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Human Body: Nervous System

The nervous system is made up of the central nervous system and peripheral nervous system. These systems work together to collect and interpret data from the body's internal and external environment and control responses.

Central Nervous System

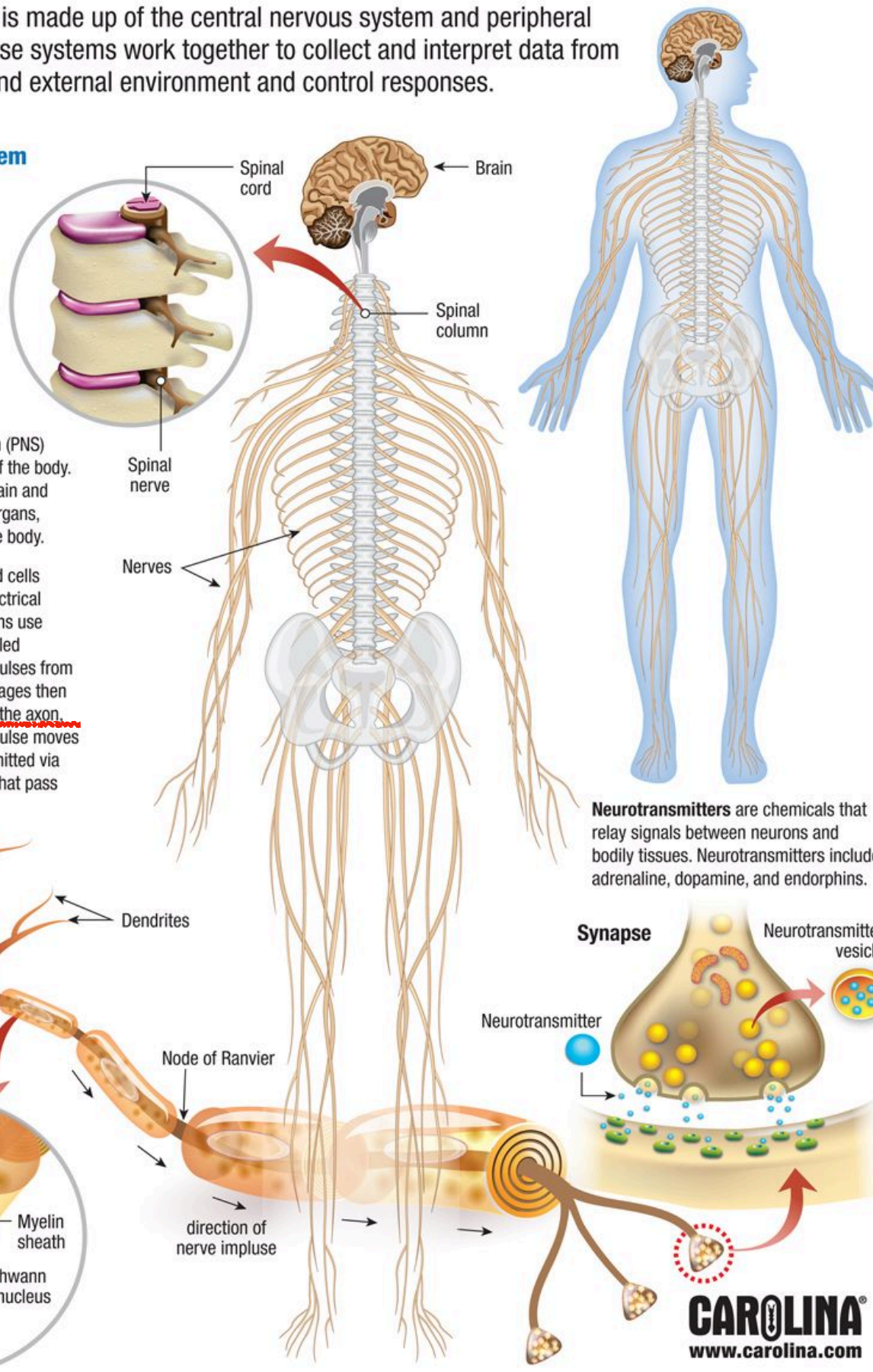
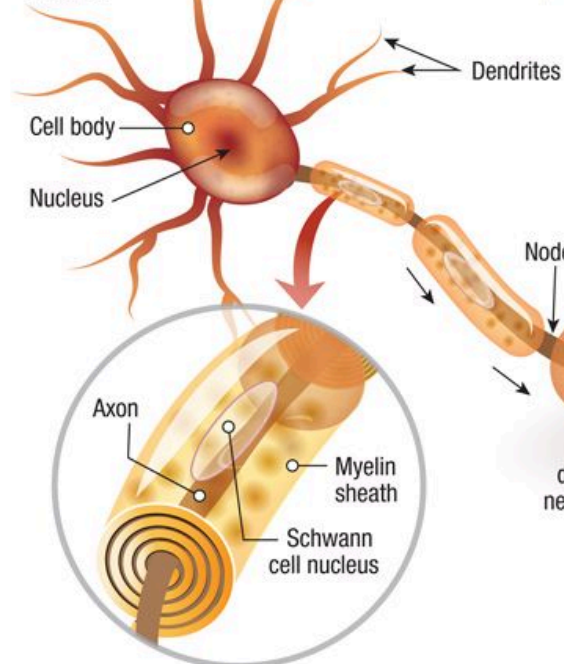
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Peripheral Nervous System

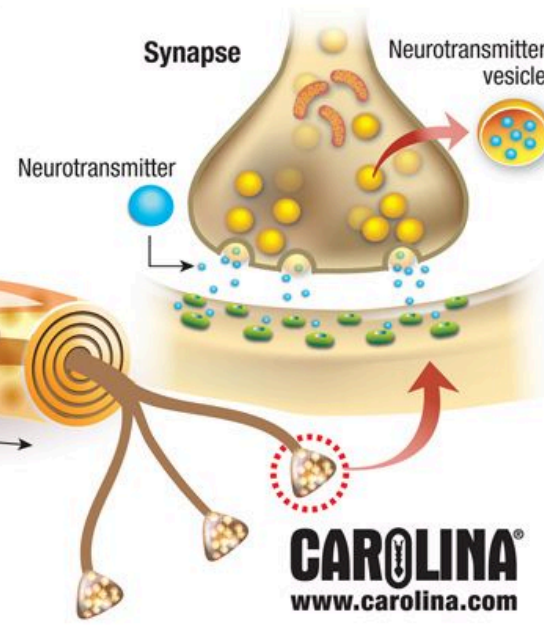
The peripheral nervous system (PNS) connects the CNS to the rest of the body. Nerves branch out from the brain and spinal cord, extending to the organs, muscles, and other parts of the body.

Neurons are highly specialized cells that transmit chemical and electrical information in the body. Neurons use short, branched extensions called dendrites to receive nerve impulses from surrounding cells. These messages then travel through the cell body to the axon, a threadlike structure. The impulse moves through the axon and is transmitted via chemical or electrical signals that pass through a synapse.

Neuron

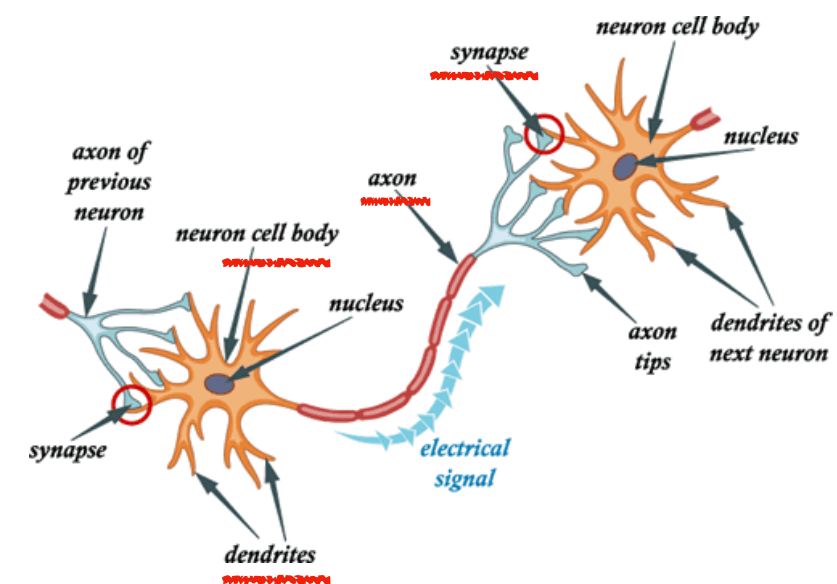


Neurotransmitters are chemicals that relay signals between neurons and bodily tissues. Neurotransmitters include adrenaline, dopamine, and endorphins.



Brain consists of $\sim 10^{11}$ neurons, building bricks for the central nervous systems

Neurons are interconnected by synapses, the complexity of the Brain is due to massive highly interconnected neurons working in parallel, One neuron receives inputs from $\sim 10^4$ others.



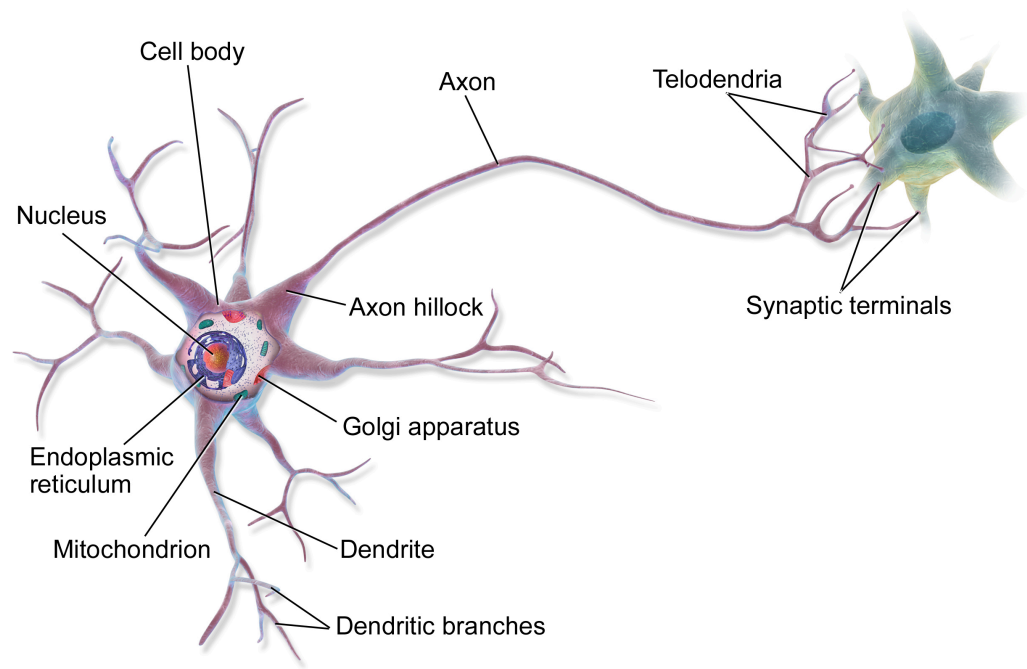
All the inputs to the neuron, are summed up. The input sum is processed by a threshold function and produces an output signal. The processing time ~ 1 ms per cycle

Brain works in both a parallel and serial way. Picture recognition of human ~ 100 ms Around 100 neurons are involved in serial Complexity requires parallel processing

Biological neural systems have high fault tolerance

People with brain injuries can perform normally




AI & Machine Learning Basics



	Supercomputer	Personal Computer	Human Brain
Computational Units	4,000 Xeon/AMD CPUs 10 ¹² transistors	4 CPUs, 10 ⁹ transistors	10 ¹¹ neurons
Storage units	10 ¹⁴ bits RAM 10 ¹⁵ bits Storage	10 ¹¹ bit RAM 10 ¹³ bit Storage	10 ¹¹ neurons 10 ¹⁴ synapses
Cycle time	10 ⁻⁹ sec	10 ⁻⁹ sec	10 ⁻³ sec
Operations/sec	10 ¹⁵	10 ¹⁰	10 ¹⁷
Memory updates/sec	10 ¹⁴	10 ¹⁰	10 ¹⁴
Weight / Space	150 tons / Basketball court	1 Kg / A4 Paper	1.5 Kg / 1/6 basketball
Power consumption	500 megawatt	100 watt	20 watt

Peripheral nervous system

- Touching a hot object
- Sensory nerves carry information about the heat to the brain
- Brain, via motor nerves, tells the muscle of the hand to withdraw
- The whole process takes less than a second

	Computing wins	<ul style="list-style-type: none">• Input and output• Information processing and memory	
	Closely matched	<ul style="list-style-type: none">• Complex movement• Vision• Language• Structured problem solving	<p>Boston Dyanmics SPOT/ATLAS</p> <p>Auditory processing Script in zoom Chat-GPT</p>
	Brain still wins	<ul style="list-style-type: none">• Creativity• Emotion and Empathy• Planning and Executive Function• Consciousness	

<https://becominghuman.ai/brains-vs-computers-f769548010f1>

Neuroscience For Kids

<http://faculty.washington.edu/chudler/neurok.html>

Biological neurons (nerve cell)

Functional and structural units of nervous system

10^{11} in human body

80% in human Brain

Input: Dendrites:

neuron receives one or more inputs through dendrites

Hypothesis: Cell Body (Soma):

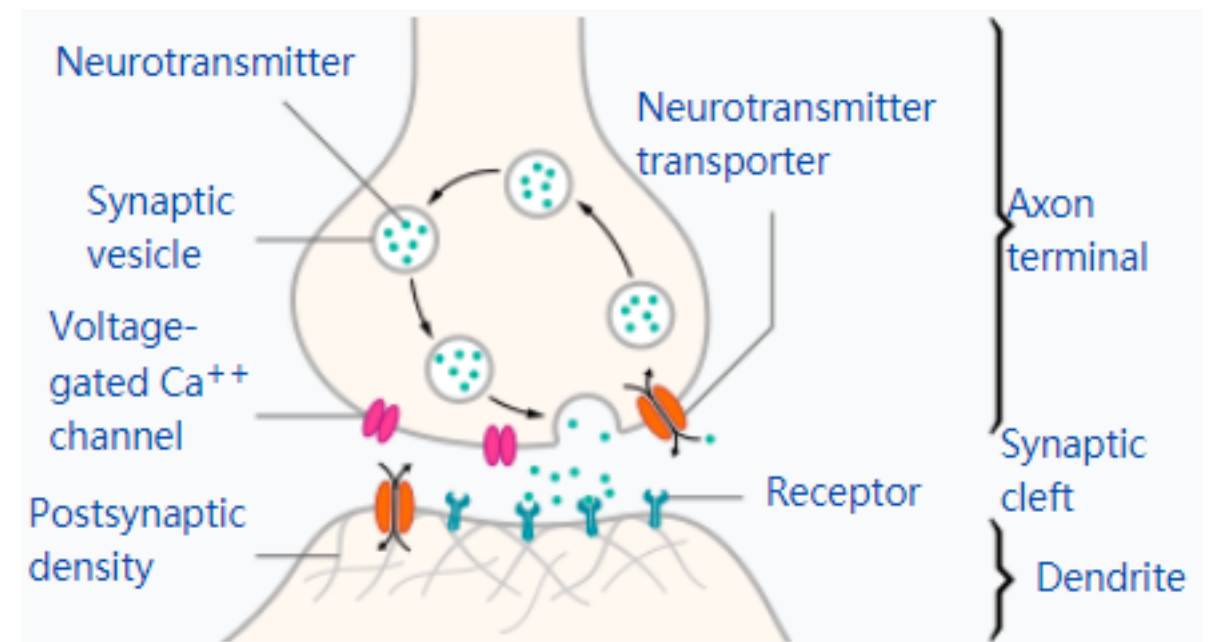
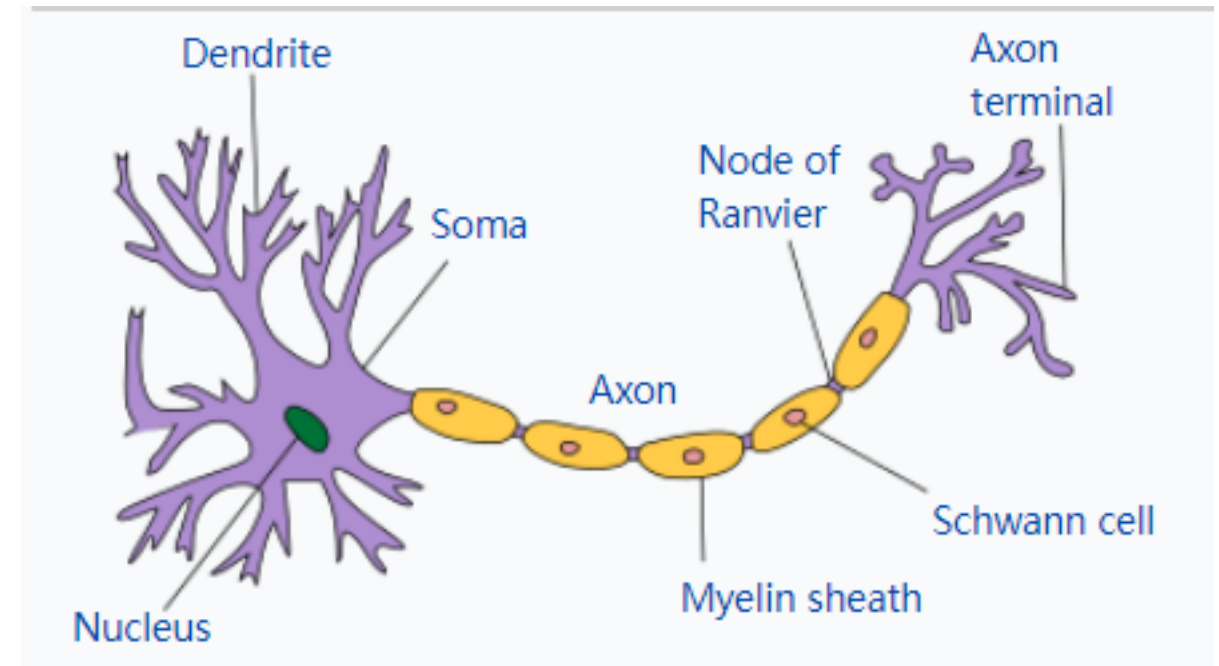
Processing the information

Output: Axon:

Send output through axon

Neurons do not touch each other, they form tiny gaps called synapses, pass electrical or chemical signal to another neuron or target cell

<https://en.wikipedia.org/wiki/Synapse>

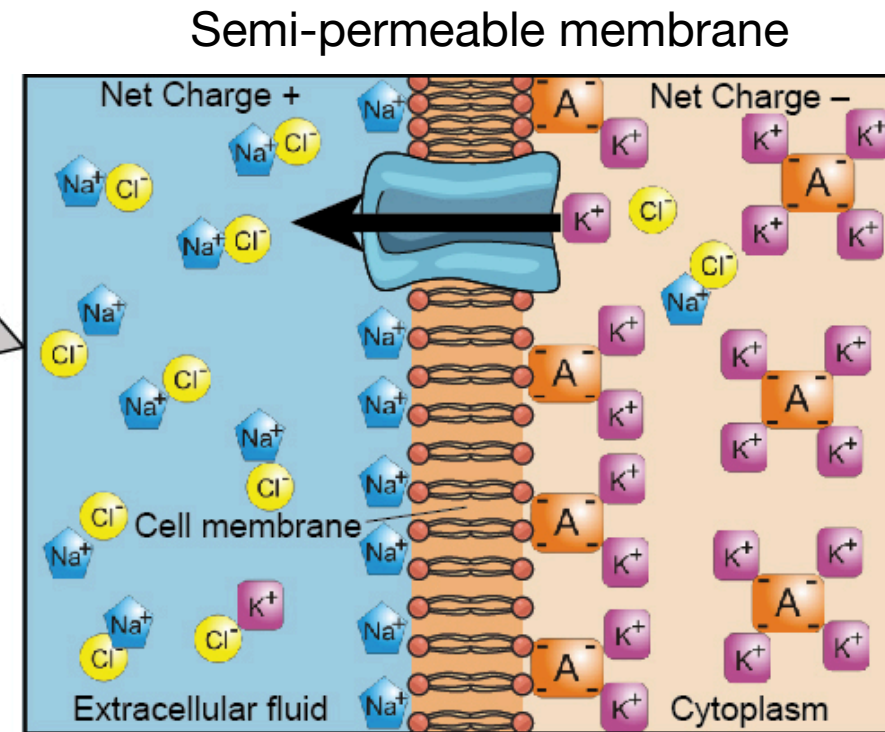
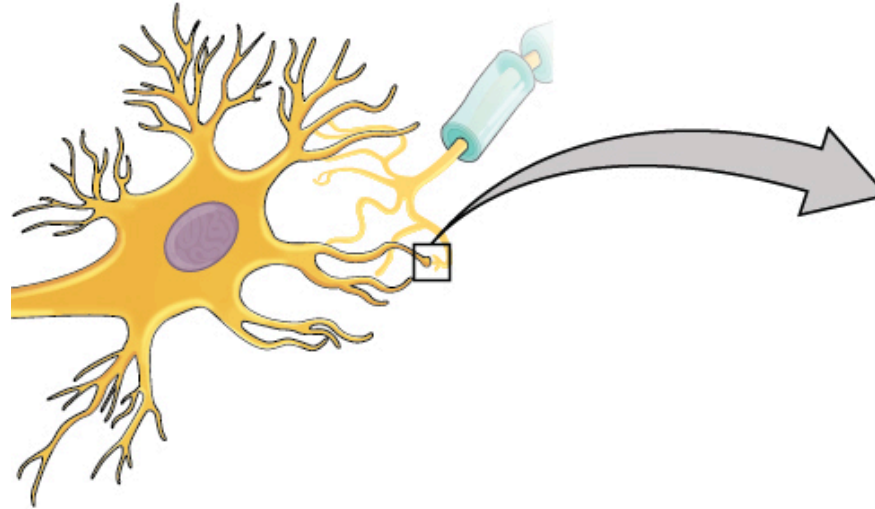


Chemical synapse

Biological neurons

Neurons send message **electrochemically**

Sodium Na^+ ion
Potassium K^+ ion
Calcium Ca^{++} ion
Chloride Cl^- ion
Protein molecules A^-



When a neuron at rest (not sending a signal)

Inside is negative relative to the outside

Resting potential about -70 mV (millivolt)

More sodium outside and more potassium inside

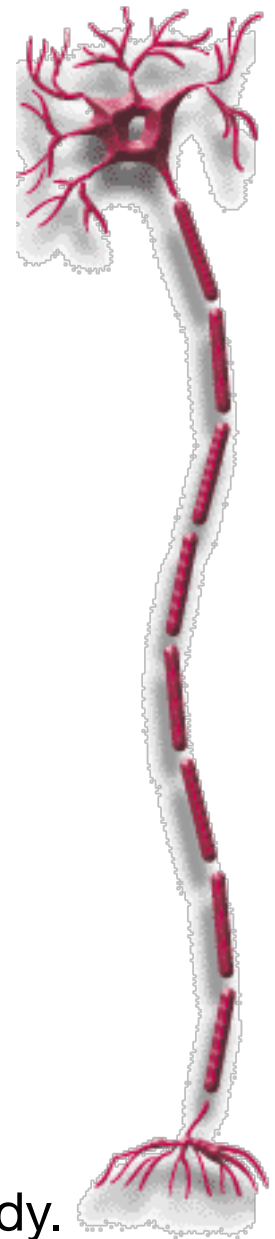
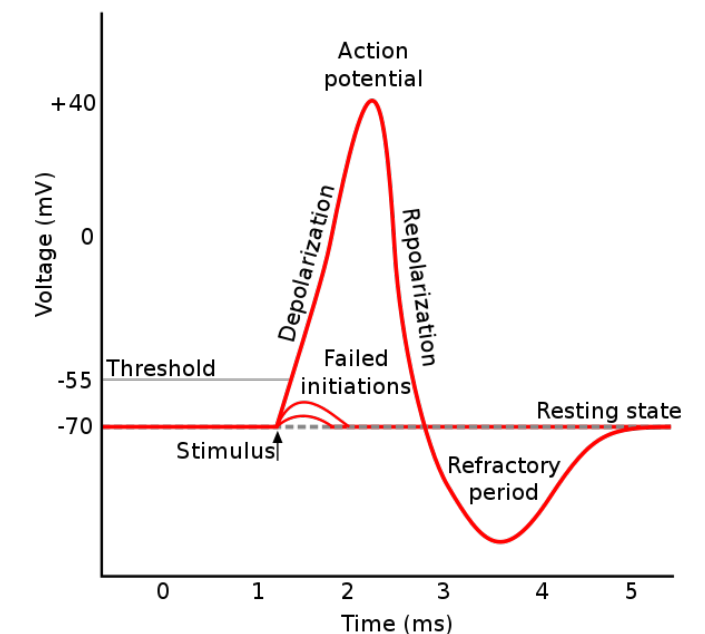
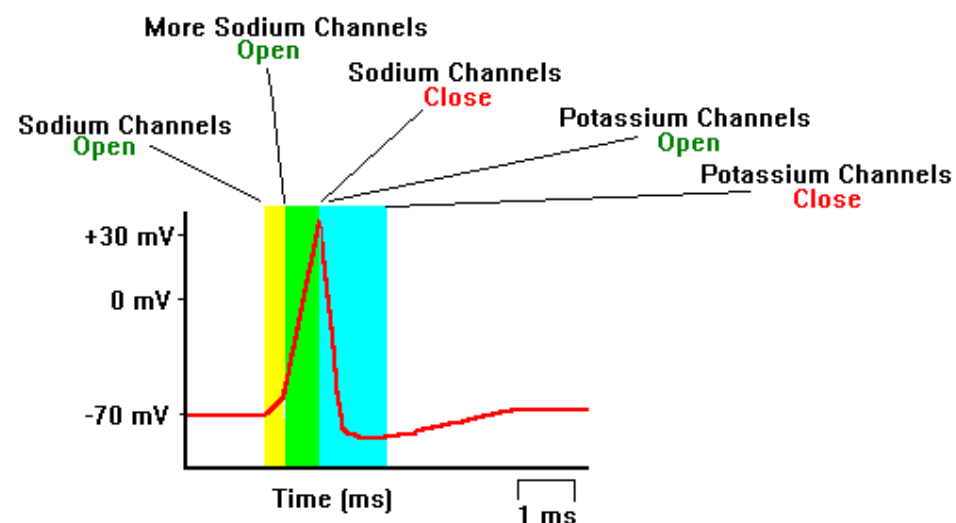
Action potential occurs when a neuron sends information down an axon, away from the cell body.

An explosion of electrical activity created by depolarizing current

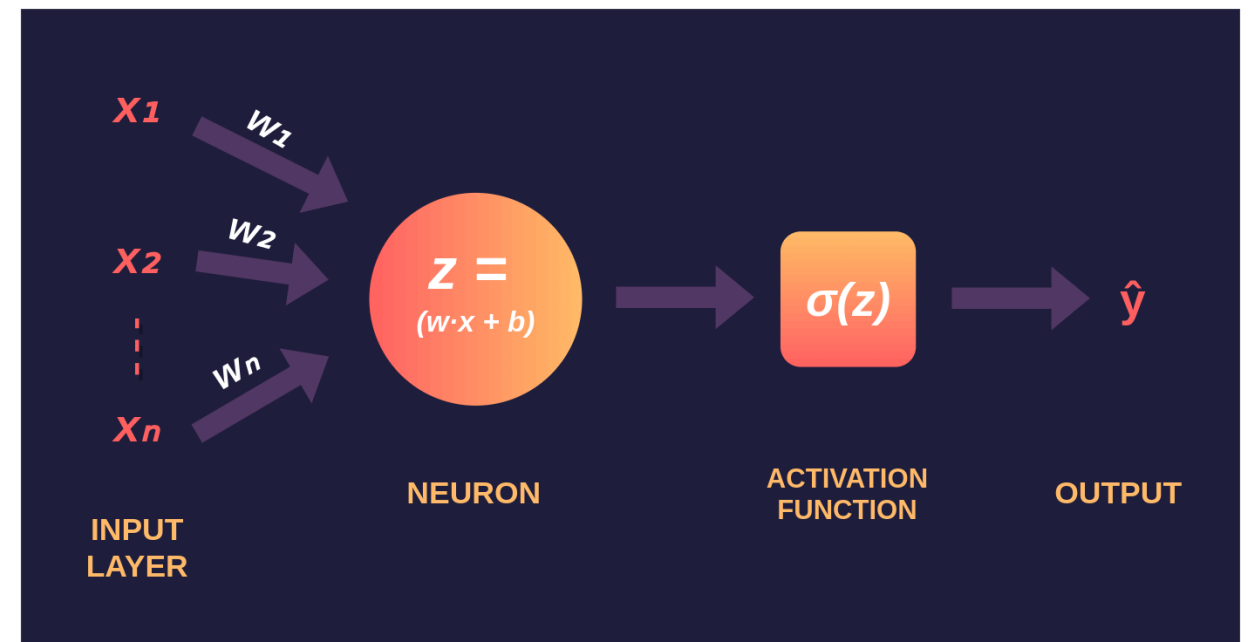
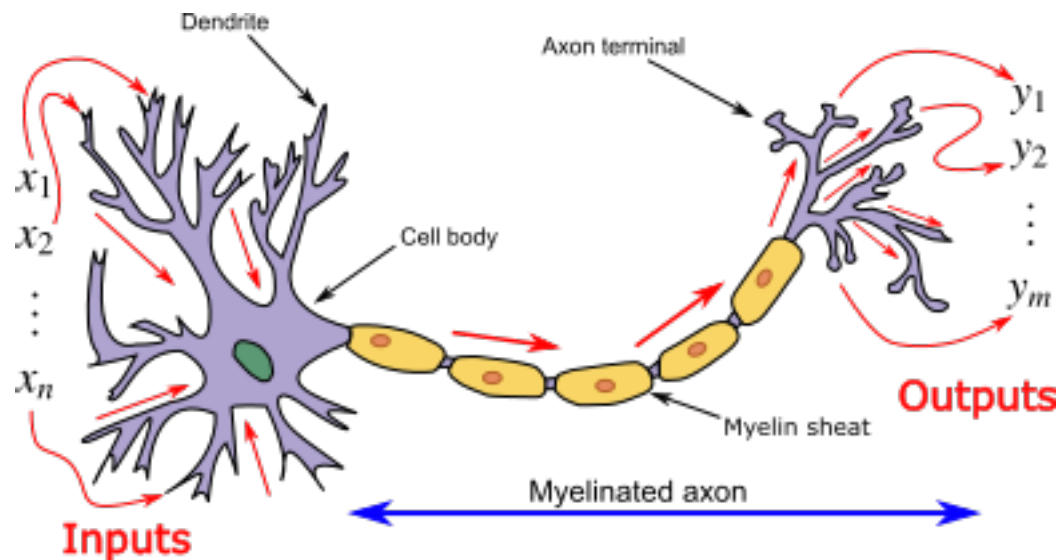
A stimulus causes the resting potential to move towards 0 mV.

Depolarisation reaches about -55 mV (threshold), neuron will fire an action potential

Spike or impulse
Depolarization
Repolarization

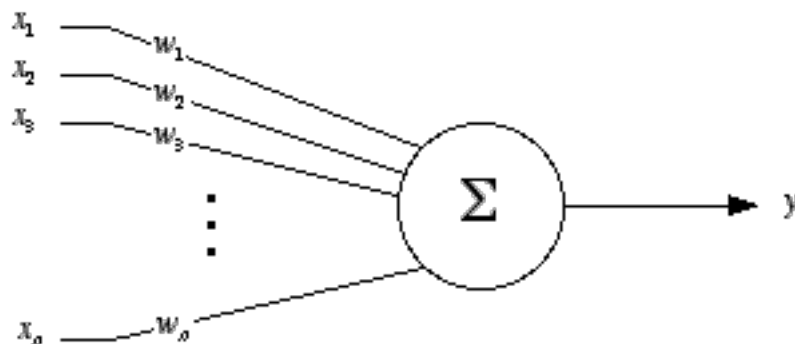


Mathematical representation for neurons: Linear model



- Brain contains neurons, each neuron can be thought of as a device having inputs and outputs.
- Inputs consist of 10^3 - 10^4 synapses on the dendritic tree, outputs consist of action potential carried by the axon sent to other neurons.
- The input currents are (roughly) summed together into the cell body, whose voltage rises and decays with the fluctuations in current.
- When the cell body voltage exceeds a certain threshold, an action potential is fired, propagates down the axon.

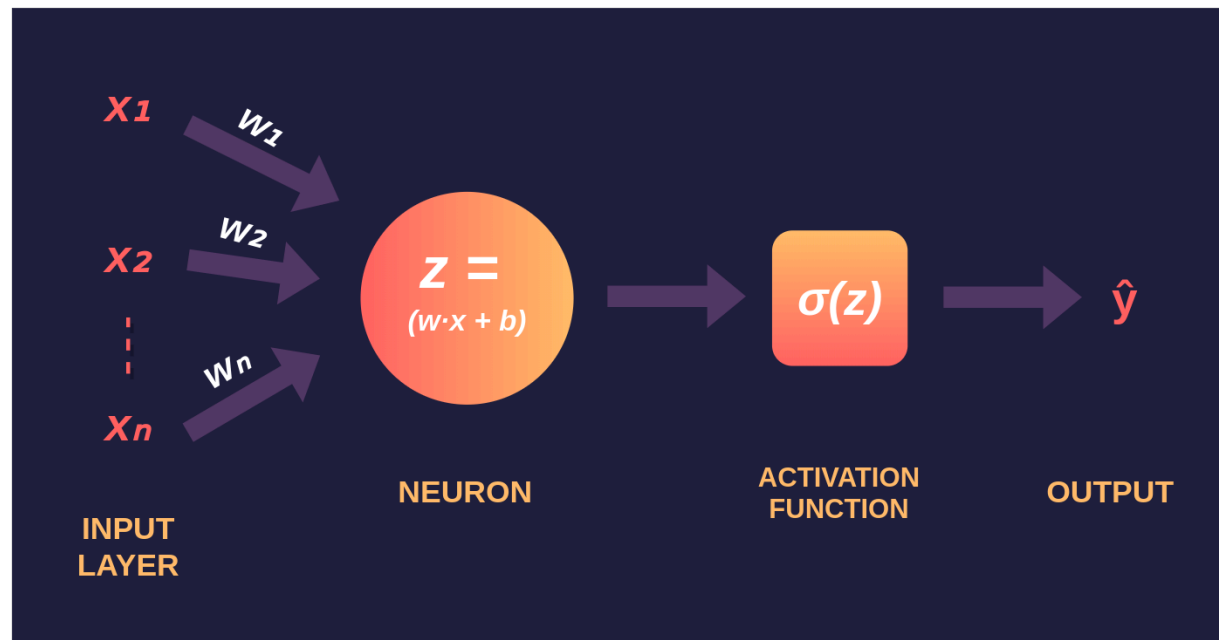
Linear neuron models



$$y = \sum_{i=1}^n w_i x_i = \theta^T x$$

w_i weight for each input

Mathematical representation for neurons: Perceptrons



- Frank Rosenblatt in 1958
- n inputs, n weights, one neuron, one output
- Passing data through via forward propagation

Binary activation function

$$y = 0 \quad \text{if} \quad \theta^T x \leq 0$$

$$y = 1 \quad \text{if} \quad \theta^T x > 0$$

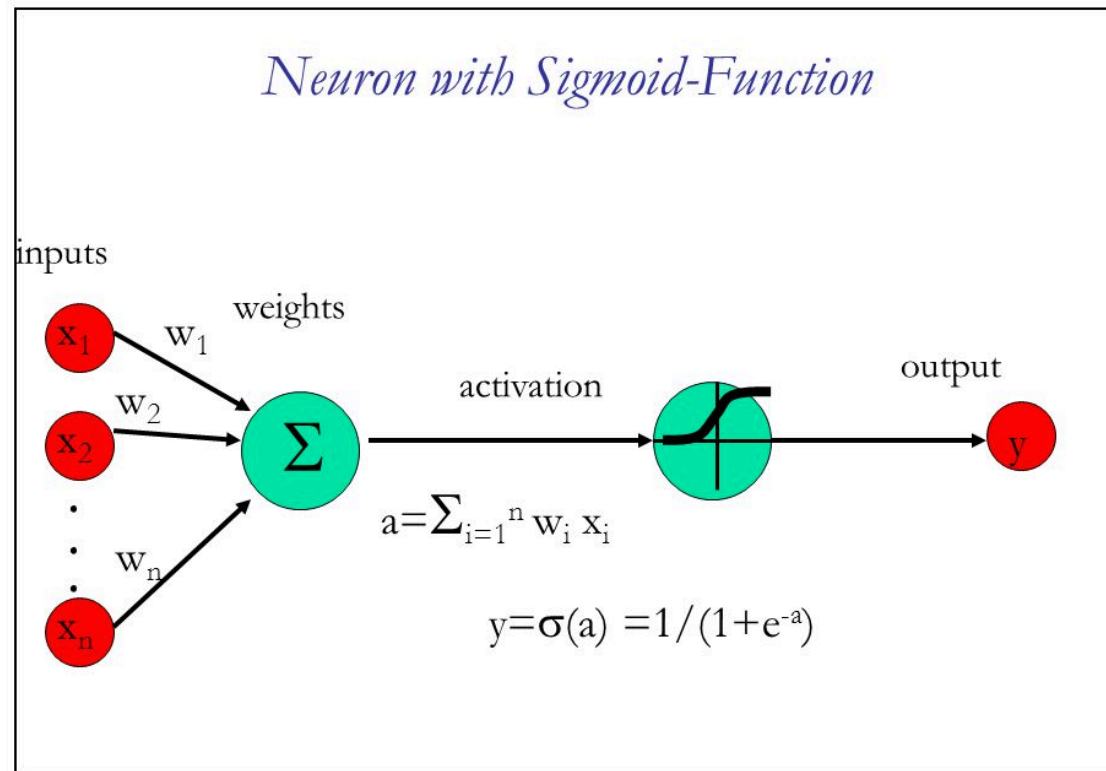
$$\begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & \cdots & x_N^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & \cdots & x_N^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(M)} & x_2^{(M)} & \cdots & x_N^{(M)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_N \end{bmatrix} = \underline{\underline{X}} \cdot \underline{\underline{\Theta}}$$

$\mathbb{R}^{M \times 1}$ vector

Logistic/Sigmoid activation function

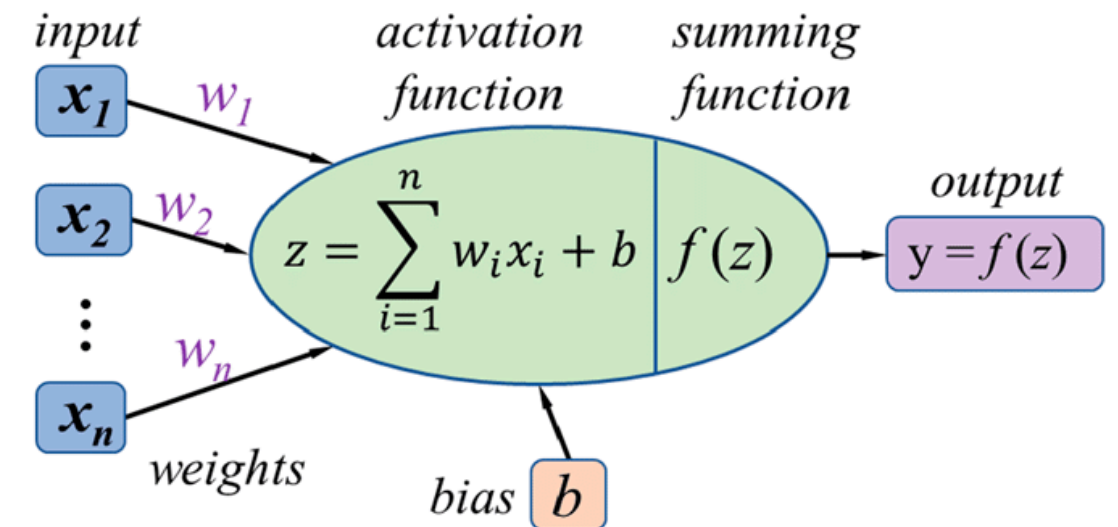
$$y = \frac{1}{1 + \exp(-\theta^T x)}$$

Perceptron

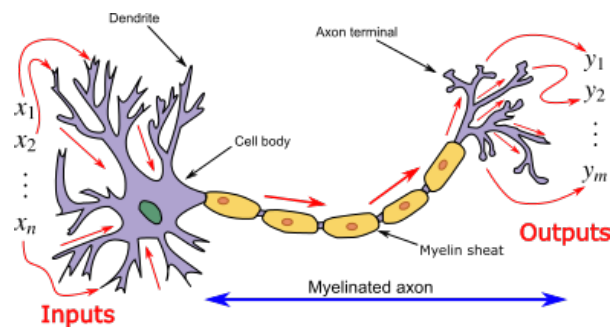
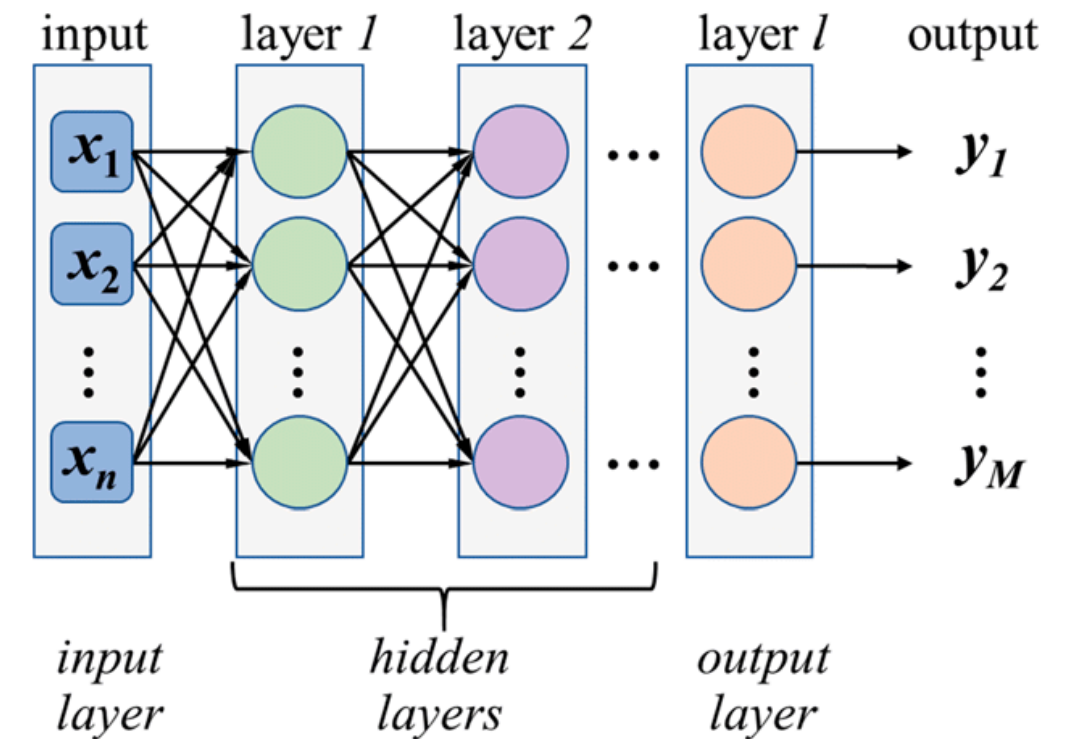


Neural network

(A) a neuron of an artificial neural network



(B) deep neural network



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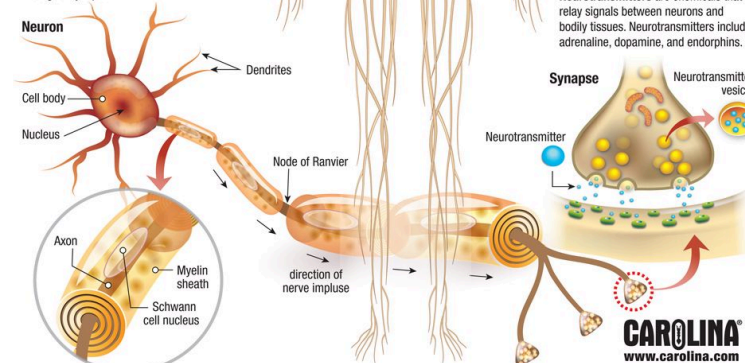
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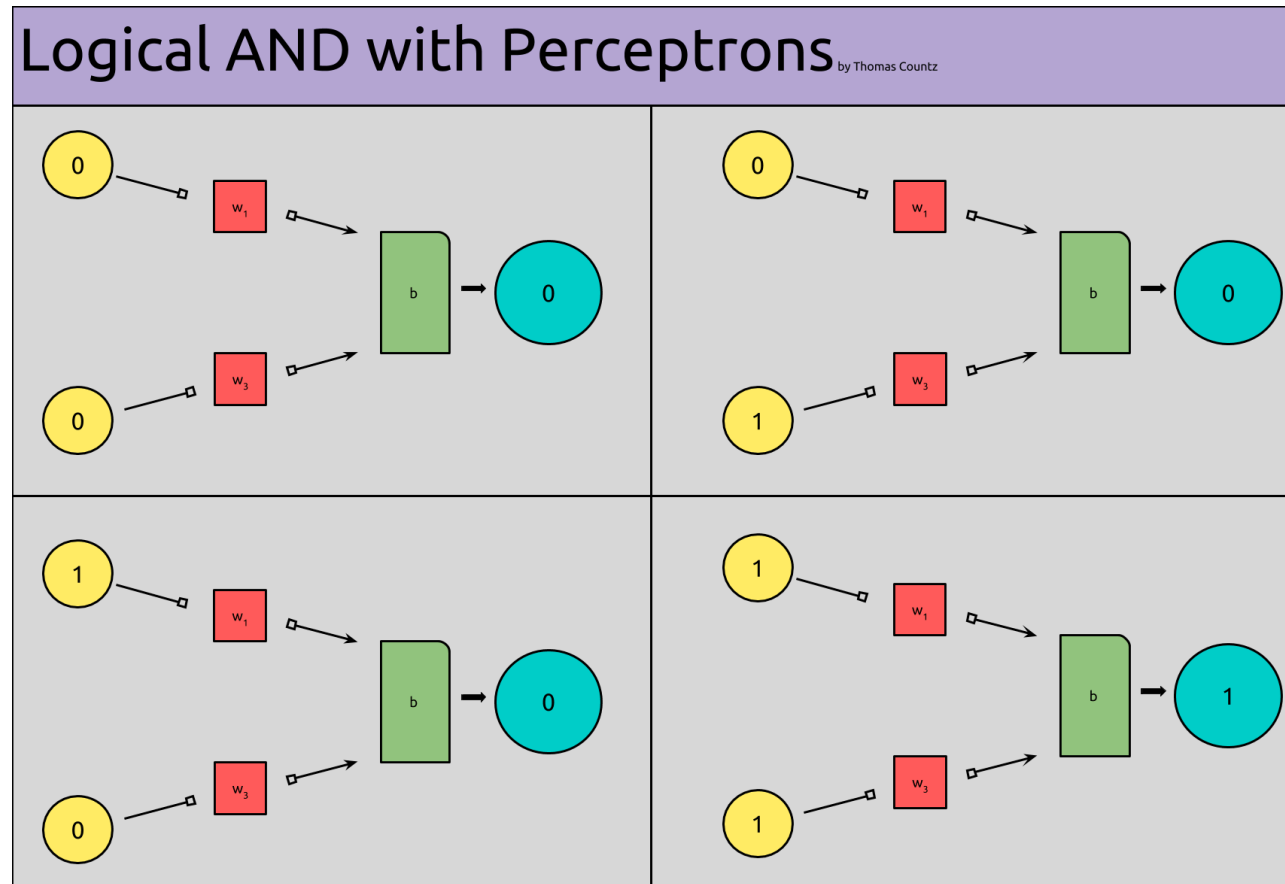
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A Simple Example of Perceptron AND



A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

$f(x)=1$ if $w.x + b > 0$

$f(x)=0$ if otherwise

y	f(x)	y - f(x)
1	1	0
0	0	0
1	0	1
0	1	-1

How to adjust the weights

- If perceptron outputs 0 ($f(x)=0$), when we want 1 ($y=1$), adjust by making $w.x+b$ larger
 $y - f(x) == 1$, then $w + x \rightarrow w$
- If perceptron outputs 1 ($f(x)=1$), when we want 0 ($y=0$), adjust by making $w.x+b$ smaller
 $y - f(x) == -1$, then $w - x \rightarrow w$
- If perceptron outputs expected value, $f(x) == y$, adjust nothing
 $y - f(x) == 0$, then $w \rightarrow w$

Since $y-f(x)$ only produces 1,-1,0, simplify $w + (y-f(x)) * x \rightarrow w$

<https://medium.com/@thomascourtz/perceptron-implementing-and-part-2-84bfb1f46597>

<https://medium.com/@thomascourtz/19-line-line-by-line-python-perceptron-b6f113b161f3>