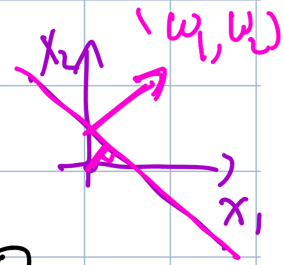
# Some high school geometry




$y = -\frac{w_1}{w_2}x - \frac{b}{w_2}$

Given a plane $\quad w_1 x_1 + w_2 x_2 + b = 0$

* distance from origin $(0,0)$ to the plane is

$\frac{w_1 0 + w_2 0 + b}{\sqrt{w_1^2 + w_2^2}} \rightarrow \boxed{\frac{b}{\sqrt{w_1^2 + w_2^2}} = \frac{b}{\|\vec{w}\|}}$

* distance from arbitrary point $(x_1, x_2)$ to the plane is

$$\boxed{\frac{|w_1 x_1 + w_2 x_2 + b|}{\|\vec{w}\|}}$$

$\sqrt{w_1^2 + w_2^2} = \|\vec{w}\|$

* $\vec{w} = (w_1, w_2)$ is the normal vector to the plane

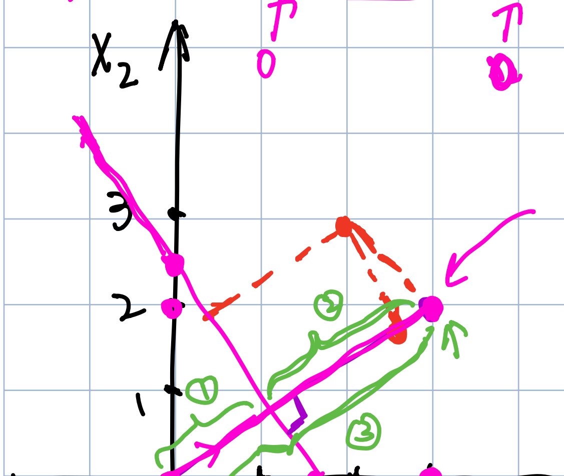
$\|\vec{w}\| = \sqrt{w_1^2 + w_2^2}$

## Example

plane $\quad 3x_1 + 2x_2 - 5 = 0$

normal vector $\vec{w} = (3, 2)$

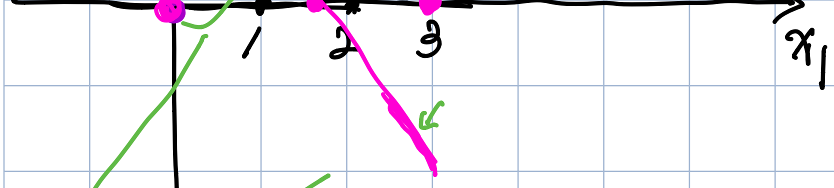
$\|\vec{w}\| = \sqrt{13}$




$(3,2) \qquad \frac{|3 \cdot 3 + 2 \cdot 2 - 5|}{\sqrt{13}} = \frac{8}{\sqrt{13}}$ ✓

$+$

$(0,0) \qquad \frac{|-5|}{\sqrt{13}} = \frac{5}{\sqrt{13}}$ ✓




$\frac{\ }{\sqrt{13}} + \frac{\ }{\sqrt{13}} = \sqrt{13} = \|w\|$

$\sqrt{9+4} = \sqrt{13}$

$(2,3)$ $\quad \frac{|3\cdot 2 + 2\cdot 3 - 5|}{\sqrt{13}} = \frac{7}{\sqrt{13}}$

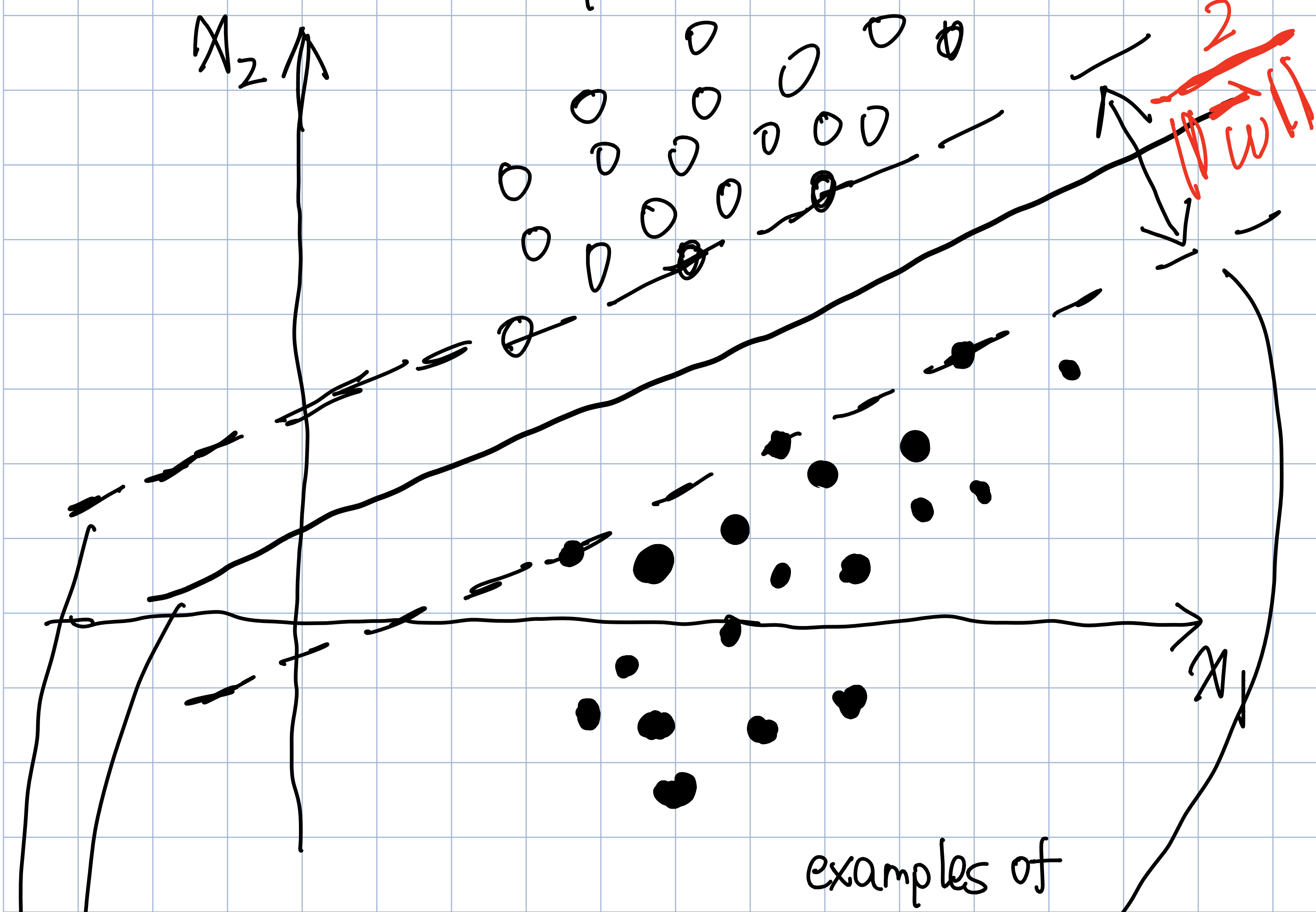
distance between $(2,3)$ and $(3,2)$ $\sqrt{1+1} = \sqrt{2}$

distance between $(2,3)$ and $2x_1 - 3x_2 = 0$ is

$$\frac{|2\cdot 2 - 3\cdot 3|}{\sqrt{13}} = \frac{5}{\sqrt{13}}$$

so $\quad \sqrt{2} = \sqrt{\left(\frac{5}{\sqrt{13}}\right)^2 + \left(\frac{1}{\sqrt{13}}\right)^2} = \sqrt{2}$

examples of $y = 1$




examples of

$$\vec{w}\cdot\vec{x}+b=0$$

$$\vec{w}\cdot\vec{x}+b=1$$

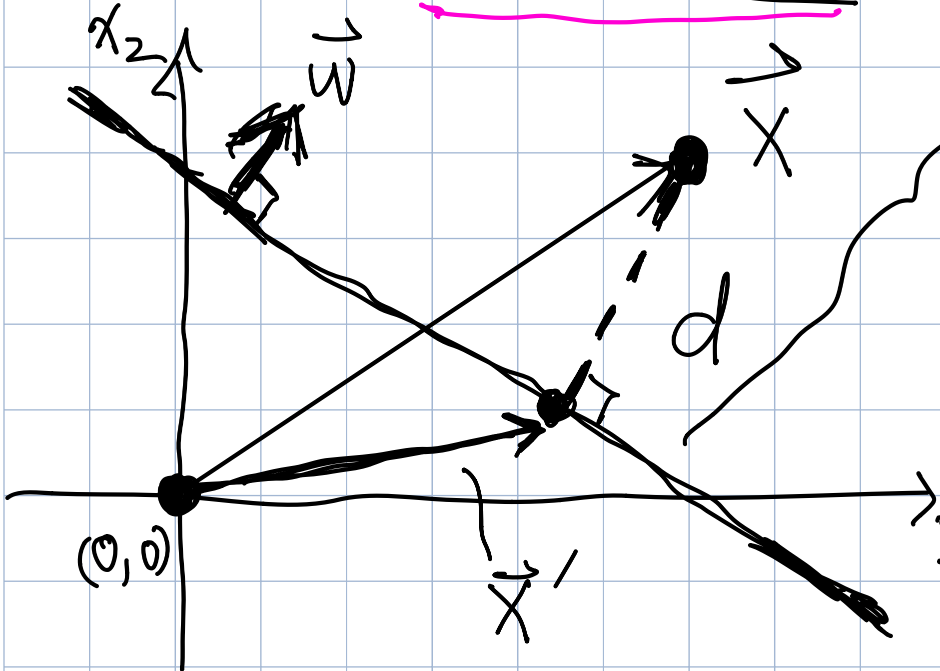
$$\vec{w}\cdot\vec{x}+b=-1$$

$y=-1$

totally m examples

proof of distance from $\vec{x}=(x_1, x_2)$ to the plane $\vec{w}\cdot\vec{x}+b=0$

$$\boxed{d=\frac{\vec{w}\cdot\vec{x}+b}{\|\vec{w}\|}}$$




$$\vec{w}\cdot\vec{x}+b=0$$

$$\vec{X}=\vec{X}'+\hat{d}$$

$$\hat{d}=\frac{\vec{W}}{\|\vec{W}\|}d \; =$$

$$\vec{w}\cdot\vec{x}'+b=0$$

$$\vec{w}\cdot(\vec{X}-\hat{d})+b=0$$

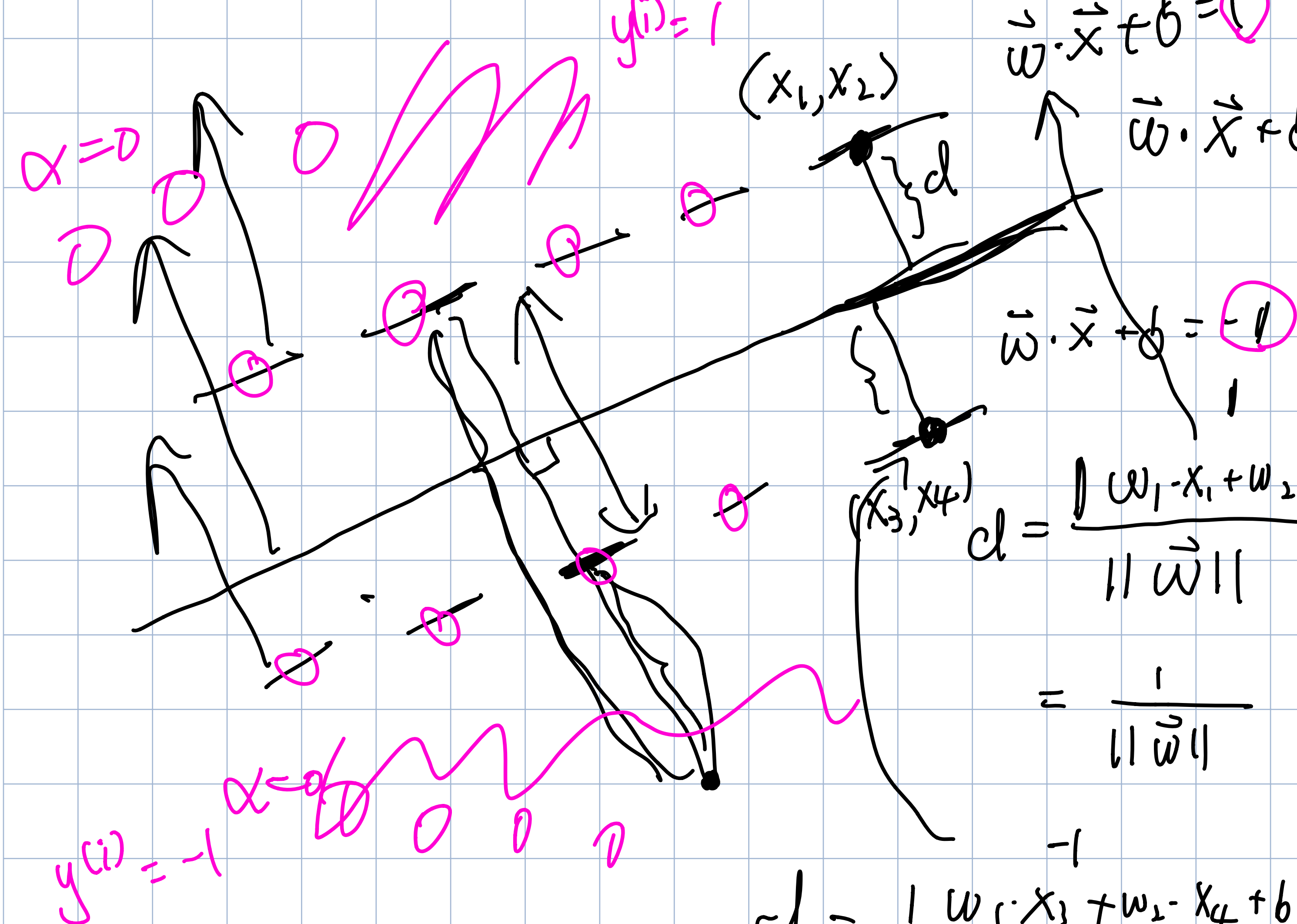
$$\vec{w}\cdot\vec{x}\quad\vec{w}\,\frac{\vec{W}}{\ }d+b=0$$

$$\vec{w}\cdot\vec{x} - \vec{w}\cdot\frac{\vec{w}}{\|\vec{w}\|}d + b = 0$$

$$d = \frac{\vec{w}\cdot\vec{x}+b}{\|\vec{w}\|}$$

$$\frac{\|\vec{w}\|^2}{\|\vec{w}\|} = \|\vec{w}\|$$

$$\vec{w}\cdot\vec{x} - \|\vec{w}\|d + b = 0$$




$$d = \frac{|w_1\cdot x_1 + w_2 x_2 + b|}{\|\vec{w}\|} = \frac{1}{\|\vec{w}\|}$$

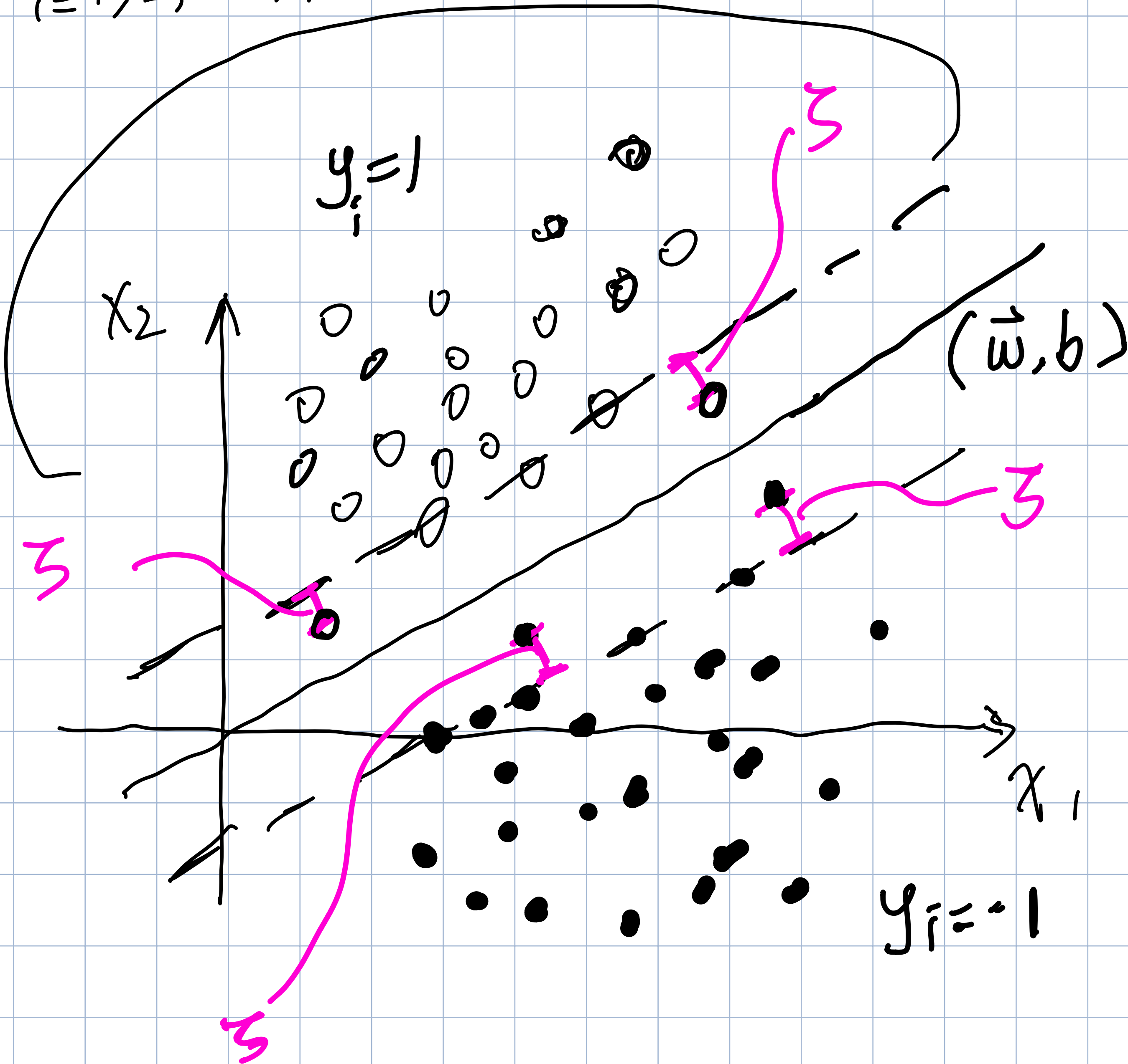
$$d = \frac{|w_1\cdot x_3 + w_2\cdot x_4 + b|}{\|\vec{w}\|} = \frac{1}{\|\vec{w}\|}$$

$$\frac{|y_i(\vec{w}\cdot\vec{x}_i + b)|}{\|\vec{w}\|} \geq M \rightarrow \frac{1}{\|\vec{w}\|}$$

$y_i=1 \quad \vec{w}\cdot\vec{x}_i + b \geq 1$

$$|y_i(\vec{w}\cdot\vec{x}_i + b)| \geq 1 \quad \left\langle\; y_i = -1 \quad \vec{w}\cdot\vec{x}_i + b \leq -1 \right.$$

$i = 1, 2, \ldots, m$




$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[\underline{y^{(i)}\log h_\theta(x^{(i)})} + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right]$$

$$J(\theta) = \frac{1}{M}\sum_{i=1}^{M} \max\left(0,\ 1 - y_i(\vec{\theta}\cdot\vec{x}_i + \theta_0)\right)$$

Hinge function

$y_i = 1$ $\quad \vec{w}\cdot\vec{x}_i + b > 1$ $\quad J(\theta) = 0$

$z$

$\vec{\theta}\cdot\vec{x}_i + \theta_0 < 1$ $\quad J(\theta) = 1 - (\vec{\theta}\cdot\vec{x}_i + \theta_0)$

$> 0$

remember in logistic regression

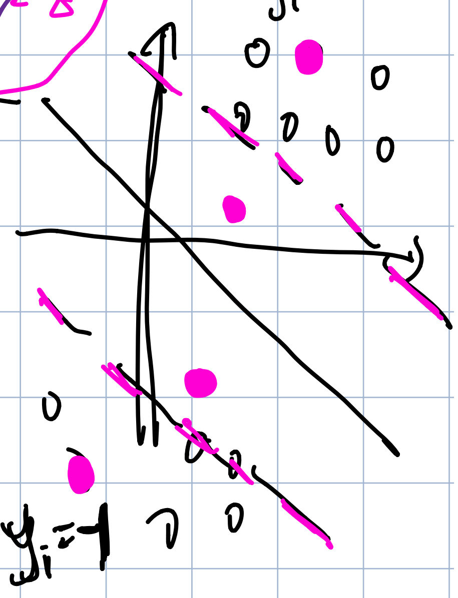
$$J(\theta) = -\log\frac{1}{1+e^{-\theta\cdot x}}$$

$= z$

$1 - z$

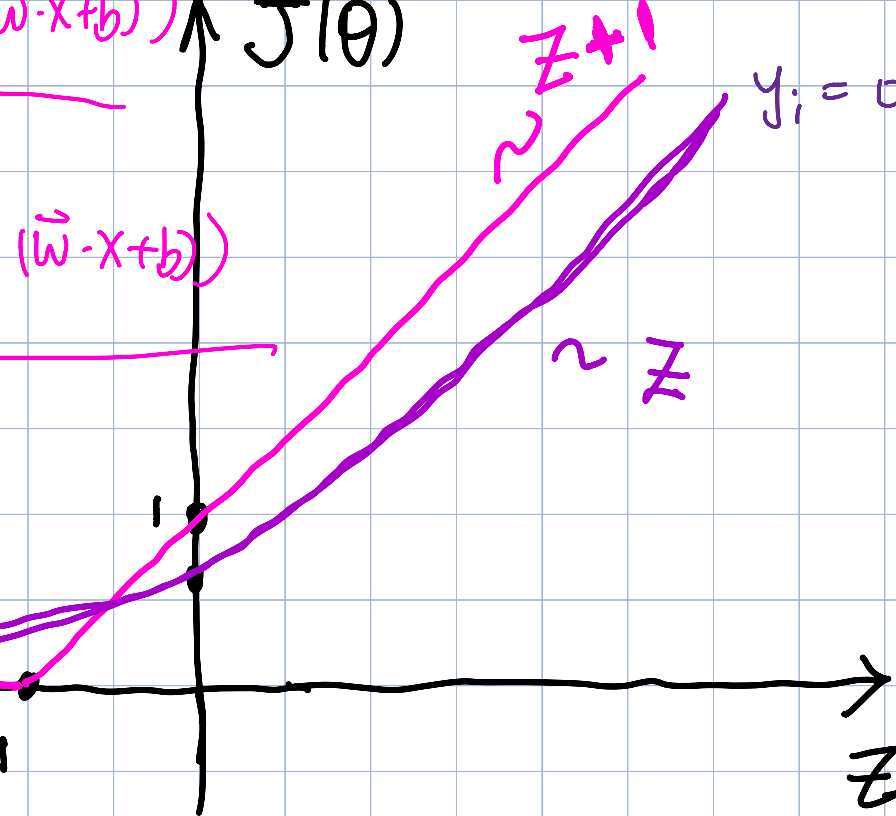
$\frac{1}{1+e^{-z}}$ $\ln e^{z}$ $\quad z$

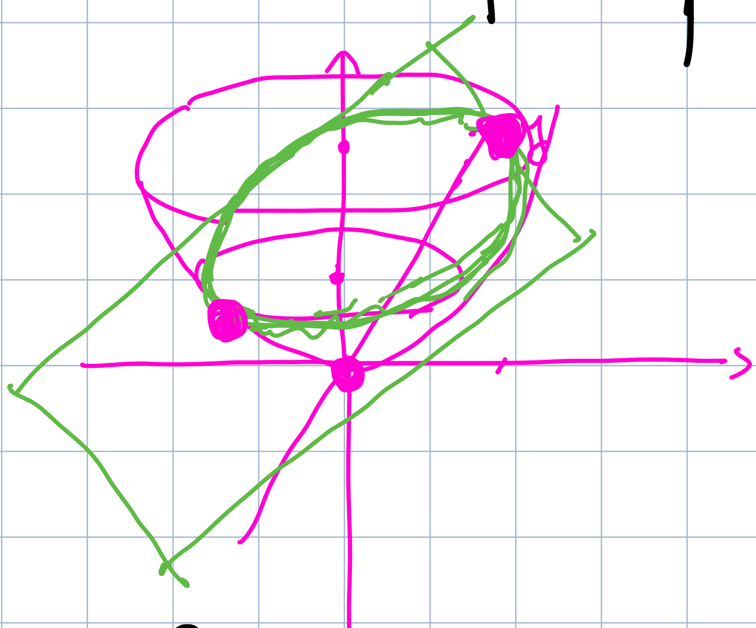
$$z = \theta\cdot x = \vec{\theta}\cdot\vec{x} + \theta_0$$

$$\frac{\lambda}{2m}\|w\|^2 + \frac{1}{m}(1-(\vec{w}\cdot x+b))$$

$$\frac{\lambda}{2m}\|\vec{w}\|^2 + \frac{1}{m}(1-y_i(\vec{w}\cdot x+b))$$




$x^2+y^2=z$

$\{x, y, \alpha\}$
↑ ↑ ↑

$$\mathcal{L} = x^2 + y^2 - \alpha(x+y-1)$$

$$\nabla \mathcal{L} = 0$$

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial x} = 0 & 2x - \alpha = 0, \quad x = \dfrac{\alpha}{2} = \dfrac{1}{2} \\ \dfrac{\partial \mathcal{L}}{\partial y} = 0 & 2y - \alpha = 0, \quad y = \dfrac{\alpha}{2} = \dfrac{1}{2} \end{cases}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \qquad x+y-1=0,$$

$$\frac{\alpha}{2}+\frac{\alpha}{2}-1=0, \quad \underline{\alpha=1}$$

## Karush-Kuhn-Tucker conditions

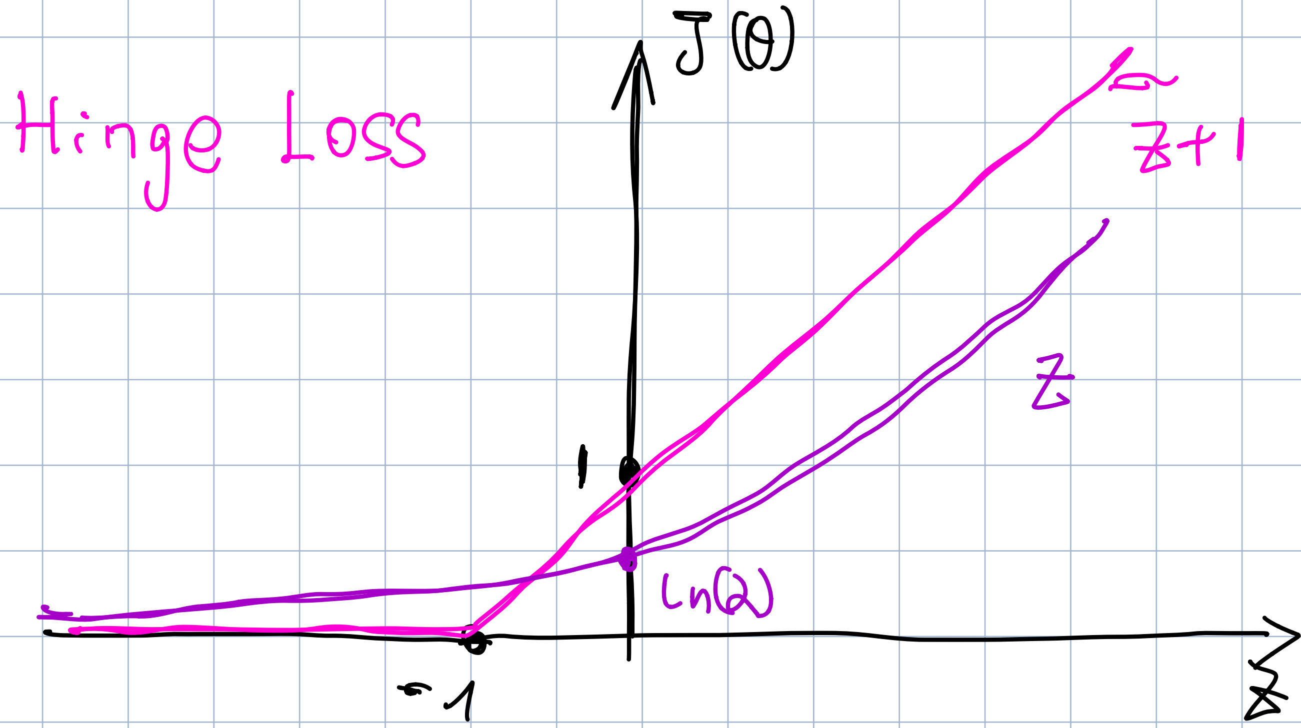
$$\mathcal{L}(\vec{w}, b, \alpha) = \frac{1}{2}\|\vec{w}\|^2 - \sum_{i=1}^{m} \alpha_i [y_i(\vec{w}_i \cdot \vec{x}_i + b) - 1]$$

$$\nabla_{\vec{w}} \mathcal{L} = \vec{w} - \sum_{i=1}^{m} \alpha_i y_i \vec{x}_i = 0$$

$$\frac{\partial \mathcal{L}}{\partial b} = \sum_{i=1}^{m} \alpha_i y_i = 0$$

$$\mathcal{L}_D(\alpha, \vec{x}_i, y_i) = \frac{1}{2}\left(\sum_{i=1}^{m} \alpha_i y_i \vec{x}_i\right)\left(\sum_{j=1}^{m} \alpha_j y_j \vec{x}_j\right) - \sum_{i=1}^{m} \alpha_i y_i \left(\sum_{j=1}^{m} \alpha_j y_j \vec{x}_j\right)\vec{x}_i - b \underbrace{\sum_{i=1}^{m} \alpha_i y_i}_{0} + \sum_{i=1}^{m} \alpha_i$$

$$\mathcal{L}_D = \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y_i y_j \vec{x}_i \cdot \vec{x}_j$$




SVM $\quad y^{(i)} = -1 \quad m = 1$

$$J(\theta) = \max\left(0,\ 1 + (\vec{\theta}\cdot\vec{x}_i + \theta_0)\right)$$

if $\vec{\theta}\cdot\vec{x}_i + \theta_0 < -1 \qquad J(\theta) = 0$

if $\vec{\theta}\cdot\vec{x}_i + \theta_0 > -1 \qquad J(\theta) = \vec{\theta}\cdot\vec{x}_i + \theta_0 + 1$

$= z + 1$

logistic $\quad y^{(i)} = 0 \quad m = 1$

$$J(\theta) = -\log\left(1 - h_\theta(x^{(i)})\right)$$

$$-\log\left(1-\frac{1}{1+e^{-\theta x}}\right)$$

$$=-\log\left(1-\frac{1}{1+e^{-z}}\right)$$

$z \to -\infty \qquad \log(1) \to 0$

$z = 0 \qquad -\log\left(\frac{1}{2}\right) = \log(2)$

$z \to \infty \qquad -\log\left(\frac{e^{z}}{1+e^{-z}}\right)$

$$= -\log(e^{-z}) = z$$