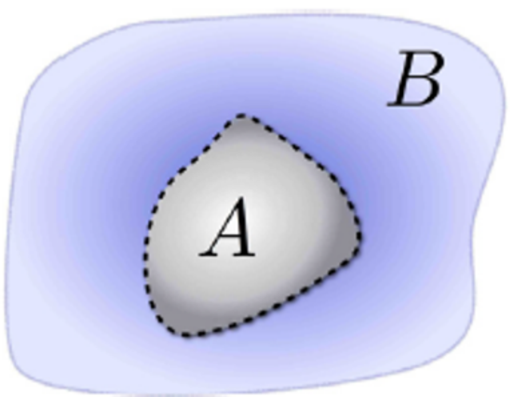
In Victorian time

A **Mother Hubbard dress** is a long, wide, loose-fitting gown with long sleeves and a high neck. It is intended to cover as much skin as possible.

# Entanglement entropy and quantum field theory

J. Stat. Mech. (2004) P06002

Pasquale Calabrese[1,3] and John Cardy[1,2]




$$\rho = |\Psi\rangle\langle\Psi|$$

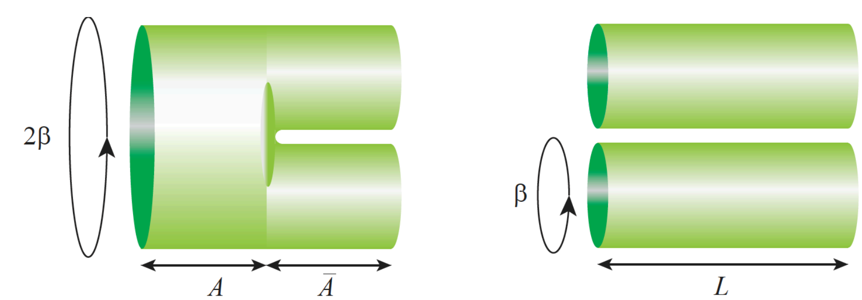
$$\rho_A = \mathbf{Tr}_B\, \rho$$

$$S_A = -\,\mathbf{Tr}_A\, \rho_A \ln(\rho_A)$$

$$S_A^{(n)} = \frac{1}{1-n} \ln(\mathbf{Tr}_A(\rho_A^{(n)}))$$

" discuss entropy in terms of the **Euclidean path integral** on an n-sheeted Riemann surface. "

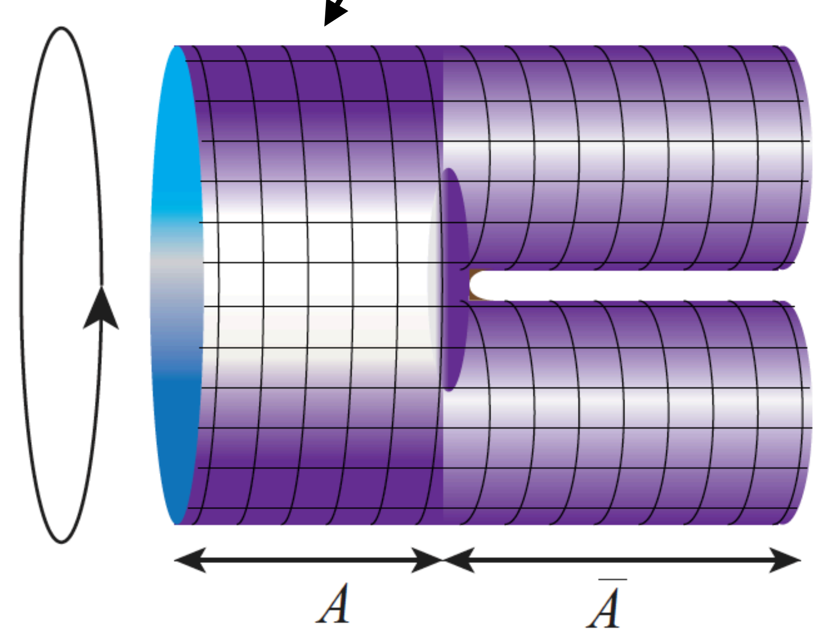
$$S_A^{(2)} = -\ln(\mathbf{Tr}_A(\rho_A^{(2)})) = -\ln(\frac{Z_A^{(2)}}{Z_\varnothing^2}) = \beta(F(Z_A^{(2)}) - F(Z_\varnothing^{(2)}))$$




" Qiu Ku is the $Z_A^{(2)}$ "

" Renyi **EE** is **the difference in free energy** between partition functions with different trace topologies " (in equilibrium)

(a)




(b)

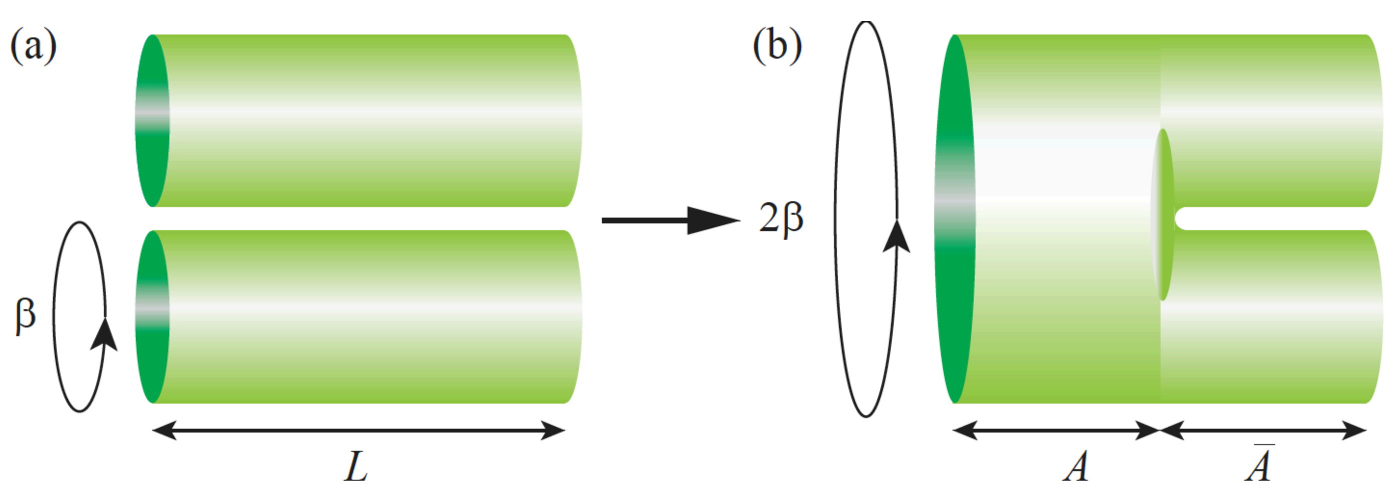
# Incremental (Qiu Ku) method

- Vincenzo Alba, PRE 95, 062132 (2017)
- Jonathan D'Emidio, PRL 124, 110602 (2020)

$$Z_A^{(2)}(\lambda) = \sum_{B\subseteq A} \lambda^{N_B}(1-\lambda)^{N_A-N_B} Z_B^{(2)}$$

$$S_A^{(2)} = -\ln\left(\frac{Z_A^{(2)}}{Z_\varnothing^{(2)}}\right) = -\int_0^1 d\lambda \frac{\partial \ln Z_A^{(n)}(\lambda)}{\partial\lambda} = = -\ln\left(\langle e^{-\beta W_A^{(2)}}\rangle\right)$$

$$Z^{(2)}(\lambda=0) = Z_\varnothing^{(2)} \qquad Z_A^{(2)}(\lambda=1) = Z_A^{(2)}$$




$$\frac{\partial \ln Z_A^{(2)}(\lambda)}{\partial\lambda} = \left\langle \frac{N_B}{\lambda} - \frac{N_A-N_B}{1-\lambda}\right\rangle_\lambda$$

$$\lambda(t_f) = 1$$

$$W_A^{(2)} = -\frac{1}{\beta}\int_{t_i}^{t_f} dt \frac{d\lambda}{dt}\left\langle \frac{N_B}{\lambda(t)} - \frac{N_A-N_B}{1-\lambda(t)}\right\rangle_{\lambda(t)}$$

$$\lambda(t_i) = 0$$

**Nonequilibrium Equality for Free Energy Differences**

C. Jarzynski*

- Phys. Rev. Lett. 78, 2690 (1997)

$$\langle W\rangle \geq \Delta F = F_B - F_A \qquad \exp(-\beta\Delta F) \equiv \langle \exp(-\beta W)\rangle = \langle \exp[-\beta\int_{t_i}^{t_f} dt\delta W(t)]\rangle = \frac{Z_f}{Z_i} \qquad S = \beta\Delta F = -\ln\left(\frac{Z_f}{Z_i}\right) = -\ln(\langle e^{-\beta W}\rangle)$$

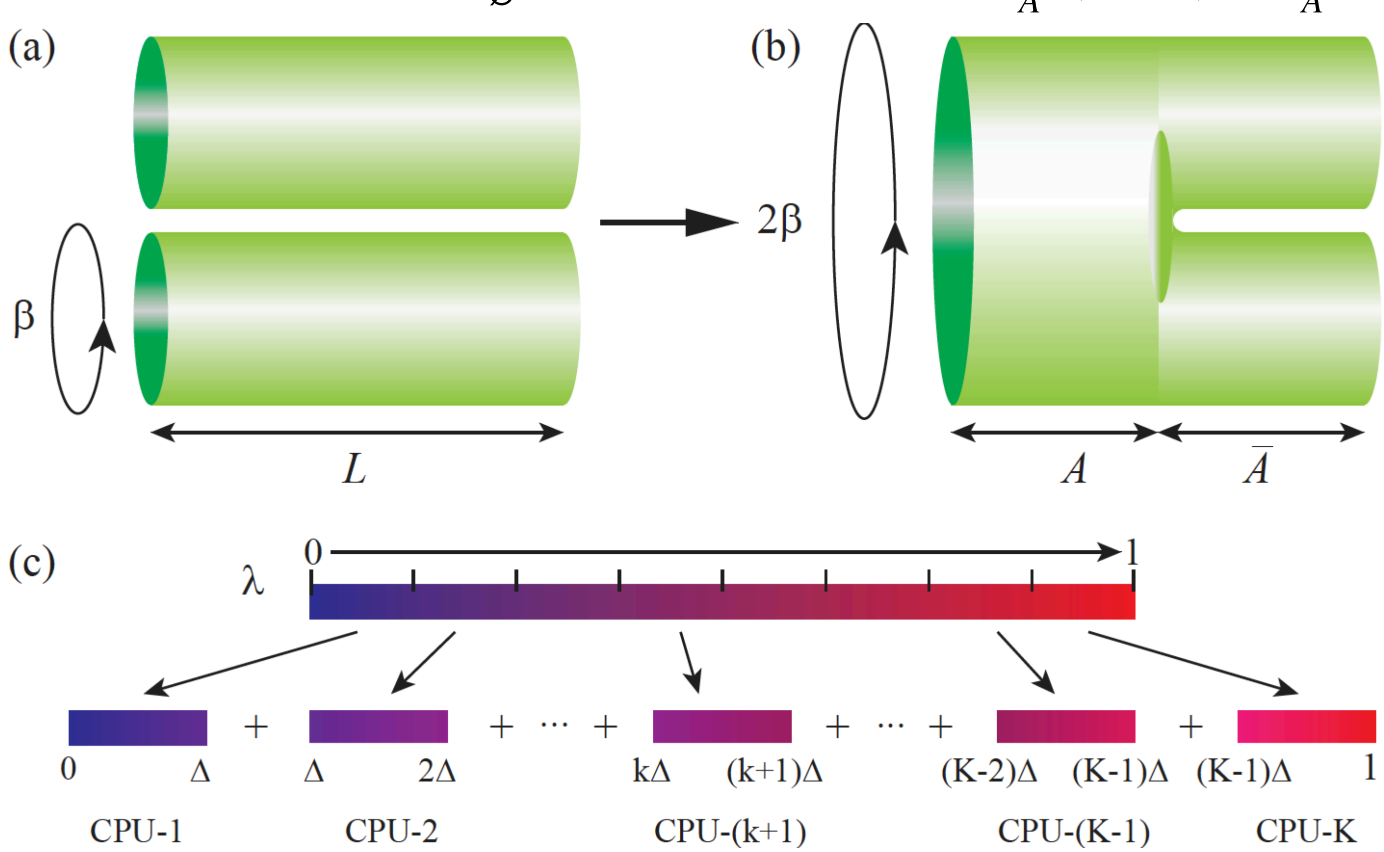
# Incremental (Qiu Ku) method

- Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)
- Jiarui Zhao, Bin-Bin Chen, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- Gaopei Pan, Yuan Da Liao, Weilun Jiang, Jonathan D'Emidio, ZYM, arXiv: 2303.14326

$$Z_A^{(2)}(\lambda) = \sum_{B\subseteq A} \lambda^{N_B}(1-\lambda)^{N_A-N_B} Z_B^{(2)}$$

$$S_A^{(2)} = -\ln\left(\frac{Z_A^{(2)}}{Z_\varnothing^{(2)}}\right) = -\int_0^1 d\lambda \frac{\partial \ln Z_A^{(n)}(\lambda)}{\partial\lambda} = -\sum_{k=1,2,\cdots,N_\lambda} \int_{(k-1)\Delta}^{k\Delta} d\lambda \frac{\partial \ln Z_A^{(2)}(\lambda)}{\partial \lambda}$$

$$Z^{(2)}(\lambda=0) = Z_\varnothing^{(2)} \qquad Z_A^{(2)}(\lambda=1) = Z_A^{(2)}$$




$$\frac{\partial \ln Z_A^{(2)}(\lambda)}{\partial\lambda} = \left\langle \frac{N_B}{\lambda} - \frac{N_A-N_B}{1-\lambda}\right\rangle_\lambda$$

$$= -\ln\left(\langle e^{-\beta W_A^{(2)}}\rangle\right) = -\sum_{k=1,2,\cdots,N_\lambda} \ln\left(\langle e^{-\beta W_{k,A}^{(2)}}\rangle\right)$$

$$\lambda(t_f) = k\Delta$$

$$W_{k,A}^{(2)} = -\frac{1}{\beta}\int_{t_i}^{t_f} dt \frac{d\lambda}{dt}\left\langle \frac{N_B}{\lambda(t)} - \frac{N_A-N_B}{1-\lambda(t)}\right\rangle_{\lambda(t)}$$

$$\lambda(t_i) = (k-1)\Delta$$

$$e^{-S_A^{(2)}} = \frac{Z(1)}{Z(0)} := \frac{Z(\lambda_1)}{Z(0)}\frac{Z(\lambda_2)}{Z(\lambda_1)}\cdots\frac{Z(\lambda_k)}{Z(\lambda_{k-1})}\cdots\frac{Z(1)}{Z(\lambda_{N_\lambda-1})}$$

Parallization does the job.
The Qiu Ku algorithm is
nostalgic, sentimental and useful

arXiv > cond-mat > arXiv:2307.10602

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Help | Advance

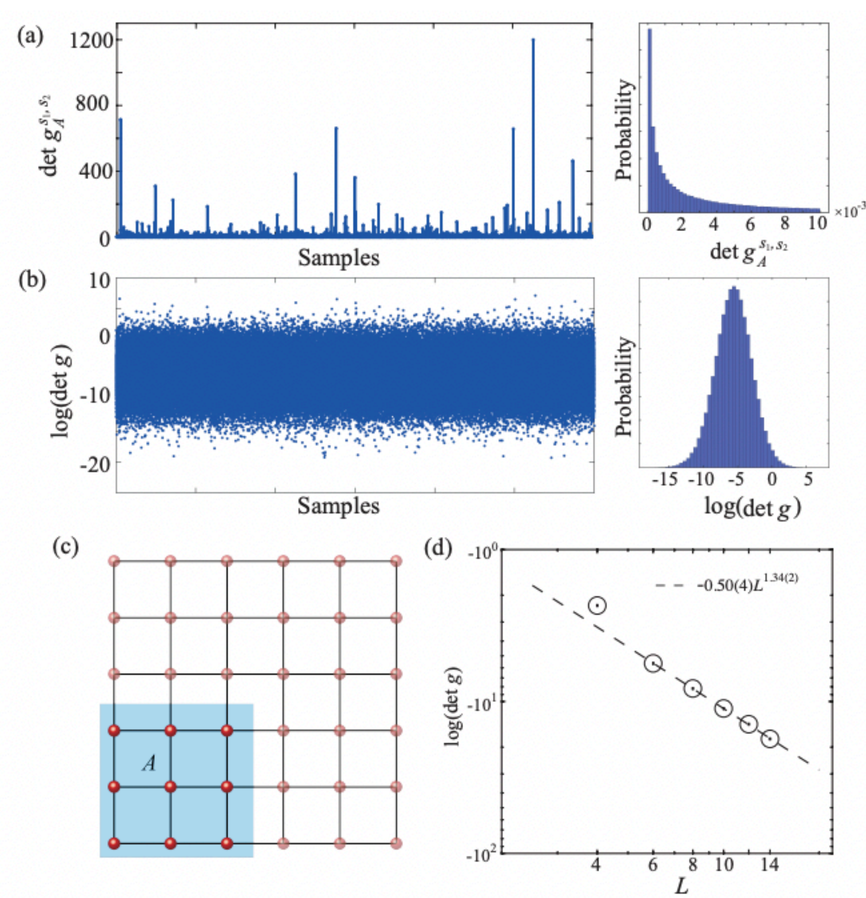
Condensed Matter > Strongly Correlated Electrons

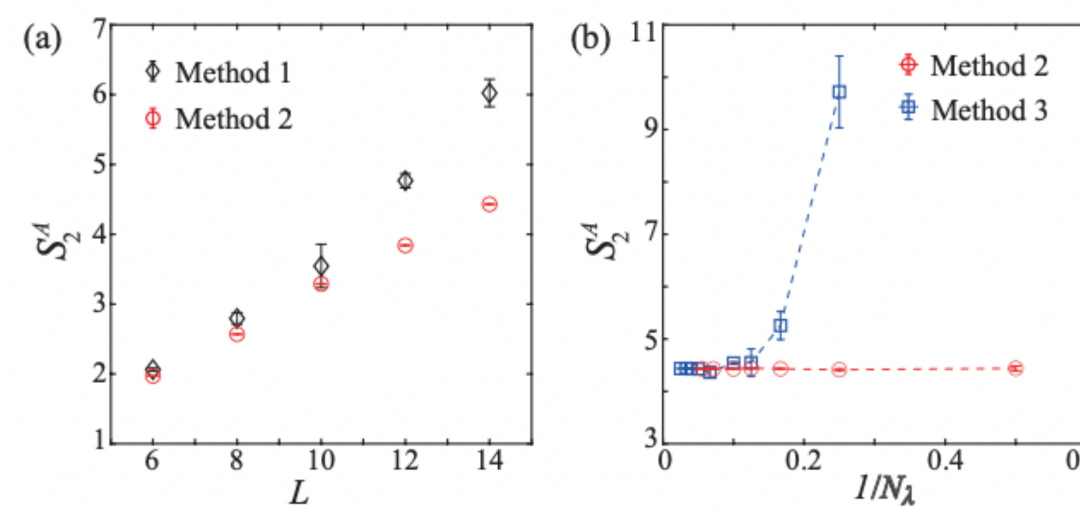
[Submitted on 20 Jul 2023]

# Controllable Incremental Algorithm for Entanglement Entropy and Other Observables with Exponential Variance Explosion in Many-Body Systems

Yuan Da Liao

Researchers in the field of physical science are continuously searching for universal features in strongly interacting many-body systems. However, these features can often be concealed within highly complex observables, such as entanglement entropy (EE). The non-local nature of these observables makes them challenging to measure experimentally or evaluate numerically. Therefore, it is of utmost importance to develop a reliable and convenient algorithm that can accurately obtain these complex observables. In this paper, with help of quantum Monte Carlo (QMC), we reveal that the



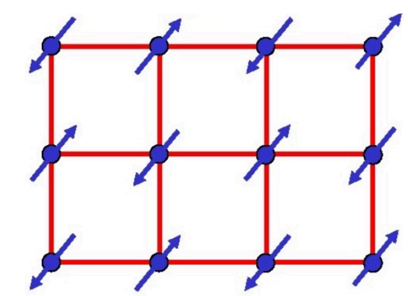



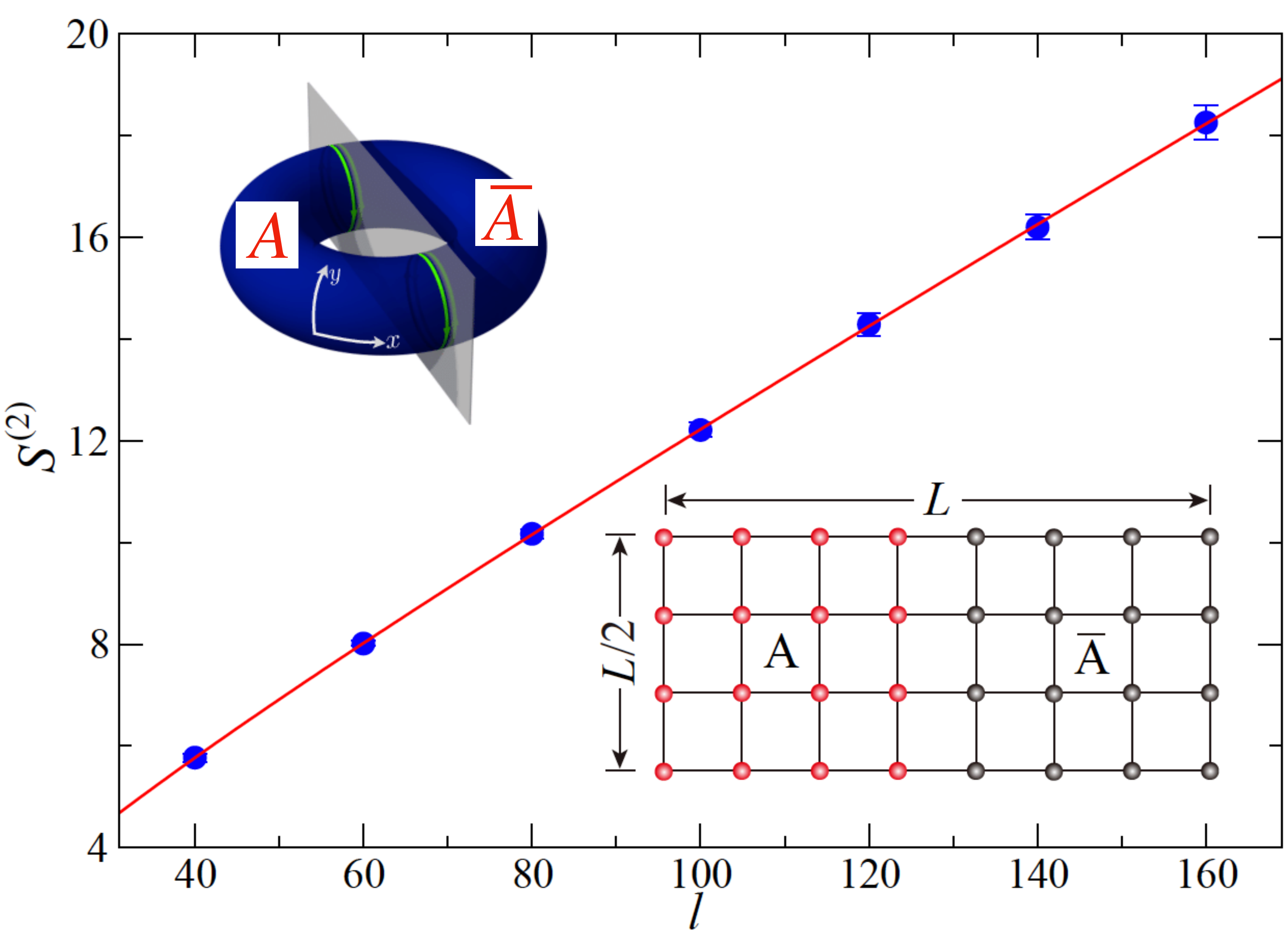
# Entanglement entropy with Qiu Ku method

## Symmetry breaking phase

- Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)
- Jiarui Zhao, Bin-Bin Chen, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, npj Quantum Materials 7, 69 (2022)

Square lattice Heisenberg model $H = J \sum_{\langle i,j \rangle} \mathrm{S}_i \cdot \mathrm{S}_j$






$$S_A^{(2)}(L) = 0.092(1)L + 1.00(9)\ln(L) - 1.63(3)$$

$N_G$ (# **Goldstone modes** ) / 2

$L \in [40,\ 160]$

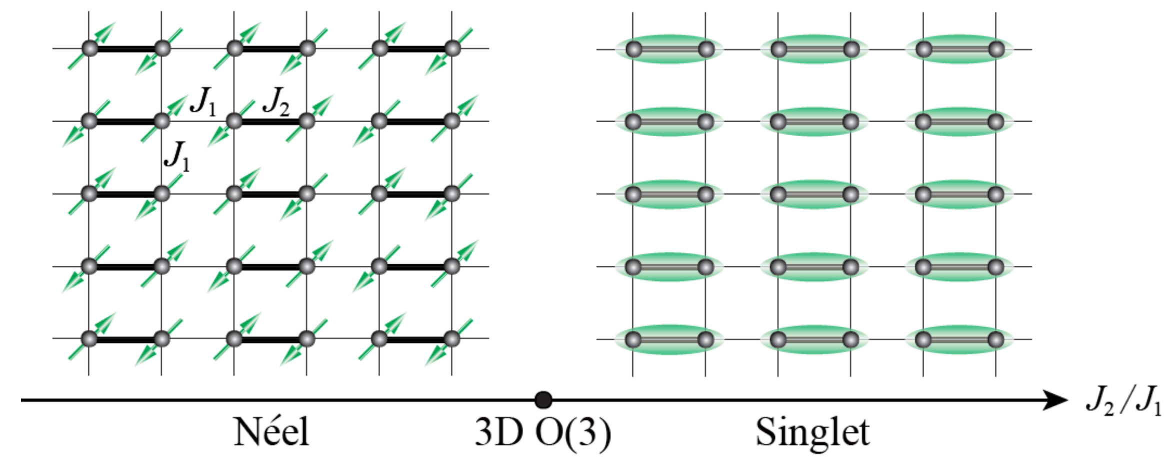
- Metlitski & Grover, arXiv:1112.5166

# Entanglement entropy with Qiu Ku method

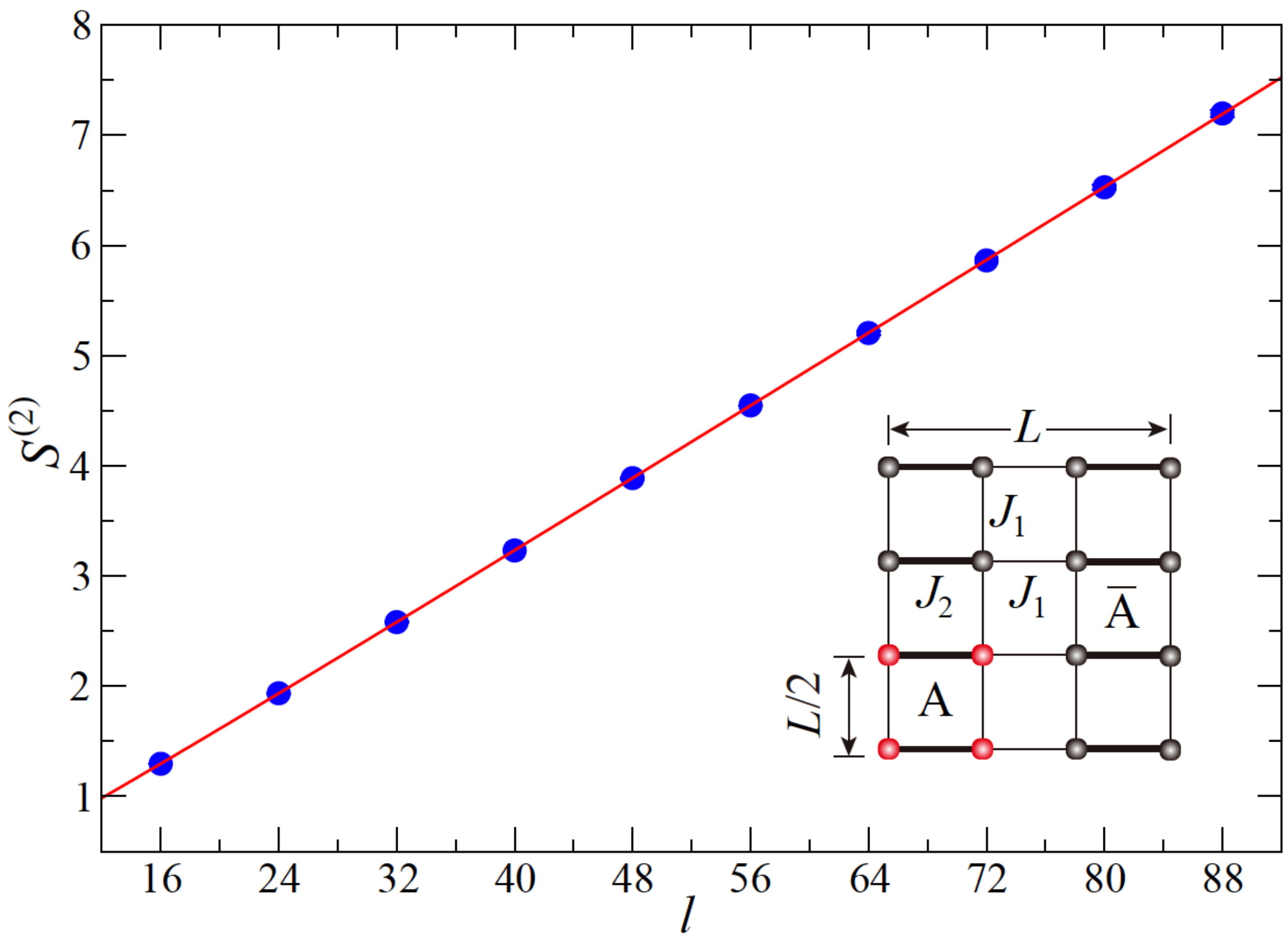
## Quantum critical points

- Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)
- Jiarui Zhao, Bin-Bin Chen, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, npj Quantum Materials 7, 69 (2022)

Columnar dimer model $$H = J_1 \sum_{\langle i,j \rangle} \mathrm{S}_i \cdot \mathrm{S}_j + J_2 \sum_{\langle i,j \rangle'} \mathrm{S}_i \cdot \mathrm{S}_j$$




$$(J_2/J_1)_c = 1.90951(1)$$




$$S_A^{(2)}(L) = 0.167(2)L - 0.081(4)\ln(L) - 0.124(7)$$

s > 0, consistent with previous works on O(n) models

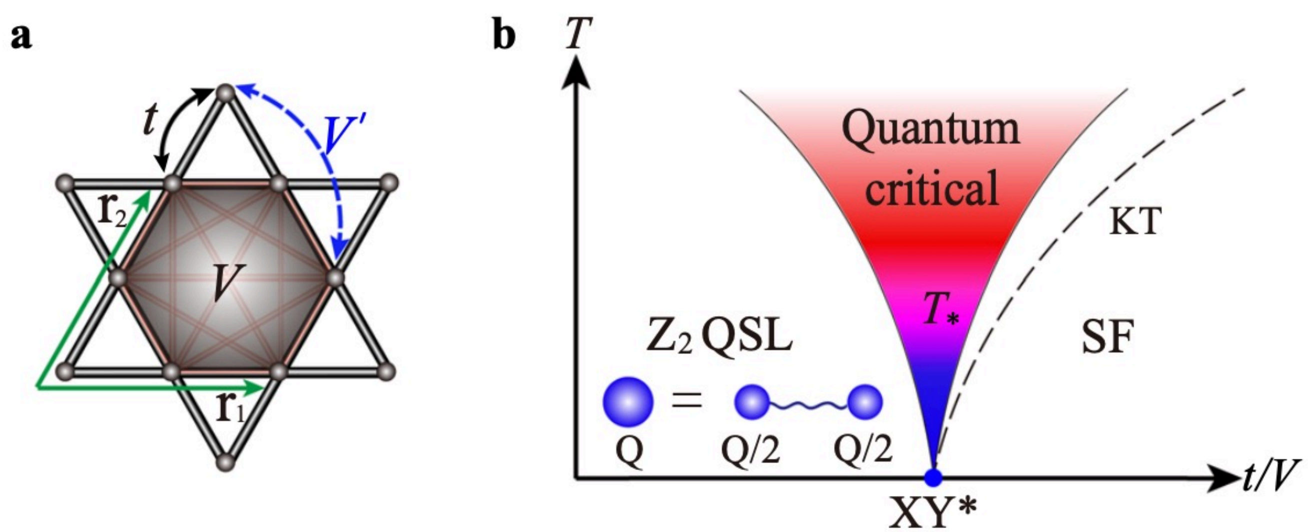
$$s = 0.077(4)$$

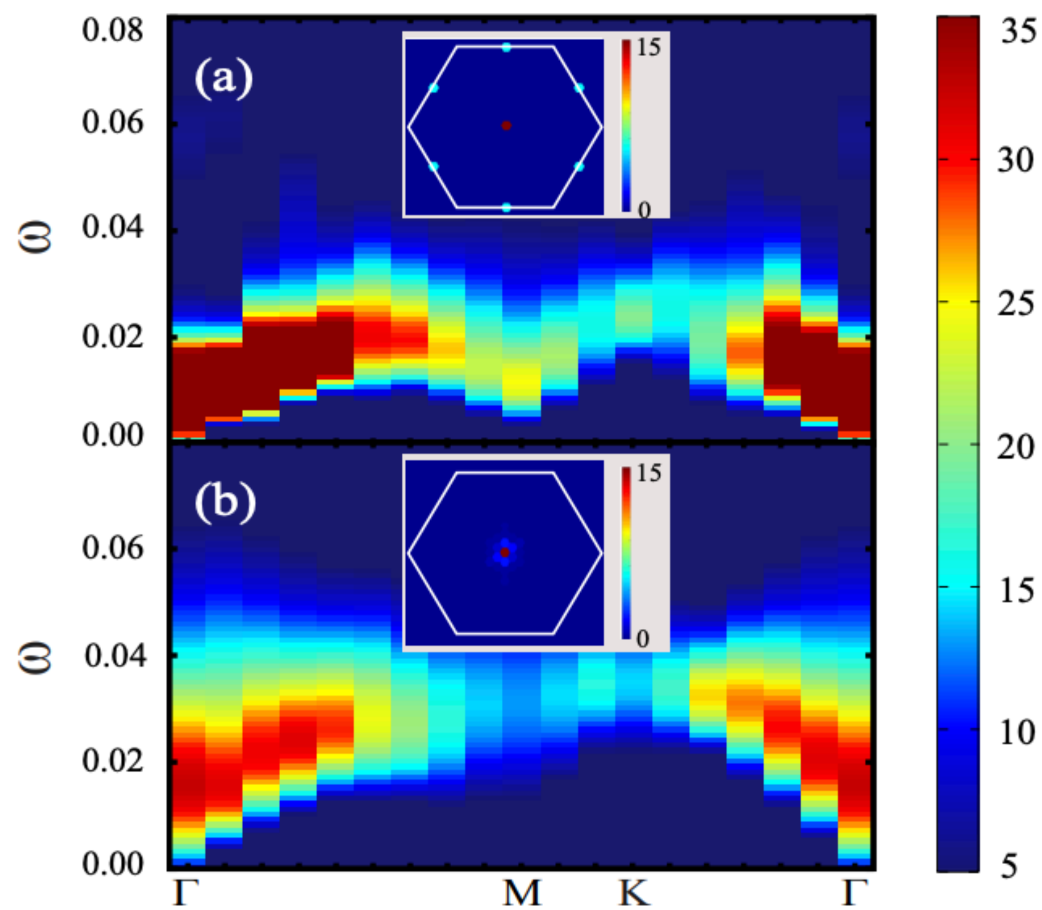
- A. Kallin, et. al, J. Stat. Mech. 2014, P06009
- J. Helmes, S. Wessel, Phys. Rev. B 89, 245120 (2014)
- J. Helmes, W. W-Krempa, R. Melko, Phys. Rev. B 94 125142 (2016)

# Entanglement entropy with Qiu Ku method

**Topological order** Yan-Cheng Wang, Meng Cheng, William Witczak-Krempa, ZYM, Nat Commun 12, 5347 (2021)

Kagome lattice frustrated spin model

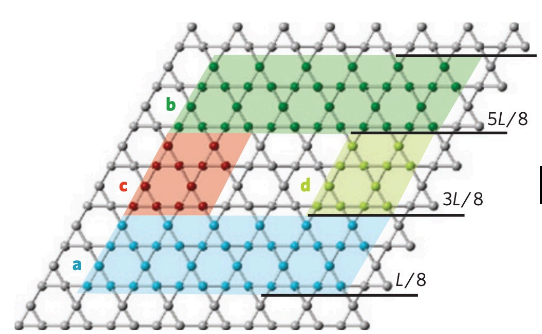



$$H = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + \mathrm{h.c.}\right) - \mu\sum_i n_i$$
$$+ V\left(\sum_{\langle ij\rangle} n_i n_j + \sum_{\langle\langle ij\rangle\rangle} n_i n_j + \sum_{\langle\langle\langle ij\rangle\rangle\rangle} n_i n_j\right)$$

Spinon and vison
Conductivity fractionalisation
Translational symmetry fractionalisation
……

- S. Isakov, Y.B. Kim, A. Paramekanti, PRL 97, 207204 (2006)
- Y.-C. Wang, et al., PRL 121, 057202 (2018)
- G.-Y. Sun, et al., PRL 121, 077201 (2018)
- J. Becker, S. Wessel, PRL 121, 077202 (2018)
- ……



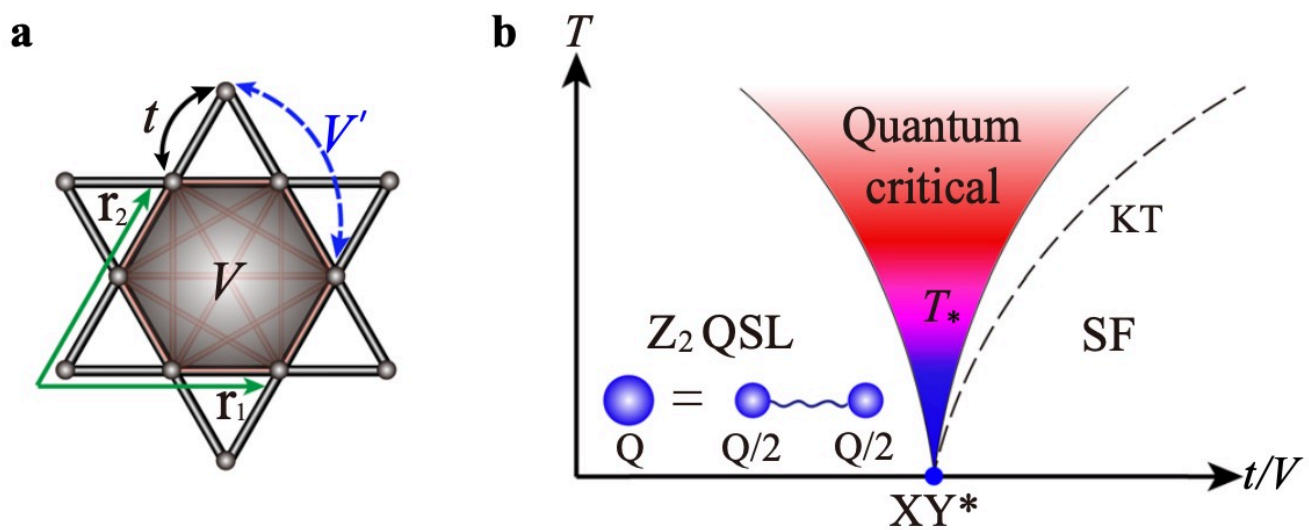

$$S(l) = al - \underline{\gamma}$$

logical entanglement entropy (TEE)

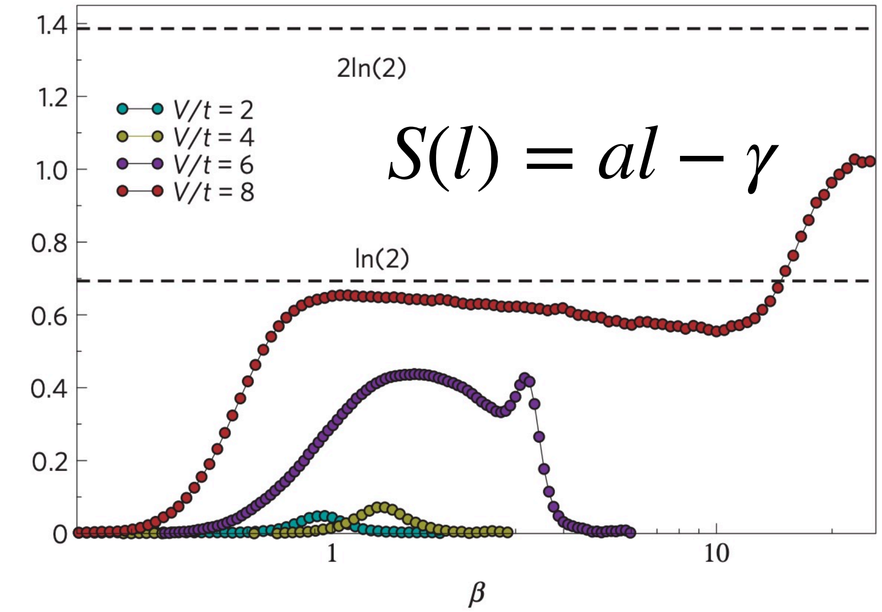
$$\gamma = \ln(\mathscr{D}) = \ln\left(\sqrt{\sum_{a\in\mathscr{C}} d_a^2}\right)$$

# Entanglement entropy with Qiu Ku method

**Topological order** Kagome lattice frustrated spin model



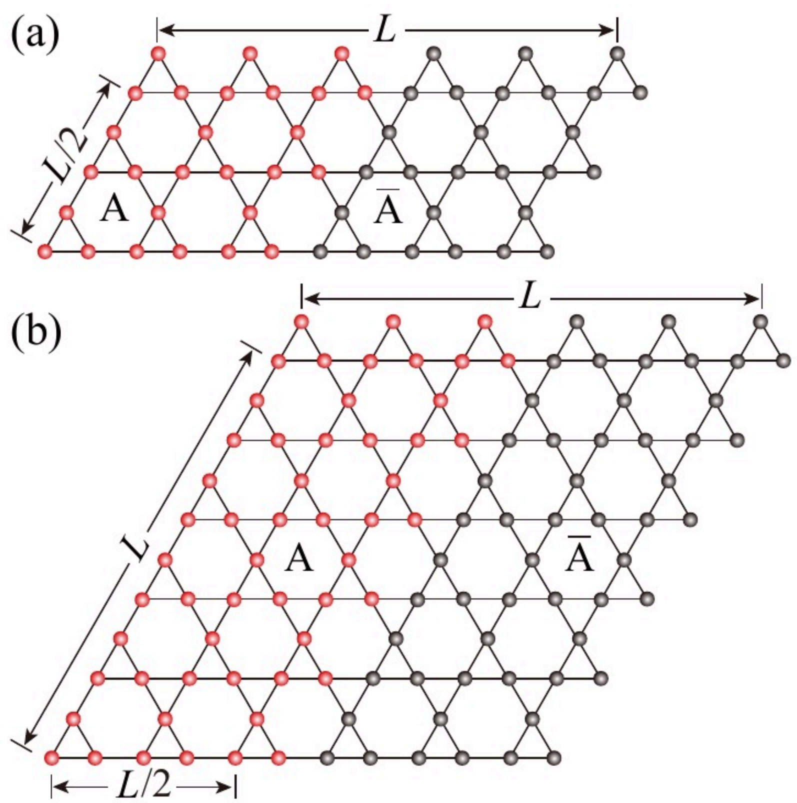

$L = 8$

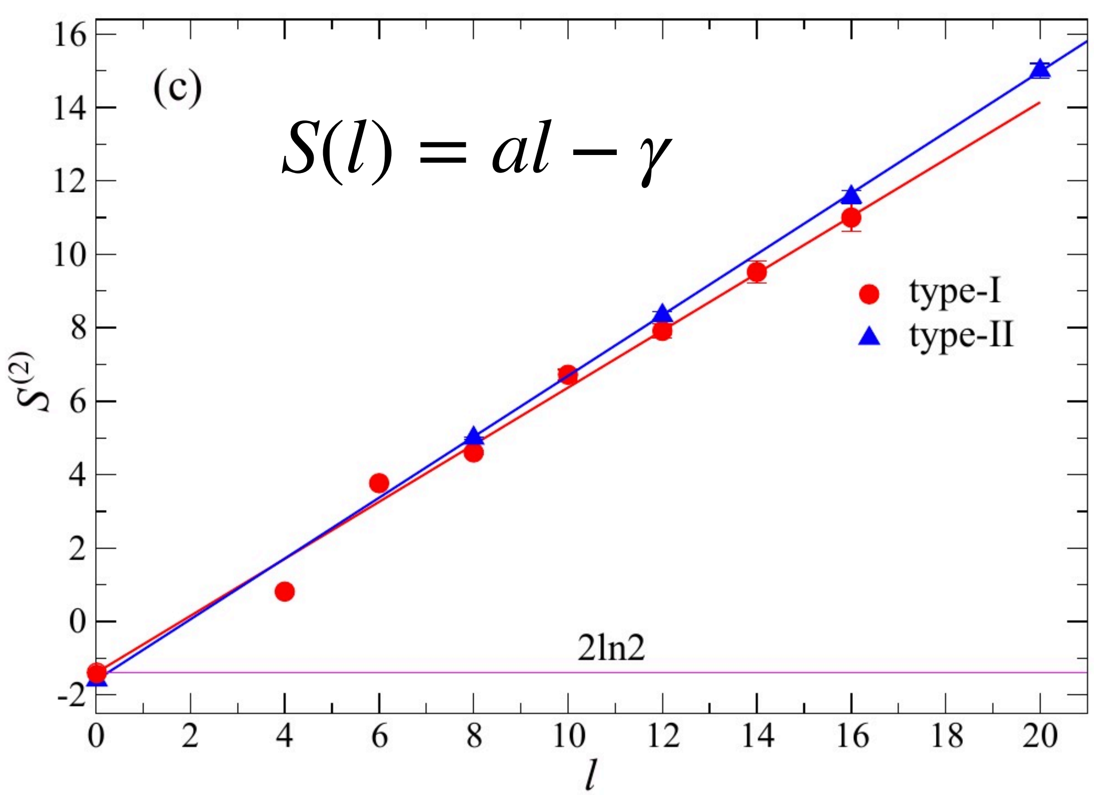



S. Isakov, M. Hastings, R. Melko, Nature Phys 7, 772 (2011)

$L = 12,14,16$

$L = 6,8,10$




Jiarui Zhao, Bin-Bin Chen, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, npj Quantum Materials 7, 69 (2022)

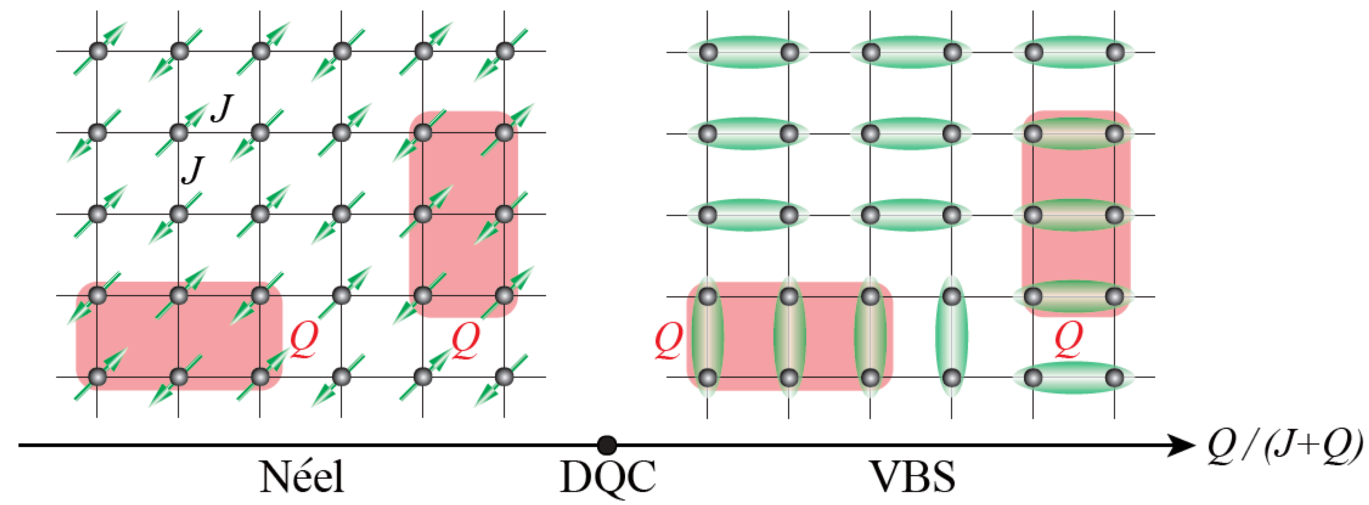
# Entanglement entropy with Qiu Ku method

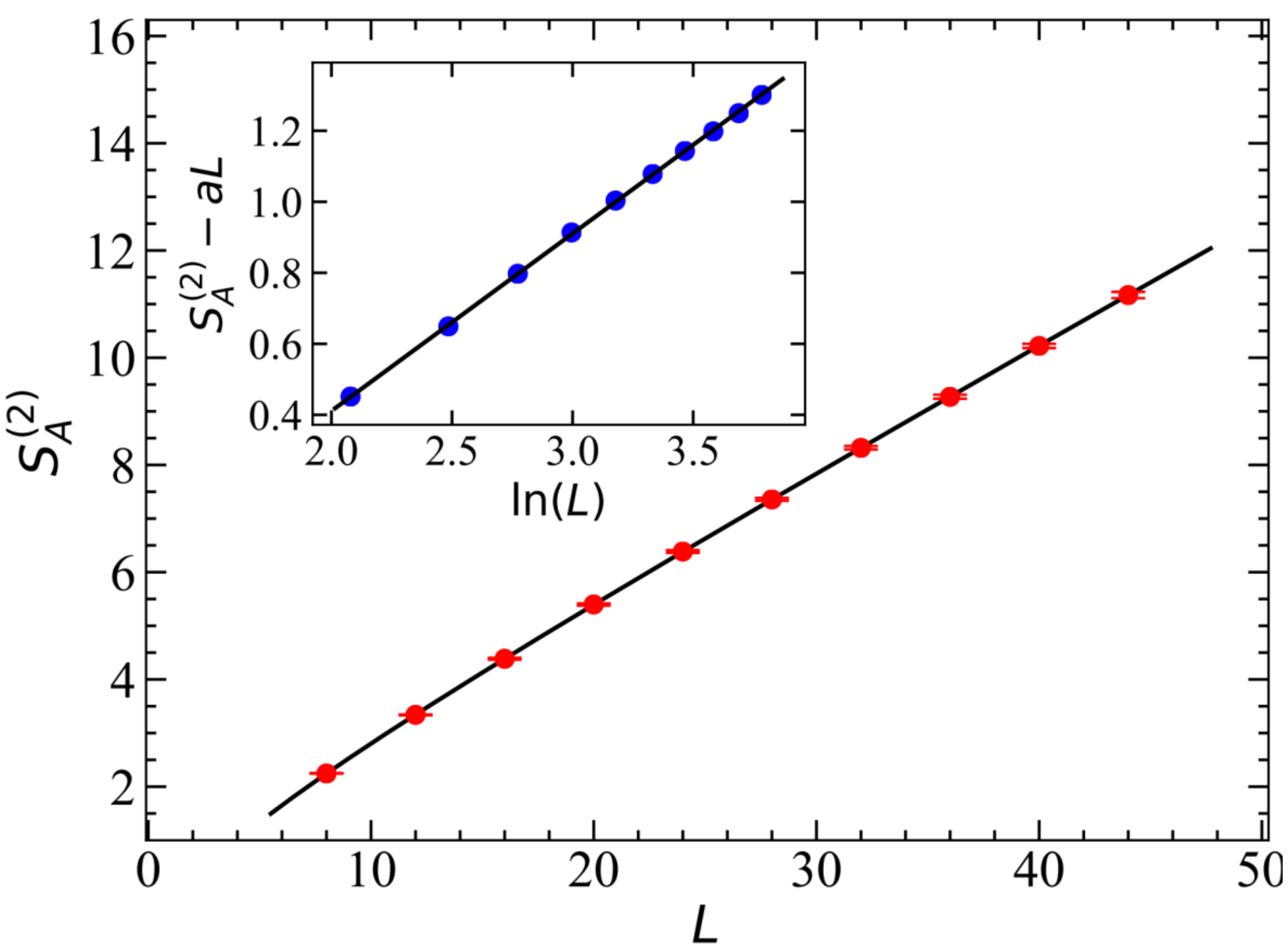
## DQCP

Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)

JQ model $H = -J\sum_{\langle i,j\rangle} P_{i,j} - Q \sum_{\langle ijklmn\rangle} P_{ij}P_{kl}P_{mn}$

$$P_{ij} = \frac{1}{4} - \mathrm{S}_i \cdot \mathrm{S}_j$$




Deconfined QCP: $(Q/(J+Q))_c = 0.59864(4)$

$$S_A^{(2)}(L) = 0.224(1)L - (-0.49(1))\ln(L) - 0.58(2)$$

$s < 0$ not a unitary CFT

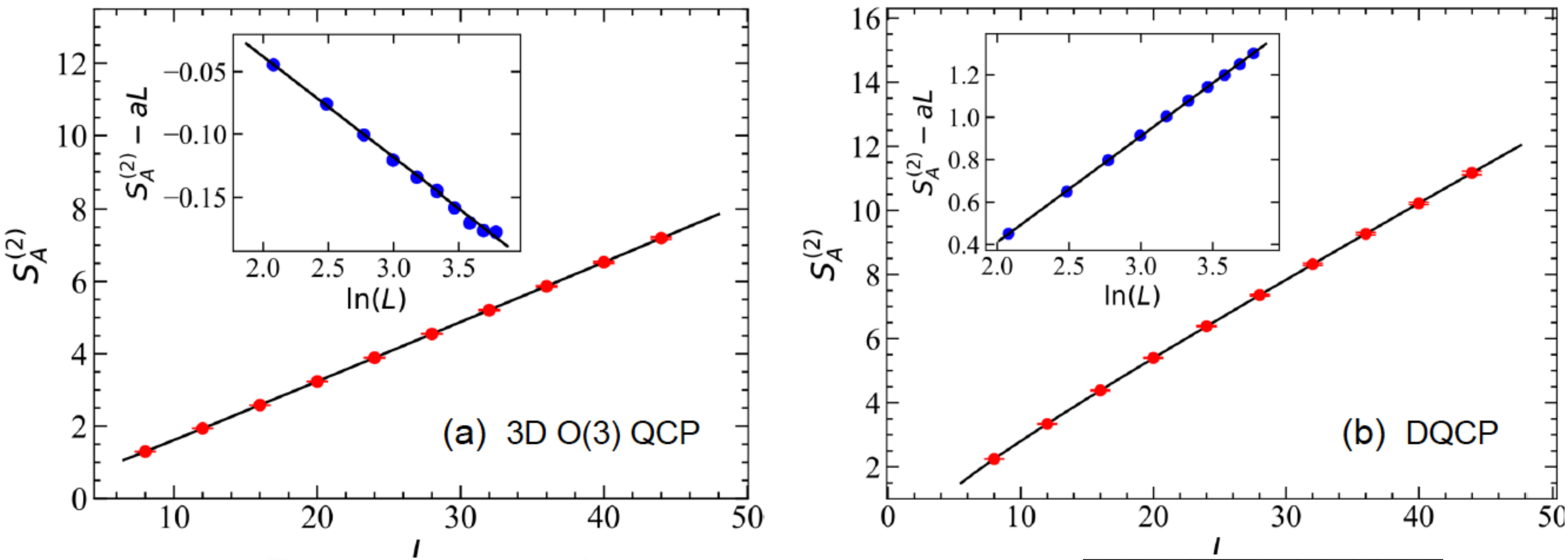
Corner corrections for Renyi EE must be positive for unitary CFTs

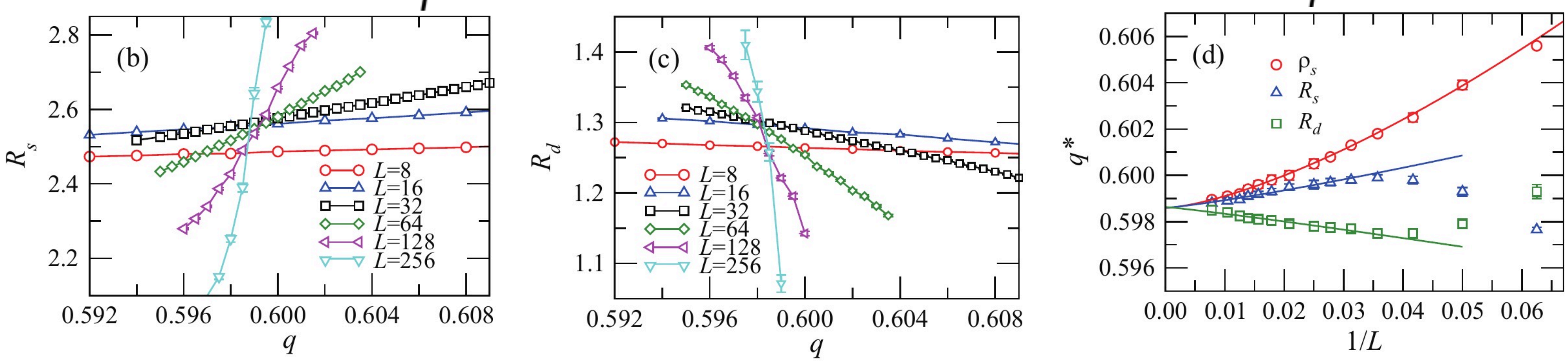
H. Casini, M. Huerta, Journal of High Energy Physics 2012, 87 (2012)

# Entanglement entropy with Qiu Ku method

Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)

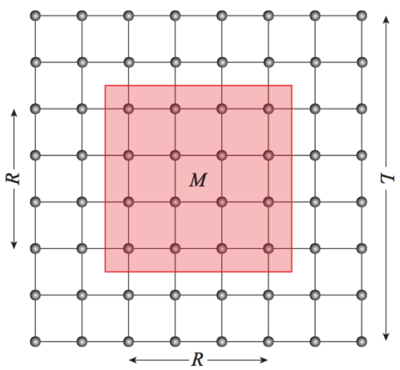
$$S_A^{(2)}(l) = al - s \ln l - b$$



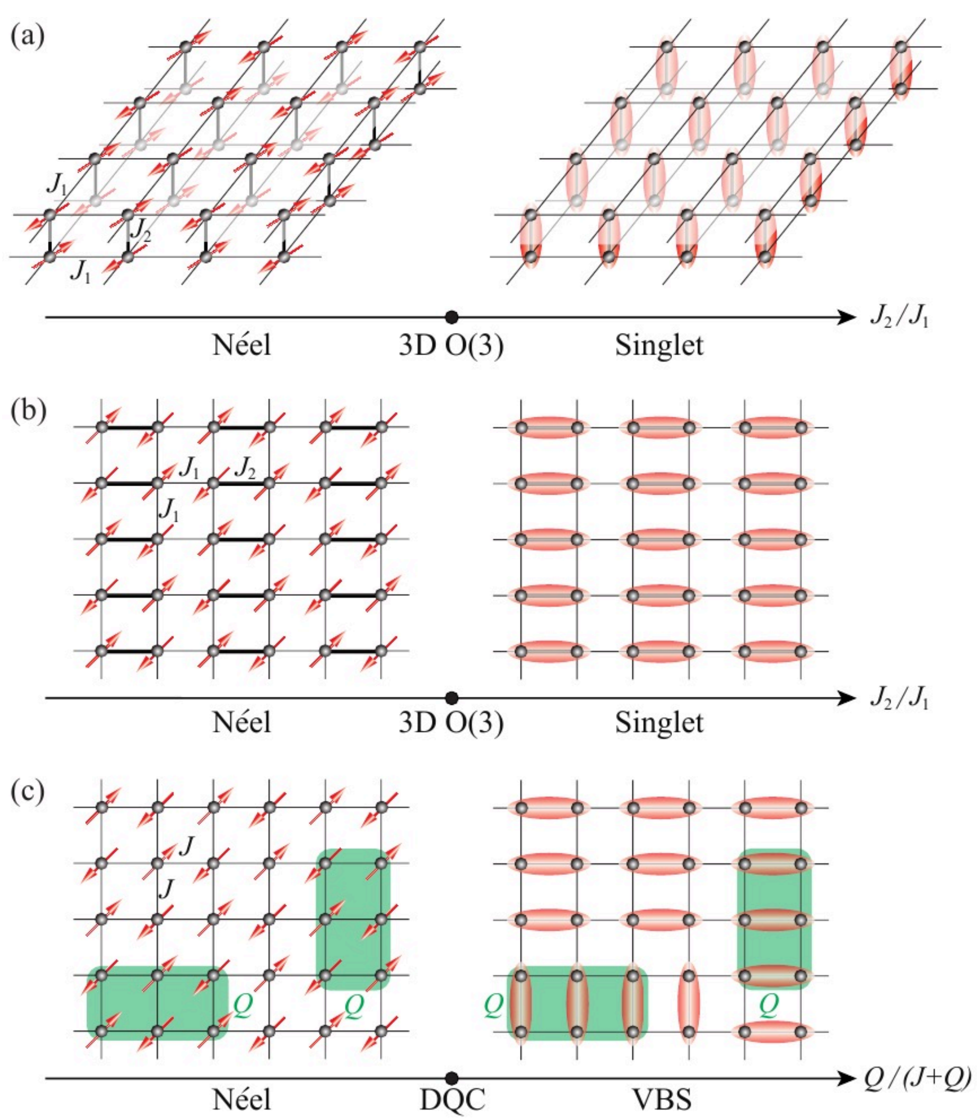
# DQCP Disorder Operator

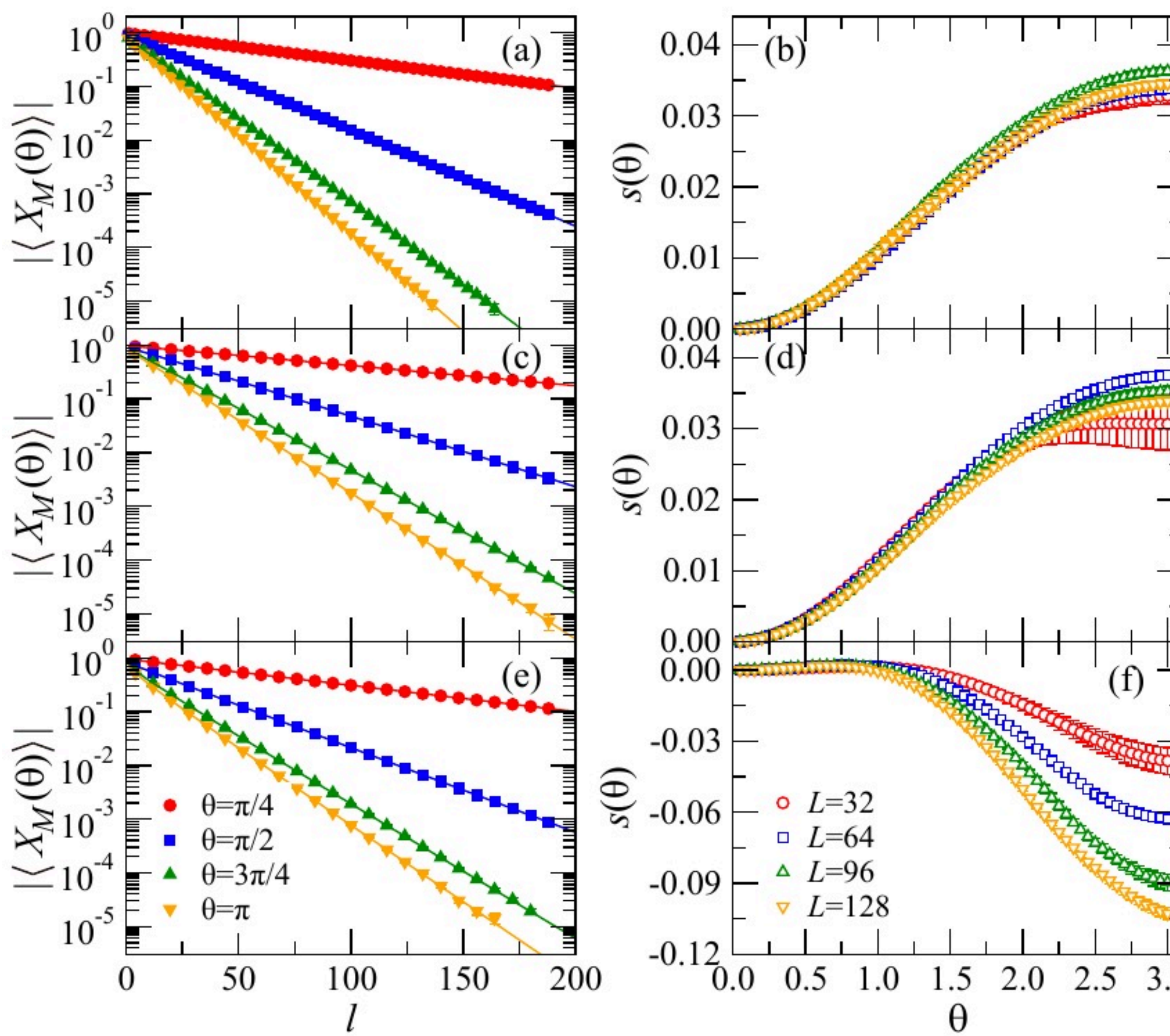
Yan-Cheng Wang, Nvsen Ma, Meng Cheng and ZYM, SciPost Phys. 13, 123 (2022)




$$X_M(\theta) = e^{i\theta \sum_{\mathbf{r}} n_{\mathbf{r}}}$$

$$-\ln |\langle X_M(\theta) \rangle| = al - s(\theta) \ln l - b$$



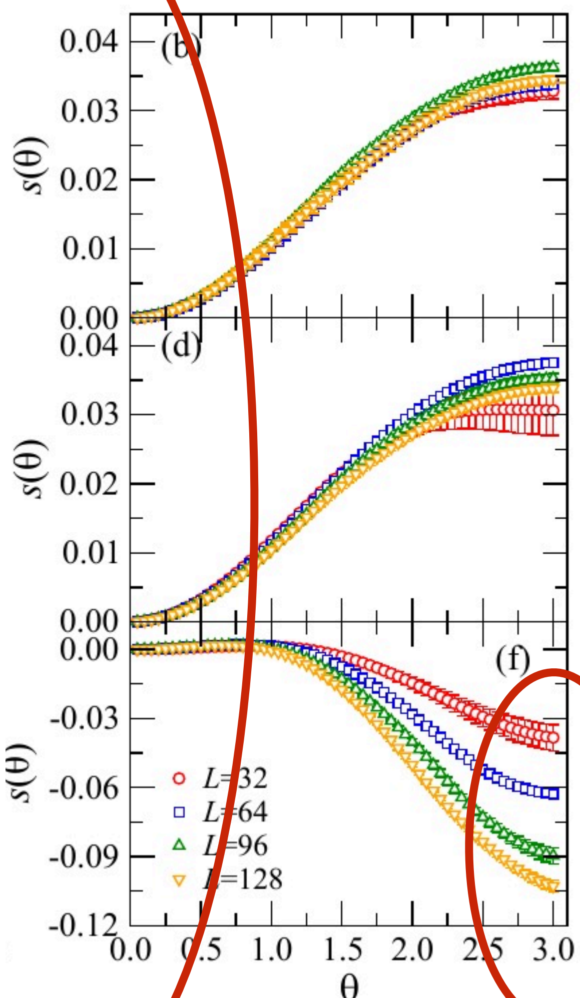



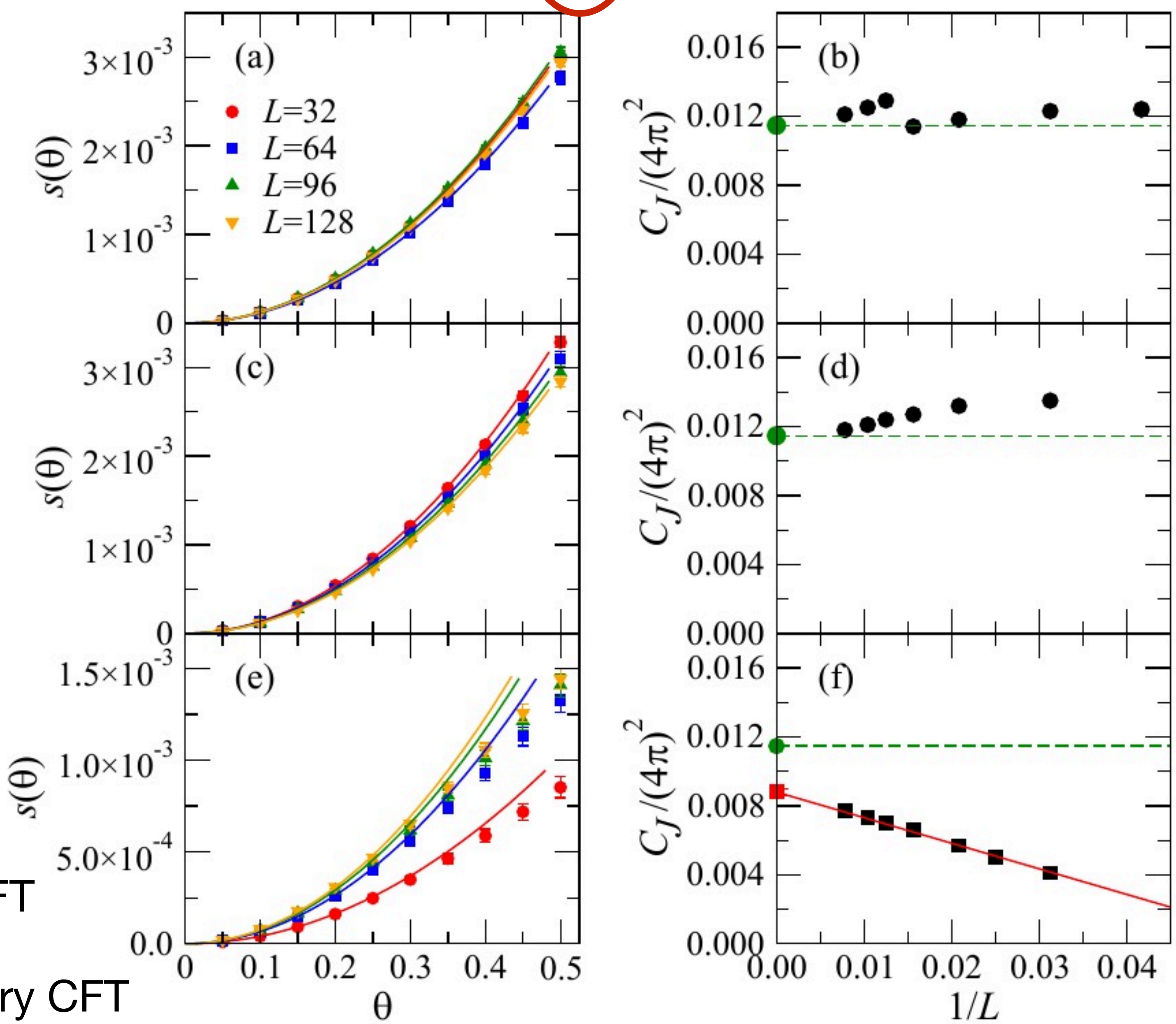
# DQCP Disorder Operator

Yan-Cheng Wang, Nvsen Ma, Meng Cheng and ZYM, SciPost Phys. 13, 123 (2022)

$$s(\theta) \approx \frac{C_J}{(4\pi)^2}\theta^2, \quad \theta \to 0$$

0.01147 O(3) CFT Bootstrap




s < 0, not a unitary CFT

Remember in EE also s < 0, not a unitary CFT

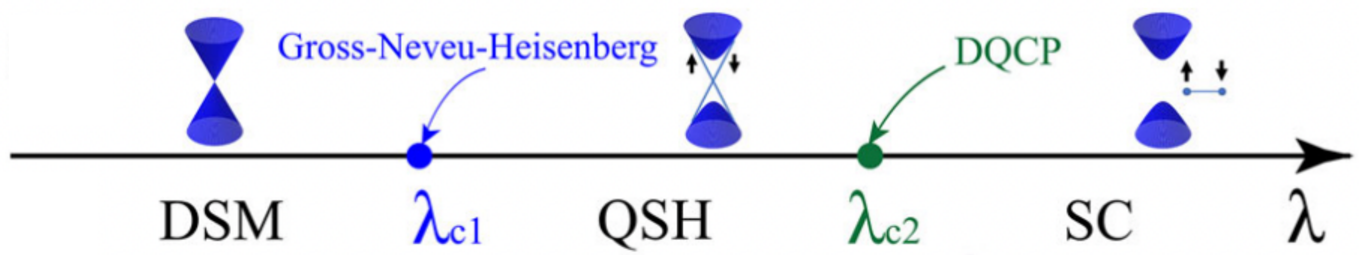
Corner corrections for Renyi EE must be positive for unitary CFTs

H. Casini, M. Huerta, Journal of High Energy Physics 2012, 87 (2012)

# Fermion Disorder Operator at Gross-Neveu and Deconfined Quantum Criticalities

Zi Hong Liu,[1] Weilun Jiang,[2,3] Bin-Bin Chen,[4] Junchen Rong,[5] Meng Cheng,[6,*] Kai Sun,[7,†]
Zi Yang Meng,[4,‡] and Fakher F. Assaad[1,§]

PRL 130, 266501 (2023)



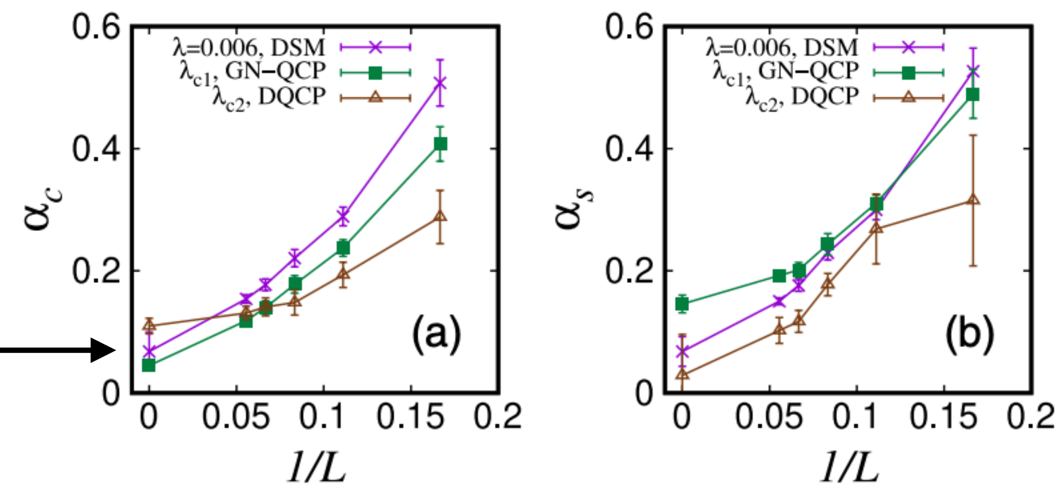

$$\hat{H} = -t\sum_{\langle ij\rangle}(\hat{c}_i^\dagger \hat{c}_j + \mathrm{H.c.}) - \lambda \sum_{\hexagon}\Big(\sum_{\langle\langle i,j\rangle\rangle\in\hexagon} \mathrm{i}\nu_{ij}\hat{c}_i^\dagger \boldsymbol{\sigma}\hat{c}_j + \mathrm{H.c.}\Big)^2$$

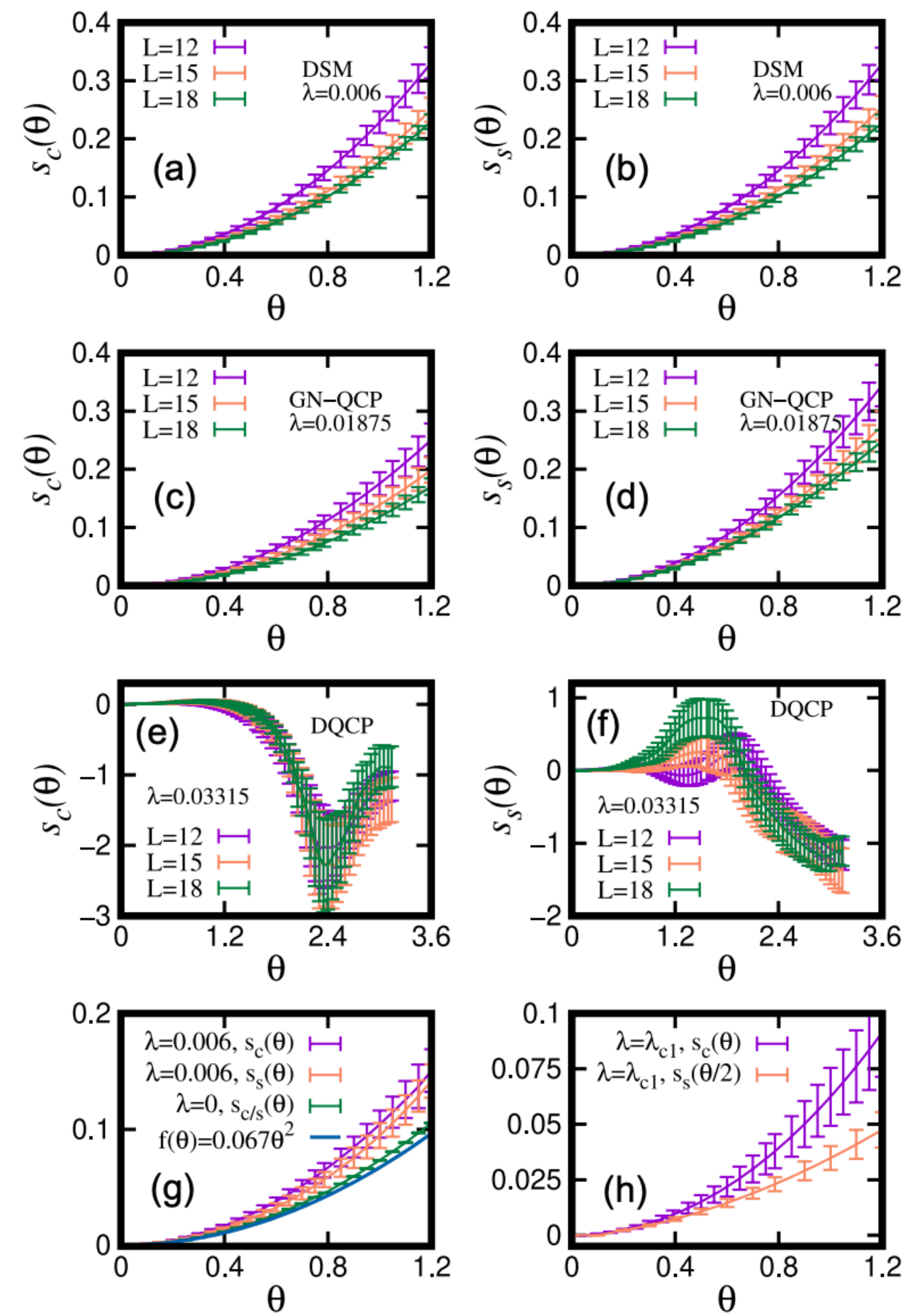
$$X_c(\theta) = \Big\langle \prod_{i\in M} e^{i\hat{n}_i\theta}\Big\rangle, \qquad X_s(\theta) = \Big\langle \prod_{i\in M} e^{i\hat{m}_i^z\theta}\Big\rangle,$$

$$-\ln|\langle X_{c/s}(\theta)\rangle| = al - s(\theta)\ln l - b$$

$$s(\theta) = \underbrace{\alpha_{c/s}}_{AN_\sigma C_J/8\pi^2} \theta^2, \quad \theta \to 0 \qquad C_J = 2 \quad \alpha = 0.066$$

0.06(1)

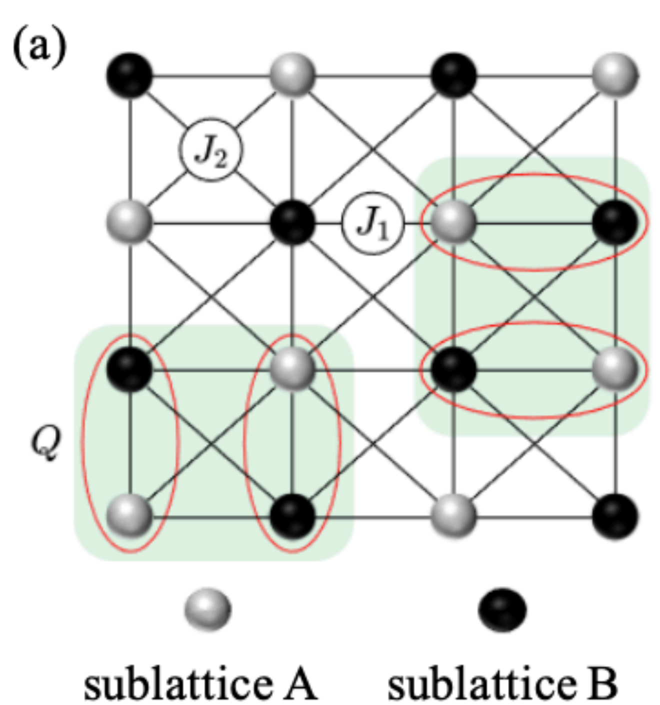


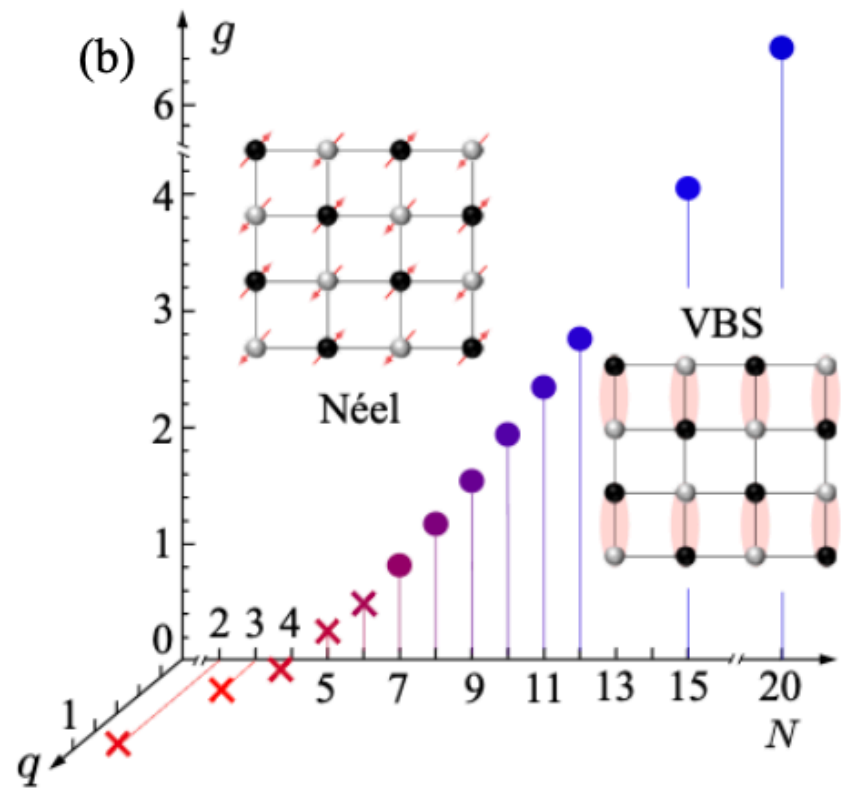



# Deconfined quantum criticality lost

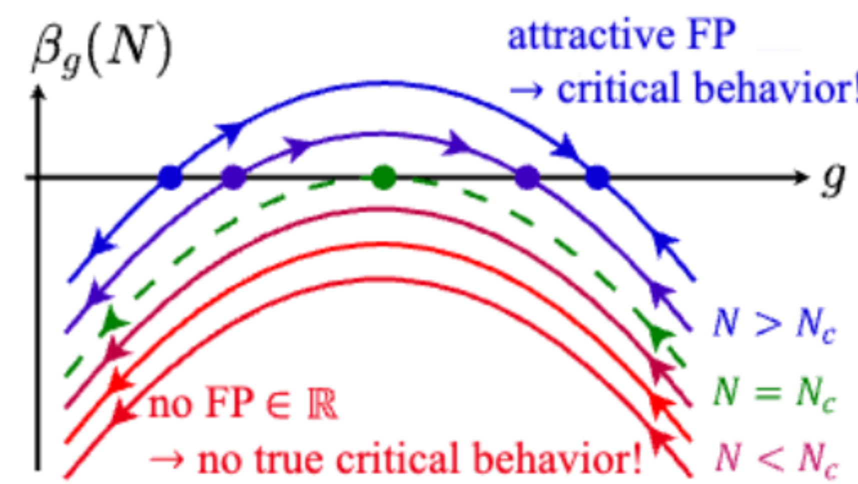
Menghan Song,[1] Jiarui Zhao,[1] Lukas Janssen,[2] Michael M. Scherer,[3] and Zi Yang Meng[1]

arXiv: 2307.02547

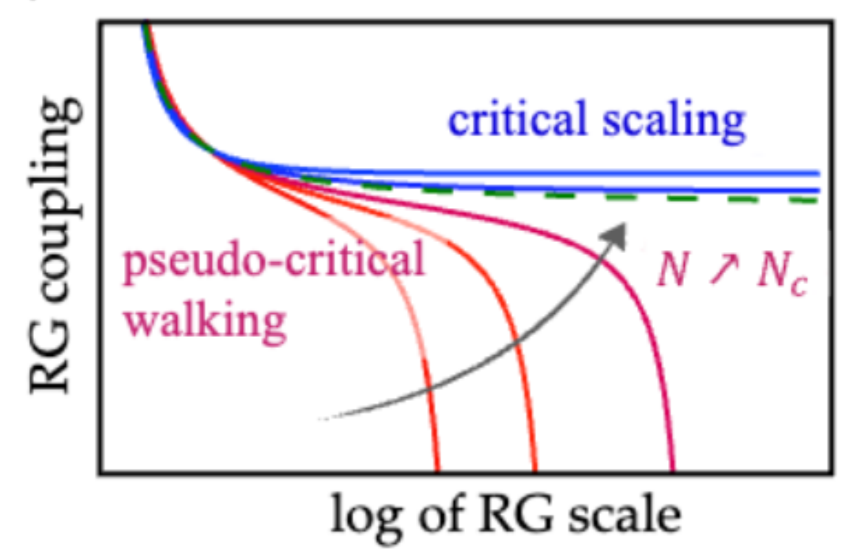



$$S_A^{(2)} = al - s\ln(l) - b$$




$$H = -\frac{J_1}{N}\sum_{\langle ij\rangle} P_{ij} - \frac{J_2}{N}\sum_{\langle\langle ij\rangle\rangle} \Pi_{ij} - \frac{Q}{N}\sum_{\langle ij\rangle,\langle kl\rangle} P_{ij}P_{kl}$$




SU(N) fundamental rep. $|\alpha\rangle_A \rightarrow U_{\alpha,\beta}|\beta\rangle_A$

SU(N) conjugate rep. $|\alpha\rangle_B \rightarrow U^*_{\alpha,\beta}|\beta\rangle_B$

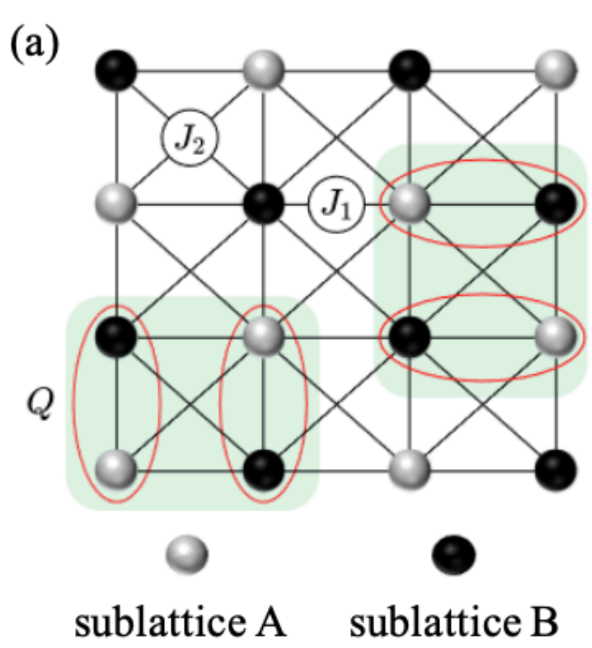
$P_{ij}$ SU(N) singlet projection

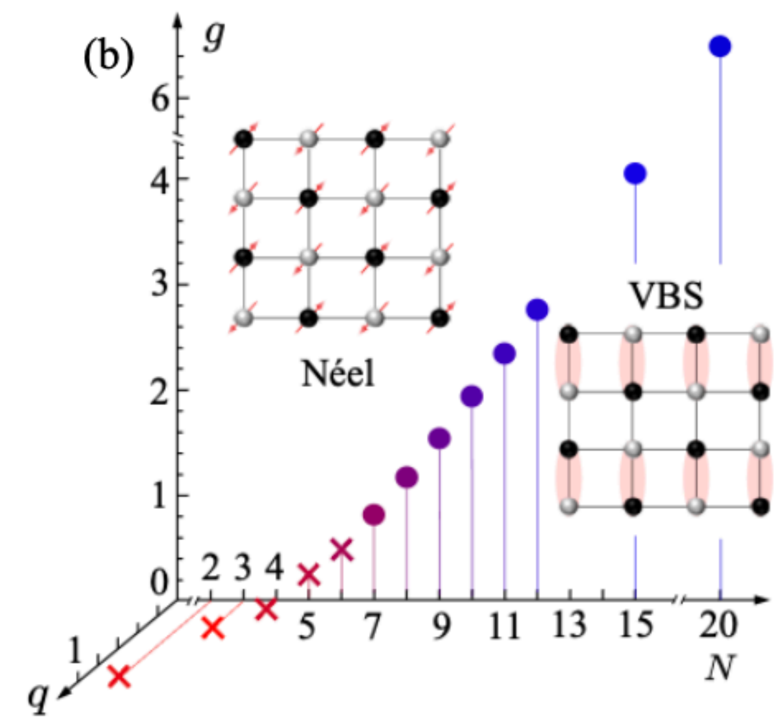
$\Pi_{ij}|\alpha\beta\rangle = |\beta\alpha\rangle$ SU(N) permutation with the same rep

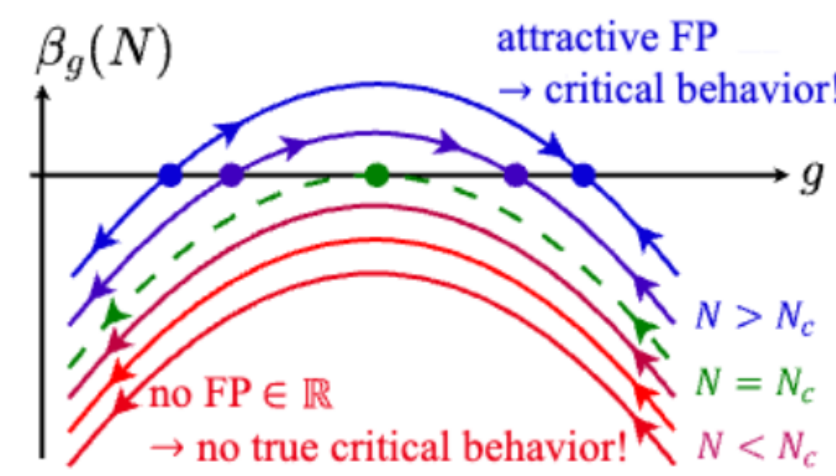
# Deconfined quantum criticality lost

Menghan Song,[1] Jiarui Zhao,[1] Lukas Janssen,[2] Michael M. Scherer,[3] and Zi Yang Meng[1]

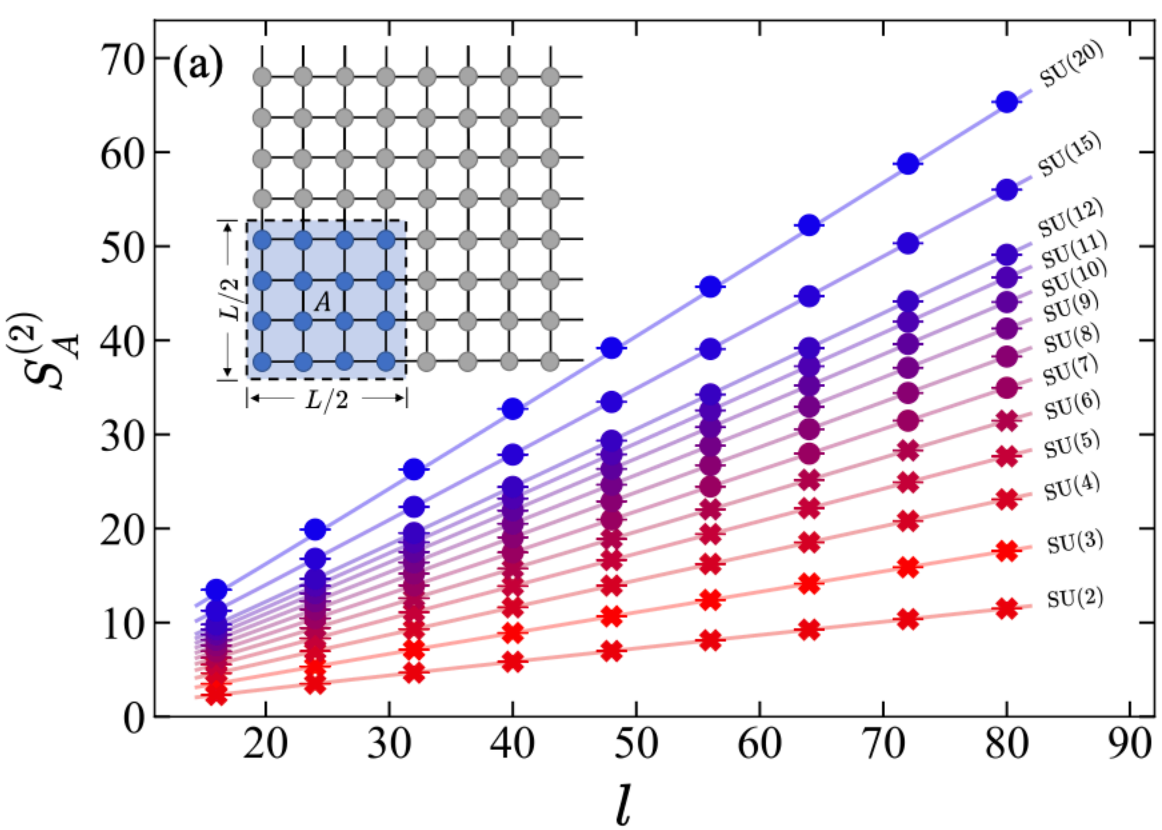
arXiv: 2307.02547

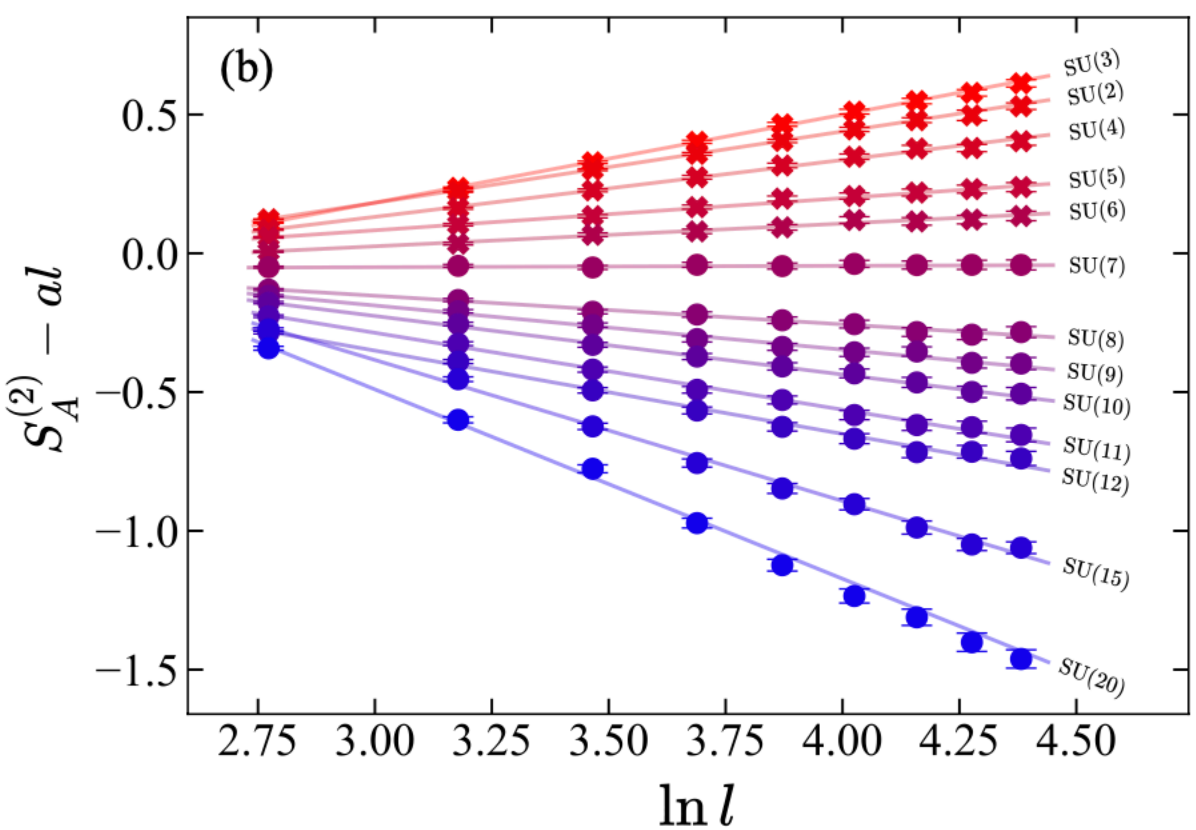


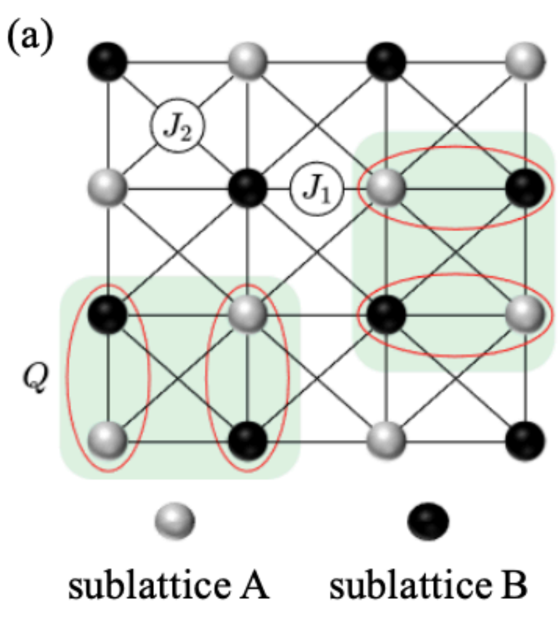

$$S_A^{(2)} = al - s\ln(l) - b$$

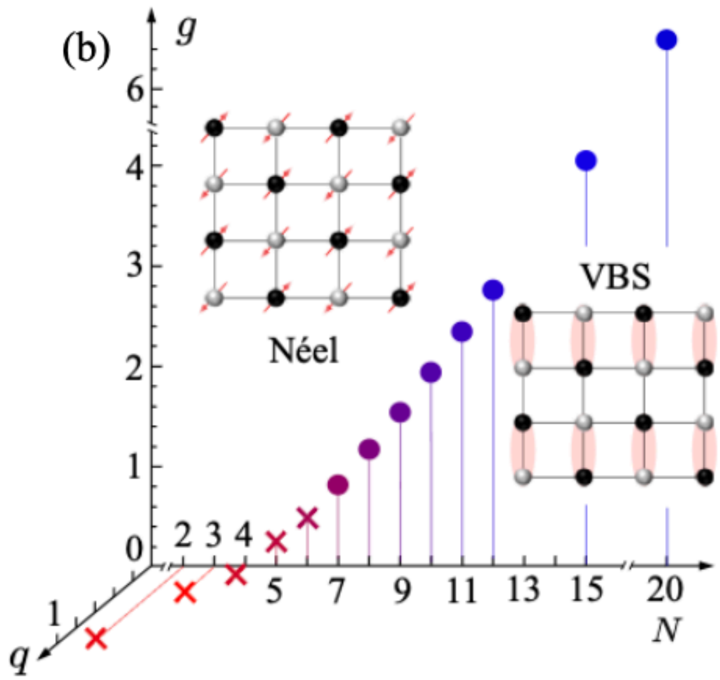


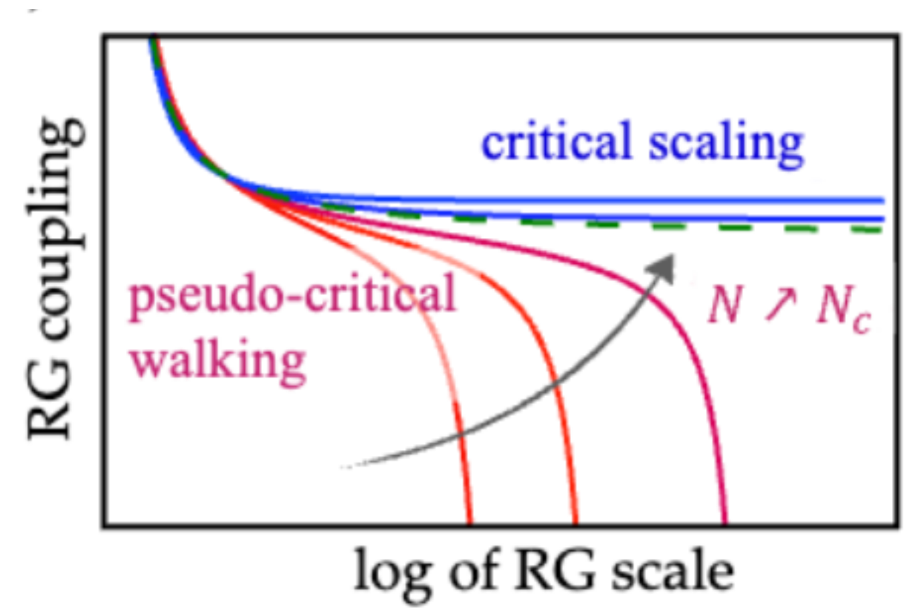
# Deconfined quantum criticality lost

Menghan Song,[1] Jiarui Zhao,[1] Lukas Janssen,[2] Michael M. Scherer,[3] and Zi Yang Meng[1]

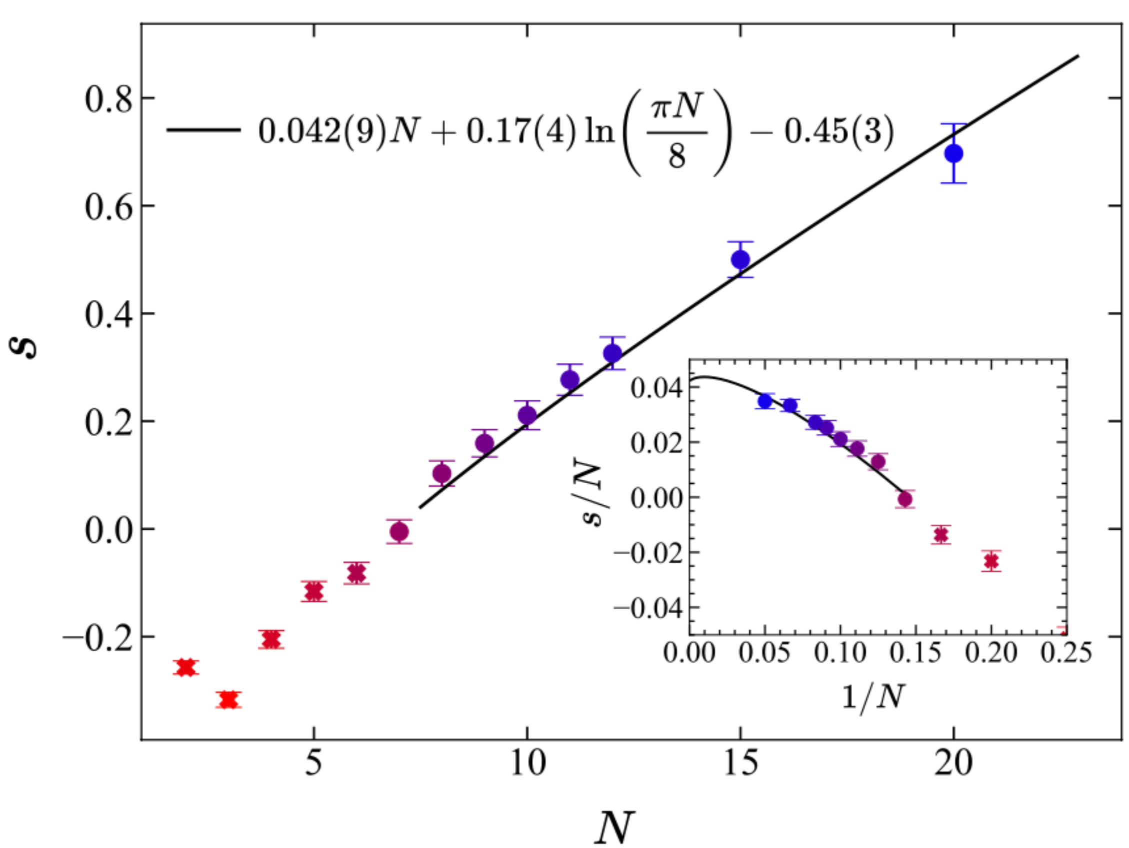


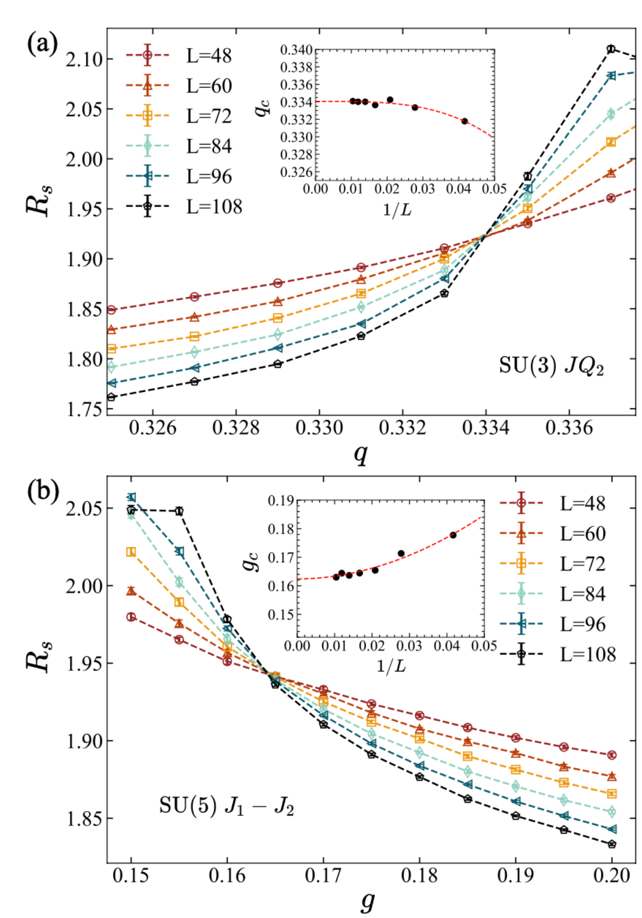



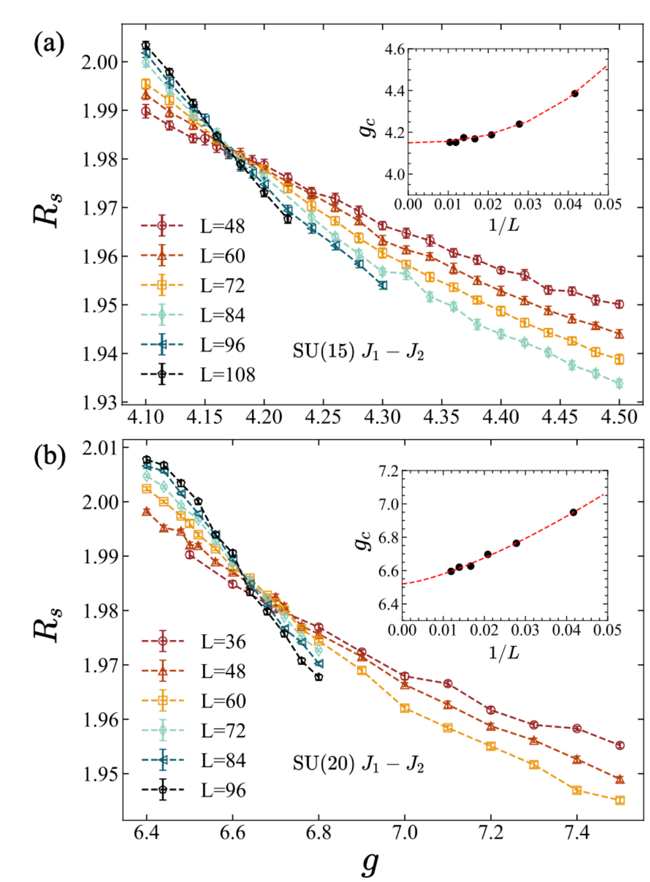

arXiv: 2307.02547




$$s(N) = a_s N + b_s \ln\left(\frac{\pi N}{8}\right) + c_s + \mathcal{O}(1/N)$$

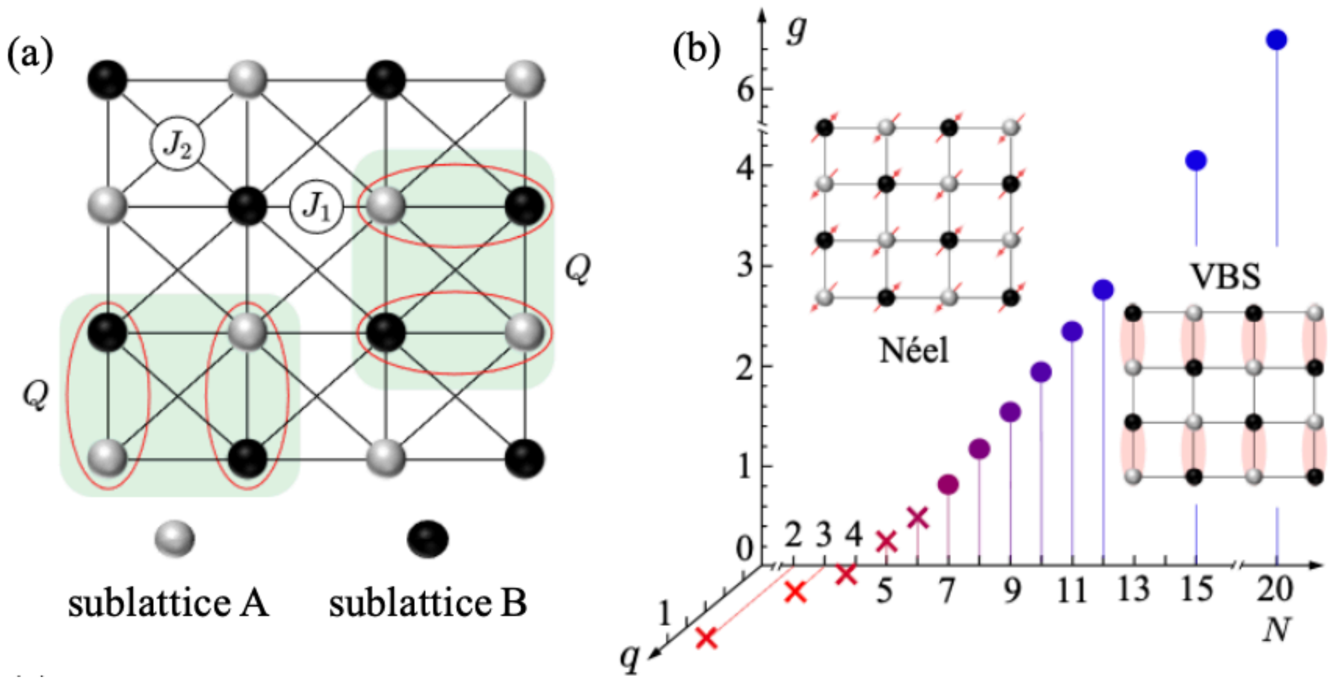


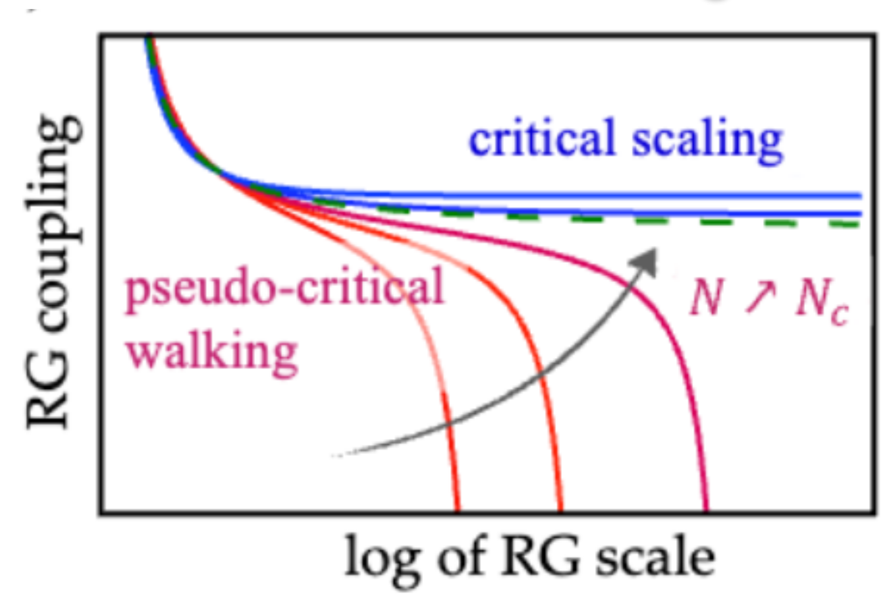



# Deconfined quantum criticality lost

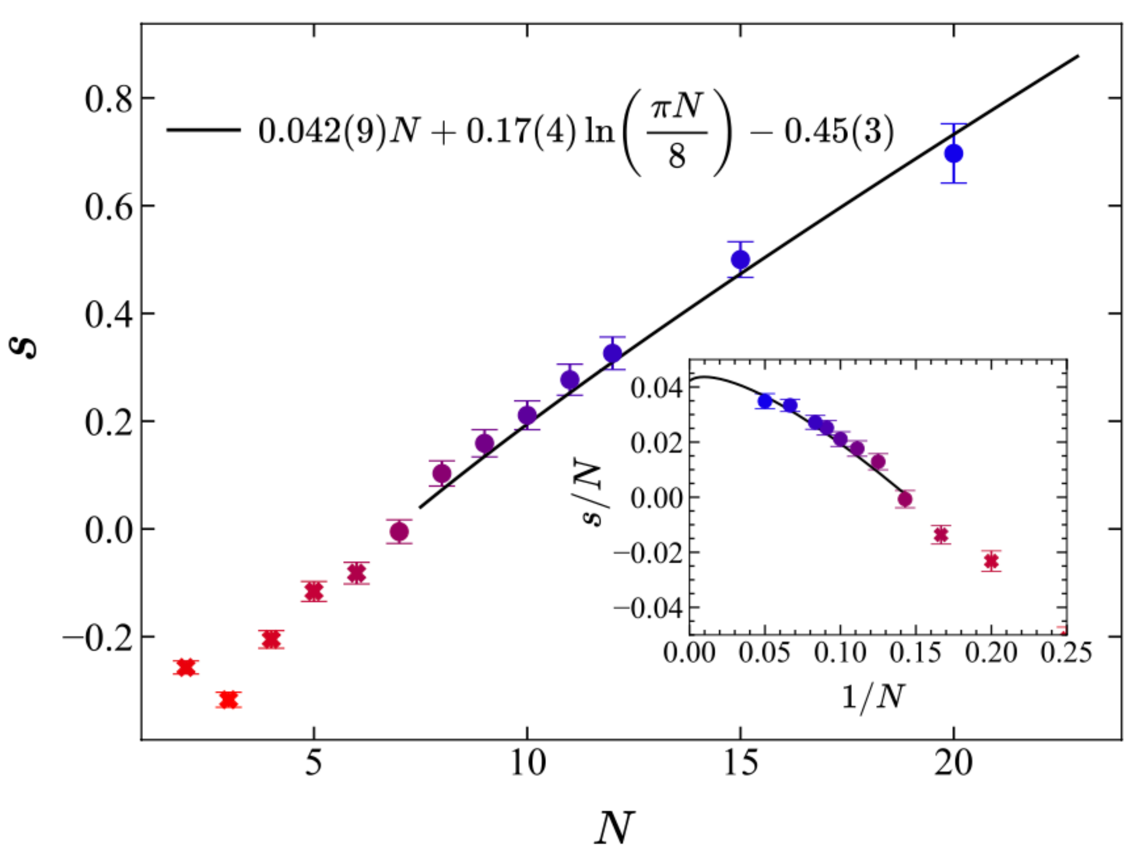
Menghan Song,[1] Jiarui Zhao,[1] Lukas Janssen,[2] Michael M. Scherer,[3] and Zi Yang Meng[1]

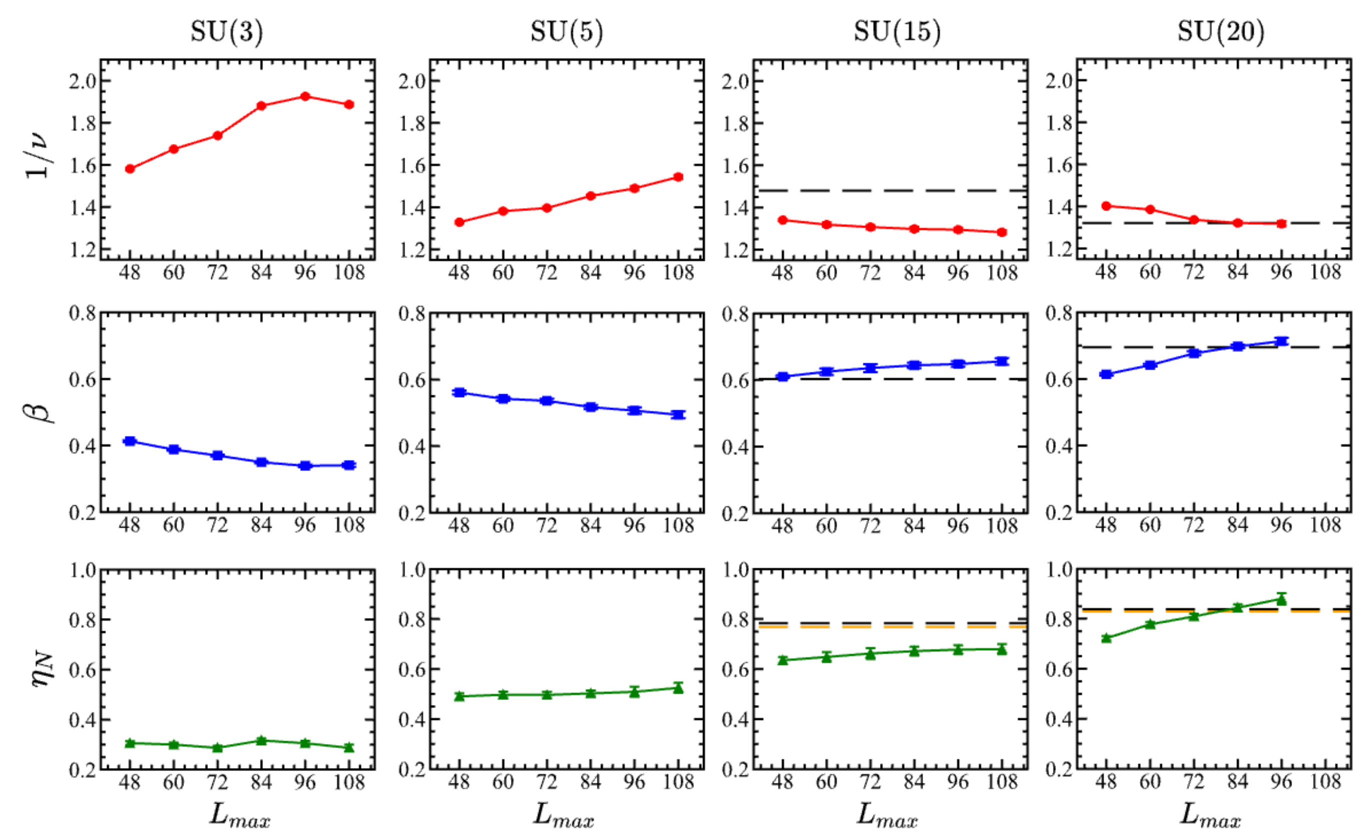



arXiv: 2307.02547




$$s(N) = a_s N + b_s \ln\left(\frac{\pi N}{8}\right) + c_s + \mathcal{O}(1/N)$$



# Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]


- M. Ippoliti, R. Mong, F. Assaad, M. Zaletel, PRB 98, 235108 (2018)
- Z. Wang, M. Zaletel, R. Mong, F. Assaad, PRL 126, 045701 (2021)


arXiv: 2307.05307

**Bin-Bin Chen (Poster)**

$$S = \frac{1}{g}\int d^3x(\nabla\hat{\varphi})^2 + S_{\mathrm{WZW}} + \cdots$$

$$H = \frac{1}{2}\int d\Omega\{U_0[\psi^\dagger(\Omega)\psi(\Omega) - 2]^2 - \sum_{i=1}^{5} u_i[\psi^\dagger(\Omega)\Gamma^i\psi(\Omega)]^2\}$$

$$\psi_{\tau\sigma}(\Omega) \qquad \Gamma^i = \{\tau_x\otimes\mathbb{I}, \tau_y\otimes\mathbb{I}, \tau_z\otimes\sigma_x, \tau_z\otimes\sigma_y, \tau_z\otimes\sigma_z\}$$

magnet monople inside a sphere $4\pi s$

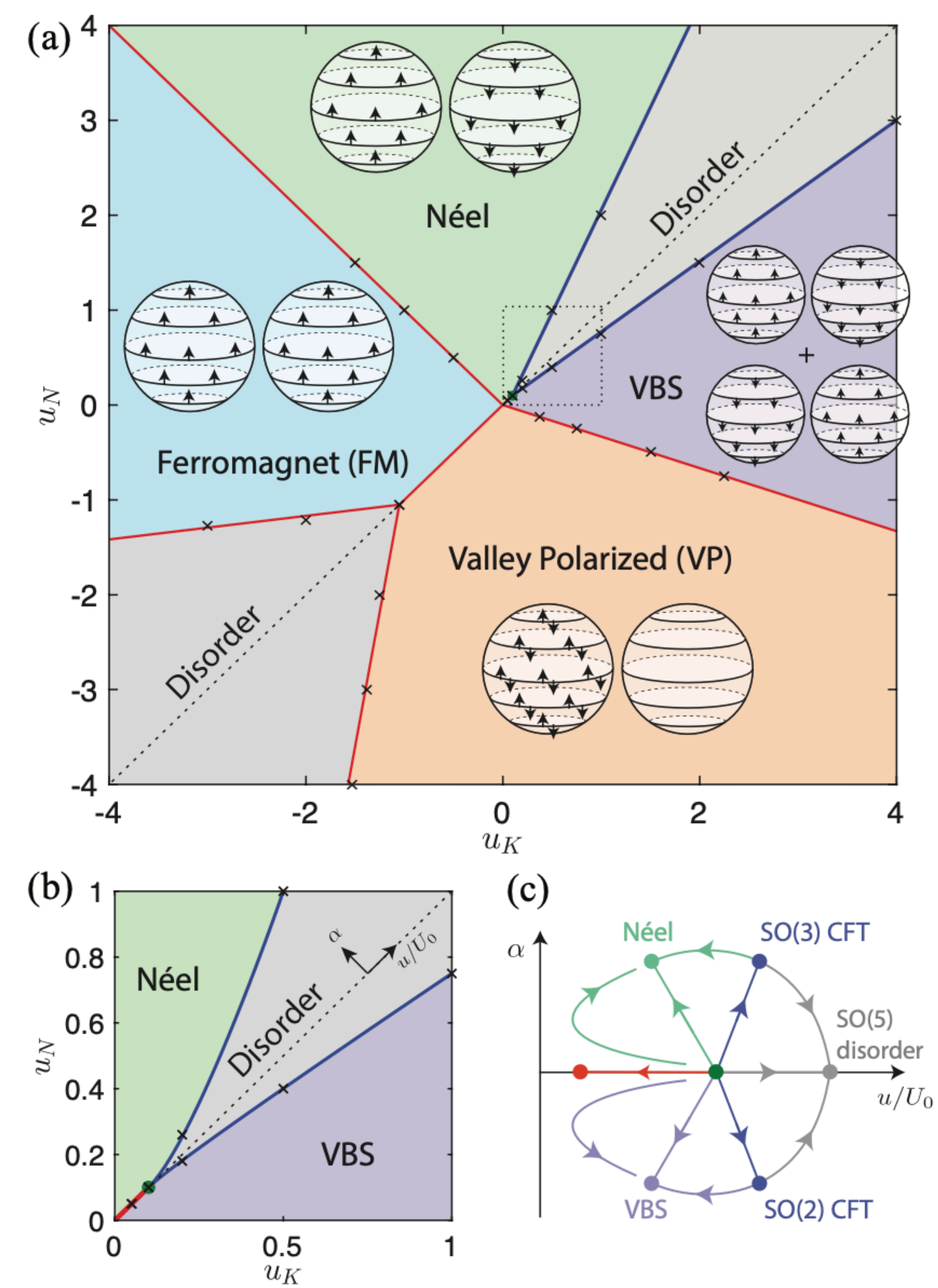
Projected to the LLL with degeneracy $N = 2s+1$

$$\psi(\Omega) = \sum_{m=-s}^{s}\Phi_m(\Omega)c_m \qquad \Phi_m(\Omega) \propto e^{im\phi}\cos^{s+m}(\frac{\theta}{2})\sin^{s-m}(\frac{\theta}{2})$$

$$\hat{H}_\Gamma = U_0\hat{H}_0 - \sum_i u_i\hat{H}_i, \text{with}$$

$$\hat{H}_i = \sum_{m_1,m_2,m} V_{m_1,m_2,m_2-m,m_1+m}\times$$

$$\left(c^\dagger_{m_1}\Gamma^i c_{m_1+m} - 2\delta_{i0}\delta_{m0}\right)\left(c^\dagger_{m_2}\Gamma^i c_{m_2-m} - 2\delta_{i0}\delta_{m0}\right)$$



# Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

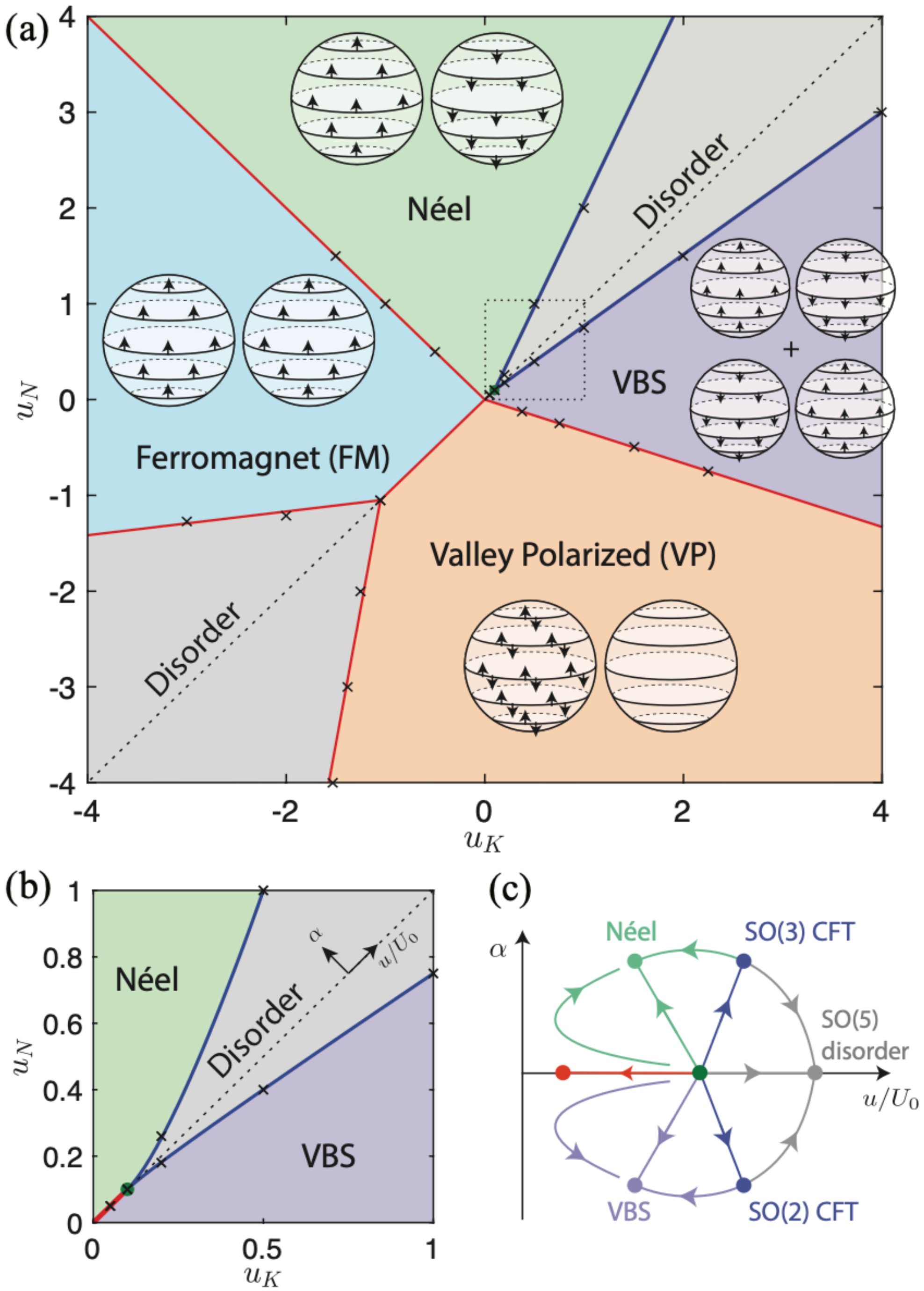
## Bin-Bin Chen (Poster)

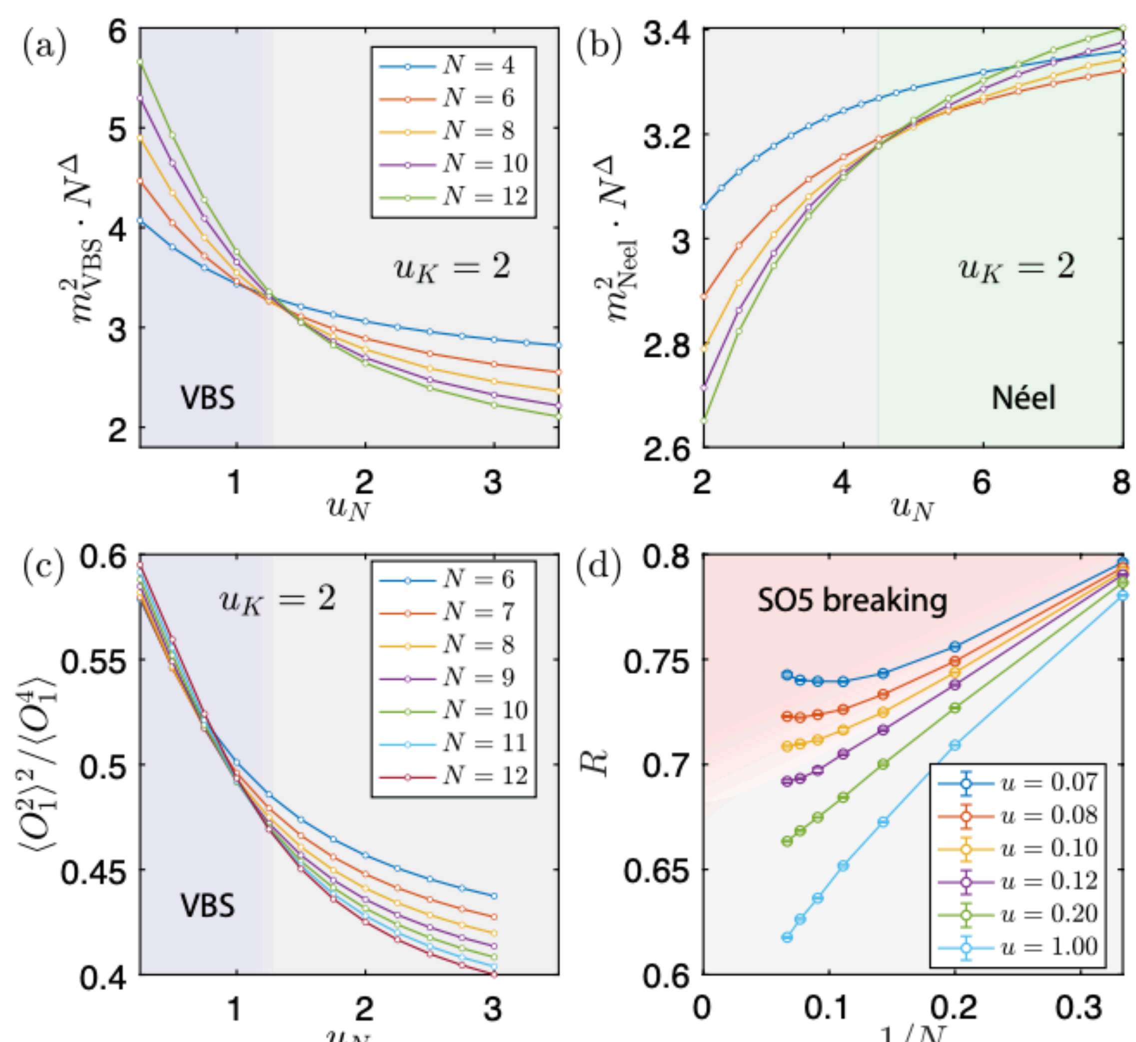
arXiv: 2307.05307

$$U_0 = 1,\ u_1 = u_2 = u_K, u_3 = u_4 = u_5 = u_N$$

$$\langle O_i \rangle = \int d\Omega \psi^\dagger(\Omega)\Gamma^i \psi(\Omega) = \sum_{m=-s}^{s} c_m^\dagger \Gamma^i c_m$$

$$m_{VBS}^2 = \frac{1}{2N^2}\langle (O_1^2 + O_2^2)\rangle \qquad m_{Neel}^2 = \frac{1}{3N^2}\langle (O_3^2 + O_4^2 + O_5^2)\rangle$$



# Phases of (2+1)D SO(5) non-linear sigma model with a topological term on a sphere: multicritical point and disorder phase

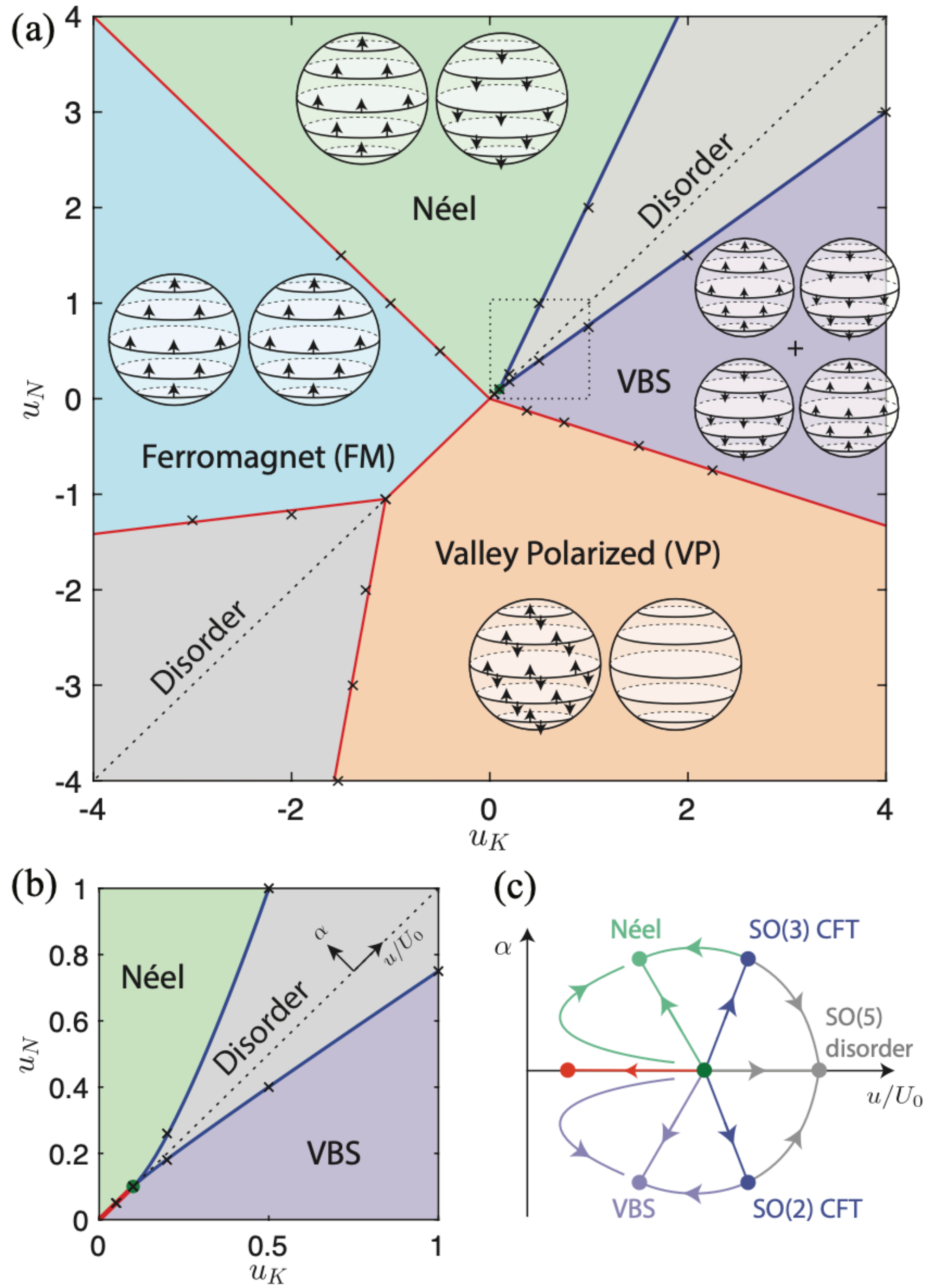
Bin-Bin Chen,[1] Xu Zhang,[1] Yuxuan Wang,[2,*] Kai Sun,[3,†] and Zi Yang Meng[1,‡]

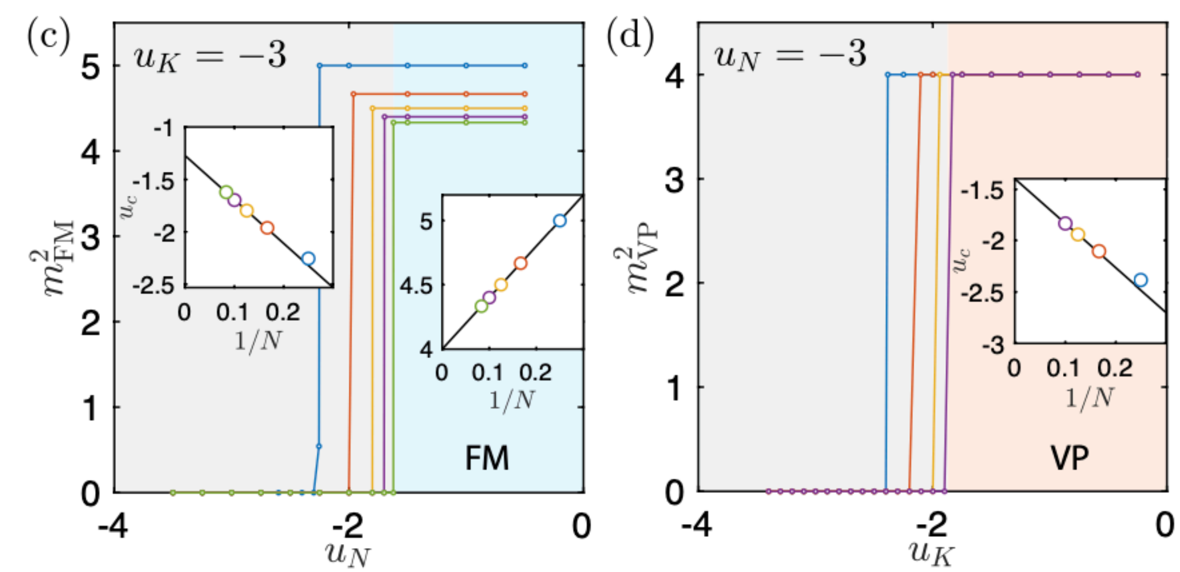
## Bin-Bin Chen (Poster)

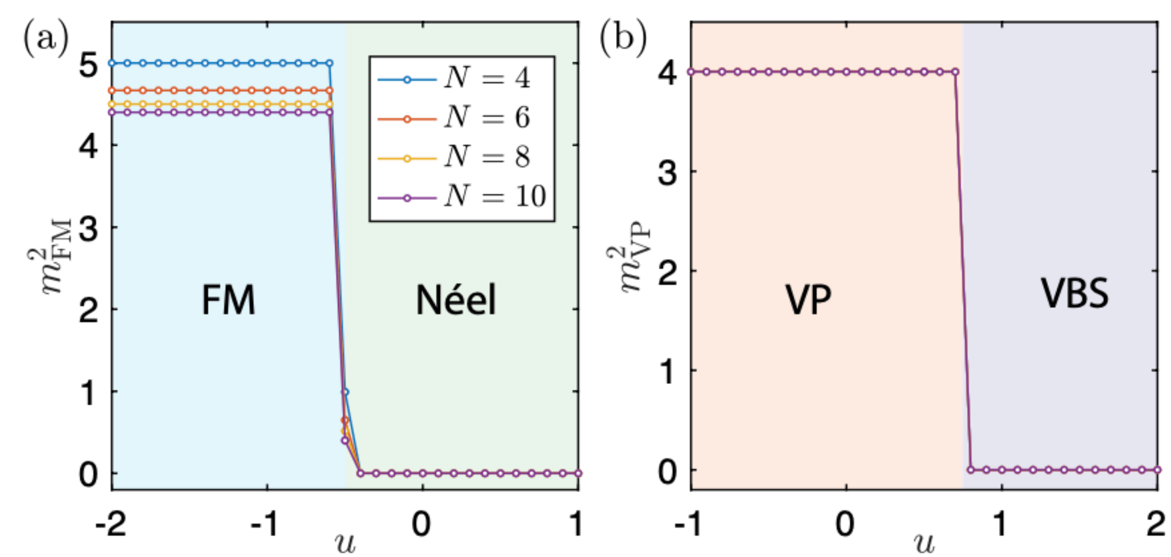
arXiv: 2307.05307

$$\langle \tilde{O}_{ij} \rangle = \int d\Omega \psi^\dagger(\Omega) L^{ij} \psi(\Omega) = \sum_{m=-s}^{s} c_m^\dagger L^{ij} c_m$$

$$m_{FM}^2 = \frac{1}{N^2}\langle(\tilde{O}_{34}^2 + \tilde{O}_{35}^2 + \tilde{O}_{45}^2)\rangle \qquad m_{VP}^2 = \frac{1}{N^2}\langle \tilde{O}_{12}^2 \rangle$$



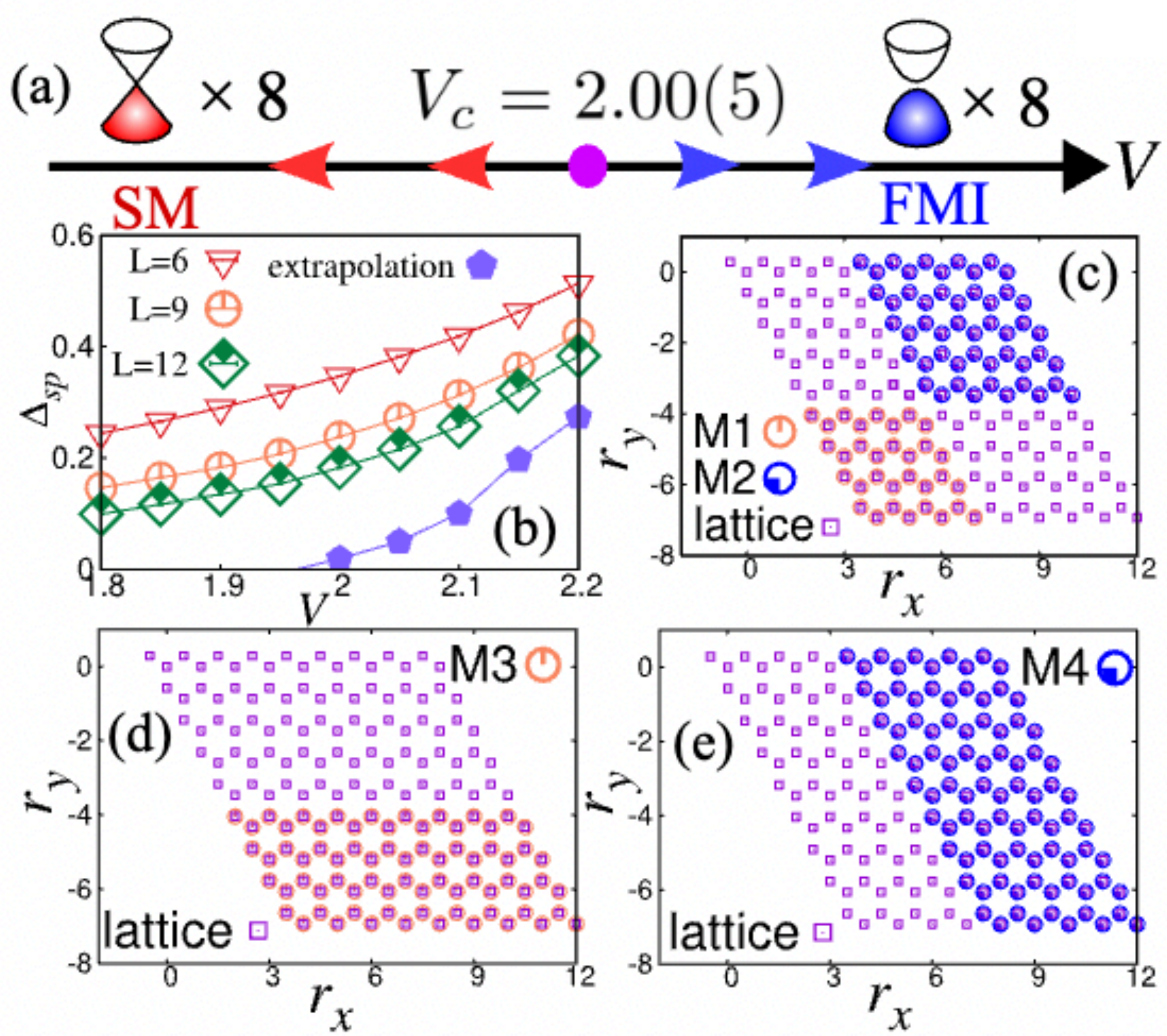
# Disorder Operator and Rényi Entanglement Entropy of Symmetric Mass Generation

Zi Hong Liu,[1] Yuan Da Liao,[2,3] Gaopei Pan,[4,5] Weilun Jiang,[6] Chao-Ming Jian,[7]
Yi-Zhuang You,[8] Fakher F. Assaad,[1,*] Zi Yang Meng,[4,†] and Cenke Xu[9,‡]

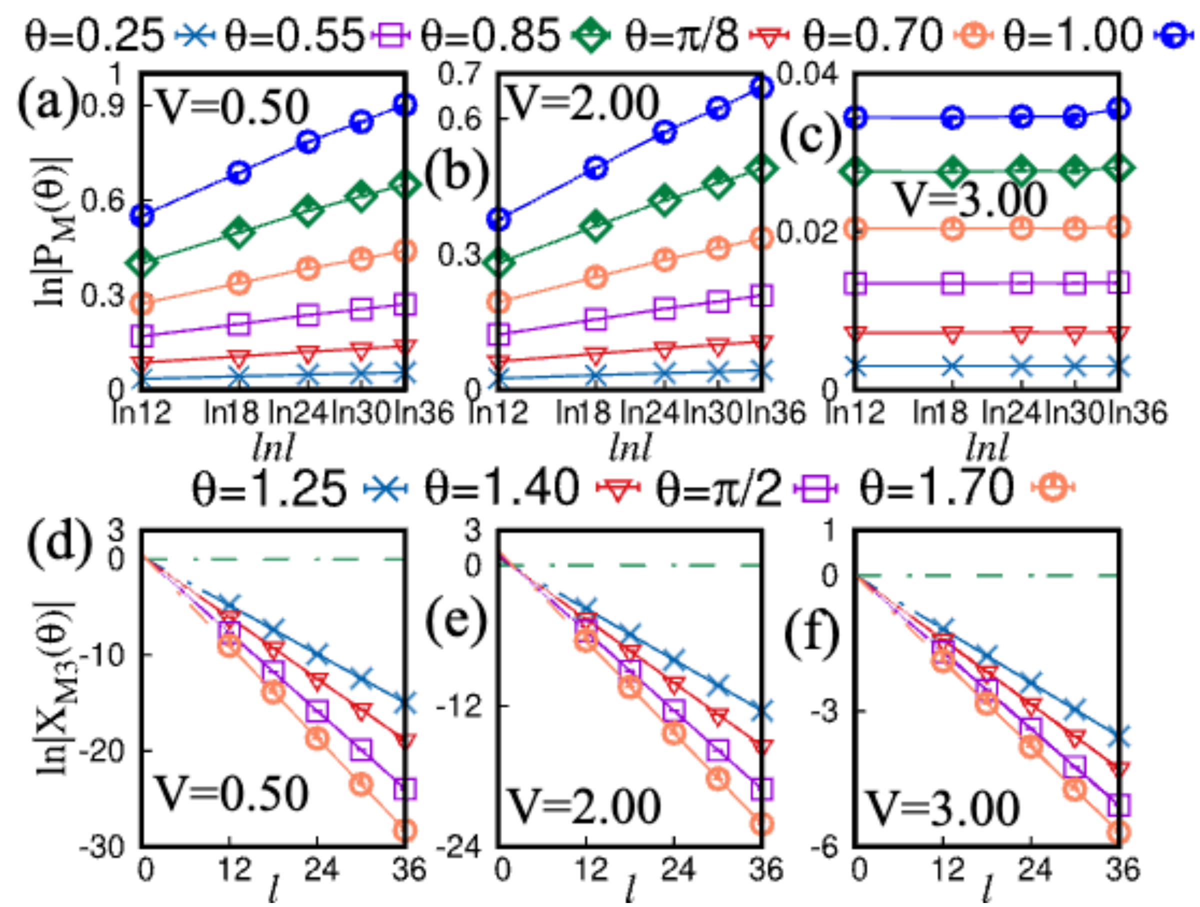
arXiv: 2308.07380

$$\hat{H} = -t \sum_{\langle ij \rangle, \alpha} (-1)^{\alpha} \left( \hat{c}^{\dagger}_{i\alpha} \hat{c}_{j\alpha} + \hat{c}^{\dagger}_{j\alpha} \hat{c}_{i\alpha} \right) + V \sum_{i} \left( \hat{c}^{\dagger}_{i1} \hat{c}_{i2} \hat{c}^{\dagger}_{i3} \hat{c}_{i4} + \hat{c}^{\dagger}_{i4} \hat{c}_{i3} \hat{c}^{\dagger}_{i2} \hat{c}_{i1} \right)$$

$$\hat{X}_M(\theta) = \prod_{i \in M} \exp(i\theta \hat{n}_i) \qquad \ln|X_M(\theta)| \sim -al + s(\theta)\ln l + c$$




$$P_M(\theta) = \left| \frac{X_{M1}(\theta) X_{M2}(\theta)}{X_{M3}(\theta) X_{M4}(\theta)} \right| \qquad \ln P_M(\theta) \sim s(\theta) \ln l$$




$$\ln|X_{M3}| \sim al + \beta(\theta)$$

# Disorder Operator and Rényi Entanglement Entropy of Symmetric Mass Generation

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arXiv: 2308.07380

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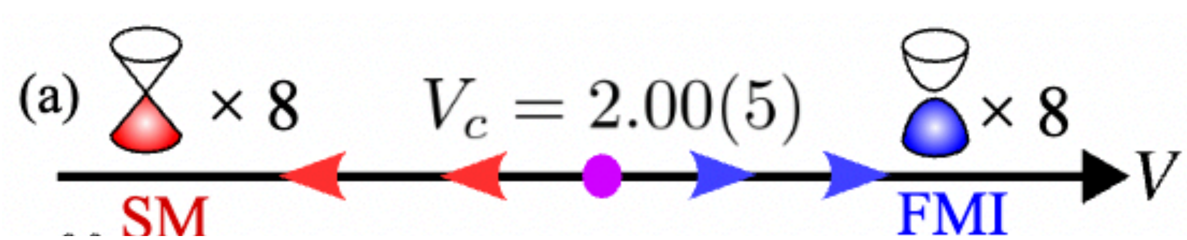
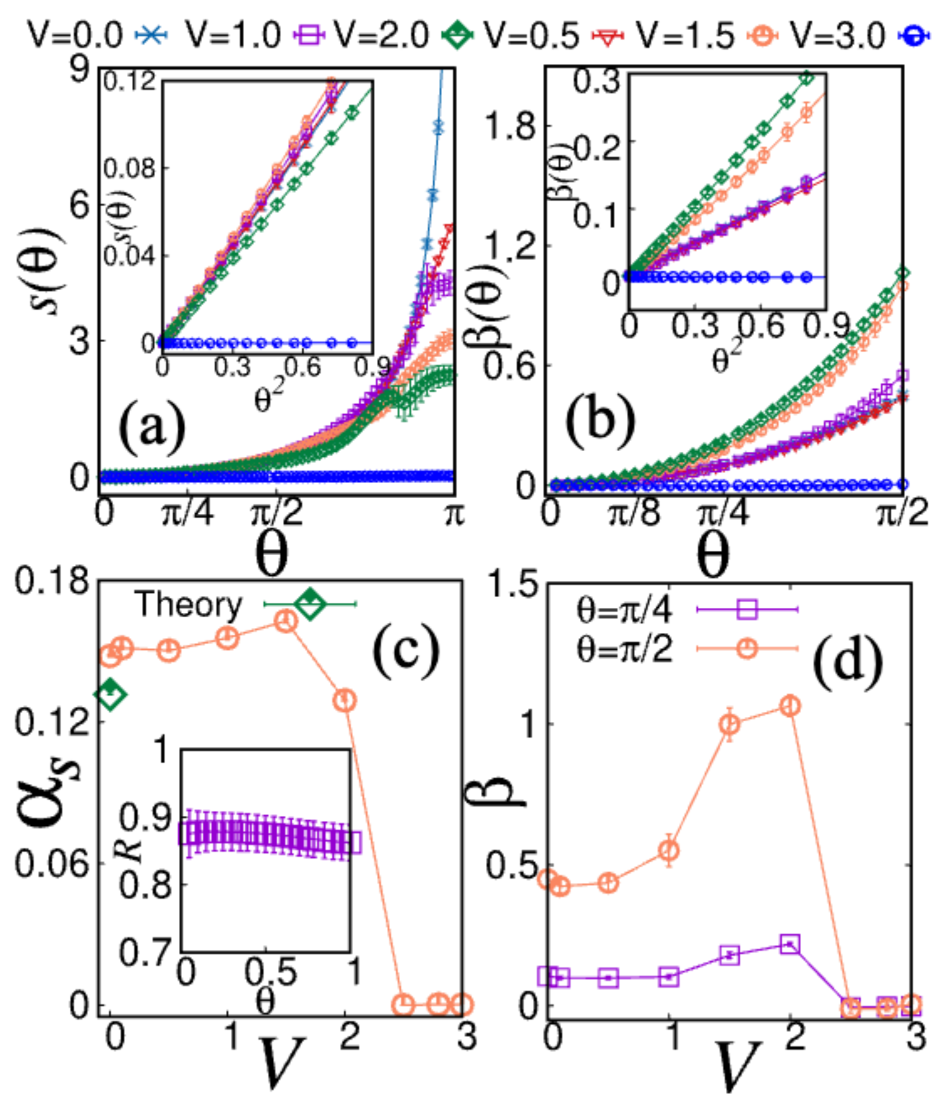
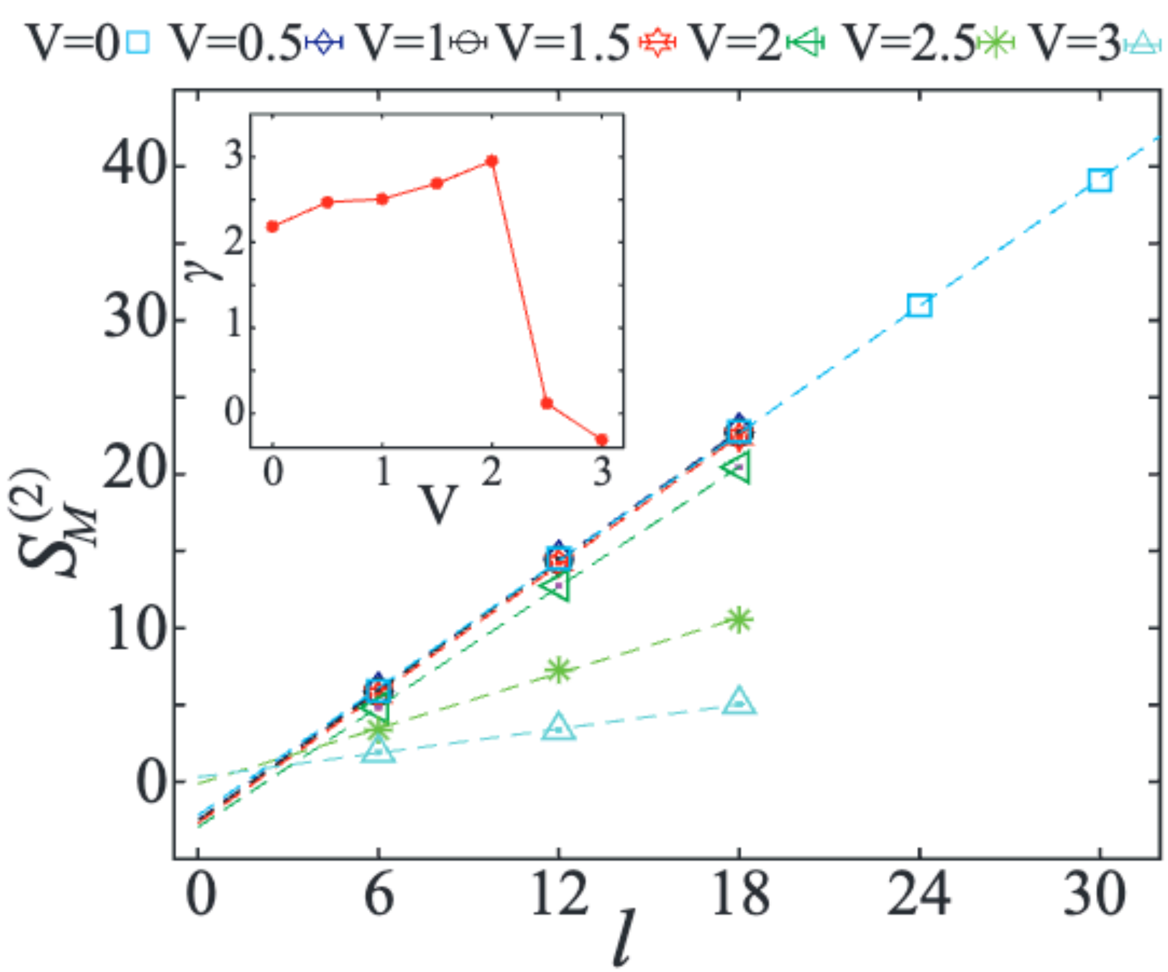



$$\ln |X_M(\theta)| \sim -al + s(\theta) \ln l + c \qquad s(\theta) \sim \alpha_s \theta^2$$

$$\ln |X_{M3}(\theta)| \sim al + \beta(\theta)$$

$$S^{(2)}_M \sim al - \gamma$$



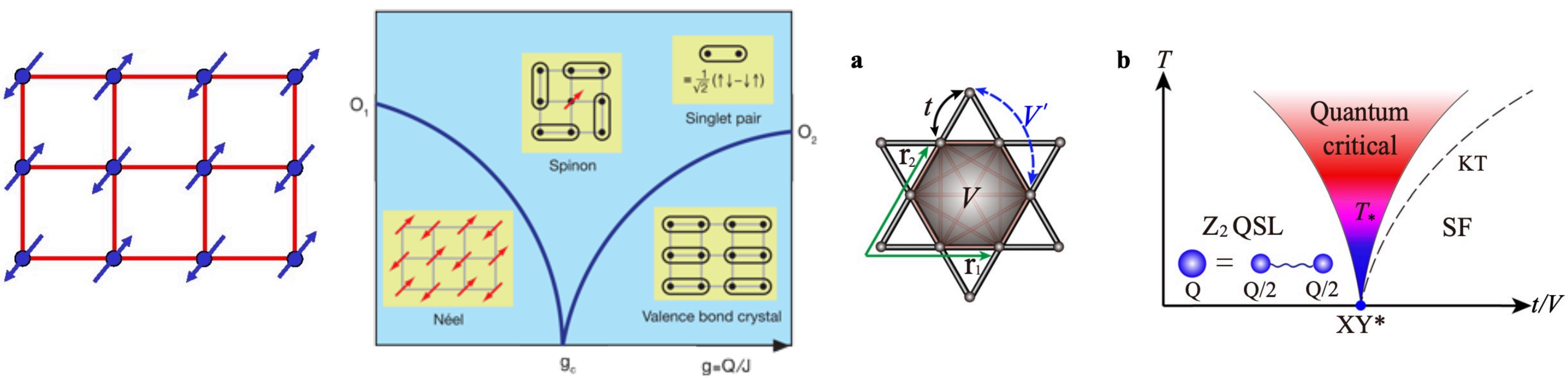
# Entanglement entropy with incremental (Qiu Ku) method

$$S_A^{(2)}(l) = al - s\ln l - b$$

- Jiarui Zhao, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, PRL 128, 010601 (2022)
- Jiarui Zhao, Bin-Bin Chen, Yan-Cheng Wang, Zheng Yan, Meng Cheng, ZYM, npj Quantum Materials 7, 69 (2022)
- Menghan Song, Jiarui Zhao, Lukas Janssen, Michael Scherer, ZYM, arXiv: 2307.02547
- Bin-Bin Chen, Xu Zhang, Yuxuan Wang, Kai Sun, ZYM, arXiv:2307.05307
- Zi Hong Liu, Yuan Da Liao, Gaopei Pan, …, Yi-Zhuang You, F. Assaad, ZYM, Cenke Xu, arXiv:2308.07380

$$-\ln|\langle X_M\rangle| = al - s\ln l - b$$

- Yan-Cheng Wang, Meng Cheng, William Witczak-Krempa, ZYM, Nat Commun 12, 5347 (2021)
- Yan-Cheng Wang, Nvsen Ma, Meng Cheng, ZYM, SciPost Phys. 13, 123 (2022)
- Weilun Jiang, Bin-Bin Chen, Zi Hong Liu, Junchen Rong, F. Assaad, Meng Cheng, Kai Sun, ZYM, SciPost Phys. (2023)
- Zi Hong Liu, Weilun Jiang, Bin-Bin Chen, Junchen Rong, Meng Cheng, Kai Sun, ZYM, F. Assaad, PRL 130, 266501 (2023)




(2+1)d SSB, O(3), Topological order Z2 QSL, GNY, FL, nFL, DQCP, SMG, …